Nuclear reactions induced by π^- at rest

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The experimental information on reactions induced by stopped π^- absorbed in nuclei is critically reviewed. Evidence for the presence of α -cluster absorptions is presented and arguments are given to show that $\approx 25\%$ of π^- absorptions are of this kind. In the case of two-nucleon absorption, the existing experimental information concerning the ratio of n-p to p-p absorbing pairs is discussed. Calculations of particle spectra and residue spallation yield distributions that, in addition to two-nucleon absorption, include α -cluster absorption are presented, and it is shown that a satisfactory reproduction of the data is achieved.

I. INTRODUCTION

Many experimental and theoretical investigations have been devoted, in recent years, to the study of reactions induced by stopped π^- . These studies have greatly improved our knowledge in this field and have lead to a qualitative understanding of the reaction mechanism; however, important aspects of the absorption process are still incompletely understood and the values to be attributed to basic parameters related to the π^- -nucleus interactions are still debated. This is partly due to the discrepancies between experimental data from different laboratories, as will be discussed in Sec. II and the following sections, and in part to the approximate theoretical models utilized and the simplifying assumptions, not always based on firm grounds, made by some authors to interpret their data.

In spite of this fact, we believe that a comprehensive consideration of the published data may lead one to a quantitative understanding of many aspects of the absorption process, without referring to a particular theoretical model. This result may be reached through comparison of data from π^- absorption on different nuclides, and of data from π^- absorption and reactions induced by protons of suitable energy.

These considerations will be presented in Sec. III of this paper. In Secs. IV and V calculations made in the framework of the exciton model, on the basis of the quantitative conclusions reached in Sec. III, will be presented and discussed and their results compared to the experimental data. It will be shown that a rather satisfactory, consistent, and comprehensive reproduction of the experimental results may be obtained. Finally, Sec. VI is devoted to the summary and conclusions.

II. EXPERIMENTAL RESULTS

Experiments on π^- absorption may be divided in two groups:

(a) Correlation experiments aimed at providing a direct information on the dynamics of the absorption process and the structure of the absorbing nucleus. These experiments include the measurement of (i) the angular correlation of nucleons, or nucleons and clusters or clusters emitted by the absorbing nucleus; (ii) the spectra of neutrons, protons, deuterons, and tritons in coincidence with a correlated nucleon; (iii) the recoil momentum distribution; (iv) the excitation energy spectrum of residual nuclei after the emission of correlated nucleons (see Ref. 1) and references therein, and Refs. 2-8).

Only a few results of this type, from Heusi *et al.*,¹ will be explicitly considered in Sec. IV of this paper to test our calculation of the initial excitation energy distributions of particle-hole states of the composite nucleus just after pion absorption.

A much greater attention will be paid, in this article, to data referring to the following:

(b) The measurements of the energy distributions of neutron and charged particle multiplicities and of the yield of the spallation residues.

These data provide information both on the pionnucleus interaction and the deexcitation mechanism of the highly excited nuclei produced in the absorption process, and show that a relevant fraction of the excitation energy is carried out by a relatively small number of fast particles.

We will not try to make an exhaustive survey of all the experimental results of this type relevant to the subject discussed here. Only a restricted sample of data will be analyzed which are thought to provide unambiguous information on the matter we are discussing.

Quite often, experimental results from different laboratories or research groups are not in a satisfactory agreement, and on the basis of the information reported it is impossible to decide with any certainty which data are considered to be more accurate. Obviously, in such conditions a greater weight must be given to the more recent results (due to the improvement of the experimental techniques) and to results on which the majority of the authors agree. When this results in the impossibility, on these bases, of deciding between partly conflicting data, all data will be considered. The choice of giving a greater weight to some data is reflected in the estimate of some basic parameters, as will be discussed in the following.

Measurements of inclusive spectra of charged particles emitted in π^- absorption have been made by several groups.^{9–17}

Due to the quite low pion fluence, in all these experiments rather thick targets have been used and the measured spectra have to be unfolded to take into account the self-absorption of low energy ejectiles in the target.¹¹ As a consequence, the low energy part of the spectra is characterized by rather conspicuous uncertainties. Also, the accurate measurement of the highest energy portions of the spectra is not easy, and in these spectral regions the experimental data are often lacking and sometimes conflicting.

We wish to call attention, in particular, to the following points:

(i) A number of authors, when comparing proton multiplicities from nuclei of different mass regions, found that the yield of emitted fast protons is highest for medium-heavy nuclei ($A \approx 40-60$).⁹⁻¹² On the other hand, the yield of fast neutrons is almost independent of A (except, possibly, the lighest nuclei),^{5,18} as shown in Fig. 1. These behaviors can be reproduced by the calculations discussed in Sec. IV, only by assuming that the ratio R of n-p to p-p pairs that can absorb the pion presents a minimum value for medium-light nuclei.

However, the multiplicity of fast protons measured by Randoll *et al.*¹⁵ in π^- absorption on ⁴⁰Ca, does not conform to the trend reported above and suggests a value of *R*, for ⁴⁰Ca, much higher than that implied by the data of Ref. 11.

(ii) A rather conspicuous yield of tritons is observed which greatly exceeds that measured in the case of proton induced reactions. Very likely, as is discussed later, a great amount of the tritons is produced in π^- absorption on α clusters.

Recent measurements of inclusive neutron spectra have been reported by several authors.^{5,14,16-23} Data from



FIG. 1. Neutron spectra from π^- absorption on various nuclei (Ref. 18).

different authors often display systematic discrepancies. A discussion of the most typical of these may be found in the papers by Cernigoi *et al.*^{16,17} and Madey *et al.*¹⁸ These discrepancies arise from the difficulty of estimating correctly the efficiency of neutron detectors for neutron energies varying from 0 to more than 100 MeV and the background contributing to the measured spectrum, and to the use, due to the low pion intensities, of rather short flight paths to measure neutron time of flights. We believe that in most experiments the uncertainties in emitted neutron energy are quite considerably underestimated. As a matter of fact, in most experiments the highest energy endpoint of measured spectra is noticeably above the maximum allowed neutron energy.

Important information is also gained through the study of spallation residue yields. When the yield of the various isotopes of an element produced in the absorption is plotted as a function of their mass, one finds that the corresponding distribution is nicely fitted by a Gaussian function. This result was established in the early 1950s by radiochemical studies of residue spallation yields made by Sugihara and Libby²⁴ and Winsberg.²⁵ More recent data have been reported by Butsev *et al.*,²⁶ Pruys *et al.*,²⁷ Abazov *et al.*,²⁸ and Orth *et al.*^{29,30}

Of special importance, in our opinion, is the measurement by this technique of the yields of residues corresponding to emission of n-n and n-p pairs from the composite nucleus.³⁰ In this way the ratio R of n-p to p-p absorbing pairs may be deduced. The relevance of this information will be discussed later in Sec. III B.

Another technique applied to measure residue spallation yields is the *in beam* measurement of the γ rays from their deexcitation.^{27,31-33} The more accurate results are obtained by utilizing this method together with the *off beam* measurement of the decay of radioactive residuals.²⁷

Other important information comes from the study of induced fission³⁴⁻³⁶ and from the observation that in the deexcitation products from π^- absorption high spin states may be excited. ^{31,32,37,38}

III. PRELIMINARY DISCUSSION OF THE EXPERIMENTAL DATA

A. Role of α -particle absorptions

Also, if experimental evidence has been reported of π^- absorption by a single nucleon, ^{16,17,21} the known results concur to strengthen the hypothesis that the pion is mainly absorbed by a small number of nucleons.

The energy distribution of the fast particles emitted after absorption and the residue isotope yield distributions indicate the dominance of π^- absorption by a nucleon pair; however, evidence in favor of π^- absorption on α substructures is provided by the rather conspicuous yield of emitted deuterons and tritons.^{4,9–13}

In the case of a dominant two nucleon absorption mechanism, the greatest part of the π^- rest mass is transformed in kinetic energy of two back to back nucleons which, on their way out of the nucleus, may interact with other nuclear constituents. If the complex particles one observes are produced through *final state in*

teractions of these primary nucleons, with mean energy \overline{E} , one may expect that their relative yield and energy distribution be quite similar to those observed in reactions induced by nucleons of energy not too different from \overline{E} .

As discussed in Ref. 39, the most suitable proton energy for comparing multiplicity energy distributions of complex particles from proton bombardment and π^- absorption is ≈ 60 MeV. In Fig. 2 the α multiplicity spectra from π^- absorption¹² and reactions induced by 62 MeV protons⁴⁰ are compared and one may easily notice the expected similarity (this fact is also discussed in Refs. 39 and 41–44). The α particle multiplicities from proton interaction are obtained by dividing the measured lab system (p, α) cross section⁴⁰ by the total reaction cross sections σ_R . For these, values calculated using a formula reported by Menet *et al.*⁴⁵ are used. For the three nuclei considered the values of σ_R are, respectively, equal to 363, 857, and 1880 mb. The multiplicities from π^- absorption are taken by Pruys *et al.*¹²

A discussion of these data and of the comparison of ³He spectra from proton and π^- induced reactions has already been given in Ref. 39. On the basis of the analysis made there, the same origin for both α and ³He in the two processes is indicated, thus supporting the hypothesis that emission of these complex particles in π^- absorption is compatible with a dominant nucleon pair absorption mechanism in which both primary nucleons are active in initiating the deexcitation intranuclear cascade.

On the other hand, the comparison of data concerning deuteron and triton emission from π^- absorption and proton bombardment reveals significant differences.

(a) While in the case of π^- absorption the multiplicity of high energy α particles and ³He is always considerably smaller than the multiplicity of high energy tritons, as shown in Fig. 3, where the multiplicities of d, t, α , and ³He emitted in π^- absorption on ¹²C, ⁵⁹Co, and ¹⁹⁷Au are reported as a function of the ejectile energy, ¹² in the case of reactions induced by ≈ 60 MeV protons the opposite occurs (see Fig. 4). In this case the yield of high energy α



FIG. 2. Comparison of α particle multiplicity distributions from π^- absorption on ¹²C, ⁵⁹Co, and ¹⁹⁷Au (Ref. 12) (solid circles), and reactions induced by ≈ 62 MeV protons on ¹²C, ⁵⁴Fe, and ¹⁹⁷Au (Ref. 40) (histograms).



FIG. 3. Multiplicities of d, t, α , and ³He, emitted after absorption of stopped π^- on ¹²C, ⁵⁹Co, and ¹⁹⁷Au (Ref. 12) as a function of ejectile energy.

particles always exceeds the triton yield and, for nuclei as light as ${}^{12}C$ and ${}^{54}Fe$, also the yield of high energy ${}^{3}He$ is significantly greater than that of tritons.⁴⁰

(b) The multiplicities of high energy tritons from π^- absorption display a weak dependence on the target nucleus mass, while, e.g., in the case of proton bombardment the multiplicity of tritons from ¹⁹⁷Au is greater, by approximately a factor 2, than that of tritons from ⁵⁴Fe, as shown in Fig. 5.

(c) In the case of processes induced by ≈ 62 MeV protons the ratio between the yield of high energy deuterons and tritons is considerably higher than that found in reactions induced by stopped π^- . In the proton case the ratio is approximately equal to 10 irrespective of the target nucleus considered⁴⁰ (this is also true at higher proton energies⁴⁶), while in the case of π^- absorption it is substantially lower, being, for all nuclei, not far from 3.^{9,10,12}

We are thus lead, quite naturally, to the conclusion that, most presumably, the mechanism for production of energetic α particles and ³He is similar in π^- and proton induced reactions, while the mechanism for production of deuterons and tritons is not.

The most simple explanation of these experimental data is to assume that, in π^- absorption, part of the deuterons and tritons are not produced through final state interactions but at the very moment of the absorption on α substructures. In fact, the ratio of the yields of fast deuterons and tritons is nearly the same one measures in π^- absorptions on ⁴He.⁴⁷ This might even imply that the predom-



FIG. 4. Spectra of d, t, α , and ³He, emitted in ≈ 62 MeV proton bombardment of ¹²C, ⁵⁴Fe, and ¹⁹⁷Au (Ref. 40).



FIG. 5. Multiplicity of tritons from π^- absorption (Ref. 12) and proton induced reactions (Ref. 40).

inant mechanism for production of fast deuterons and tritons is α particle absorption.

However, lower energy deuterons and tritons are probably produced also by final state interactions of the primary nucleons excited in the absorption process or are evaporated from the equilibrated system at the end of the deexcitation cascade of intranucleon interactions. The experimental data, as shown in Fig. 6, indicate that the amount of this contribution decreases by increasing the absorbing nucleus mass. In the simplifying hypothesis that it might be considered unimportant in the case of the heaviest nuclei, if one further assumes that π^- absorption



FIG. 6. Multiplicity distributions of deuterons and tritons from π^- absorption on ¹²C (solid circles), ⁵⁹Co (open circles), and ¹⁹⁷Au (pluses) (Ref. 12).

occurs on α substructures at the nuclear surface and that only one-half of the deuterons and tritons may come out of the nucleus (the ones directed outward from the nuclear surface that do not need to go through a thick slab of nuclear matter) from the measured d and t multiplicities for a heavy nucleus like gold¹² and the known branching ratios for d and t production in π^- absorption on ⁴He [equal, respectively, to $(58\pm7)\%$ and $(19.4\pm1.8)\%$ (Ref. 47)], one may try to estimate the percentage of absorptions on α substructures. This amounts to $\approx 25\%$, a value that, for our neglecting d and t production by final state interactions, must be considered an upper limit of the true value.

The occurrence of π^- absorptions on α clusters was first suggested to interpret the quite notable amount of correlated emissions of neutrons and tritons in π^- absorption on ${}^{12}C$,⁴ and from the fact 4,9,11 that the deuteron and triton spectra are well reproduced by calculations made by Kolybashov⁴⁸ in the hypothesis of this reaction mechanism. However, most of the authors did not consider this evidence conclusive since it could not be excluded that the data could also be interpreted by hypothesizing two nucleon absorption followed by final state interactions of one of the primary nucleons and, in fact, calculations based on this hypothesis afford a satisfactory reproduction of deuteron spectra.^{17,49} Nevertheless, there is a general consensus about the fact that in a fraction of cases, of the order of that above reported, π^- absorption occurs on an α clusters (see, for instance, Refs. 1, 50, and 51).

The value reported above for the probability of α particle absorption is the same one obtains on the basis of the yield of n-t coincidences measured recently by Heusi *et al.*¹ for π^- absorption on ¹²C (≈ 0.011 per stopped π^-) if one considers that (a) the branching ratio for n-t production is ≈ 0.19 , (b) only in about one-half of the cases, as suggested above, the triton may leave the nucleus without being absorbed, and (c) also in the case of the correlated primary neutron the probability of being emitted without suffering final-state interactions is smaller than unity and of the order of 0.5. Taking into account all these factors, the amount of α -cluster absorption results in ≈ 0.23 per stopped π^- .

B. Ratio of np to pp pairs that can absorb the pion

In the case of π^- absorption by a nucleon pair a basic quantity to be known is the ratio *R* between the numbers, N_{np} and N_{pp} , of neutron-proton and proton-proton pairs that may absorb the pion. Estimates of *R* range from the so called statistical value

$$R_{\text{stat}} = \frac{2N}{Z-1} , \qquad (1)$$

valid in the hypothesis that any possible proton-proton or neutron-proton pair may contribute to the π^- absorption, to values notably greater, as predicted by calculations including rescattering terms in the absorption mechanism.⁵² The situation is summarized in the paper by Heusi et al.¹

Theoretical models of π^- -nucleon interaction are not considered sufficiently accurate to provide reliable *a priori* estimates of R, so the usually adopted procedure is to deduce R directly by the data.

At first, it was assumed that R could be measured as the ratio of the yields of correlated back to back neutronneutron and neutron-proton emissions, in coincidence experiments.^{2,3} Subsequent work^{1,5,53} suggested that, except for the lighest nuclei, even in the case of a 180° correlated emission of nucleons, final state interactions are far from negligible and may significantly decrease the value measured for the ratio of n-n to n-p pairs. Thus, R may be reliably measured, in coincidence experiments, only in the case of the lighest nuclei, especially if, in addition to measuring a 180° correlated emission of n-n and n-p pairs, one imposes the further condition that the sum of the emitted nucleon energies cannot differ, by more than a given amount, from the total energy delivered in the absorption.¹

Values reported for the ratios of the yields of n-n to n-p coincidences¹⁻³ are reported in Fig. 7.

Orth *et al.*,³⁰ to deduce R, measured the yields of twoneutron and one-neutron-one-proton-out products with radiochemical techniques. These residual nuclei are produced only when the two nucleons escape the nucleus, leaving behind an excitation energy lower than the binding energy of the next least bound nucleon. By this measurement, values of R which increase with A and are rather higher than those obtained through the analysis of different experimental data are obtained. These authors interpret their data by assuming that "the yields of two-



FIG. 7. Values of R deduced from (a) coincident emission of n-n and n-p pairs [open (Ref. 1), solid inverted (Ref. 2), and solid triangles (Ref. 3); the rectangle for $A \approx 6-16$ represents the ensemble of the values obtained in Ref. 2 for these light nuclei]; (b) the measured yields of two-neutron-out and one-neutron-oneproton-out residuals, diamonds (Ref. 30). The arrow above the diamond, in the case of $A \ge 70$ nuclei, indicates how much R might differ by the measured ratio. Assuming 25% of α -cluster absorption, for all these nuclei R could be as high as 25. In absence of α -cluster absorption the value of R would be intermediate between the value reported by Orth et al. (Ref. 30) and 25. Analogous information cannot be given here for ²⁶Mg, where only the emission of preequilibrium particles was calculated; (c) the yield of highest energy neutrons and protons [solid (Ref. 12) and open circles (Ref. 17)]; (d) the analysis of inclusive neutron and proton spectra [solid (Ref. 11) and open squares (Ref. 12)].

nucleon-out residual products ($\Delta A = 2$) reflect the neutron and proton densities at the surface" because production of these nuclei may occur only when the primary nucleons did not undergo final state interactions. This is by no means an obvious assumption. In fact, most of the models adopted to predict fast particle emission in nuclear reactions lead to the conclusion that a nucleon may escape the nucleus without sharing its energy with other nuclear constituents also if it is produced quite deeply in the nuclear volume. If this is true, the experimental data of Orth et al.³⁰ should be somewhat sensitive also to absorptions occurring in the nuclear interior, and the measured ratios of the yields of two-nucleon-out products, though related to R, do not constitute a *direct* measure of this quantity. As will be discussed in Sec. VE, the calculations made using the exciton model predict a value of the ratio, R^* , between two-neutron-out and oneneutron-one-proton-out products noticeably smaller than the value of R utilized as an input datum.

This is due to the following facts.

(i) These residuals may be produced in a non-negligible amount, in a two nucleon absorption mechanism, also when one fast nucleon is emitted which leaves a slightly excited residual that deexcites by evaporation of only another nucleon. This effect, especially in the case of not too heavy nuclei, where proton evaporation is far from negligible, leads to a substantial increase in the rate of one-neutron-one-proton emissions,

(ii) a fraction of the absorptions occurs on α clusters. This absorption mode alone would lead to R^* values of the order of unity, varying from ≈ 0.7 to ≈ 2 according to the nucleus considered.

Though the theoretical prediction of the yield of twonucleon-out products is not very accurate, as will be discussed in Sec. V E, calculations of this kind seem to indicate that the data of Orth *et al.* are consistent with Rvalues that are almost A independent and whose values may be as high as 25.

Another approach widely adopted consists of deducing the value of R by the ratio of measured inclusive multiplicities of fast protons and neutrons. In this case consideration of only the highest energy part of measured multiplicity distributions^{12,17,54} may lead to incorrect estimates of R because of the experimental uncertainty affecting this part of the measured spectra. When considering a greater portion of the measured nucleon spectra, one has to take into account that interactions of primary nucleons on their way out of the nucleus modify the ratio between the number of primary protons and neutrons,

$$\frac{n_{\rm p}}{n_{\rm n}} = \frac{1}{2R+1}$$
, (2)

just after π^- absorption. The value of R one deduces may thus depend on the particular model and on the calculation parameters one utilizes for evaluating the spectra of emitted fast nucleons. Another source of error may be due to the systematic uncertainty on the absolute values of the measured multiplicities, which often differ significantly from one author to the other. In the analyses of the spectra, based on the use of the exciton model, that are discussed in the following sections of this paper, R is treated as a free parameter and it is found to display a quite notable A dependence, being substantially greater in the case of medium-heavy and heavy nuclei ($A \ge 100$) than in the case of nuclei with $A \approx 40-60$. As discussed in Sec. II, this is a consequence of the A dependence of the magnitude of the multiplicities of fast protons emitted in π^- absorption, an experimental finding on which not all authors agree.¹⁵

The *R* values measured by the different techniques that have been discussed and the values obtained in our analysis of the proton spectra and the radiochemical data by Orth *et al.*³⁰ are reported in Fig. 7.

The *R* values obtained from consideration of proton and neutron spectra and coincidence experiments refer, in most of the cases, to nuclei which have not been investigated by Orth *et al.*,³⁰ with the exception of $A \approx 26$ and 115 nuclei. In these two cases a strict numerical agreement between values from the different types of data has not been found, so the possible *A* dependence of *R*, suggested by the figure, should be checked by further experiments.

The discussion of the experimental results in this section may help to fix some basic input parameters for calculations of the cross sections of the reactions induced by π^- absorption. In the next section calculations based on the exciton model will be described.

IV. THEORY

The calculations described in this section are done in the framework of the exciton model^{55,56} and are an improvement of those already published by our group.^{27,36,57,58}

 π^- absorption is assumed to excite states of a simple configuration in the composite nucleus. These states may either evolve toward more complex states, through twobody residual interactions, or the nucleus may deexcite by emission of one fast particle in the continuum. The process is repeated until an equilibrated system is created which further decays by evaporation. At any stage of this deexcitation cascade, the probability of a given process is evaluated as the ratio between the decay rate corresponding to that event and the total decay rate of the composite nucleus.

The initial configuration which gives rise to the deexciting interaction cascade depends on the assumed absorption mode. As a result of the discussion reported in the preceding sections, we will consider two dominant absorption modes, by assuming that in 75% of the cases the pion is absorbed by a pair of nucleons and, in 25% of the cases, by an α cluster. In two nucleon absorption, the excitation energy is divided between two *one-particle-onehole* pairs. In the case of α cluster absorption, the hypothesis is made that this process may occur only in the outer edge of the nucleus and then it may be assumed that a negligible energy is provided to the hole degrees of freedom.

A. Two nucleon absorption

The width for pion absorption is given by

$$d^{9}\Gamma = 2\pi |M|^{2} \delta(\mathbf{P} - \mathbf{p}_{1} - \mathbf{p}_{2}) \delta(E_{i} - E_{f}) d\mathbf{P} d\mathbf{p}_{1} d\mathbf{p}_{2} .$$
(3)

P, \mathbf{p}_1 , and \mathbf{p}_2 are, respectively, the recoil momentum of the core and the momenta of the primary nucleons after absorption,

$$E_i = Q = (m_{\pi^-} + {}^Z A_N - {}^{Z-1} A_{N+1})c^2 + B_{\pi^-}$$
(4)

is the energy delivered in the absorption $(m_{\pi^-}, {}^ZA_N)$, and ${}^{Z^{-1}}A_{N+1}$ are the pion rest mass and the masses of the nucleus before and after absorption; B_{π^-} is the π^- binding energy in the mesic atom), and

$$E_f = E_1 + E_2 + E_R , (5)$$

where E_1 , E_2 , and E_R are, respectively, the energies given to the primary nucleons and the recoiling core.

The transition matrix element M depends in first approximation only on the sum of the momenta of the absorbing nucleons.^{59,60} A widely used expression for it is

$$|M|^2 \approx e^{-(P/\gamma)^2}, \qquad (6)$$

where P is the sum of the absorbing nucleon momenta and γ is proportional to the variance of the two-nucleon momentum distribution. A justification of expression (6) may be given in the harmonic oscillator shell model⁵⁹ or by assuming that the two absorbing nucleons may be selected at random from the Fermi sea, as we also gave in our previous calculations. In this case

$$\gamma \approx \left(\frac{4}{5}\right)^{1/2} p_F \quad , \tag{7}$$

where p_F is the Fermi momentum.⁶¹ By assuming for ϵ_F the value of 20 MeV, $\gamma = 174.25$ MeV/c.

To evaluate the energy distribution of each of the two one-particle-one-hole configurations, excited in the absorption, we used a Monte Carlo approach. These distributions are well fitted by a Gaussian expression,

$$P(E)dE = Ce^{-(Q/2 - E)^2/2\sigma^2} dE , \qquad (8)$$

where $\sigma \approx 23$ MeV.

To check if the approximations introduced allow one to reproduce satisfactorily the excitation energy distributions of the initial particle-hole states, in Fig. 8, in the case of π^- absorption on ¹²C, the measured spectra of neutrons and protons in coincidence with the accompanying primary nucleon emitted at 180° with energy $E \ge 15-20$ MeV (Ref. 1) are compared with those calculated by our model. The agreement appears sufficiently good for the calculation of inclusive particle spectra and spallation residue yields.⁶²

A more *indirect* check of the results of our calculation is afforded by the analysis of the angular correlation of the primary n-n pairs¹ shown in Fig. 9. Also in this case a reasonable reproduction of the data is obtained.

The composite nucleus deexcites by means of two independent cascades characterized by the oneparticle-one-hole initial configuration.^{27,36,57,58} The ratio R between the number of absorbing n-p and p-p pairs was



FIG. 8. Spectra of nucleons emitted in coincidence with a second nucleon with $E \ge E^*$, at 180°, after π^- absorption on ¹²C. Left, n-n coincidences, $E^* = 20$ MeV; right, n-p coincidences, $E^* = 15$ MeV. The measured spectra (Ref. 1) are given by open (neutrons) and black (protons) circles. The calculated spectra are given by the solid and dashed lines.

treated as a free parameter. During the preequilibrium deexcitation phase, neutrons, protons, and α 's can be emitted. The α particles are supposed to be emitted as a consequence of the interaction of excited nucleons with preformed α substructures. The possibility of up to four preequilibrium emissions in sequence is considered; how-



FIG. 9. Angular correlation of primary n-n pairs emitted after π^- absorption on ¹²C. Experimental values (Ref. 1) are given by the open circles, the calculated distribution by the solid line.

ever, one particle emission in each cascade is the most likely occurrence. The decay rates for particle emission are reported elsewhere; $^{58,63-65}$ those for exciton-exciton two body residual interactions are reported in Table I and are calculated as described in Ref. 63.

The expression of the decay rates utilized to describe

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(MeV)	$\boldsymbol{W}_{\mathrm{eq}}^{1,1}$	$W_{\rm eq}^{2,2}$	$W_{eq}^{3,3}$	$W_{\mathrm{eq}}^{4,4}$	$W_{\rm eq}^{5,5}$	$W_{\mathrm{eq}}^{6,6}$	$W_{\mathrm{eq}}^{7,7}$	$W_{\mathrm{eq}}^{8,8}$	$W_{\rm eq}^{9,9}$	$W_{eq}^{10,10}$
20	0.82	0.53	0.41	0.34	0.29	0.26	0.24	0.22	0.21	0.20
24	0.93	0.73	0.56	0.46	0.40	0.36	0.33	0.30	0.28	0.26
28	1.02	0.92	0.73	0.60	0.52	0.47	0.43	0.39	0.36	0.34
32	1.10	1.09	0.91	0.76	0.66	0.60	0.54	0.49	0.46	0.43
36	1.16	1.25	1.09	0.93	0.80	0.73	0.66	0.60	0.56	0.52
40	1.20	1.39	1.27	1.11	1.01	0.91	0.81	0.73	0.67	0.62
44	1.24	1.51	1.44	1.28	1.16	1.06	0.96	0.87	0.79	0.74
48	1.26	1.61	1.60	1.46	1.35	1.22	1.13	1.01	0.93	0.86
52	1.28	1.70	1.75	1.63	1.52	1.40	1.30	1.18	1.07	0.99
56	1.29	1.78	1.88	1.79	1.70	1.59	1.49	1.35	1.23	1.13
60	1.30	1.85	2.00	1.95	1.88	1.77	1.68	1.53	1.39	1.29
64	1.31	1.91	2.12	2.10	2.05	1.95	1.87	1.71	1.57	1.44
68	1.31	1.96	2.22	2.24	2.23	2.15	2.06	1.90	1.75	1.61
72	1.32	2.00	2.31	2.37	2.38	2.34	2.25	2.09	1.93	1.79
76	1.32	2.05	2.40	2.50	2.54	2.50	2.43	2.28	2.12	1.97
80	1.32	2.08	2.48	2.61	2.70	2.71	2.61	2.47	2.31	2.15
84	1.32	2.11	2.55	2.72	2.82	2.87	2.78	2.65	2.50	2.34
88	1.31	2.14	2.61	2.82	2.92	2.99	2.95	2.83	2.68	2.53
92	1.31	2.17	2.67	2.92	3.05	3.14	3.11	3.00	2.87	2.72
96	1.31	2.19	2.73	3.01	3.14	3.26	3.26	3.17	3.05	2.90
100	1.30	2.21	2.78	3.09	3.26	3.38	3.40	3.34	3.23	3.09
104	1.30	2.23	2.83	3.17	3.33	3.48	3.53	3.49	3.40	3.27
108	1.30	2.25	2.87	3.25	3.43	3.58	3.65	3.64	3.56	3.44
112	1.29	2.26	2.91	3.31	3.53	3.67	3.78	3.79	3.73	3.62
116	1.29	2.27	2.95	3.38	3.62	3.75	3.89	3.92	3.88	3.79
118	1.28	2.28	2.97	3.41	3.67	3.79	3.94	3.99	3.96	3.88

TABLE I. Decay rates for exciton-exciton interactions utilized in the present work (units of 10²² s).

the evaporation phase of the deexcitation may be found in Ref. 64, where the computational technique (of Monte Carlo type) is also described.

B. α -cluster absorption

If one assumes that absorption on α clusters occurs at the surface of the nucleus, the following approximations seem to be reasonable.

(a) A negligible amount of excitation energy is, initially, provided to the hole degrees of freedom.

(b) If a deuteron or a triton is produced, they may survive and come out of the nucleus only if they are directed outward (this happens in approximately one-half of the cases); otherwise, they soon dissolve in their constituent nucleons with a statistical partition of the energy among them.

The branching ratios for the possible final channels,

$$\pi^- + \alpha = \mathbf{p} + 3\mathbf{n} , \qquad (9)$$

$$\pi^- + \alpha = \mathbf{t} + \mathbf{n} , \qquad (10)$$

$$\pi^- + \alpha = \mathbf{d} + 2\mathbf{n} , \qquad (11)$$

are the experimental ones renormalized to unity (25%, 19%, and 56%, respectively).

In the case of reaction (9) a statistical partition of the excitation energy (=Q) among the four nucleons is assumed and a deexcitation cascade characterized by a four particle (three-neutron-one-proton) initial configuration is considered.

In the case of reactions (10) and (11), if the triton or the deuteron are emitted, a cascade, characterized, respectively, by one neutron and two neutron initial configurations, is calculated. To evaluate, in these cases, the initial excitation energy distribution for each cascade, one could calculate the primary triton and deuteron energy distributions with the formulas reported by Kolybashov,⁴⁸ however, in agreement with previous considerations, we decided to utilize for them the distributions measured in the case of a heavy nucleus like ¹⁹⁷Au.¹² If the triton or the deuteron dissolve in their constituents, a cascade characterized by three-neutron–one-proton is considered, with excitation energy equal to Q.

C. Calculation parameters

All the parameters entering the calculation except the ratio R between n-p and p-p absorbing pairs have been fixed *a priori*. These are the following.

(a) The percentage of absorptions on α substructures, taken equal to 25%, as previously discussed.

(b) The decay rates for exciton-exciton interaction, calculated as discussed in Ref. 63 and tabulated in Table I.

(c) The single nucleon state densities $g(\epsilon)$ that characterize the particle-hole state densities, which are those corresponding to the Fermi gas for energies below the Fermi energy ϵ_F (=20 MeV), and are taken to be constant (= $A/13.16 \text{ MeV}^{-1}$) for energies above. The state densities have been calculated by a recursion technique which allows one to take correctly into account the limitation in

the hole excitation energy due to the finite depth of the potential well. 63

(d) The ratio between Φ_{α} , the density of performed α particles, and g_{α} , the α state density, necessary to evaluate the decay rates for α emission,⁶⁵ which is taken equal to $\approx 0.9 A^{-1}$ MeV.

(e) The inverse cross sections appearing in the expression of the decay rates for particle emission during the preequilibrium deexcitation cascade, calculated as reported in Ref. 67.

(f) The inverse cross sections in the decay rates for particle emission during the evaporation phase calculated by the semiclassical expression and the parameters reported in Ref. 64.

(g) Experimental nucleon and α particle binding energies, ⁶⁸ or, when unmeasured, those calculated by means of Myers and Swiatecki mass formula.⁶⁹

(h) Pairing energies from Nemirowsky and Adam-chuck. $^{70}\,$

D. Dependence of the calculated results on the percentage of absorptions on α substructures

In Figs. 10(a), 10(b), and 10(c) the neutron spectrum for π^- absorption on 27 Al, and the proton spectra for π^- absorption on, respectively, 40 Ca and 197 Au, are reported. The theoretical spectra are calculated either for 75% two-nucleon and 25% α -cluster absorptions, or by assuming only one type of absorption mechanism. In the first case, contributions resulting from the dissolution of the α into four nucleons (3n-1p) and the emission of, respectively, a deuteron (2n-d) and a triton (n-t) are separately reported.

The contribution of two-nucleon absorptions to the theoretical neutron spectrum from ²⁷Al is calculated for R = 2.8; however, the result depends weakly on the values assumed for this parameter.

The spectrum calculated for $25\% \alpha$ -cluster absorption (solid line) differs very slightly from that corresponding to two nucleon absorption alone (dotted line). Also, if the comparison with the experimental neutron spectra, discussed in the next section, always favors the calculations which take into account α -cluster absorption, independently on the considered absorbing nucleus, the conclusion one draws from Fig. 10(a) is that the shape and absolute value of neutron spectra depend very weakly on the percentage of α -cluster absorptions, at least for reasonable values of this quantity, and, at the same time, exclude the dominance of absorptions on α clusters which leads to spectra which are too soft (dashed line).

In the case of 40 Ca, the proton spectrum calculated considering only α particle absorption (dashed line histogram) overestimates slightly that measured by Schlepuetz *et al.*¹¹ for proton energies ranging from ≈ 15 to 50 MeV, while the spectrum calculated for two nucleon emission only, with R = 4, is barely distinguishable from that evaluated taking into account 25% of α -cluster absorption (solid line histogram). The conclusion is that consideration of proton spectra alone does not allow one to deduce—with reasonable accuracy—a small percentage of absorptions on α clusters, if R is small. 10

10

10

particles





tions from α absorption and different reaction paths [dissolution in four nucleons (3n-1p), neutron emission accompanied by fast deuteron (2n-d), and fast triton emission (n-t)] are explicitly shown; (ii) two-nucleon absorption only, dotted line, and (iii) α -cluster absorption only, dashed line. (b) Comparison of the experimental inclusive spectrum of protons from π^- absorption on ⁴⁰Ca, open (Ref. 9) and solid (Ref. 11) circles with spectra calculated assuming the following: (i) 75% two-nucleon absorption (R=4) and 25% α -cluster absorption, solid line histogram. The contributions from α absorption and different reaction paths are explicitly shown; (ii) α -cluster absorption only, dashed line histogram. (c) Same as (b) for the inclusive proton spectrum from π^- absorption on ¹⁹⁷Au. In the evaluation of the two-nucleon absorption contribution, the value R=9 was utilized.



FIG. 11. Spallation residue yields for π^- absorption on ⁵⁹Co. The experimental cross sections (Ref. 27) are given by the open circles. The theoretical distributions are calculated by assuming 75% two nucleon and 25% α cluster absorption (solid line histograms), two nucleon absorption alone (dotted line histograms), and α cluster absorption alone (dashed line histograms). In two nucleon absorption R was taken to be equal to 2.46. The dependence of the calculations on R is shown in Fig. 17.

In the case of ¹⁹⁷Au, the spectrum calculated considering only α -cluster absorption, dashed line histogram, significantly overestimates the measured one. Qualitatively, this indicates the presence of two nucleon absorptions and a quite high value for *R*. However, the analysis of the proton spectrum alone does not allow, also in this case, an unambiguous determination of both *R* and the percentage of α -cluster absorption. It would be possible to obtain a satisfactory reproduction of the measured absorption the measured absorption ab

ering 25% of α -cluster absorptions. To conclude, the analysis of neutron and proton spectra shows that two-nucleon absorption is more important than α -cluster absorption. Proton spectra may be reproduced also without considering α -cluster absorption by using R values lower than those necessary when this absorption mode is taken into account. Neutron spectra are reproduced slightly better by considering a small percentage of α absorption, but even in this case it is impossible to deduce with reasonable accuracy the importance of this absorption mechanism.

spectrum by considering only two nucleon absorption

with an R value lower than that (=9) obtained by consid-

Similar conclusion are also reached through the analysis of spallation residue yields. In Fig. 11 a typical result is shown. The spallation residue yields predicted by assuming only two nucleon absorption (dotted line histogram) differ slightly from those calculated by assuming a 25% probability of α cluster absorption (solid line histogram), and both afford a reasonable reproduction of the data. Then, it appears impossible to utilize data of this kind to deduce a small percentage of α cluster absorptions. On the other hand, the yield distributions calculated by assuming only α absorption (dashed line histogram) in the case of (π^- ,xn) and (π^- ,pxn) residues differ quite considerably from the experimental ones.

According to our calculations, one must resort to consideration of α -cluster absorptions to explain emission of fast deuterons and tritons, and the yield of neutron-triton coincidences, as discussed in Sec. III A, but not to reproduce neutron and proton spectra and spallation residue yields.

V. COMPARISON OF EXPERIMENTAL DATA WITH THEORY

A. Neutron spectra

Experimental and theoretical neutron spectra are compared in Figs. 10(a) and 12–14. Except if explicitly stated, all the calculations discussed in this section are made by assuming 75% and 25% of, respectively, two-neutron and α -cluster absorptions.

In the case of ¹²C, the data are from Refs. 5 and 18. The experimental spectra differ quite considerably below ≈ 60 MeV. The possible origin of the discrepancy is discussed by Madey *et al.*¹⁸ The arrow shows the maximum energy available for the emitted neutrons. Also taking into account the experimental energy resolution, which allows some neutrons to be detected with apparent energy above this limit, it appears likely that the experimental spectra are somewhat overestimated at the high energy side. The theoretical spectrum corresponds to R = 3.8, the value obtained by a best fitting of the proton spectrum; however, as previously stated the neutron spectra depend very weakly from the value chosen for this parameter. The code we dispose of was not considered sufficiently accurate to evaluate the evaporative contribution in the case of a nucleus as light as carbon, which may really dissolve into smaller pieces as soon as the energy delivered in the absorption is no longer concentrated on the primary nucleons, so only the preequilibrium contribution is shown, which is in satisfactory agreement with the data from Madey *et al.*¹⁸

Due to the weak A dependence of neutron spectra, shown in Fig. 1, both the measured neutron spectra from ⁵⁹Co (Ref. 5) and natural copper (Ref. 18) are reported in Fig. 13. The two sets of experimental data are in better agreement among themselves than in the case of ¹²C. The calculated spectrum holds for both ⁵⁹Co and Cu. The calculation is made for $R = R_{\text{stat}} = 2.46$, the value obtained by a best fitting of proton spectrum on ⁵⁹Co. Also in this case the spectrum from Madey is reproduced quite satisfactorily, also taking into account that experimental and calculated spectra differ by about 30% at energies around 20 MeV.

In Fig. 14 experimental spectra from ¹⁹⁷Au, ⁵ ¹⁸¹Ta and Pb, ¹⁸ and ¹⁶⁵Ho (Ref. 23) are reported. The calculated



FIG. 12. Comparison of experimental and theoretical neutron and proton spectra for π^- absorption on ¹²C. The neutron data, in the upper part of the figure, are from Ref. 5, solid points, and Ref. 18, open points. The calculated spectrum is given by the solid line. The proton data, in the lower part of the figure, are from Ref. 12, solid points, an Ref. 13, solid line. The calculated spectrum is given by the solid line histogram.

spectrum is for ¹⁹⁷Au and corresponds to R=9. Also in this case the experimental results are rather satisfactorily reproduced except for energies around 20 MeV where, also in this case, a disagreement of up to about 30% is found between theoretical prediction and data. Both in this and in the previous case it is not easy to quantify exactly the amount of disagreement. Part of the disagreement could be due to a systematic overestimation of the experimental neutron yield, or to a systematic underestimation of the theoretical yield all over the spectrum. Nevertheless, though the qualitative features of the experimental data are correctly reproduced, there is definite evidence of some underestimation in the theoretical neutron yield at energies around about 20 MeV. Two possible origins for the disagreement are (a) an underestimation of secondary nucleon emission, and (b) the approximation of neglecting the possibility of providing energy to the holes in the case of α -cluster absorption. We found it impossi-

10 10 10 MeV particles stopped π^- 10 10 10 0 50 100 E (MeV)



FIG. 13. Comparison of experimental and theoretical neutron, proton, and α -particle spectra for π^- absorption on ⁵⁹C. The neutron data, in the upper part of the figure, are from Ref. 5, open points, and Ref. 18 referring to Cu, solid points. The calculated spectrum is given by the solid line. The proton data, in the middle part of the figure, are from Ref. 12, open points. Calculated spectra are given by the solid (R=2.46) and dashed line (R=9.5) histograms. The α -particle data, lower part of the figure, are given by the solid points (Ref. 12). The calculated spectrum is given by the solid line is given by the solid line.

FIG. 14. Comparison of experimental and theoretical neutron, proton, and α -particle spectra for π^- absorption on ¹⁹⁷Au and heavy nuclei. The neutron data, in the upper part of the figure, are from Ref. 5 (¹⁹⁷Au, open squares), Ref. 18 (Ta, solid points; Pb, triangles), and Ref. 23 (¹⁶⁵Ho, open points). The calculated spectrum is given by the solid line. The proton data, in the middle part of the figure, are from Ref. 12, open points. Calculated spectra are given by the solid (R=9) and dashed line (R=18) histograms. The α -particle data, lower part of the figure, are given by the solid points (Ref. 12). The calculated spectrum is given by the solid points (Ref. 12).



FIG. 15. Total neutron multiplicities (lower part) and preequilibrium neutron multiplicities (upper part) for π^- absorptions on various nuclei. The data are from Ref. 5, open squares, Ref. 14, solid triangles, Ref. 18, open circles, Ref. 22, solid squares, and Ref. 27, diamonds. Calculated values are given by the solid circles connected by the dashed line.

ble to increase, with acceptable variations of the calculation parameters, the emission of secondary neutrons. As far as the second possibility is concerned, it is, at present, under consideration.

Measured total multiplicities and multiplicities of preequilibrium neutrons deduced from the experimental data by subtraction of the evaporation component evaluated by means of the Le Couteur formula⁷¹ are reported in Fig. 15 together with the theoretical estimates. The absolute value and the A dependence of these multiplicities are well reproduced.



FIG. 16. Experimental (Ref. 14), solid circles with error bars, and theoretical, solid triangles, total neutron multiplicities (lower part) and multiplicities of neutrons with energy exceeding 20 MeV (upper part) for π^- absorptions on nickel isotopes.

Neutron multiplicities have been measured by Isaak et al. in the case of Ni isotopes¹⁴ to check the theoretical prediction⁵⁸ of a strong dependence of evaporated neutron and proton multiplicities on N-Z for these isotopes. In Fig. 16 the experimental neutron multiplicities and the multiplicities of neutrons with $E \ge 20$ MeV are compared with the theoretical prediction, which may be seen to afford an accurate reproduction of these data.



FIG. 17. Proton spectra from Ca [upper part of the figure: open points (Ref. 9), solid points (Ref. 11), solid line (Ref. 15)]; In and Cd [middle part: open points (Ref. 9), solid points (Ref. 11)]; Bi [lower part: open points (Ref. 9), solid points (Ref. 11)]. For Ca, theoretical spectra corresponding to three different values of R are reported: R=4, solid line histogram; R=8, dashed line; R=20, dotted line. The calculated spectrum for In-Cd corresponds to R=11. For Bi, the solid line histogram corresponds to R=7.5, the dashed to R=18.

B. Proton spectra

The results of analyses of inclusive proton spectra, for nuclei ranging from ¹²C to ²⁰⁹Bi, are reported in Figs. 10(a), 10(b), 12–14, and 17. The absolute value of proton yield depends on the value assumed for R. This dependence is quite sensible in the highest energy part of the spectra where primary protons mostly contribute. In the intermediate energy range ($\approx 20-50$ MeV) the dependence is weaker due to the relevant contribution of secondary emissions to proton yield and the fact that most of the secondary protons are knocked out by primary neutrons. So, R also influences the spectral shape, though not in a very sensible way.

For each spectrum, by a χ^2 procedure, the value of R which provides the best reproduction of the data was selected. In all cases, 25% of α -cluster absorptions was assumed. As was discussed in Sec. IV D, consideration of two-nucleon absorption alone would lead to R values somewhat smaller than those deduced when α -cluster absorption is considered, especially when high values are found for this parameter.

The spectrum of protons from ${}^{12}C$ has been measured by Pruys *et al.*¹² and Mechtersmeyer *et al.*¹³ The two sets of experimental data, reported in Fig. 12, are in reasonable agreement, except in the highest energy part. The theoretical calculation satisfactorily reproduces both set of data, being halfway between the two measured spectra at the highest energy side. The best fit *R* value is equal to 3.8, a numerical value in good agreement with that found in coincidence experiments.^{1,3}

The spectrum of protons from ⁵⁹Co, ¹² shown in Fig. 13, is reproduced less satisfactorily. The best fit *R* value is equal to about R_{stat} (=2.46). The theoretical yield is lower than the exprimental one in the energy interval $\approx 15-40$ MeV, with a maximum discrepancy of about 30%. Also shown in the figure is the result of a calculation corresponding to the much higher value R=9.5. Also, if the spectral shape is reproduced more satisfactorily, the calculated yield is too low, being smaller than the experimental one by approximately a factor of 2.

The spectrum of protons from ¹⁹⁷Au (Ref. 12) is shown in Fig. 14. The calculation corresponding to R=9 reproduces satisfactorily the highest energy part of the spectrum, but underestimates the measured yield below ≈ 30 MeV. Also shown in the figure is the result of a calculation corresponding to the higher value R=18. In this case one may appreciate how the value obtained for R depends on the value reported for the absolute proton multiplicity: if the absolute proton yield measured by Pruys *et al.*¹² is reduced by about 20%, a very satisfactory agreement with the calculation corresponding to the higher R value is obtained.

The analysis of the two previous spectra, from Pruys *et al.*, 12 could indicate a systematic underestimation of the measured proton yield at energies where secondary emission mostly contributes, a result qualitatively similar to that already found in the analysis of neutron spectra from medium-heavy and heavy nuclei. However, this conclusion is not substantiated by the analysis of proton spectra from different laboratories.^{9,11} In fact, the

analysis of the spectra of protons from ⁴⁰Ca, ¹¹⁵In, and ²⁰⁹Bi measured by Schlepuetz et al.¹¹ seems to indicate that the calculated spectra reproduce accurately the measured ones in the energy interval where a systematic disagreement with the data from Pruys et al.¹² was found (see Fig. 17). This is also true when the spectra from Ca, Cd, and Pb, measured by Budyashov et al.,9 are considered. These authors did not report the absolute yield of the measured proton multiplicities that we have arbitrarily normalized to the values measured by Schlepuetz et al.,¹¹ for the same or neighboring nuclei. These normalized proton spectra are also reported in Fig. 17. The best reproduction of the experimental spectra was obtained in the case of Ca, Cd-In, and Pb-Bi with values of R equal to, respectively, 4, 11, and 7.5 (solid line histograms). In the case of Ca, the experimental proton spectrum reported by Randoll et al.¹⁵ and theoretical spectra corresponding to R = 8 and 20 (the dashed and dotted line histograms, respectively) are also reported. The yield measured by Randoll et al. is lower by approximately a factor of 2 than that reported by Schlepuetz et al., and the spectrum shape cannot be reproduced all over the proton energy interval, even by taking R = 20. In the case of Pb/Bi, the theoretical spectrum corresponding to R = 18is also shown (dashed line histogram).

Isaak *et al.*¹⁴ have investigated, for Ni isotopes, the variation with N - Z of the total proton multiplicities and the multiplicities of the fast protons ($E \ge 20$ MeV). Their results are reported in Fig. 18, where it is shown how accurately the theory may reproduce these data. The calculations have been made by assuming $R = R_{\text{stat}}$.

The analysis of proton spectra indicates a substantial agreement between theory and experiment. However, the lack of agreement between data from different laboratories does not allow one to conclude if the agreement extends all over the proton energy interval or if some systematic disagreement exists which may indicate the need for fur-

stopped π

stopped π

0.5

0

2

0



ther refinements in the theoretical model. In many instances, as shown in Figs. 14 and 17, the value deduced for R depends quite sensitively on the measured absolute yield of emitted protons.

C. α -particle spectra

In Figs. 13 and 14 the comparison between experimental¹² and theoretical α -particle multiplicity distributions is shown for, respectively, ⁵⁹Co and ¹⁹⁷Au. The theory reproduces rather satisfactorily, both in shape and absolute value, the data with essentially the same parameters used for evaluating alpha spectra in proton induced reactions on the same nuclei at $E_p \approx 70$ MeV.^{42,72}

D. Spallation residue yield distributions

In Figs. 19–21 experimental and theoretical distributions of the yields of spallation residues are compared in the case of 59 Co, 27 209 Bi, 27,32 and 181 Ta. 29

In Sec. IV D it was already shown that the calculated distributions depend weakly on the presence of some contribution from α -cluster absorption. In this subsection we show how much the calculated distributions depend on the value assumed for R. In Fig. 19, where ⁵⁹Co is discussed, the solid line histograms correspond to R=9.5 and the dashed line histograms to R=2.46, the value from the best fit of inclusive proton spectrum. The calculated distributions depend very little on the value assumed for R, except, essentially, in the case of one-proton-one-neutron-out products. This is confirmed also by the results obtained in the case of 209 Bi in Fig. 20. Here the solid line histogram corresponds to R=18 and the dashed to R=7.5, the value from the best fit of the inclusive pro-

ton spectrum. In the case of ¹⁸¹Ta (Fig. 21) the calculated distributions correspond to R = 18.5.

In all the cases considered, a very satisfactory agreement between theory and experiment has been found, except perhaps in the case of 3pxn residues from π^- absorption on ¹⁸¹Ta. Nevertheless, the discrepancy between data and theoretical prediction is, also in this case, within the limits of the accuracy expected *a priori* in predicting the experimental distributions, and, most presumably, the major source of disagreement has to be found in the difficulty of evaluating accurately the branchings for rare events in the evaporation phase of the deexcitation process.

Satisfactory results, that are not discussed here, have also been found in the case of other nuclei. These improved calculations confirm the conclusions reached in our previous works.^{27,58}

E. Yield of two-neutron and one-neutron-one-proton-out products

These data are those reported by Orth *et al.*³⁰ measuring the ratio R^* between the yields of two-neutron-out and one-proton-one-neutron-out products for nuclei ranging from ²⁶Mg to ¹⁷⁴Yb.

As anticipated in Sec. III B, the calculation shows that (i) in a rather sizable fraction of the cases two nucleon emission occurs also following α -cluster absorption, and (ii) even in the case of two-nucleon absorption, not always the two emitted nucleons are *both* primary nucleons. As a consequence, R^* does not represent a direct measure of R.

The small yield of 1p-1n residuals cannot be reproduced very accurately by our model with the use of a set of average parameters, like the ones we utilize. Also, the



FIG. 19. Comparison of experimental (Ref. 27), open circles, and theoretical distributions of the yields of the spallation residues from π^- absorption on ⁵⁹Co. In evaluating the contribution of two-nucleon absorption, two values of *R* have been considered: 9.5, solid line histograms, and 2.46, dashed line histograms.



FIG. 20. Comparison of experimental [open circles (Ref. 27), solid triangles (Ref. 32)] and theoretical distributions of the yields of the spallation residues from π^- absorption on ²⁰⁹Bi. In evaluating the contribution of two nucleon absorption, two values of *R* have been considered: 18, full line histograms, and 7.5, dashed line histograms.

TABLE II. Comparison of experimental (Ref. 30) and theoretical ratios of the yield of two-neutron-out and oneneutron-one-proton-out products in stopped π^- absorption. In the first column the absorbing nucleus is reported; in the second the experimental value; in the third and fourth the theoretical values corresponding to R=25, assuming, respectively, 25% of α -cluster absorption and two-nucleon absorption alone.

Nucleus	R_{expt}^*	R *'	R *"		
⁷⁴ Ge	11.0±0.6	10.7±4.3	14.6± 5.8		
⁹⁷ Mo	12.1±1.2	13.2±5.3	21.3 ± 8.5		
114 Cd	12.4±1.0	9.4±3.8	13.5± 5.4		
¹⁴² Ce	16.2 ± 0.9	11.5±4.6	18.3 ± 7.3		
¹⁷⁴ Yb	$18.2{\pm}0.9$	17.1±6.8	28.5±11.4		

statistical uncertainty which affects the numerical values of these cross sections, calculated by a Monte Carlo procedure, is far from negligible. Then, the 1p-1n residue production rate cannot be estimated with an accuracy greater than $\approx 30-50$ %. If one further takes into account the uncertainty in the calculated yield of 2n products, one may conclude that it is quite unrealistic to think that one may deduce accurate values for R from the analysis of these data. Nevertheless, some qualitative conclusions may be reached. The results by Orth *et al.*³⁰ seem consistent with R values systematically higher than those obtained from coincidence experiments, in the case



FIG. 21. Comparison of experimental (Ref. 29), open circles, and theoretical distributions (R = 18.5) of the yields of the spallation residues from π^- absorption on ¹⁸¹Ta. C means cumulative production of a given residue.

of light nuclei, and from the analysis of inclusive proton spectra. In Table II the values calculated for R^* either in presence and absence of α -cluster absorptions, by using as input datum R = 25 for all the nuclei investigated by these authors (except ²⁶Mg, for which the expected uncertainty of the theoretical prediction is too high) are reported. One may conclude that such a high value of R is consistent with the measured values of R^* in the case of 25% α cluster absorptions and that the value of R one deduces from these data depends quite sensitively on this percentage, being much smaller than the value above reported in presence of only two nucleon absorption.

VI. SUMMARY AND CONCLUSIONS

The experimental information on reactions induced by absorption of π^- at rest has been reviewed. The uncertainties in experimental results which may prevent one from reaching an accurate quantitative understanding of the absorption and deexcitation mechanism have been discussed.

The analysis of the data establishes the dominant contribution of the two-nucleon absorption mechanism; however, $\approx 25\%$ of absorptions on α clusters seems necessary to explain the relevant deuteron and triton emission. Calculations of particle spectra and spallation residue yield distributions made in the framework of the exciton model have been shown to be able to reproduce the existing data satisfactorily.

Though some of the conclusions reached in this work have been anticipated in previous papers, we deem to have provided here additional evidence for them and to have presented calculations which take account of α -cluster absorption more accurately than what has been previously done.

Further investigations are necessary, in our opinion, to obtain more accurate estimates of R (the ratio between the number of neutron-proton and proton-proton pairs that may absorb the pion in the case of two-nucleon absorption) and to establish its variation with A.

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