PHYSICAL REVIEW C VOLUME 36, NUMBER 2 AUGUST 1987

Neutron and proton matrix elements for transitions in ^{39}K from pion inelastic scattering

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(Received 31 October 1986)

Cross sections have been measured for π^+ and π^- inelastic scattering at 180 MeV for low-lying quadrupole and octupole transitions in ^{39}K. A distorted-wave impulse-approximation analysis of the data using collective-model transition densities yields neutron and proton octupole matrix elements which are in good agreement with the corresponding electromagnetic matrix elements. Disagreement between the two probes is observed for the $E2$ transition to the 2.52-MeV $\frac{1}{2}^+$ first excited state in ^{39}K.

Previous studies have shown that, in the energy region of the $\Delta_{3,3}$ resonance, collective states are strongly excited in pion inelastic scattering. A unique feature of pion scattering arises from the fact that in this energy region the $\pi^+ + \mathrm{p}(\pi^- + \mathrm{n})$ elastic scattering amplitude is three times larger than the corresponding $\pi^+ + \mathrm{n}(\pi^- + \mathrm{p})$ amplitude. Therefore comparison of π^+ and π^- inelastic scattering provides an excellent means of obtaining information about neutron and proton (isoscalar and isovector) multipole matrix elements in nuclear transitions. Large π^+/π^- (or π^-/π^+) cross-section ratios have already been observed in pion scattering on p-shell nuclei for transitions involving pure proton or pure neutron excitations.[1–3]

Using empirically determined transition densities from inelastic electron scattering, a static distorted-wave impulse-approximation (DWIA) model, in which intermediate delta propagation is ignored, gives excellent agreement with data for pion inelastic scattering to collective states.[4–7] In Ref. 8 it was found that proton and neutron quadrupole matrix elements (M_p and M_n, respectively) for the 2_1^+ states extracted from pion inelastic scattering were in good agreement with the same quantities deduced from electromagnetic data.

Recent studies have reported anomalous strengths of $M1$ and $M3$ moments in ^{39}K. Measurements of the electroexcitation of the first excited state in ^{39}K at The National Institute for Nuclear and High-Energy Physics (NIKHEF) (Ref. 9) showed a significantly enhanced $B(M1)$ strength for this state $[B(M1)=0.014 \pm 0.006\mu_n^2]$. This result is surprising because the extreme shell model would predict that this transition is almost pure $1d_{3/2}^{-1} \rightarrow 2s_{1/2}^{-1}$ proton single particle, and thus the $M1$ transition is l forbidden. Second-order core polarization and meson-exchange current corrections[10] were suggested as possible explanations for this anomaly. A similar anomaly was observed in the $M3$ moment in ^{39}K.[11,12]

In this letter we report the results of an analysis of cross sections for π^+ and π^- inelastic scattering to low-lying states in ^{39}K. Data were taken using the Energetic Pion Channel and Spectrometer (EPICS) at the Clinton P. Anderson Meson Physics Facility (LAMPF). Cross sections were measured at an incident pion kinetic energy of 180 MeV. The target material was natural potassium metal (93.3% in ^{39}K) of area 20×12 cm^2 and areal density 100 mg/cm^2. As potassium is a highly reactive material, the target was kept immersed in oil until immediately prior to installation in the scattering chamber. Scattering angles, measured using a set of wire chambers at the entrance to the spectrometer were compared with trajectories measured in a similar set of detectors in the focal plane to identify and reject pion decays inside the spectrometer dipoles. In addition, a set of veto scintillators separated by graphite wedges, after the spectrometer, were used to reject muons. The system was fine tuned by placing an aluminum absorber in front of the first scintillator. The thickness was chosen to stop the pions but allow the muons to pass through to one of the veto scintillators. Absolute normalizations were obtained by measuring π-p scattering from a polyethylene (CH_2) target of areal density 73 mg/cm^2 and comparing the yields with cross sections calculated from the π-nucleon phase shifts of Rowe, Salomon, and Landau.[13] Relative pion beam normalization was taken from an ion chamber mounted directly in the beam, inside the scattering chamber. This procedure was consistent with normalizations taken from the LAMPF primary beam toroid. Relative normalization between π^+ and π^- is known to be better than $\pm 3\%$ and the absolute normalization is known to $\pm 10\%$. The variation of spectrometer acceptance across the focal plane was measured by pion scattering from ^{12}C, keeping the scattering angle fixed and varying the spectrometer field to cover an outgoing pion momentum range of $\pm 8\%$ of the spectrometer central momentum. Figure 1 shows a typical missing mass spectrum obtained with π^- at an incident energy of 180 MeV and laboratory angle of 35°. The spectrum has been corrected for variation of spectrometer acceptance. The peaks were fitted using a Gaussian shape with exponential tails.

Calculations of the angular distributions were performed using a modified version of the computer code DWPI (Ref. 7) which ties DWPI to the MINUIT optimizer package. The Kisslinger[18] form of the optical-model potential was used, with parameters obtained from pion-nucleon phase shifts evaluated at an energy 28 MeV below the center-of-mass energy in the pion-nucleus system.[19] The collective model has been used to obtain the radial shape of the transition density but with different proton and neutron deformation parameters (β_p and β_n, respectively). These parameters were varied to simultaneously reproduce the experimental π^+ and π^- cross sections. The resulting transition densities were integrated to obtain the multipole matrix elements:

$$M^{\lambda}=\int_0^{\infty}\rho_{tr}(r)r^{\lambda+2}dr$$

and

$$\rho_{tr}(r)=-(\beta c)d\rho(r)/dr ,$$

where $\rho(r)$ is the neutron or proton ground state density taken as a three-parameter Fermi distribution,

$$\rho(r)=\rho_0(1+wr^2/c^2)/\{1+\exp[(r-c)/a]\} ,$$

with values of $c=3.743$ fm, $a=0.585$ fm, and $w=-0.201$ fm obtained from electron-scattering data on ^{39}K.[20] In all calculations the neutron ground-state density was assumed to have the same shape as the proton ground-state density. Figure 2 shows the measured inelastic differential cross sections to the low-lying states in ^{39}K together with the DWIA calculations. The $E3$ transitions are quite well reproduced by the theory over the entire range of the angular distributions. The collective model fails, however, to account for the absence of a minimum in the $E2$ angular distribution for the transition to the 2.52-MeV $\frac{1}{2}^+$ state.

Assuming that the pion-nucleus interaction is dominated by the π-nucleon $\Delta_{3,3}$ resonance, the relationship between the matrix elements and the peak cross section is approximately given by

$$\sigma^+=K^+(3M_p+M_n)^2 ,$$
$$\sigma^-=K^-(M_p+3M_n)^2 ,$$

where K^+ and K^- are constants obtained from the DWIA calculations. The solutions for the 2.81-MeV $\frac{7}{2}^-$ state are illustrated graphically in Fig. 3. The bands labeled $\sigma(\pi^+)$ and $\sigma(\pi^-)$ represent the values of M_n and M_p consistent with the π^+ or π^- data. The intersections of the two $\sigma(\pi^+)$ and $\sigma(\pi^-)$ bands indicate the values of M_n and M_p resulting from a simultaneous fit to the π^+ and π^- data. The bands labeled EM represent the electromagnetic measurements for this state and its mirror in ^{39}Ca. There are two solutions for the neutron and proton amplitudes: in phase and out of phase.

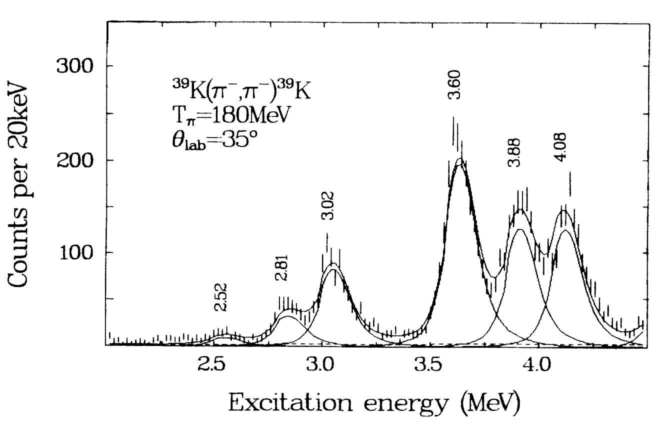

FIG. 1. Fitted missing mass spectrum for 180 MeV π^- scattering from ^{39}K at 35° laboratory angle.

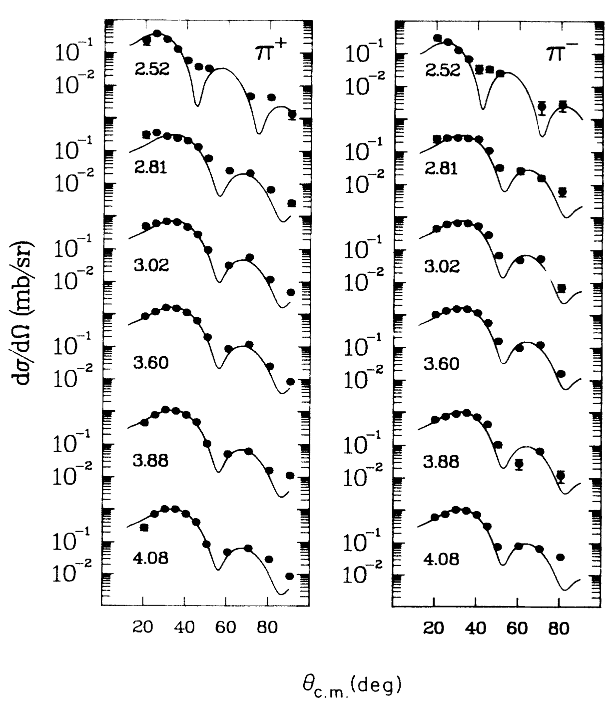

FIG. 2. Differential cross sections for π^+ and π^- inelastic scattering at 180 MeV for the quadrupole and octupole transitions in ^{39}K. Solid lines are DWIA calculations with collective-model transition densities.

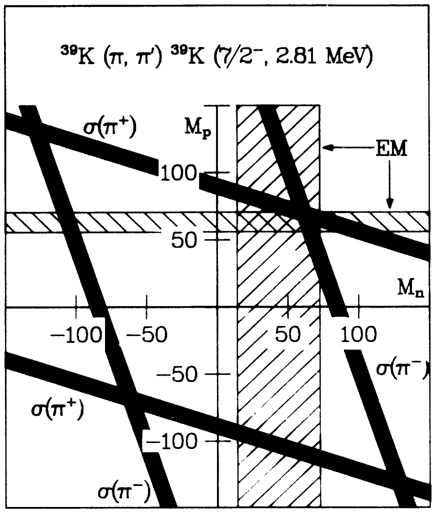

FIG. 3. M_p vs M_n multipole matrix elements derived for the 2.81 MeV $\frac{7}{2}^-$ state in ^{39}K. The intersections of the bands labeled $\sigma(\pi^+)$ and $\sigma(\pi^-)$ represent the solutions for M_p and M_n that simultaneously fit the π^+ and π^- data. The bands labeled *EM* correspond to electromagnetic measurements in the mirror nucleus pair.

Since the low-lying states are predominantly isoscalar, the solution with M_n and M_p in phase is assumed. Matrix elements for all states were obtained by varying M_n and M_p to give the best fit to the entire range of the angular distributions. The deduced values are listed in Table I.

The matrix elements obtained from this simple analysis are compared with electromagnetic measurements[14,21] in Table I. Also listed are shell-model predictions for M_n and M_p for the 2.52-MeV state.[22] The electromagnetic values of M_p were extracted from values of $B(E\lambda)\downarrow$ using the relation

$$M_p = [B(E\lambda)\times(2J_i+1)]^{1/2} ,$$

where J_i is the angular momentum of the excited state. The electromagnetic values of M_n in ^{39}K were taken to be equal to M_p values in ^{39}Ca, derived in the same way from $B(E\lambda)$ measurements. Assuming charge symmetry, the neutron matrix element $M_n(T_z)$ measured in a $T_z = T$ nucleus is equal to the electromagnetic matrix element $M_p(-T_z)$ measured in its mirror nucleus. The relatively large uncertainties in M_n from electromagnetic probes arise mainly from the reported uncertainties in the mixing ratios of the corresponding γ-decay transitions in ^{39}Ca, rather than from uncertainties in lifetimes.

The agreement between the present study and the electromagnetic measurements is quite good for the $E3$ transitions, the values mainly agreeing within uncertainties. The pion data indicate that all the low-lying octupole transitions in ^{39}K have M_p approximately equal to M_n, a feature which is not clear from the electromagnetic data. However, a significant disagreement exists between the pion and electromagnetic values of M_p for the first excited state, 21.6 ± 1.4 e fm^2 compared with 8.2 ± 1.5 e fm^2. Pion data yield a ratio $M_p/M_n=2.1$, whereas the electromagnetic data give only a lower limit of 1. The theoretical calculations[22] predict a larger ratio of 3.8. Based on a simple shell model one would expect a large M_p/M_n ratio for the transition to the 2.52-MeV $\frac{1}{2}^+$ state since it can be considered as almost pure $1d_{3/2}^{-1}\rightarrow 2s_{1/2}^{-1}$ proton single particle transition. In fact, in the model of Ref. 22, M_p/M_n for a single hole is just e_p/e_n, the ratio of effective charges. Other than through the effective charge, neutron contributions to this state (or higher positive-parity states) require at least $2\hbar\omega$ excitations since $1\hbar\omega$ neutron space contributes only to negative-parity states.

In an attempt to understand this disagreement a calculation has been done for the 2.52-MeV $\frac{1}{2}^+$ transition using the Argonne pion inelastic scattering code ARPIN (Ref. 23) which performs microscopic distorted-wave impulse-approximation calculations. Distorted waves were generated by a modified version of the momentum-space elastic scattering code PIPIT.[24] The transition density was derived from a pure single-particle-hole $(1d_{3/2})(2s_{1/2})^{-1}$ component. The calculated cross section (not shown in Fig. 2) for the $\Delta S=1$ ($\Delta L=2$, $\Delta J=1$) spin-flip transition was found to be 1–2 orders of magnitude smaller than the

TABLE I. Matrix elements in ^{39}K from pion inelastic scattering compared with results from electromagnetic measurements and theoretical predictions.

			Current analysis		Electromagnetic		Theory[d]	
E_x (MeV)	J^{π}[a]	$E\lambda$	M_p (e fm$^\lambda$)	M_n (e fm$^\lambda$)	M_p[b] (e fm$^\lambda$)	M_n[c] (e fm$^\lambda$)	M_p (e fm$^\lambda$)	M_n (e fm$^\lambda$)
2.52	$\frac{1}{2}^+$	$E2$	21.6±1.4	10.3±0.7	8.2±1.5	≤8.2	9.16	2.38
2.81	$\frac{7}{2}^-$	$E3$	68.8±4.5	61.5±4.0	63.0±7.5	43.3±29.2		
3.02	$\frac{3}{2}^-$	$E3$	146.2±9.5	137.3±8.9	118.7±91.2	245.0±110.8		
3.60	$\frac{9}{2}^-$	$E3$	139.5±9.1	128.1±8.3	144.1±3.2	289.8±91.9		
3.88[e]	$\frac{5}{2}^-$	$E3$	123.9±8.1	103.6±6.7				
4.08[f]	$\frac{7}{2}^-$	$E3$	87.2±5.6	82.4±5.4	≤95.5			

[a]References 14, 15, and 16.
[b]Reference 21.
[c]Reference 14.
[d]Reference 22.
[e]Triplet, 3.88 MeV $\frac{5}{2}^-$; 3.938 MeV $\frac{3}{2}^+$; 3.944 MeV $\frac{11}{2}^-$.
[f]Triplet, 4.08 MeV $\frac{3}{2}^-$; 4.095 MeV $\frac{1}{2}^+$; 4.126 MeV $\frac{7}{2}^-$.

measured cross section. This prediction seems to be consistent with the measured cross section of the few previously reported $M1$ excitations in pion scattering.[25–28] Thus the possible contribution from the anomalous enhanced $B(M1)$ strength reported for this state seems to be too small to account for the disagreement between the electromagnetic and pion values of the proton matrix element. It could, however, be responsible for the filling in of the minimum in the angular distribution.

In summary, neutron and proton multipole matrix elements for transitions to low-lying states in ^{39}K have been extracted from comparison of π^+ and π^- inelastic scattering cross sections at $T_\pi = 180$ MeV. The matrix elements for the octupole transitions are in good agreement with those obtained using electromagnetic probes. We find that the average ratio of M_p/M_n obtained from the pion data for the $E3$ transitions is 1.11 ± 0.05. The value of M_p derived from the present study for the $E2$ transition to the first excited state is larger by a factor of 2 than the corresponding electromagnetic measurement.

This work was supported in part by the United States Department of Energy, the Robert A. Welch Foundation, and the National Science Foundation.

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