# Structure of S* from nuclear production experiments

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The three models for the structure of S*, where it is considered as a member of the $q\bar{q}$ $L=1$ nonet, or as a member of the lightest $q^2\bar{q}^2$ nonet or as a $K\bar{K}$ molecule, can be distinguished by studying the $A$ dependence of the production cross sections at high energies of the inclusive reaction $\pi + A \rightarrow S^* + X$ and of the exclusive reaction $\pi + A \rightarrow S^* + N + (A-1)$. The energy dependence of the inclusive cross section can provide an independent test for the $K\bar{K}$ molecular model where we find a depletion in the cross section at energies corresponding to the Y* resonances of the $\bar{K}$-nucleon system.

## I. INTRODUCTION

In the conventional quark models the $0^+$ mesons S*(975) and δ(980) are assigned to the $^3P_0$ $q\bar{q}$ states. However, such an assignment fails to explain the widths and decay modes of these mesons. On the basis of the standard quark model, the large branching ratio (22%) of S* into $K\bar{K}$ is completely unexplainable given that it lies below the $K\bar{K}$ threshold, as it is the small widths of these resonances. The degeneracy of the S* and δ with the $K\bar{K}$ threshold seems totally accidental in this picture. These peculiarities can be naturally explained by considering these mesons as $q^2\bar{q}^2$ singlets.[1] In the MIT bag model such a state is expected to have a size similar to the ordinary $q\bar{q}$ mesons. On the other hand, nonrelativistic potential quark models support the existence of a deuteron-like two meson states. Within these quark models there is evidence that such a molecular state occurs only for the $K\bar{K}$ system which makes the S* and δ of special interest.[2]

In this work we show how nuclear production experiments can be used in order to distinguish among these three models for S* and δ, as already was suggested by Lenz.[3] Before we describe our method we would like to point out that there are alternative approaches in order to investigate the structure of these mesons. We mention here that the rate for photon decay of S* and δ seems to support the molecular picture for S* and δ,[4] but because of the large uncertainties involved in the estimation of the decay rate, the $q\bar{q}$ alternative cannot be completely ruled out. Indications for molecular structure are also obtained by looking at such decay modes as the $\iota(1140) \rightarrow K\bar{K}\pi$.[5]

In contrast to the other approaches, we nowhere require a detailed knowledge of the wave function of S* or δ. Rather, we use purely geometrical arguments related to the very different spatial extension of S* and δ in the $K\bar{K}$ molecular picture as compared to the other two alternatives, and the fact that we have twice as many quarks in the $q^2\bar{q}^2$ and $K\bar{K}$ models as compared to the usual $q\bar{q}$ mesons. The very large size of a weakly bound molecular state leads to an anomalously small S*-nucleon elastic cross section which produces a different $A$ dependence for the inclusive production cross section through the large breakup probability of the produced meson, S* or δ, in the nucleus. We expect the S*-nucleon total cross section for a $2q2\bar{q}$ structure to be larger than that of ordinary $q\bar{q}$ mesons.[6] We show that this leads to a different $A$ dependence for the exclusive production cross section on nuclei. Therefore, a simultaneous knowledge of both the exclusive and inclusive production cross sections will provide information about the internal structure of these mesons. For the rest of this work we will concentrate on the $I=0$ S* resonance although our conclusions hold equally well for the δ. For this investigation the S* must be produced with incident pions of momentum of at least $\sim 3$ GeV/$c$, so that we are in a region where the $\bar{K}$-nucleon elementary cross sections have a smooth energy dependence.

In a complementary way, the study of the energy dependence of the total inclusive production cross section can provide a test for the $K\bar{K}$ model. For this model the $\bar{K}$ constituent of S* will produce the well-known $\bar{K}$-nucleon resonances and therefore the production cross section will have an energy dependence strongly related to the $\bar{K}$-nucleon resonances. The main observation is that at incident $\bar{K}$ momentum of 1 GeV/$c$ the $\bar{K}$-nucleon total cross section has a broad resonance which we expect to show up as a depletion in the S* production cross section, thus providing a definitive signature for the $K\bar{K}$ model. For such a study we need pions with momenta in the range of 2.5–3.5 GeV/$c$.

At these high energies the Glauber multiple scattering series describes very well scattering of a composite projectile with the nucleus at small momentum transfer.[7] Thus we use it to describe reliably the final and initial state interactions.

The contents of this paper are divided into three main sections: In Sec. II we describe our calculation for the inclusive and exclusive production cross section as a function of the number of nucleons, $A$, in the target. In Sec. III we obtain the energy dependence of the inclusive cross section on $^{16}$O. In Sec. IV we give our results and conclusions.

## II. CALCULATIONS

### A. S*-nucleon interaction

In this section we discuss the S*-Nucleon (S*N) interaction for the various models of S* and show how information about the internal structure of S* can be obtained from the qualitative behavior of the S*N cross sections. For the case of $q\bar{q}$ and $q^2\bar{q}^2$ models, on the level of S*N scattering, we treat the S* as a one-constituent meson. The S*N scattering amplitude is parametrized by

$$f_{S^*N}(\Delta)=\frac{ik\sigma_{S^*N}}{4\pi}e^{-\Delta^2\beta^2_{S^*N}/2}, \tag{2.1a}$$

with $\sigma_{S^*N}=\sigma^{tot}_{S^*N}(1+\alpha_{S^*N})$. The elastic section is given by

$$\sigma^{el}_{S^*N}=\frac{|\sigma_{S^*N}|^2}{16\pi\beta^2_{S^*N}} . \tag{2.1b}$$

For the $K\bar{K}$ model we obtain the S*N scattering amplitude using Glauber theory, which has been very successfully applied to describe high energy hadron-nucleus scattering. By analogy to hadron-deuteron scattering, the S*N scattering amplitude in the S*N c.m. is given by

$$\bar{f}_{S^*N}(\Delta)=\bar{f}_{KN}(\Delta)S(\Delta/2)+\bar{f}_{\bar{K}N}(\Delta)S(\Delta/2)-\frac{1}{2\pi ik_{c.m.}}\int d^2q\, S(q)\bar{f}_{KN}(\mathbf{\Delta}/2+\mathbf{q})\bar{f}_{\bar{K}N}(\mathbf{\Delta}/2-\mathbf{q}) , \tag{2.2}$$

where $\bar{f}_{KN},\bar{f}_{\bar{K}N}$ are the free KN, $\bar{K}N$ amplitudes in the S*N c.m., respectively, $k_{c.m.}$ is the momentum in the S*N c.m., and the S* form factor, $S(\Delta)$, is given by

$$S(\Delta)=\int |\psi_{S^*}(r)|^2e^{i\Delta\cdot r}d^3r . \tag{2.3}$$

The momentum transfer $\mathbf{\Delta}=\mathbf{k}_i-\mathbf{k}_f$ is assumed to be completely transverse and $\psi_{S^*}$ is the S* wave function. In writing (2.2) we have used the high energy approximation of nearly forward scattering to relate the free KN scattering amplitude in the KN c.m. frame to the scattering amplitude, $\bar{f}_{KN}$, in the S*N c.m. frame. For the KN scattering amplitudes we take a Gaussian parametrization, namely

$$f_{KN}(\Delta)=\frac{ik\sigma_{KN}}{4\pi}e^{-\beta^2_{KN}\Delta^2/2}, \tag{2.4}$$

with $\sigma_{KN}=\sigma^{tot}_{KN}(1+i\alpha_{KN})$. Taking a Gaussian wave function of width $R_S$ for the S*, we obtain for the total and elastic S*N cross sections, $\sigma^{tot}_{S^*N}$ and $\sigma^{el}_{S^*N}$, the following expressions,

$$\sigma^{tot}_{S^*N}=\sigma^{tot(1)}_{S^*N}+\delta\sigma \tag{2.5a}$$

and

$$\sigma^{el}_{S^*N}=\sigma^{el(1)}_{S^*N}+\delta\sigma' , \tag{2.5b}$$

where

$$\sigma^{tot(1)}_{S^*N}=\sigma^{tot}_{KN}+\sigma^{tot}_{\bar{K}N} \tag{2.6a}$$

and

$$\sigma^{el(1)}_{S^*N}=\frac{|\sigma_{KN}+\sigma_{\bar{K}N}|^2}{16\pi(\beta^2_{KN}+R^2_S/8)} . \tag{2.6b}$$

$\sigma^{tot(1)}_{S^*N}$ and $\sigma^{el(1)}_{S^*N}$ are the cross sections due to one scattering of the incident nucleon N on K or on $\bar{K}$, and $\delta\sigma,\delta\sigma'$ are corrections due to shadowing. For values of $R_S$ similar to the deuteron radius, these correction terms are on the few percent level. We give their expressions for completeness, although they are not essential for this qualitative discussion:

$$\delta\sigma=-\mathrm{Re}\left[\frac{\sigma_{KN}\sigma_{\bar{K}N}}{8\pi(\beta^2_{KN}+R^2_S/4)}\right]$$

and

$$\delta\sigma'=-(\sigma^{el}_{KN}\sigma^{tot}_{\bar{K}N}+\sigma^{tot}_{KN}\sigma^{el}_{\bar{K}N})\frac{\beta^2_{KN}}{2\pi(\beta^2_{KN}+R^2_S/4)(3\beta^2_{KN}+R^2_S/4)}+\sigma^{el}_{KN}\sigma^{el}_{\bar{K}N}\frac{\beta^2_{KN}}{2\pi(\beta^2_{KN}+R^2_S/4)^2} ,$$

with

$$\sigma^{el}_{KN}=\frac{|\sigma_{KN}|^2}{16\pi\beta^2_{KN}} ,$$

where we have set $\beta_{KN}=\beta_{\bar{K}N}$.

From (2.5) and (2.6) it is clear that $\sigma^{tot}_{S^*N}$ for the $K\bar{K}$ model is expected to be typically twice as large as that for a usual meson. On the other hand, $\sigma^{el}_{S^*N}$ is expected to be anomalously small because of the large spatial extension of S*. Physically, this means that the S* can easily break up by a collision and has very little probability of propagating in the nucleus. Thus the ratio $r=\sigma^{el}_{S^*N}/\sigma^{tot}_{S^*N}$ will be abnormally small compared with the corresponding ratio for a $q\bar{q}$ meson. In the case of the $q^2\bar{q}^2$ model we expect $\sigma^{tot}_{S^*N}$ to be given approximately by $\sigma^{tot(1)}_{S^*N}$ and therefore it should also be twice that for a $q\bar{q}$ meson, whereas $\sigma^{el}_{S^*N}$ is given approximately by $\sigma^{el(1)}_{S^*N}$ with $R_S\sim0$ and therefore we expect it to be anomalously large. This will lead to a ratio $r$ which in this case will be abnormally large. For the $q\bar{q}$ model we expect both $\sigma^{tot}_{S^*N}$ and $\sigma^{el}_{S^*N}$ to be those of usual mesons.

In Fig. 1 we show $\sigma^{tot}_{S^*N}$ versus $\sigma^{inel}_{S^*N}=\sigma^{tot}_{S^*N}-\sigma^{el}_{S^*N}$ calculated within this simple model. The experimental values of KN and $\pi$N cross sections at 3 GeV/$c$ are shown for comparison. As a representative of the $q\bar{q}$ or the $q^2\bar{q}^2$ picture of S*, we evaluated the cross sections using (2.1), where we take, for the range parameter, $\beta^2_{S^*N}=6$ GeV$^{-2}$, which is a typical value for the usual $q\bar{q}$ mesons or a little larger value of 9 GeV$^{-2}$ with $\alpha_{S^*N}$ set to zero. For comparison with the $K\bar{K}$ model we also show the re-

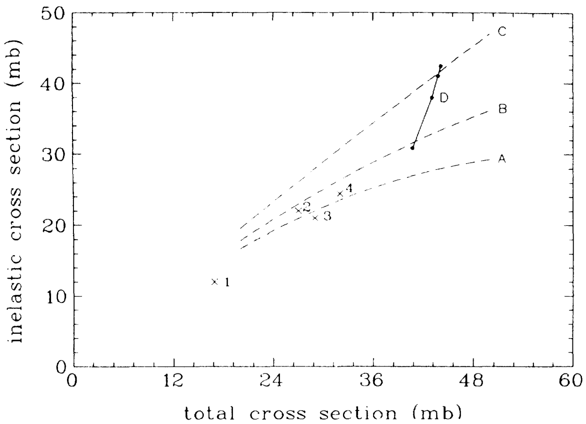


FIG. 1. Inelastic S*N cross section vs total S*N cross section ($\sigma^{\rm inel}_{\rm S^*N}$ vs $\sigma^{\rm tot}_{\rm S^*N}$). Graphs $A,B,C$ are computed for a one-constituent S* using the amplitude given by (2.1) with the range parameter $\beta^2_{\rm S^*N}=6$, 9, and 40 GeV$^{-2}$, respectively. $D$ is computed for a two-constituent S* with the S* radius $R_{\rm S}$ ranging from 1 fm (lowest point on the curve) to 4 fm (uppermost point on the curve). The crosses correspond to experimental cross sections as follows: (1) $\rm K^+p$, (2) $\rm K^-p$, (3) $\pi^+$p, and (4) $\pi^-$p.

sult for the extreme case of $\beta^2_{\rm S^*N}=40$ GeV$^{-2}$, which would be the effective range of the S*N amplitude (2.2) for a large S* radius. As inputs for the amplitudes in the $\rm K\overline{K}$ model of S*, we take the following parameters,[8]

$$\sigma^{\rm tot}_{\rm KN}=17.2 \text{ mb}, \quad \sigma^{\rm tot}_{\rm \overline{K}N}=27.4 \text{ mb} , \tag{2.7a}$$

and

$$\beta^2_{\rm KN}=\beta^2_{\rm \overline{K}N}=5.8 \text{ GeV}^{-2} , \tag{2.7b}$$

with the size of S* ranging from $R_{\rm S}=1$–4 fm. We take $\alpha_{\rm KN}=0$ since the results are not sensitive to the value of $\alpha_{\rm KN}$. Clearly, simultaneous knowledge of $\sigma^{\rm inel}_{\rm S^*N}$ and $\sigma^{\rm tot}_{\rm S^*N}$ enables us to distinguish among the $q\overline{q}$, $q^2\overline{q}^{\,2}$, and $\rm K\overline{K}$ molecular pictures of S*. The large inelastic cross sections (40 mb) together with the large total cross section suggest the weakly bound $\rm K\overline{K}$ structure. Large total cross sections together with large elastic cross sections support the $q^2\overline{q}^{\,2}$ picture.

### B. S* production on nuclei

#### 1. *Inclusive production*

We study the reaction $\pi+{\rm A}\to{\rm S}^*+{\rm X}$. The basic production reaction in the case where $\pi^+$ are used is $\pi^++{\rm n}\to{\rm S}^*+{\rm p}$. Since we are also interested in the exclusive reaction where the p from the production process is detected, it is experimentally easier to use $\pi^+$. From now on we will therefore specifically refer to S* production using $\pi^+$. Within the Glauber theory, the scattering amplitude of an elementary projectile with a nucleus is given by[9]

$$F_{fi}(\Delta)=\frac{ik}{(2\pi)}\int d^2b\, e^{i\Delta\cdot b}\left\langle \psi^f_A \left| \sum_{j=1}^{A}\gamma_{\rm p}(\mathbf{b}-\mathbf{s}_j)e^{i\Delta_L z_j}\prod_{z_i<z_j}[1-\gamma_\pi(\mathbf{b}-\mathbf{s}_i)]\prod_{z_k>z_j}[1-\gamma_{\rm S^*}(\mathbf{b}-\mathbf{s}_k)]\right|\psi^i_A\right\rangle . \tag{2.8}$$

The incoming momentum of the projectile is taken along the $z$ axis and $\mathbf{s}_i$ are the nucleon coordinates perpendicular to the $z$ axis. The profile functions $\gamma_\alpha$ are related to the free elastic scattering amplitudes of $\pi$N and S*N and to the production amplitude of $\pi{\rm N}\to{\rm S^*N}$ by

$$\gamma_\alpha(b)=\frac{1}{2\pi ik}\int e^{-i\Delta\cdot b}f_\alpha(\Delta)d^2\Delta . \tag{2.9}$$

$\Delta_L$ is the longitudinal momentum transfer given by $\Delta_L\simeq(m^2_{\rm S^*}-m^2_\pi)/2k$.

For the nuclear state we take an uncorrelated product of single particle wave functions (s.p.w.'s).

$$\psi^\alpha_A=\prod_{j=1}^{\rm A}\phi^\alpha(r_j)=\prod_{j=1}^{\rm A}\phi^\alpha_j \quad \text{where } \alpha=i,f . \tag{2.10}$$

The inclusive cross section is given by summing over all the final nuclear states using the closure approximation and integrating over the angular distribution of S*,

$$\sigma_{\rm incl}=\int\sum_f |F_{fi}(\Delta)|^2 d\Omega . \tag{2.11}$$

In the what follows we neglect $\pi^+$ charge exchange processes since they are small. Therefore no coherent S* production is possible. Since we use uncorrelated wave functions, this means that only diagonal terms enter in (2.10).[7]

In this case the $z$ integrations can be carried out explicitly to give

$$\sigma_{\rm incl}=\frac{n}{A}\int d^2b\,\Omega_{\rm p}(b)\sum_{k=0}^{A-1}[\Gamma_\pi(b)]^k[\Gamma_{\rm S^*}(b)]^{A-1-k} , \tag{2.12a}$$

where

$$\Gamma_\alpha(b)=1-M_\alpha(b)-M^*_\alpha(b)+\Omega_\alpha(b) , \tag{2.12b}$$

with

$$\Omega_\alpha(b)=\int d^2s\, T_{\rm N}(s)\gamma_\alpha(\mathbf{b}-\mathbf{s})\gamma^*_\alpha(\mathbf{b}-\mathbf{s}) , \tag{2.12c}$$

$$M_\alpha(b)=\int d^2s\, T_{\rm N}(s)\gamma_\alpha(\mathbf{b}-\mathbf{s}) , \tag{2.12d}$$

and

$$T_{\rm N}(s)=\int |\phi^i(r)|^2dz=\int\rho_{\rm N}(r)dz , \tag{2.12e}$$

where $n$ and $A$ is the number of neutrons and nucleons, respectively. In order to obtain the qualitative behavior of the inclusive cross section, we take the range of the elementary interactions to be small compared with the nuclear size. We can then approximate $\Gamma_\alpha$ by

$$\Gamma_\alpha(b) \simeq 1-\sigma_\alpha^{\text{inel}} T_{\text{N}}(b) \ . \tag{2.13}$$

Therefore we expect $\sigma_{\text{incl}}$ to essentially depend on the elementary inelastic cross sections of $\pi$N and S*N.

The treatment given so far applies only when the projectile has no composite structure. This would be applicable in the case of the $q\bar{q}$ and $q^2\bar{q}^2$ models of S*. For the K$\overline{\text{K}}$ molecular model we must properly take into account the composite structure of S* in the final state interaction. In the Glauber theory we can properly take this into account by replacing

$$G_{fi} = \prod_{z_k > z_j} \langle \phi_k^f \mid [1-\gamma_{\text{S}^*}(\mathbf{b}-\mathbf{s}_k)] \mid \phi_k^i \rangle \tag{2.14}$$

by

$$G_{fi} = \left\langle \text{S}^* \left| \prod_{z_k > z_j} \langle \phi_k^f \mid [1-\gamma_{\text{KN}}(\mathbf{b}-\mathbf{s}_k-\mathbf{s}/2)][1-\gamma_{\overline{\text{K}}\text{N}}(\mathbf{b}-\mathbf{s}_k+\mathbf{s}/2)] \mid \phi_k^i \rangle \right| \text{S}^* \right\rangle , \tag{2.15}$$

where $s$ is the relative S* coordinate perpendicular to the $z$ axis.

We also can approximately take into account the composite structure of S* by using the effective S*N amplitude given by (2.2). Physically, this approximation means that we neglect processes where recombination of K,$\overline{\text{K}}$ takes place after the S* has been broken up by a collision. By (2.2) we obtain the S*N scattering amplitude in the S*N c.m. frame. In the high energy approximation the scattering amplitude in the c.m. and laboratory frames is related by[10]

$$\frac{1}{K_{\text{lab}}} f_{\text{lab}}(K'_{\text{lab}}, K_{\text{lab}}) \simeq \frac{1}{K_{\text{c.m.}}} f_{\text{c.m.}}(K'_{\text{c.m.}}, K_{\text{c.m.}}) \ , \tag{2.16}$$

where $K_{\text{lab}}$ ($K'_{\text{lab}}$) and $K_{\text{c.m.}}$ ($K'_{\text{c.m.}}$) are the initial (final) momenta in the lab and c.m. frames, respectively. Therefore the effective S*N amplitude can be trivially obtained from (2.2) by replacing $k_{\text{c.m.}}$ and $\bar{f}_{\text{KN}}$ by their laboratory values $k$ and $f_{\text{KN}}$, respectively. Using this effective S*N amplitude, we obtain

$$G_{fi}^{\text{eff}} = \prod_{z_k > z_j} \langle \phi_k^f \mid \gamma_{\text{S}^*}^{\text{eff}}(\mathbf{b}-\mathbf{s}_k) \mid \phi_k^i \rangle \ , \tag{2.17}$$

with

$$\gamma_{\text{S}^*}^{\text{eff}} = \int d^2s \, T_{\text{S}^*}(s)[1-\gamma_{\text{KN}}(\mathbf{b}-\mathbf{s}_k-\mathbf{s}/2)] \times [1-\gamma_{\overline{\text{K}}\text{N}}(\mathbf{b}-\mathbf{s}_k+\mathbf{s}/2)]$$

and

$$T_{\text{S}^*}(s) = \int dz \, \psi_{\text{S}^*}(r)\psi_{\text{S}^*}^*(r) \ .$$

Using $G_{fi}^{\text{eff}}$ the inclusive cross section is given by (2.12). By comparing Eqs. (2.15) and (2.17), the inclusive cross section for a two constituent S* is obtained by making the substitution

$$[\Gamma_{\text{S}^*}(b)]^n \rightarrow \int d^2s \, d^2s' T_{\text{S}^*}(s) T_{\text{S}^*}(s') \left[ \int d^2s_k T_{\text{N}}(s_k)[1-\gamma_{\text{KN}}(\mathbf{b}-\mathbf{s}_k-\mathbf{s}/2)][1-\gamma_{\overline{\text{K}}\text{N}}(\mathbf{b}-\mathbf{s}_k+\mathbf{s}/2)] \times [1-\gamma_{\text{KN}}^*(\mathbf{b}-\mathbf{s}_k-\mathbf{s}'/2)][1-\gamma_{\overline{\text{K}}\text{N}}^*(\mathbf{b}-\mathbf{s}_k+\mathbf{s}'/2)] \right]^n . \tag{2.18}$$

We have calculated $\sigma_{\text{incl}}$ using both (2.17) and (2.18) for the case of the harmonic oscillator nuclear wave function. For an S* radius $\sim 3$ fm we found only a $\sim 2\%$ difference in $\sigma_{\text{incl}}$ for heavy nuclei. The error decreases with the S* radius. Therefore in what follows we use the effective S*N interaction to describe the K$\overline{\text{K}}$ model of S*, which is accurate enough for our present discussion.

### 2. *Exclusive production*

We consider the reaction $\pi^+ + \text{A} \rightarrow \text{S}^* + \text{p} + (\text{A}-1)$, where we go to a definite final nuclear state and we observe the outgoing proton. Here we assume that at these high energies production of S* will be dominated by a knock-out of the nucleon. In this work we are mainly interested in the final state interaction of S* with the nucleus. We shall therefore neglect the final state interaction of the outgoing proton. In a more realistic calculation this can be corrected by using a distorted proton wave function instead of a plane wave. The differential cross section is given by[11]

$$\frac{d^5\sigma_{\text{excl}}}{d\Omega_{\text{S}^*} d\Omega_{\text{p}} dp_{\text{S}^*}} = \frac{p_{\text{S}^*}^2}{v_{\text{lab}}} \mid T_{fi} \mid^2 2\pi\delta(E_i - E_f) \frac{d^3k_{\text{p}}}{(2\pi)^3(2\pi)^3} \ , \tag{2.19}$$

where the $T$ matrix is related to the scattering amplitude by

$$T = -\frac{2\pi v_{\text{lab}}}{k} f \ .$$

$k_{\text{p}}$ is the momentum of the knocked out proton and $E_i, E_f$ are the initial and final energies in the lab. Using the Glauber formalism we obtain

$$\frac{d^5\sigma_{\text{excl}}}{d\Omega_{S^*}d\Omega_p dp_{S^*}} = \frac{p_{S^*}^2 v_{\text{lab}}}{(2\pi)^2} \left| \int d^2b\, e^{i\Delta\cdot\mathbf{b}} \langle \psi_{A-1}^f k_p \,|\, \gamma_p(\mathbf{b}-\mathbf{s}_i) e^{i\Delta_L z_i} \times \prod_{z_j<z_i} [1-\gamma_\pi(\mathbf{b}-\mathbf{s}_j)] \prod_{z_k>z_i} [1-\gamma_{S^*}(\mathbf{b}-\mathbf{s}_k)] \,|\, \psi_A^i \rangle \right|^2 \frac{d^3k_p}{(2\pi)^3} \delta(E_i-E_f) . \tag{2.20}$$

In order to obtain the qualitative behavior of $\sigma_{\text{excl}}$, we take the initial and final state interactions in (2.20) to be the same. In this case there is no $z$ ordering and (2.20) can be written as

$$\frac{d^5\sigma_{\text{excl}}}{d\Omega_{S^*}d\Omega_p dp_{S^*}} = \frac{p_{S^*}^2 v_L}{(2\pi)^2} \left| \int d^2b\, e^{i\Delta\cdot\mathbf{b}} \langle k_p \,|\, \gamma_p(\mathbf{b}-\mathbf{s}_i) e^{i\Delta_L z_i} \,|\, \phi^i(r_i) \rangle [1-M_{S^*}(b)]^{A-1} \right|^2 \frac{d^3k_p}{(2\pi)^3} \delta(E_i-E_f) . \tag{2.21}$$

Taking the range of the elementary interaction to be small compared with the nuclear size and an imaginary forward amplitude, we can approximate $M_{S^*}(b)$ by

$$M_{S^*}(b) \simeq 1 - \frac{\sigma_{S^*N}^{\text{tot}}}{2} T_N(b) . \tag{2.22}$$

Therefore $\sigma_{\text{excl}}$ should depend strongly on the elementary total cross sections. In integrating over the energy $\delta$ function, we neglect nuclear recoil and take nonrelativistic kinematics for the outgoing proton. The exclusive production cross section falls off rapidly for $(\Delta_L - k_p^z)R_A > 1$ because of the oscillating exponential factor, where $R_A$ is the nuclear radius and $k_p^z$ is the longitudinal proton momentum. Therefore we consider kinematics where $\Delta_L = k_p^z$.

The momentum dependence of the differential cross section as given by (2.20) depends strongly on the orbital wave function of the nucleon involved in the production process. However, if we integrate (2.20) and normalize with the corresponding cross section in the impulse approximation, we expect the resulting total exclusive cross section to depend weakly on the particular nuclear wave function of the production nucleon. Therefore, in this case, we are justified in using an uncorrelated product of $s$ state s.p.w.'s for $\psi_A^i$ and $\psi_{A-1}^f$. In line with our calculation for the inclusive process, we use the effective S*N amplitude to describe the S* final state interaction.

## III. ENERGY DEPENDENCE OF THE INCLUSIVE S* PRODUCTION CROSS SECTION

In this section we investigate the energy dependence of the total S* nuclear production cross section. As we mentioned in the Introduction, we expect a very different energy dependence for the $K\bar{K}$ model as compared with the $q\bar{q}$ standard quark assignment. This is because of the very different final state interaction of the $\bar{K}$ component of S*. Near a $\bar{K}$-nucleon resonance, there is a time delay of S* in the nucleus, resulting in a depletion in the outgoing S* flux. Therefore we expect the S* production cross section to show a depletion at energies corresponding to the $\bar{K}$-nucleon resonances. On the other hand, if S* is a pure $q\bar{q}$ state, then we expect no such correlation. The $\Lambda(1520)$ resonance in the $\bar{K}$-nucleon system does not show up because it is smeared out by Fermi motion.[12] Therefore we look at the broad Y* resonances present at a $\bar{K}$ lab momentum of $\sim 1$ GeV/$c$. This means that the S* must be produced with a momentum of $\sim 2$ GeV/$c$.

Since the $\bar{K}$-nucleon amplitudes are very strongly energy dependent, we must take into account both the Fermi motion associated with the internal momenta of S* and of the nucleus. For this we follow the procedure for Fermi averaging developed by Lenz.[13]

In this formulation we need to obtain the Fermi-averaged scattering amplitude in the laboratory frame in terms of the free c.m. $\bar{K}$-nucleon amplitude $f_{\text{c.m.}}$. In the high energy approximation using (2.15), we obtain, for the scattering amplitude in the laboratory frame,[13]

$$\frac{1}{k_i}\langle \mathbf{k}_f,\mathbf{p}_f \,|\, f_{\text{lab}} \,|\, \mathbf{k}_i,\mathbf{p}_i \rangle \simeq \frac{1}{K_{\text{c.m.}}} \delta^3(\mathbf{k}_f+\mathbf{p}_f-\mathbf{k}_i-\mathbf{p}_i) f_{\text{c.m.}} \left[ E_{\text{tot}} - H_{A-1} - \frac{(\mathbf{k}_i+\mathbf{p}_i)^2}{2M^*};\ \frac{\mathbf{k}_f M}{M^*} - \alpha\mathbf{p}_f;\ \frac{\mathbf{k}_i M}{M^*} - \alpha\mathbf{p}_i \right] , \tag{3.1}$$

where

$$M^* = E + M, \quad \alpha = E/M^* . \tag{3.2}$$

$E_{\text{tot}}$ is the total energy in the lab frame, $E$ is the projectile energy, $M$ is the mass of the target particles, and $\mathbf{k}_i,\mathbf{k}_f,\mathbf{p}_i,\mathbf{p}_f$ are the initial and final projectile and target momenta in the lab frame, respectively. For the nuclear ground state, $\psi_A$, we take a product of single particle harmonic oscillator (HO) wave functions,

$$\psi_A = \prod_{j=1}^{A} \phi_j(r_j) . \tag{3.3}$$

The appropriate scattering amplitude of a projectile with a bound nucleon is given by

$$\frac{1}{k_i}\langle \psi_A,\mathbf{k}_f \mid f_{\text{lab}} \mid \mathbf{k}_i,\psi_A\rangle \simeq \frac{1}{K_{\text{c.m.}}} \int d^3p_j \phi_j^*(\mathbf{\Delta}+\mathbf{p}_j)\phi_j(\mathbf{p}_j) f_{\text{c.m.}} \left[E+M+\epsilon-\frac{[\mathbf{p}_j+(\mathbf{\Delta}+\mathbf{Q})/2]^2}{2M^*}\right], \tag{3.4}$$

where $\epsilon$ is the binding energy, $\mathbf{\Delta}=\mathbf{k}_i-\mathbf{k}_f$ is the momentum transfer, and $\mathbf{Q}=\mathbf{k}_i+\mathbf{k}_f$.

We now proceed to generalize (3.4) in order to take into account the composite structure of S*. As was discussed in Sec. II, it is sufficient for our considerations to obtain the effective elastic S*N amplitude in the S*N c.m. This can be obtained from (3.4) with the nucleon being the projectile of incoming momentum $-\mathbf{k}_i^{S^*N}$ and S* the target having momentum $\mathbf{k}_i^{S^*N}$. We only Fermi average the $\overline{\text{K}}$N amplitudes, the KN amplitudes having a smooth energy dependence. Since at high energies the most important contribution comes from the forward amplitude, we use the forward c.m. $\overline{\text{K}}$N amplitude and neglect the spin-flip amplitude since it vanishes in the forward direction. With these approximations the Fermi-averaged $\overline{\text{K}}$N scattering amplitude is given by

$$\frac{1}{k_i^{S^*N}}\langle \psi_{S^*},-\mathbf{k}_i^{S^*N} \mid f_{\overline{K}N}(E_{S^*N}) \mid -\mathbf{k}_f^{S^*N},\psi_{S^*}\rangle$$

$$\simeq \frac{1}{K_{\text{c.m.}}}\int \frac{d^3q_i}{(2\pi)^3}\frac{d^3q_f}{(2\pi)^3}(2\pi)^3\delta^3(\mathbf{q}_i-\mathbf{q}_f-\mathbf{\Delta}/2)\psi_{S^*}^*\left[\frac{\mathbf{q}_i+\mathbf{q}_f}{2}-\frac{\mathbf{\Delta}}{4}\right]\psi_{S^*}(\mathbf{q}_i) f_{\text{c.m.}}^{\overline{K}N}[\omega(\mathbf{q}_i)], \tag{3.5}$$

where

$\omega^2=(E_N+E_{\overline{K}})^2-(\mathbf{k}_i^{S^*N}/2+\mathbf{q}_i)^2$ is the $\overline{\text{K}}$N c.m. energy ,

$$E_{\overline{K}}=\left[\left[\frac{\mathbf{k}_i^{S^*N}}{2}-\mathbf{q}_i\right]^2+m_{\overline{K}}^2\right]^{1/2},$$

$E_N=[(k_i^{S^*N})^2+M^2]^{1/2}$, and $E_{S^*N}$ is the S*N c.m. energy. We have neglected the S* binding energy $\epsilon$. Using a Gaussian for the S* wave function, we obtain

$$\overline{f_{\overline{K}N}}\equiv\langle \psi_{S^*},-\mathbf{k}_f^{S^*N} \mid f_{\overline{K}N} \mid -\mathbf{k}_i^{S^*N},\psi_{S^*}\rangle \simeq \frac{k_i^{S^*N}}{K_{\text{c.m.}}}\left[\frac{8}{\pi^3}\right]^{1/2} e^{-\Delta^2R_S^2/16}\int d^3q_i e^{-2q_i^2} f_{\text{c.m.}}^{\overline{K}N}(\omega)\Bigg|_{\Delta=0}, \tag{3.6}$$

where we have taken the forward $\overline{\text{K}}$N c.m. scattering amplitude $f_{\text{c.m.}}^{\overline{K}N}(\omega)$. The S*N amplitude is then given by

$$f_{S^*N}(E_{S^*N},\Delta)\simeq\overline{f_{\overline{K}N}}+\frac{k_i^{S^*N}}{K_{\text{c.m.}}}f_{KN}(\overline{\omega})e^{-\Delta^2R_S^2/16}, \tag{3.7}$$

where $\overline{\omega}=\omega\mid_{q_i=0}$. In (3.7) we have neglected the second scattering term. The error of neglecting this term is very small ($\sim 1\%$) for the K$\overline{\text{K}}$ molecular S* because of its large extension.

We take $f_{S^*N}$ to be the appropriate two body scattering amplitude to be used for the Glauber multiple scattering series, i.e., we take $\mathbf{\Delta}$ to be completely transverse and $\mathbf{Q}=2\mathbf{k}_i^{S^*N}$ along the $z$ axis. In order to calculate the total production cross section, we need to Fermi average $f_{S^*N}$ over the nuclear ground state. This we can immediately do using (3.4), where S* is now the projectile, with $f_{S^*N}$ being the appropriate c.m. amplitude. Since, for the Fermi averaging of the scattering amplitude, the relevant quantity is the Fermi momentum, we take an oscillator parameter, $R_A'$, which gives the correct Fermi momentum deduced from electron scattering experiments.[14] Using single particle $s$-state harmonic oscillator wave functions, we obtain, for the forward scattering amplitude,

$$\frac{1}{k_i}\overline{F}_{S^*N}\Bigg|_{\Delta=0}=\frac{1}{k_i^{S^*N}(\pi^3)^{1/2}}\times\int d^3p\, e^{-p^2}f_{S^*N}\times\left[E+M-\frac{(\mathbf{p}/R_A'+\mathbf{Q}/2)^2}{2M^*}\right], \tag{3.8}$$

with

$$R_A'=(\tfrac{3}{2})^{1/2}\frac{1}{k_F},$$

where $M^*=E+M$, $p$ is the rescaled nucleon momentum, and $k_F$ is the Fermi momentum. Using $\overline{F}_{S^*N}$ we can immediately write down the profile function needed as an input to the Glauber scattering series. For the calculation of the production amplitude in the Glauber theory, it is important to use nuclear wave functions which give the correct nuclear rms radius. For this part of the calculation, for $p$-shell nuclei, we use $s$- and $p$-state HO wave functions with an oscillator parameter, $R_A$, which reproduces the nuclear rms radius.

The inclusive production cross section as a function of the S* energy is then given by

$$\sigma(E)=\int d^2b\,dz\Big[2\tilde{\Omega}_p^{(s)}(\mathbf{b},z)[1-\tilde{\Gamma}_\pi^{(s)}(\mathbf{b},z)-\tilde{\Gamma}_{S^*}^{(s)}(\mathbf{b},z)]^3[1-\tilde{\Gamma}_\pi^{(p)}(\mathbf{b},z)-\tilde{\Gamma}_{S^*}^{(p)}(\mathbf{b},z)]^{A-4}$$
$$+\frac{A-4}{2}\tilde{\Omega}_p^{(p)}(\mathbf{b},z)[1-\tilde{\Gamma}_\pi^{(s)}(\mathbf{b},z)-\tilde{\Gamma}_{S^*}^{(s)}(\mathbf{b},z)]^4[1-\tilde{\Gamma}_\pi^{(p)}(\mathbf{b},z)-\tilde{\Gamma}_{S^*}^{(p)}(\mathbf{b},z)]^{A-5}\Big], \tag{3.9}$$

where

$$\tilde{\Omega}_{\rm p}^{(i)}(\mathbf{b},z)=\int d^2s\,\rho_i(\mathbf{s},z)\gamma_{\rm p}(\mathbf{b}-\mathbf{s})\gamma_{\rm p}^*(\mathbf{b}-\mathbf{s})\ ,$$

$$\tilde{\Gamma}_{\pi}^{(i)}(\mathbf{b},z)=\int_{-\infty}^{z} dz'd^2s\,\rho_i(\mathbf{s},z')[\gamma_{\pi}(\mathbf{b}-\mathbf{s})+\gamma_{\pi}^*(\mathbf{b}-\mathbf{s})-\gamma_{\pi}^*(\mathbf{b}-\mathbf{s})\gamma_{\pi}^*(\mathbf{b}-\mathbf{s})]\ ,$$

$$\tilde{\Gamma}_{\rm S^*}^{(i)}(\mathbf{b},z)=\int_{z}^{\infty} dz'd^2s\,\rho_i(\mathbf{s},z')[\gamma_{\rm S^*}(\mathbf{b}-\mathbf{s})+\gamma_{\rm S^*}^*(\mathbf{b}-\mathbf{s})-\gamma_{\rm S^*}(\mathbf{b}-\mathbf{s})\gamma_{\rm S^*}^*(\mathbf{b}-\mathbf{s})]\ ,$$

and $\rho_s$ and $\rho_p$ are $s$ and $p$ state HO nuclear densities.

## IV. RESULTS AND CONCLUSIONS

We have taken a Gaussian parametrization for the elementary $\pi$N elastic scattering amplitude of the same form as for the KN amplitudes given by (2.4) with the parameters[8]

$$\sigma_{\pi\rm N}^{\rm tot}=31.9\ \text{mb},\quad \alpha_{\pi\rm N}=0,\quad \beta_{\pi\rm N}^2=6.16\ \text{GeV}^{-2}\ . \qquad (4.1)$$

To describe the final state interaction of S* for the $q\bar{q}$ and $q^2\bar{q}^2$ models, we used the scattering amplitudes given in (2.1) with

$$\sigma_{\rm S^*N}^{\rm tot}=\frac{\sigma_{\rm KN}^{\rm tot}+\sigma_{\rm \bar{K}N}^{\rm tot}}{2}$$

and

$$\sigma_{\rm S^*N}^{\rm tot}=\sigma_{\rm KN}^{\rm tot}+\sigma_{\rm \bar{K}N}^{\rm tot}\ ,$$

respectively. The cross sections $\sigma_{\rm KN}^{\rm tot}$ and $\sigma_{\rm \bar{K}N}^{\rm tot}$ are given by (2.7) and we take $\alpha_{\rm S^*N}=0$. For the range parameter $\beta_{\rm S^*N}^2$ we take 5.8 and 10 GeV$^{-2}$. For the K$\bar{\rm K}$ model we use the effective amplitude (2.2) with the parameters given in (2.7) and $\alpha_{\rm KN}=\alpha_{\rm \bar{K}N}=0$ with the S* radius $R_{\rm S}=3$ fm. In order to show the sensitivity of our results on the S* radius, we also take $R_{\rm S}=1$ fm, although this does not correspond to any of the three proposed models.

Nothing is known about the range of the S* production amplitude for which we have taken a Gaussian parametrization. We assumed the same range parameter $\beta_{\rm p}$ as for $\beta_{\pi\rm N}$. The elementary production cross section $\sigma_{\rm p}$ enters as an overall normalization factor. For the calculation of the inclusive production cross section, we take a Woods-Saxon single particle nuclear density given by

$$\rho_{\rm N}(r)=\frac{\rho_0}{1+e^{(r-r_0)/t}}\ , \qquad (4.2)$$

with $r_0=1.14A^{1/3}$ fm and $t=0.545$ fm.

In Fig. 2 we show the inclusive production cross section normalized with the corresponding cross section in the impulse approximation, as a function of the mass number $A$. The normalization is chosen so that with no initial and final state interactions we obtain the neutron number. The inelastic cross sections $\sigma_{\rm S^*N}^{\rm inel}$ are shown for each case. For the larger inelastic cross sections we obtain a weaker $A$ dependence. This is what we expect since for the large $\sigma_{\rm S^*N}^{\rm inel}$ the S* must be produced near the nuclear surface in order to emerge from the nucleus. Whereas the inclusive cross section is strongly dependent on $\sigma_{\rm S^*N}^{\rm inel}$, it depends only very weakly on $\sigma_{\rm S^*N}^{\rm tot}$ or $\sigma_{\rm S^*N}^{\rm el}$ if $\sigma_{\rm S^*N}^{\rm inel}$ is kept constant. Thus by the $A$ dependence of the inclusive S* production, we can deduce the S*N inelastic cross section.

For the calculation of the exclusive production cross section, we take $s$-state HO nuclear wave functions, since

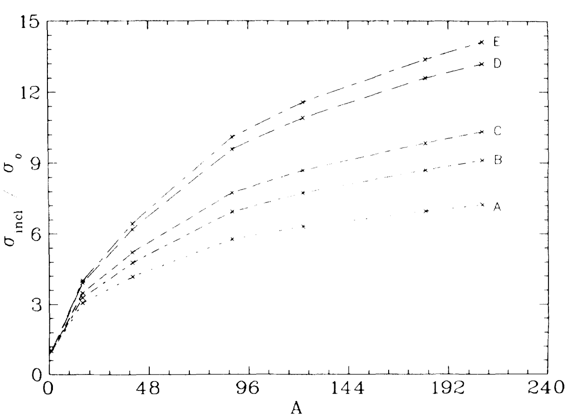


FIG. 2. $A,B$ are calculated for a two-constituent S*; $A$ with $R_{\rm S}=3$ fm corresponding to $\sigma_{\rm S^*N}^{\rm tot}=43.8$ mb and to $\sigma_{\rm S^*N}^{\rm inel}=41$ mb (K$\bar{\rm K}$ molecule); $B$ with $R_{\rm S}=1$ fm corresponding to $\sigma_{\rm S^*N}^{\rm tot}=40.7$ mb and to $\sigma_{\rm S^*N}^{\rm inel}=30.5$ mb. $C,D,E$ are calculated for a one-constituent S*. $C$ corresponds to $\sigma_{\rm S^*N}^{\rm tot}=44.6$ mb and $\sigma_{\rm S^*N}^{\rm inel}=27.5$ mb with range parameter $\beta_{\rm S^*N}^2=5.8$ GeV$^{-2}$ and it is representative of the $q^2\bar{q}^2$ model. $D,E$ correspond to $\sigma_{\rm S^*N}^{\rm tot}=22.3$ mb, typical of a $q\bar{q}$ meson; $D$ with $\sigma_{\rm S^*N}^{\rm inel}=19.8$ mb and $\beta_{\rm S^*N}^2=10$ GeV$^{-2}$; $E$ with $\sigma_{\rm S^*N}^{\rm inel}=18.04$ mb and $\beta_{\rm S^*N}^2=5.8$ GeV$^{-2}$.

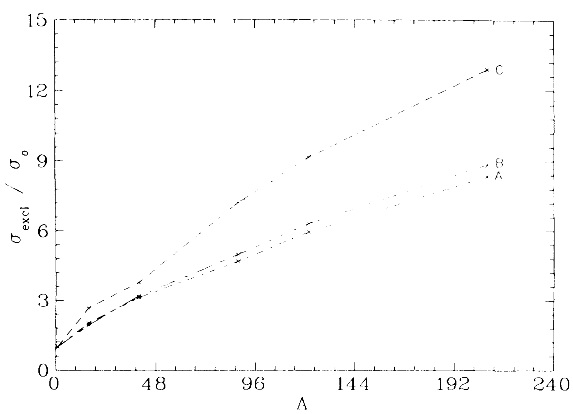


FIG. 3. $A,C$ are calculated as representatives of the $q^2\bar{q}^2$ and $q\bar{q}$ models, respectively, with the range parameter $\beta_{\rm S^*N}^2=5.8$ GeV$^{-2}$; $A$ with $\sigma_{\rm S^*N}^{\rm tot}=44.6$ mb; $C$ with $\sigma_{\rm S^*N}^{\rm tot}=22.3$ mb. $B$ corresponds to a K$\bar{\rm K}$ model with $R_{\rm S}=3$ fm and $\sigma_{\rm S^*N}^{\rm tot}=43.8$ mb.

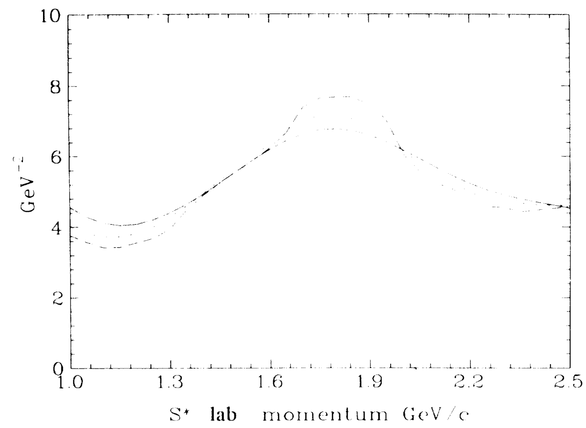


FIG. 4. $K^- + {}^{12}C$ total cross section. The solid line is drawn through the experimental points (Ref. 17). The dashed line is the result of a Glauber calculation using a Fermi-averaged forward $K^-N$ scattering amplitude.

a more realistic calculation will require improving the proton wave function for which we take a plane wave. We have calculated the inclusive production cross section using HO wave functions instead of the more realistic density (4.2), and we have found the same qualitative $A$ behavior. Therefore we expect the essential $A$ dependence of the exclusive cross section to be well reproduced with HO nuclear wave functions. The oscillator parameter $R_A$ is adjusted to give the nuclear rms radius. In Fig. 3 we show the normalized exclusive production cross section as a function of $A$ for an incoming pion momentum of 4 GeV/$c$. We adopted the same normalization convention as for the inclusive production cross section. We show the results for the three models of S*. We take $R_S = 3$ fm for the $K\bar{K}$ molecule. We find that the exclusive production cross section is strongly dependent on $\sigma^{\mathrm{tot}}_{S^*N}$ and only weakly dependent on $\sigma^{\mathrm{inel}}_{S^*N}$. Therefore, we obtain the same $A$ dependence for the

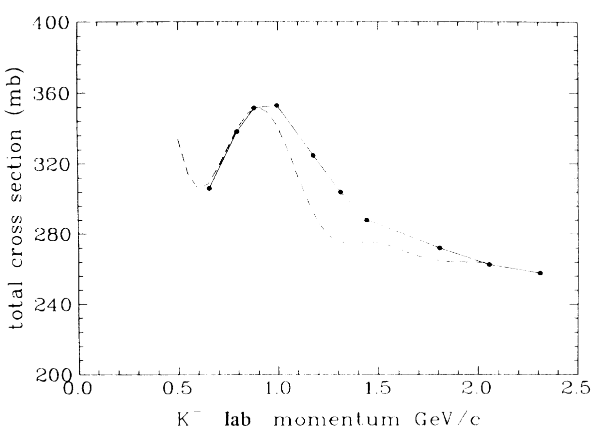


FIG. 5. The dashed line is the imaginary part of the elementary forward $K^-N$ plus $K^+N$ scattering amplitudes. The dotted line is the imaginary part of the forward $K^-N$ plus $K^+N$ scattering amplitudes Fermi averaged over the S* internal momenta (forward S*N scattering amplitude). The solid line is the imaginary part of the S*N scattering amplitude Fermi averaged over the nuclear momenta of $^{16}$O. For this calculation the $K\bar{K}$ molecular model of S* is used.

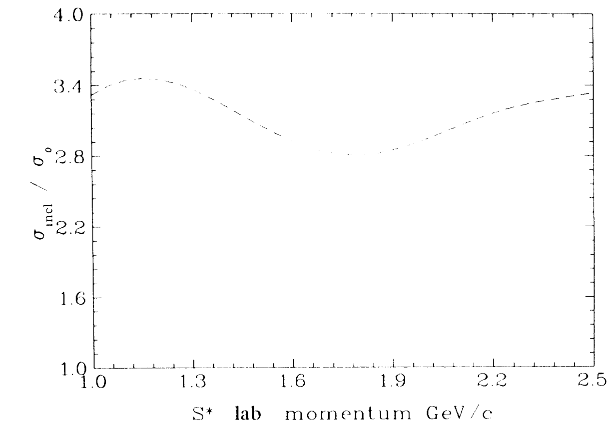


FIG. 6. Inclusive S* production cross section on $^{16}$O, for the case where S* is a weakly bound $K\bar{K}$ state.

$q^2\bar{q}^2$ and $K\bar{K}$ models since $\sigma^{\mathrm{tot}}_{S^*N}$ in these two models is approximately the same, although $\sigma^{\mathrm{inel}}_{S^*N}$ is very different. For the same reason we expect the exclusive production cross section to be insensitive to the S* radius since $\sigma^{\mathrm{tot}}_{S^*N}$ depends weakly on $R_S$. Therefore, from the $A$ dependence of the exclusive production cross section, we can extract $\sigma^{\mathrm{tot}}_{S^*N}$. Having extracted $\sigma^{\mathrm{tot}}_{S^*N}$ and $\sigma^{\mathrm{inel}}_{S^*N}$ we can use the arguments given in Sec. II A to distinguish the three models.

For the calculation of the energy dependence of the inclusive production cross section, we used the KN amplitudes of the BGRT collaboration[15] and of Ref. 16. In order to check how good our approximation of Fermi averaging using the forward c.m. amplitudes is, we computed the total $K^-$ $^{12}$C cross section in the energy region of interest. We used $s$-state HO wave functions with an oscillator parameter $R'_A = 1.36$ fm to Fermi average the forward $K^-N$ scattering amplitude. This amplitude is then used to define the profile function $\gamma(\mathbf{b})$ and the scattering amplitude is computed in the Glauber theory with $^{12}$C considered as a closed $P_{3/2}$-shell nucleus with $R_A = 1.64$ fm. In Fig. 4 we show the results of our calculation as compared to experiment. Although we get a resonance width which is too narrow, the general features are well reproduced for the purpose of our present discussion. In Fig. 5 we show the effective S*N amplitude $(1/k_i)\bar{F}_{S^*N}\,|_{\Delta=0}$ Fermi averaged both over the internal momenta of S* with $R_S = 3$ fm and over the $^{16}$O internal momenta using $s$-state HO wave functions, with $R'_A = 1.44$ fm. In Fig. 6 the total normalized production cross section for $^{16}$O, considered a closed $p$-shell nucleus, is shown where we took $R_A = 1.76$ fm. Clearly, the depletion seen in the production cross section in the resonance region is a definitive signature for the $K\bar{K}$ molecular model of S*.

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