Pion double charge exchange and nuclear correlations

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Single- and double-charge-exchange reactions of pions on nuclei are analyzed by means of an elementary model based on the first- and second-order impulse approximations. The model is applied to the description of transitions induced between the isobaric analog states of ^{14}C and ^{14}O and ^{14}N at 50 MeV incident pion energy. We find that the model, with only the familiar nuclear orbital wave functions as input, successfully explains the angular distributions observed in both the single and double analog state transitions. In the case of the double-charge-exchange reaction a particularly significant role in the process is played by the position correlation of the two valence neutrons in ^{14}C. Double-charge-exchange measurements, we show, furnish a direct means of detecting this correlation, which is an important feature of any nuclear shell model. Calculating the effect of this correlation requires summing the nuclear transition amplitudes over all accessible intermediate states. To restrict the summation to the intermediate analog state is to omit the effect of correlation. The double-charge-exchange probability also has some sensitivity to the spatial dependence of the pion-nucleon interaction at short distances. We find that it indicates an approximate radius for the interaction of about 0.7 fm. We have evaluated corrections to the impulse approximations due to nuclear scattering and absorbtion and find that they make only modest quantitative changes in the analysis based on plane waves.

I. INTRODUCTION

Although the double-charge-exchange process is one of the rarer forms of pion reactions, it is no longer difficult to detect and has recently been receiving considerable study.[1,2] Several of the more interesting features of the process are the consequences of its second-order nature. If a positive pion, for example, entering a nucleus, is to emerge as a negative pion simply by exchanging charge with the nucleons present, then the process must take place in at least two stages—converting two different neutrons to protons. This picture of the reaction as a sequence of two single-charge-exchange processes suggests that it may be sensitive to the correlation properties of pairs of nucleons within the nucleus. We shall show that double charge exchange is indeed sensitive to the relative position distribution of the nucleons lying outside closed shells and furnishes the first direct probe of those correlations. The reaction, we shall see, offers some information about the range of the fundamental pion-nucleon interaction as well.

Double-charge-exchange measurements carried out recently at TRIUMF,[1] and at the Clinton Anderson Laboratory[2] have presented an interesting puzzle. Observations of the double-analog-state transition $\pi^+ {}^{14}\mathrm{C}(0^+) \rightarrow \pi^- {}^{14}\mathrm{O}(0^+)$ at 50 MeV incident pion energy have revealed a cross section larger than that predicted by optical model calculations[2] and a strong forward peak in the angular distribution. The forward peak was perhaps the greater surprise in view of the known properties of the single charge-exchange process. At 50 MeV there is a destructive interference between the s and p wave contributions to the amplitude for pion-nucleon charge exchange that leads to a dramatic minimum in the forward direction of an otherwise relatively flat cross section. The double-charge-exchange process had proved to be most probable, in other words, in the angular range in which the fundamental single-charge-exchange process was quite improbable. The single-charge-exchange processes implicit in double charge exchange tend to take place in connection with scattering processes at finite angles. The convolution of two such finite-angle scatterings could indeed have an angular distribution with a smoothed-out dip in the forward direction, but that would not explain the observed forward peak.

A possible resolution of the puzzle and a challenge to the sequential picture of the double exchange process has been presented by Miller[3] who has suggested that the process depends on the existence of di-nucleon structures within the nucleus. A pion striking one of these can exchange two units of charge in a single step. The angular distribution of the emerging pions is then just the form factor for the distribution of di-nucleons and that form factor could easily be forward-peaked.

It is our purpose to show in the present paper that the paradox can be resolved without invoking any such mechanism. All that is required is a careful analysis of coherent multiple scattering. We shall present a model that allows us to discuss both single- and double-charge-exchange processes by means of the most elementary approximations. We are enabled thereby to gain simple analytical insights into the dependence of the angular

distributions of the scattered pions on the properties of the nuclear wave functions and the πN charge-exchange amplitudes.

We shall base our initial approach on the first and second order impulse approximations. Our purpose in doing this is to take full advantage of the insight made available by the relative weakness of the πN interaction at low energies. We thus begin by neglecting the effects of pion absorbtion and elastic multiple scattering and thereby simplify the analysis considerably. Once we have outlined the properties of the charge-exchange processes by means of this approximation, we undertake the inclusion, in approximate terms, of the effects of absorbtion and elastic scattering. Taking account of these effects, we show, does not materially alter any of the conclusions reached without them.

The charge-exchange processes we shall study are induced by positive pions incident on ^{14}C and lead either to the analog state in ^{14}N or to the ground state of ^{14}O as final states. In such analog-state transitions the nuclear isospins are simply rotated. The nuclear transitions are otherwise essentially elastic; the configuration space and spin wave functions of the nucleons undergo no change. The nucleus ^{14}C may be accurately pictured as consisting of a closed core, which is essentially ^{12}C, plus two valence nucleons. Within the context of the lower order impulse approximation, the single-charge-exchange reaction takes place in a single collision process and double charge exchange involves no more than two successive collisions. Since in the analog transitions these processes take place elastically, they cannot involve any core excitations. The core then plays a completely passive role. We therefore focus our attention for the present entirely on the two valence nucleons, which are the only ones that can play an active role.

The basis elements of our model and its application to the analysis of the angular distributions of the charge-exchange reactions are presented in the following two sections. The object of these sections is to demonstrate that by using the same fundamental form for the πN charge-exchange amplitude in the analysis of both single and double charge exchange, it is possible to give simultaneous descriptions of the two processes that are in accord with the experimental data. Section IV is then devoted to a more detailed discussion of the mechanism of the double-charge-exchange reaction. Its purpose is to discuss the properties of the nuclear wave function, of the intermediate nuclear states, and of the features of the πN interaction that are most responsible for determining the magnitude and shape of the observed differential cross section. The modifications made by pion absorbtion and scattering effects are then estimated in Sec. V.

II. SINGLE-CHARGE-EXCHANGE REACTION

We begin by discussing the single-charge-exchange process $\pi^+ {}^{14}\mathrm{C}(0^+)\rightarrow\pi^0 {}^{14}\mathrm{N}(0^+)$. If we denote the spin-dependent charge-exchange amplitude of the jth nucleon by $F_j(\mathbf{k}_f,\mathbf{k}_i)$, where $\mathbf{k}_i$ and $\mathbf{k}_f$ are the initial and final pion momenta, we can write the amplitude for the exchange of one unit of charge with the two valence neutrons as

$$F_{\mathrm{SCX}}(\mathbf{k}_f,\mathbf{k}_i)=\langle \pi^0\,{}^{14}\mathrm{N}\,|\,F_1(\mathbf{k}_f,\mathbf{k}_i)+F_2(\mathbf{k}_f,\mathbf{k}_i)\,|\,\pi^+\,{}^{14}\mathrm{C}\rangle\ . \tag{2.1}$$

The charge-exchange amplitudes F_j contain familiar retardation phase factors depending on the positions of neutrons 1 and 2. We can specify these and the spin- and isospin-dependent parts of the amplitudes as well by writing

$$F_j(\mathbf{p}'\mathbf{p})=e^{i(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}_j}(\boldsymbol{\tau}_j\cdot\mathbf{T}_\pi)\times[f(\mathbf{p}',\mathbf{p})+i\boldsymbol{\sigma}_j\cdot(\mathbf{p}\times\mathbf{p}')g(\mathbf{p}',\mathbf{p})],\quad j=1,2\,. \tag{2.2}$$

In this expression $\mathbf{r}_1$ and $\mathbf{r}_2$ are the positions of the two valence nucleons, while $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are their Pauli spin operators and $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_2$ are their respective isospin operators. The pion isospin operator is represented by $\mathbf{T}_\pi$ and $f(\mathbf{p}',\mathbf{p})$ and $g(\mathbf{p}',\mathbf{p})$ are the spin-independent and spin-flip amplitudes, respectively.

The wave function of the ^{14}C nucleus is assumed to separate into a wave function for two valence neutrons in the p-shell orbit $\langle\mathbf{r}_1,\mathbf{r}_2\,|\,1,2\rangle$ and a wave function for the core. Since the core plays a passive role we can omit further mention of its wave function. The two p-shell nucleons, having total angular momentum zero, can only be in either of two states, the spin singlet $^1S_0$ or the spin triplet $^3P_0$ state. The most general state available to them can thus be written as

$$|\,1,2\rangle=\alpha\,|\,{}^1S_0\rangle+\beta\,|\,{}^3P_0\rangle\ , \tag{2.3}$$

with $|\alpha|^2+|\beta|^2=1$. We shall employ as shell model wave functions the harmonic oscillator parametrizations of the S and P states,

$$\langle\mathbf{r}_1,\mathbf{r}_2\,|\,{}^1S_0\rangle=N_S e^{-(r_1^2+r_2^2)/2b^2}(\mathbf{r}_1,\mathbf{r}_2)\chi^0(1,2)\eta^-(1,2) \tag{2.4}$$

and

$$\langle\mathbf{r}_1,\mathbf{r}_2\,|\,{}^3P_0\rangle=N_P e^{-(r_1^2+r_2^2)/2b^2} i(\mathbf{r}_1\times\mathbf{r}_2)\cdot\frac{\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2}{2}\chi^0(1,2)\eta^-(1,2)\ , \tag{2.5}$$

where $\chi^0(1,2)$ is the spin-singlet wave function of the two valence neutrons, $\eta^-(1,2)$ is the $T_z=-1$ component of their isospin-triplet wave function, and $N_S=2/\sqrt{3}\pi^{-3/2}b^{-5}$ and $N_P=\sqrt{2/3}\pi^{-3/2}b^{-5}$ are the normalization coefficients. The wave functions of $^{14}\mathrm{N}(0^+)$ and $^{14}\mathrm{O}(0^+)$, which are related by rotation in isospin space, are identical to those of Eq. (2.4) and Eq. (2.5) in their coordinate and spin dependences and can be obtained by replacing the isospin wave function $\eta^-(1,2)$ by $\eta^0(1,2)$ and $\eta^+(1,2)$, respectively. For a $J=0\rightarrow J=0$ transition, such as the one we are considering, the terms linear in the target nucleon spins make no contribution to the single-charge-exchange amplitude.

That amplitude may be written as

$$F_{\mathrm{SCX}}(\mathbf{Q})=2f(\mathbf{k}_f,\mathbf{k}_i)S(Q)\ , \tag{2.6}$$

where $\mathbf{Q}=\mathbf{k}_f-\mathbf{k}_i$ is the momentum transfer and

$$S(Q)=\langle 12\mid e^{i\mathbf{Q}\cdot\mathbf{r}_1}\mid 12\rangle=\langle {}^1S_0\mid e^{i\mathbf{Q}\cdot\mathbf{r}_1}\mid {}^1S_0\rangle$$
$$=\langle {}^3P_0\mid e^{i\mathbf{Q}\cdot\mathbf{r}_1}\mid {}^3P_0\rangle=(1-Q^2b^2/6)e^{-Q^2b^2/4} \tag{2.7}$$

is the form factor for the single-particle density distribution of either of the two valence neutrons. While the wave functions for the ${}^1S_0$ and ${}^3P_0$ states tend to correlate the positions of the neutrons in rather different ways, it is easy to verify that their single particle densities take the identical form

$$\rho(r)=3/2\pi^{-3/2}b^{-5}r^2e^{-r^2/b^2} \tag{2.8}$$

characteristic of p-shell orbitals and that is, of course, why the two form factors in Eq. (2.7) are the same. It follows then that the single-charge-exchange cross section is independent of the amount of mixing of the S and P states in the wave function of the two valence neutrons.

The factorization of the amplitude for the single-charge-exchange reaction represented by Eq. (2.6) makes it clear that the narrow forward dip in the elementary charge-exchange cross section will be preserved in the nuclear charge-exchange process. The form factors $S(Q)$ typically vary much more slowly as a function of momentum transfer Q.

Single-charge-exchange measurements on ^{14}C at 50 MeV do clearly show the presence of the forward dip as do measurements on several other nuclei as well.[4] It is worth emphasizing that the forward dip structure could be altered substantially or annihilated entirely if the effects of elastic scattering and absorbtion within the nucleus were appreciable. That fact alone furnishes a reassuring indication that our neglect of these effects by using the impulse approximation remains fairly accurate.

In Fig. 1 we display the results of our calculations of

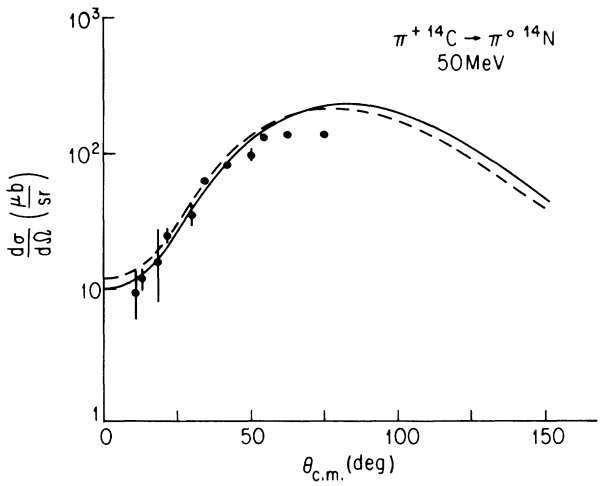

FIG. 1. Comparison of the experimental differential cross section for $\pi^+ {}^{14}\mathrm{C}(0^+)\rightarrow\pi^0 {}^{14}\mathrm{N}(0^+)$ at 50 MeV, and the theory predictions. The dashed and solid curves correspond to the plane wave and modified impulse approximations described in Sec. I.

the angular distribution for the single-charge-exchange cross section. The dashed curve was obtained by using the impulse approximation, as we have noted. The solid curve, on the other hand, follows from a calculation including the effects of pion scattering and absorbtion corrections that we shall discuss in Sec. V. The corrections involved are evidently quite small.

The parametrization we have used, in the above calculation for the πN charge exchange amplitude f takes the familiar form

$$f(\mathbf{p}',\mathbf{p})=a_1+b_1(\mathbf{p}\cdot\mathbf{p}')\ , \tag{2.9}$$

where the coefficients a_1 and b_1 can be obtained from the compilation of phase shifts for π-N scattering.[5] We note that since our calculation is carried out in the target Breit frame, the coefficients a_1 and b_1 differ from the corresponding values of the expansion coefficients a_1^{cm} and b_1^{cm} of the πN amplitude in the center-of-mass frame through a Galilean transformation that is described, for example, by Brown, Jennings, and Rostokin.[6] If we follow their prescription for a nucleon at rest, we find that the transformation changes the value of a_1 from -0.13 fm^{-1} $(=a_1^{\mathrm{cm}})$ to -0.18 fm^{-1}, while leaving $b_1=b_1^{\mathrm{cm}}$. We have used in our impulse approximation calculation an adjusted effective value $a_1=-0.21$ fm^{-1} which represents a further change from the free value by 17%. Such an adjustment seems to be well within the range of the ambiguities associated with the Galilean transformation of πN amplitudes into the frame of reference of a bound but rapidly moving nucleon.[7] An additional motivation for adjusting the value of the isovector component of the πN amplitude results from the inclusion of the effect of the Coulomb field effects in the phase shift analysis of πN scattering.[8]

III. DOUBLE-CHARGE-EXCHANGE REACTION

We make use of the second-order impulse approximation in treating the double-charge-exchange reaction. The two stages of the process will each be assumed to involve the same effective πN amplitude that fits the single-charge-exchange data.

Our expression for the transition amplitude for the double-charge-exchange reaction $\pi^+ {}^{14}\mathrm{C}(0^+)\rightarrow\pi^0 {}^{14}\mathrm{O}(0^+)$ in the second-order impulse approximation is given by the matrix element, between the initial and final nuclear states, of an operator $\tilde{F}(\mathbf{Q},\mathbf{r}_1,\mathbf{r}_2)$ that represents the double-collision amplitude for pion scattering from the pair of nucleons located at $\mathbf{r}_1$ and $\mathbf{r}_2$,

$$F_{\mathrm{DCX}}(\mathbf{Q})=\langle \pi^0 {}^{14}\mathrm{O}\mid\tilde{F}(\mathbf{Q},\mathbf{r}_1,\mathbf{r}_2)\mid\pi^+ {}^{14}\mathrm{C}\rangle\ . \tag{3.1}$$

The operator $\tilde{F}$ is given by

$$\tilde{F}(\mathbf{Q},\mathbf{r}_1,\mathbf{r}_2)=-\frac{1}{2\pi^2}\int d^3p\,[F_1(\mathbf{k}_f,\mathbf{p})G(\mathbf{p})F_2(\mathbf{p},\mathbf{k}_i)$$
$$+F_2(\mathbf{k}_f,\mathbf{p})G(\mathbf{p})F_1(\mathbf{p},\mathbf{k}_i)]\ . \tag{3.2}$$

In this expression

$$G(\mathbf{p})=(k^2-p^2+i\epsilon)^{-1} \tag{3.3}$$

is the propagator for the neutral pion wave in its passage from the first collision to the second.

It is worth emphasizing that the matrix element (3.1) and the operator (3.2) make no reference to any particular intermediate states through which the nucleus may pass in the interval between the two collision events. That is because a summation over a complete set of intermediate nuclear states (often called the closure approximation) is contained implicitly in the expressions presented. They render account, in other words, of transitions to all accessible intermediate states, including isosinglet states and states that fail grossly to conserve energy. We shall discuss the special role of the analog intermediate state at a later point.

The intermediate nuclear states are not the only unconstrained element in the expression (3.2). The π^0 propagates only a short distance between collisions and thus is not constrained to be on its mass shell. In order to evaluate the integral (3.2) then, we need to know not only the values of the πN scattering amplitude on the mass shell (i.e., for $|\mathbf{p}|=|\mathbf{p}'|=|\mathbf{k}_i|\equiv k$) (but its values "half-off-shell" as well (i.e., for $|\mathbf{p}|=k$ and $|\mathbf{p}'|\neq k$ or for $|\mathbf{p}'|=k$ and $|\mathbf{p}|\neq k$). In the absence of a complete theory of the πN interaction, only the on-shell (i.e., energy conserving) part of the πN amplitude can be determined from existing phase shift analysis of experimental scattering data. We shall treat the off-shell dependence of the πN interaction phenomenologically by parametrizing the isovector components of the πN collision matrix entering Eq. (3.2) as

$$F_j(\mathbf{p}',\mathbf{p})=e^{i(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}_j}(\tau_j\cdot\mathbf{T}_\pi) \times[f(\mathbf{p}',\mathbf{p})+i\sigma_j\cdot(\mathbf{p}\times\mathbf{p}')g(\mathbf{p}',\mathbf{p})]h(p,p') , \quad j=1,2 , \tag{3.4}$$

where the function $h(pp')$ represents the off-shell form factor with the constraint

$$h(k_i,k_f)=h(k_f,k_i)=1 . \tag{3.5}$$

The form factor $h(p,k)$, on the other hand, presumably decreases for $p\to\infty$, as p recedes from the mass shell. We shall assume for convenience that $h(p,k_i)$ can be represented by the frequently used expression[9]

$$h(p,k)=h(k,p)=\frac{\Lambda^2+k^2}{\Lambda^2+p^2} . \tag{3.6}$$

Such a form factor serves the function of describing the finite size and shape of the π-N interaction. The parameter Λ it contains may be interpreted as the reciprocal of its range. Indeed, in the absence of such a form factor, the scattering amplitude (3.4) describes, in effect, point interactions that are known to lead to unphysical and even divergent descriptions of scattering at higher energies.

A convenient way of discussing the effect of form factor (3.6) is to combine it with the free-pion propagator (3.3) by defining an effective propagator

$$\tilde{G}(p)=G(p)h^2(p,k) . \tag{3.7}$$

Although the effects of free-particle propagation and interaction are logically quite separate, the effective propagator $\tilde{G}$ can be thought of as an average propagator for finite-sized particles. In configuration space it takes the form

$$\tilde{G}(r)=\frac{1}{(2\pi)^3}\int e^{i\mathbf{p}\cdot\mathbf{r}}\tilde{G}(p)d\mathbf{p} =-\frac{e^{ikr}}{4\pi r}+\frac{e^{-\Lambda r}}{4\pi r}\left[1+\frac{k^2+\Lambda^2}{2\Lambda}r\right] , \tag{3.8}$$

while $G(r)$ is simply the first of these terms. The propagators $G(r)$ and $\tilde{G}(r)$ differ only in their behavior for $r\to 0$, where $G(r)$ becomes infinite while $\tilde{G}(r)$ tends to a finite value. The role of the propagator is an important one in the matrix element (3.1) since it lends heavy emphasis to configurations in which the two neutrons are close together. The effective propagator $\tilde{G}(r)$ moderates that emphasis to a degree, but still provides that double charge exchange is most probable when the separations are small.

By substituting the target wave functions of Eq. (10) and the πN scattering amplitudes given by Eqs. (2.2) and (2.3) into Eq. (3.2), we cast the expression for the double-charge-exchange amplitude into the form

$$F_{\mathrm{DCX}}(\mathbf{Q})=|\alpha|^2F^S_{\mathrm{DCX}}(\mathbf{Q})+|\beta|^2F^P_{\mathrm{DCX}}(\mathbf{Q}) +2i\alpha\beta F^{SP}_{\mathrm{DCX}}(\mathbf{Q}) \simeq|\alpha|^2F^S_{\mathrm{DCX}}(\mathbf{Q})+|\beta|^2F^P_{\mathrm{DCX}}(\mathbf{Q}) . \tag{3.9}$$

The two terms in this expression labeled by S and P are contributed individually by the $^1S_0$ and $^3P_0$ components of the wave function of the two valence nucleons. The remaining interference term is contributed only by collision induced double spin flip processes and vanishes for zero momentum transfer. We find this term to have an upper bound of about 2% of the cross section. We have included that term in all of our calculations but will not discuss its structure in detail here. The absence of other interference terms representing $S\to P$ and $P\to S$ transitions follows from considerations of rotational invariance and parity conservation.

Each of the two valence nucleons receives a certain recoil momentum during the double collision process, but their overall wave function remains intact. The amplitude for this part of the process is a two-particle form factor that is implicitly contained in the matrix element (3.1) and takes fairly different forms for the $^1S_0$ and $^3P_0$ states. For the $^1S_0$ state it is

$$S_s(\mathbf{q}_1,\mathbf{q}_2)=\langle{}^1S_0|e^{i\mathbf{q}_1\cdot\mathbf{r}_1+i\mathbf{q}_2\cdot\mathbf{r}_2}|{}^1S_0\rangle =[1-\tfrac{1}{6}b^2(q_1^2+q_2^2)+\tfrac{1}{12}b^4(\mathbf{q}_1\cdot\mathbf{q}_2)^2] \times e^{-b^2(q_1^2+q_2^2)/4} , \tag{3.10}$$

while for the $^3P_0$ state the corresponding expression is

$$S_p(\mathbf{q}_1,\mathbf{q}_2)=\langle{}^3P_0|e^{i\mathbf{q}_1\cdot\mathbf{r}_1+i\mathbf{q}_2\cdot\mathbf{r}_2}|{}^3P_0\rangle =[1-\tfrac{1}{6}b^2(q_1^2+q_2^2)+\tfrac{1}{24}b^4(\mathbf{q}_1\times\mathbf{q}_2)^2] \times e^{-b^2(q_1^2+q_2^2)/4} \tag{3.11}$$

For its latter state we must also define a second species of form factors associated with the possibility of spin flips. We write

$$S'_p(\mathbf{q}_1,\mathbf{q}_2)=\langle {}^3P_0 \mid e^{i\mathbf{q}_1\cdot\mathbf{r}_1+i\mathbf{q}_2\cdot\mathbf{r}_2}(\boldsymbol{\sigma}_1\cdot\mathbf{A})(\boldsymbol{\sigma}_2\cdot\mathbf{B}) \mid {}^3P_0\rangle$$
$$=\{\tfrac{1}{3}(\mathbf{A}\cdot\mathbf{B})+\tfrac{1}{24}b^2(\mathbf{A}\cdot\mathbf{B})(\mathbf{q}_1\times\mathbf{q}_2)^2-\tfrac{1}{12}b^4[\mathbf{A}\cdot(\mathbf{q}_1\times\mathbf{q}_2)][\mathbf{B}\cdot(\mathbf{q}_1\times\mathbf{q}_2)]\}e^{-b^2(q_1^2+q_2^2)/4}, \quad (3.12)$$

where $\mathbf{A}=\mathbf{k}_i\times\mathbf{p}$, $\mathbf{B}=\mathbf{p}\times\mathbf{k}_f$, and $\mathbf{A}\cdot\mathbf{B}=(\mathbf{k}_f\cdot\mathbf{p})(\mathbf{p}\cdot\mathbf{k}_i)-p^2(\mathbf{k}_i\cdot\mathbf{k}_f)$.

By making use of these form factors and carrying out the appropriate spin algebra we can write the explicit matrix elements (3.1) for the double-charge-exchange process in the ${}^1S_0$ and ${}^3P_0$ states as

$$F^S_{\mathrm{DCX}}(\mathbf{Q})=-\frac{1}{2\pi^2}\int d^3p\tilde{G}(p)[a_1^2+2a_1b_1(\boldsymbol{l}\cdot\mathbf{p})+b_1^2(\mathbf{k}_f\cdot\mathbf{p})(\mathbf{p}\cdot\mathbf{k}_i)-c_1^2(\mathbf{A}\cdot\mathbf{B})]S_s(\mathbf{q}_1,\mathbf{q}_2) \quad (3.13)$$

and

$$F^P_{\mathrm{DCX}}(\mathbf{Q})=-\frac{1}{2\pi^2}\int d^3p\tilde{G}(p)\{[a_1^2+2a_1b_1(\boldsymbol{l}\cdot\mathbf{p})+b_1^2(\mathbf{k}_f\cdot\mathbf{p})(\mathbf{p}\cdot\mathbf{k}_i)]S_p(\mathbf{q}_1,\mathbf{q}_2)+c_1^2S'_p(\mathbf{q}_1,\mathbf{q}_2)\}, \quad (3.14)$$

respectively. The vector $\boldsymbol{l}$ in these expressions is defined as $\boldsymbol{l}=\frac{1}{2}(\mathbf{k}_i+\mathbf{k}_f)$, and the parameter c_1 represents the constant spin-flip amplitude $g=c_1$.

With our choice of harmonic oscillator shell-model wave functions and the off-mass-shell form factor given by Eq. (3.6), it is possible to carry the integrations out analytically to a last stage that involves error functions of complex argument.[10] Anticipating the possibility of using other forms for the function $h(p,k)$, however, we have chosen to carry out the last integration, over the absolute value of p numerically.

The theory we have described to this point contains two partially unknown parameters, the inverse range Λ, and either of the two linear combination coefficients, α or β. Since the ${}^1S$ state and the ${}^3P$ state lead to rather different angular distributions, comparison of our calculated results with the shape of the experimental angular distribution leads to an estimate of the parameters α and β. The parameter Λ on the other hand influences mainly the magnitude of the cross section. Decreasing Λ tends to raise the cross section slightly since it raises the maximum modulus of the function (3.8).

The angular distribution we have calculated for the choice $|\alpha|=0.85$, $|\beta|=0.52$ (which correspond to 14% of $p_{3/2}$ and 86% of $p_{1/2}$ configurations), and $\Lambda=3.5$ fm^{-1} is compared with the experiment data[1,2] in Fig. 2. In this figure, the dashed curve follows from the calculation based on the second order impulse approximation. The solid curve represents the differential cross section calculated with pion scattering and absorbtion corrections which we find, similarly as in the case of the single-charge-exchange reaction, to be quite small. Our method of calculating these corrections will be presented

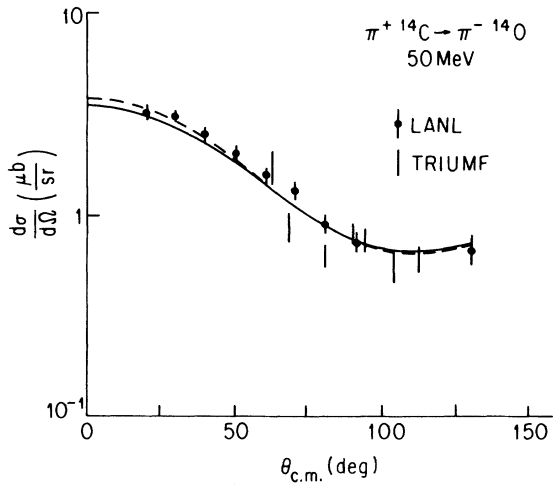

FIG. 2. Comparison of the experimental differential cross section for $\pi^+ {}^{14}C(0^+)\rightarrow\pi^- {}^{14}O(0^+)$ at 50 MeV, and the results of the calculations described in Sec. IV. The dashed and solid curves correspond to the plane wave and modified impulse approximations, respectively.

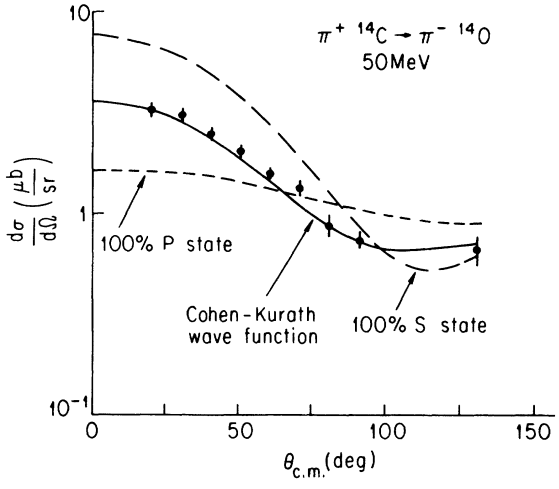

FIG. 3. Comparison of angular distributions for the double-charge-exchange reaction $\pi^+ {}^{14}C(0^+)\rightarrow\pi^- O(0^+)$ at 50 MeV, and the theory predictions based on pure ${}^1S_0$-state wave function for the target valence nucleons (dashed curve), that employing pure ${}^3P_0$-state wave function (dotted curve) and that in which the Cohen-Kurath wave function was used (solid curve).

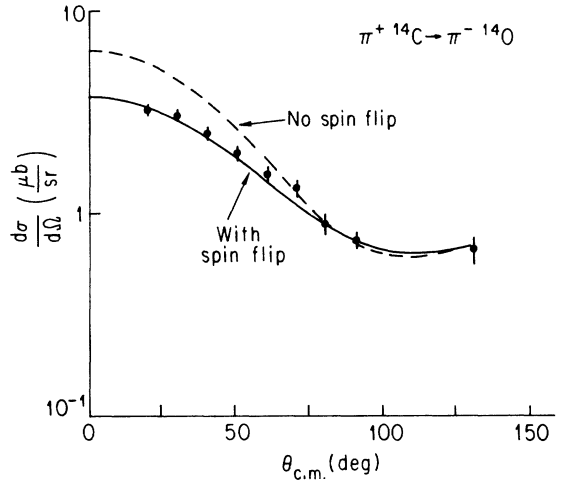

FIG. 4. Comparison of angular distributions for the double-charge-exchange reaction $\pi^+ {}^{14}C(0^+) \rightarrow \pi^- {}^{14}O(0^+)$ at 50 MeV. The dashed curve corresponds to the omission of spin-dependent terms from the πN amplitudes. The solid curve represents the angular distribution that includes all spin-dependent terms.

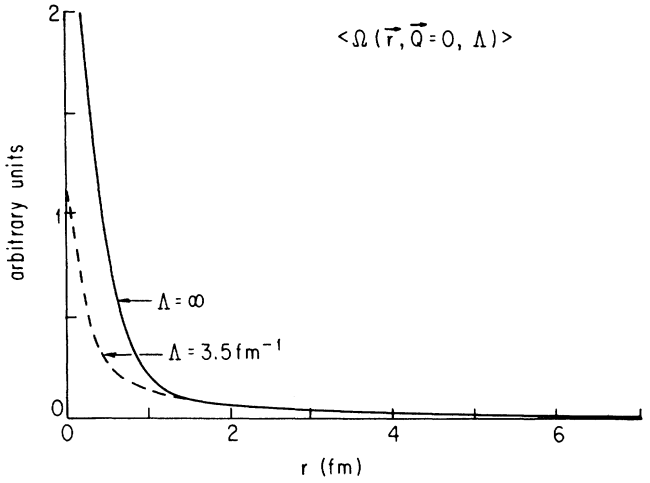

FIG. 6. Comparison of the functions $\Omega(\mathbf{r},\mathbf{Q}=0;\Lambda)$ for two values of the parameter Λ. The solid and the dashed curves correspond to $\Lambda = 3.5$ fm^{-1} and $\Lambda = \infty$, respectively.

in Sec. V. The calculated results, we must emphasize, are based on the same πN scattering amplitudes we used earlier in our description of the single-charge-exchange process. While the graph shows that the theory can indeed predict a forward peak in the angular distribution, the close agreement shown with experiment must be considered in conjunction with the values of the parameters we have reached. In fact, the values we have found for $|\alpha|$ and $|\beta|$ happen to be consistent within about 2% with the theoretical values found for ^{14}C by Cohen and Kurath,[11,12] who have used an intermediate coupling scheme to calculate the shell model wave function. The value $\Lambda = 3.5$ fm^{-1} corresponds to a root-mean-square radius (mean radius) for the charge-exchange interaction of about $\sqrt{6}/\Lambda \cong 0.7$ fm.

In Figs. 3–8 we illustrate the sensitivity of the calculated angular distribution for double charge-exchange to various elements that have entered the calculations. It is particularly significant, as we shall see, that the $^1S_0$ and $^3P_0$ states furnish different angular distributions. We have shown these angular distributions separately in Fig. 3. The cross section for the $^1S_0$ state is larger and much more strongly forward-peaked than that coming from the $^3P_0$ state. We shall discuss the reason for these results in the next section.

The contribution of the spin-dependent terms is isolated in Fig. 4, which shows the angular distributions reached

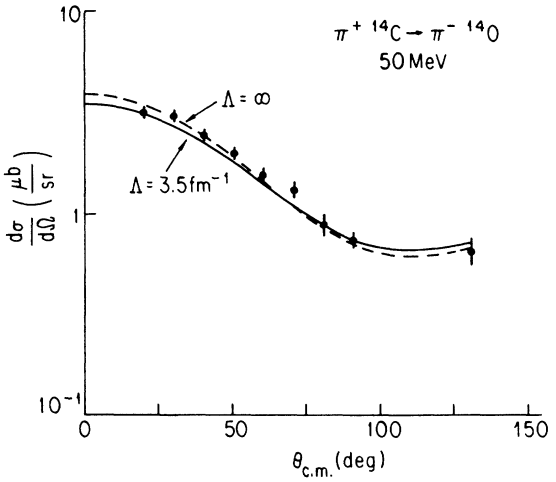

FIG. 5. Comparison between the angular distributions for the double-charge-exchange reaction for the reaction $\pi^+ {}^{14}C(0^+) \rightarrow \pi^- {}^{14}O(0^+)$ at 50 MeV, resulting from the calculation employing $\Lambda = \infty$ (dashed curve) with that resulting from the complete calculation ($\Lambda = 3.5$ fm), the same as that in Fig. 2 (solid curve).

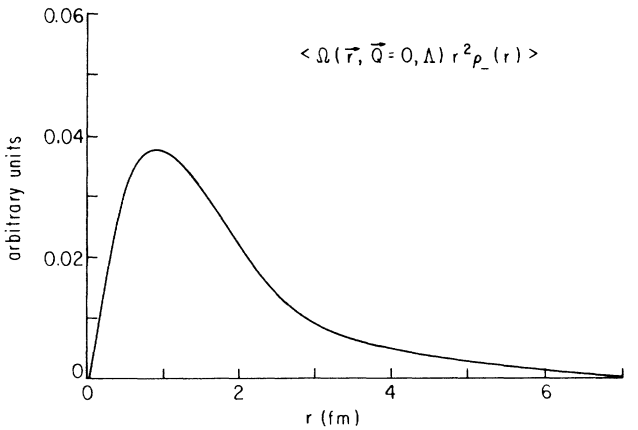

FIG. 7. The function $|\Omega(\mathbf{r},\mathbf{Q}=0;\Lambda)| r^2 \rho_-(r)$ averaged over orientations of $\mathbf{r}$ calculated with $\Lambda = 3.5$ fm^{-1}, where $\rho_-(r)$ is the probability density distribution of the relative coordinate of the valence nucleons.

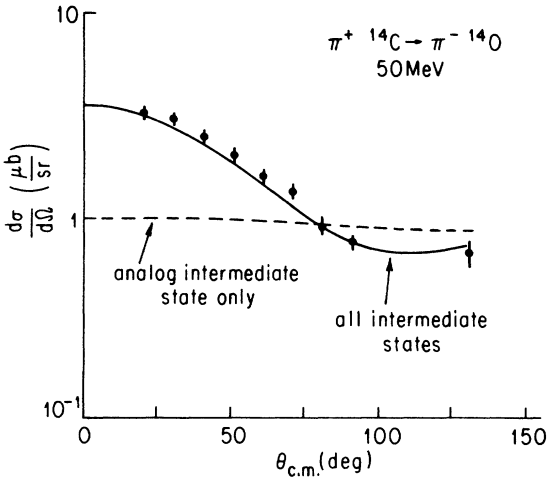

FIG. 8. Comparison of angular distributions for $\pi^+ {}^{14}C(0^+) \rightarrow \pi^- {}^{14}O(0^+)$ at 50 MeV. The dashed curve results from the restriction that the analog state be the only intermediate state, while the solid curve results from including all accessible intermediate nuclear states.

with and without them. Inclusion of the spin-flip terms leads to a slight flattening of the differential cross section and a decrease of its value at small angles of about 40%.

The sensitivity of the differential cross section to the parameter Λ is illustrated in Fig. 5. Alongside the angular distribution for $\Lambda = 3.5$ fm^{-1} we have plotted the one for $\Lambda = \infty$, which corresponds to assuming pointlike πN interactions. The latter assumption raises the cross section slightly and gives somewhat heavier emphasis to the ${}^1S_0$ state contribution than that of the ${}^3P_0$ state.

IV. INTERPRETATION OF ANGULAR DISTRIBUTIONS

What are the central elements of the double-charge-exchange process? We hope by simplifying and rephasing several features of the foregoing calculations to make them a good deal clearer. We can begin by neglecting altogether the dynamical effects of spin dependence and the effects of nuclear absorbtion and scattering. Although these effects make perceptible quantitative differences, they do not seem to influence greatly the nature of double charge-exchange.

An important feature of our results, on the other hand, is that the ${}^1S_0$ and ${}^3P_0$ states lead to quite different angular distributions for the double-charge-exchange process. If they do that, it cannot be because of their single particle density functions, which we have already noted are both the same. That fact alone is enough to suggest that the difference must arise from the different correlation properties implicit in the two states. The constraints on the angular momenta in their wave functions and the requirements of overall antisymmetry do indeed impose certain long-range correlations on the nucleon coordinates. The ${}^3P_0$ wave function of Eq. (2.5) for example is spatially antisymmetric and vanishes for $\mathbf{r}_1 = \mathbf{r}_2$, or more generally whenever $\mathbf{r}_1$ and $\mathbf{r}_2$ are parallel or antiparallel. It is largest in modulus for $\mathbf{r}_1$ and $\mathbf{r}_2$ perpendicular. The ${}^1S_0$ wave function of Eq. (2.4), on the other hand, is largest in modulus for $\mathbf{r}_1$ and $\mathbf{r}_2$ parallel or antiparallel, and vanishes when they are perpendicular. It does not inhibit the close approach, $\mathbf{r}_1 \approx \mathbf{r}_2$, of the two nucleons at all.

The role of the pion Green's function in the calculation is also a significant one. Its purpose is to describe the propagation of the neutral pion present in the intermediate state from the site of the first collision to the site of the second. Its amplitude for going that distance is greatest, of course, when the distance is small. The free particle Green's function $G(r)$ is proportional to $1/r$. The effective Green's function $\tilde{G}(r)$ given by Eq. (3.8) tempers the singularity at small distances $\sim 1/r$, but still provides that the amplitude for double charge exchange is largest when the nucleons approach each other most closely.

In view of this sensitive dependence on $r = |\mathbf{r}_1 - \mathbf{r}_2|$, it is useful to express the wave functions and the integrals in the scattering amplitude in terms of the relative coordinate vector $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and the center of mass, or centroid coordinate vector $\mathbf{R} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)$ for the two nucleons. The double scattering amplitude (2.12) when expressed in this way can be written as

$$\tilde{F}(\mathbf{Q},\mathbf{r}_1,\mathbf{r}_2) = \tilde{F}(\mathbf{Q},\mathbf{R}+\tfrac{1}{2}\mathbf{r},\mathbf{R}-\tfrac{1}{2}\mathbf{r}) = -8\pi e^{i\mathbf{Q}\cdot\mathbf{R}}\Omega(\mathbf{r},\mathbf{Q},\Lambda) , \tag{4.1}$$

in which the function Ω is given by

$$\Omega(\mathbf{r},\mathbf{Q};\Lambda) = e^{il\cdot\mathbf{r}}[a_1^2 + 2a_1 b_1 il\cdot\nabla_r - b_1^2(\mathbf{k}_f\cdot\nabla_r)(\mathbf{k}_i\cdot\nabla_r)]\tilde{G}(r) . \tag{4.2}$$

These expressions permit us to write the double-charge-exchange amplitudes for the ${}^1S_0$ and ${}^3P_0$ states as the expectation values

$$F^S_{\mathrm{DCX}}(\mathbf{Q}) = -8\pi\langle e^{i\mathbf{Q}\cdot\mathbf{R}}\Omega(\mathbf{r},\mathbf{Q};\Lambda)\rangle^S , \tag{4.3}$$

and

$$F^P_{\mathrm{DCX}}(\mathbf{Q}) = -8\pi\langle e^{i\mathbf{Q}\cdot\mathbf{R}}\Omega(\mathbf{r},\mathbf{Q};\Lambda)\rangle^P , \tag{4.4}$$

respectively. These expressions show that the coordinates $\mathbf{R}$ and $\mathbf{r}$ play grossly different roles in the calculation. The matrix elements are simply Fourier transforms of the distribution of the centroid coordinate $\mathbf{R}$, but they are integrals over the relative coordinate $\mathbf{r}$ weighted with the function Ω.

If only S-wave scattering were present in the πN interaction ($b_1 = 0$) the function Ω would assume the simple form

$$\Omega(\mathbf{r},\mathbf{Q},\Lambda) = a_1^2\tilde{G}(r) . \tag{4.5}$$

A graph of this function for the value $\Lambda = 3.5$ fm^{-1} is shown in Fig. 6, where it may be compared with the free particle Green's function $G(r)$. For this case, at least, it is clear that the integration over the relative coordinate $\mathbf{r}$ is simply weighted with a function that heavily favors small r and is independent of the momentum transfer $\mathbf{Q}$. When the p waves are taken into account in the πN interaction the function Ω assumes the more general form given by Eq. (4.2), but its dependence on the coordinate $\mathbf{r}$ is not changed in nature. The function does begin to depend, al-

though rather weakly, on the momentum transfer $\mathbf{Q}$, but it still gives heaviest emphasis to small nucleon separations r.

Since the dependence of $\Omega(\mathbf{r},\mathbf{Q},\Lambda)$ on the direction of $\mathbf{Q}$ is not too important, it is useful to simplify the expression by averaging over the directions of $\mathbf{Q}$, which is equivalent, according to Eqs. (5.1) and (5.2), to averaging over the orientations of $\mathbf{r}$. Some measure of the weighting of the internucleon distances r for which the double-charge-exchange process takes place can then be gained by plotting this averaged function Ω multiplied by r^2 times the distribution of distances r implicit in the shell-model wave function we have used. This plot is shown in Fig. 7. It indicates that most of the double-charge-exchange processes occur for distances smaller than 2 fm and that the most probable distance for their occurrence is about 1.2 fm.

We should also note that our findings concerning the sensitivity of the double-charge-exchange reaction to the properties of the two-nucleon wave function appear to be consistent with the results obtained recently by Gibbs, Kaufmann, and Siegel[13] who work within an advanced numerical framework allowing them to evaluate the Watson multiple scattering series in the fixed scatterer approximation (the calculations of Ref. 13 are based on the Monte Carlo integration over the 42-dimensional space of target nucleons).

The fact that the dependence of Ω on $\mathbf{Q}$ is not very strong shows that the angular distribution of the emerging pions is determined mainly by the average value of $e^{i\mathbf{Q}\cdot\mathbf{R}}$. It is governed, in other words, by the form factor for the distribution of the centroid coordinates $\mathbf{R}$. That coordinate is distributed in practice over the entire volume of the nucleus, and so the angular distribution is inevitably somewhat forward peaked.

Some further insight into these effects may be gained by examining more closely the two-particle densities associated with the $^1S_0$ and $^3P_0$ states. When expressed in terms of the coordinates $\mathbf{R}$ and $\mathbf{r}$ these are

$$\rho_S(\mathbf{R},\mathbf{r})=N_S^2[R^4-r^2R^2/2+r^4/16]e^{-(2R^2+r^2/2)/b^2} \tag{4.6}$$

and

$$\rho_P(\mathbf{R},\mathbf{r})=N_P^2[R^2r^2-(\mathbf{r}\cdot\mathbf{R})^2]e^{-(2R^2+r^2/2)/b^2}, \tag{4.7}$$

respectively. The P-state density of course vanishes quadratically as $r\to 0$, an expression, as we have noted, of the exclusion principle. The P-state contribution to double charge exchange is therefore suppressed by the requirement that the nucleons be close together. Some double charge exchange does take place in the P state, however, and its angular distribution is given by the form factor of a density proportional to $R^2\exp(-2R^2/b^2)$.

The S-state density, on the other hand, contains a term proportional to R^4 and it remains finite for $r\to 0$. The contribution to that term is greatly enhanced by the weighting coming from the propagator $\bar{G}$ or the function Ω, and it therefore dominates the double-charge-exchange amplitudes. Its contribution to the scattering amplitude is a term proportional to the form factor of $R^4\exp(-2R^2/b^2)$. This integral, because it contains the factor R^4 rather than R^2, contains contributions from larger radii than the integral for the P state. It therefore causes the angular distribution for the $^1S_0$ state to be much more sharply peaked near the forward direction than the corresponding angular distribution for the $^3P_0$ state.

Our calculation, as we have emphasized, has summed from the beginning over the contributions of all accessible intermediate nuclear states. But the analog-state transition we have discussed is essentially elastic in nature and so it is only natural to ask what is the special role, if any, of the analog state itself as an intermediate state. The nuclear wave functions enter the process through a two-particle form factor that for a wave function Ψ_0 takes the form

$$S(\mathbf{q}_1,\mathbf{q}_2)=\int \Psi_0^*(\mathbf{r}_1\mathbf{r}_2)e^{i\mathbf{q}_2\cdot\mathbf{r}_2}e^{i\mathbf{q}_1\cdot\mathbf{r}_1}\Psi_0(\mathbf{r}_1\mathbf{r}_2)d\mathbf{r}_1d\mathbf{r}_2 . \tag{4.8}$$

This integral can be expressed explicitly as a sum over a complete set of intermediate state wave functions Ψ_i by writing it as

$$S(\mathbf{q}_1,\mathbf{q}_2)=\sum_i \int \Psi_0^*(\mathbf{r}_1\mathbf{r}_2)e^{i\mathbf{q}_2\cdot\mathbf{r}_2}\Psi_i(\mathbf{r}_1\mathbf{r}_2)d\mathbf{r}_1d\mathbf{r}_2 \times \int \Psi_i^*(\mathbf{r}_1'\mathbf{r}_2')e^{i\mathbf{q}_i\cdot\mathbf{r}_1}\Psi_0(\mathbf{r}_1'\mathbf{r}_2')d\mathbf{r}_1'd\mathbf{r}_2' \tag{4.9}$$

and that is indeed a form which we would have encountered had we begun by being more explicit about taking all intermediate states into account. The particular question raised about the role of the intermediate analog state amounts to the comparison of the form factor (4.8) with the single term

$$S_0(\mathbf{q}_1,\mathbf{q}_2)=\int |\Psi_0^*(\mathbf{r}_1\mathbf{r}_2)|^2e^{i\mathbf{q}_2\cdot\mathbf{r}_2}dr_1dr_2 \times \int |\Psi_0(\mathbf{r}_1'\mathbf{r}_2')|^2e^{i\mathbf{q}_1\cdot\mathbf{r}_1'}dr_1'dr_2' \tag{4.10}$$

of the summation (4.9). We should note carefully therefore that the form factor (4.8) is a form factor for the two particle density $\rho(\mathbf{r}_1\mathbf{r}_2)$,

$$S(\mathbf{q}_1,\mathbf{q}_2)=\int e^{i\mathbf{q}_1\cdot\mathbf{r}_1+i\mathbf{q}_2\cdot\mathbf{r}_2}\rho(\mathbf{r}_1\mathbf{r}_2)dr_1dr_2 , \tag{4.11}$$

while the single term S_0 is a product of form factors for a single particle distribution

$$S_0(\mathbf{q}_1,\mathbf{q}_2)=\int e^{i\mathbf{q}_1\cdot\mathbf{r}_1}\rho(\mathbf{r})d\mathbf{r}\int e^{i\mathbf{q}\cdot\mathbf{r}'}\rho(\mathbf{r}')d\mathbf{r}' . \tag{4.12}$$

If the wave function Ψ_0 were a product wave function, that is if the particle coordinates $\mathbf{r}_1$ and $\mathbf{r}_2$ were entirely uncorrelated, its two expressions S and S_0 would be the same. But as soon as the coordinates $\mathbf{r}_1$ and $\mathbf{r}_2$ become correlated, that is $\rho(\mathbf{r}_1,\mathbf{r}_2)\neq\rho(\mathbf{r}_1)\rho(\mathbf{r}_2)$, they may become quite different. The analog states we have been describing in fact contain significant correlations, and we can see that restricting the summation to the analogue intermediate state, as is implicitly done in an optical model analysis, amounts to ignoring them altogether. Indeed, when we project the intermediate nuclear state in our calculations onto the analog state as in Eq. (4.10), the angular distribution that results is shown in Fig. 8. It corre-

sponds to omitting the correlation that tends to bring the nucleons together and therefore lowers the contribution of the dominant term contributed by the S state. It annihilates the strong forward peak in the angular distribution.

V. EFFECTS OF PION SCATTERING AND ABSORBTION

The analyses of the single- and double-charge-exchange reactions we have presented in the foregoing sections have been based on the assumption that the initial and final pion states are plane waves. The pion wave function may in fact deviate greatly from a plane wave within a nucleus, because of the effects of absorbtion and elastic scattering. It is important to estimate the changes these effects introduce, and we have carried out a number of calculations to that end. A fair estimate of the effects of pion scattering and absorbtion on the single-charge amplitude can be obtained by replacing the incident and final pion plane wave functions in Eq. (2.1) by the solutions of the Klein-Gordon equation within the nucleus. We take these to be the solutions obeying the appropriate outgoing and incoming wave conditions for an optical model interaction suited to the description of elastic scattering of pions. This procedure leads to the following modification of Eq. (2.6):

$$F_{\mathrm{SCX}}(\mathbf{Q})=2f(\mathbf{k}_f,\mathbf{k}_i)\tilde{S}(Q)\ , \tag{5.1}$$

where

$$\tilde{S}(Q)=\int d^3r\, e^{i\mathbf{Q}\cdot\mathbf{r}}\phi^{(-)*}(\mathbf{r},\mathbf{k}_f)\rho(r)\phi^{(+)}(\mathbf{r},\mathbf{k}_i) \tag{5.2}$$

is the effective form factor modified by the pion absorbtion and scattering corrections. The function $\rho(r)$ in Eq. (5.2), we recall, represents the single-particle density of the valence nucleons, and the functions $\phi^{(+)}(\mathbf{r},\mathbf{k}_i)$ and $\phi^{(-)}(\mathbf{r},\mathbf{k}_f)$ represent the incident and scattered pion waves, respectively.

We carried out a number of computations of the functions $\phi^{(+)}(\mathbf{r},\mathbf{k}_i)$ and $\phi^{(-)}(\mathbf{r},\mathbf{k}_f)$ for the pion wave equations containing optical potentials of the form

$$V(r)=v_0f(r)+v_1f(r)^2+iv_2\exp(-r^2/\gamma_2^2)$$

with

$$f(r)=(1+\delta_0r^2+\delta_1r^4)\exp(-r^2/\gamma_0^2)\ ,$$

in which the various coefficients are treated as adjustable parameters. Fitting the elastic π-nucleus scattering cross section narrows the choice of these parameters considerably, but does not determine them uniquely. One fit to the elastic scattering data of π^+ by ^{14}C at 50 MeV, for example, correspond to the choices $v_0=0.01$, $v_1=0.28$, $\delta_0=0.006$, $\delta_1=0.25$, $\gamma_0=1.85$, $v_2=-0.015$, and $\gamma_2=1.44$ (all values in fermi units). The differential cross section for the single charge exchange obtained with these parameters in the one shown in Fig. 1. It evidently differs from the impulse approximation far less than the gross departures of the functions $\phi^{(+)}(\mathbf{r},\mathbf{k}_i)$ and $\phi^{(-)}(\mathbf{r},\mathbf{k}_f)$ from the plane wave behavior might indicate. We have computed several other solutions for the parameters in $V(r)$ and found them to lead to ratios $\tilde{S}(Q)/S(Q)$ ranging between 0.9 and 1.15 in modulus, and thereby, to lead to

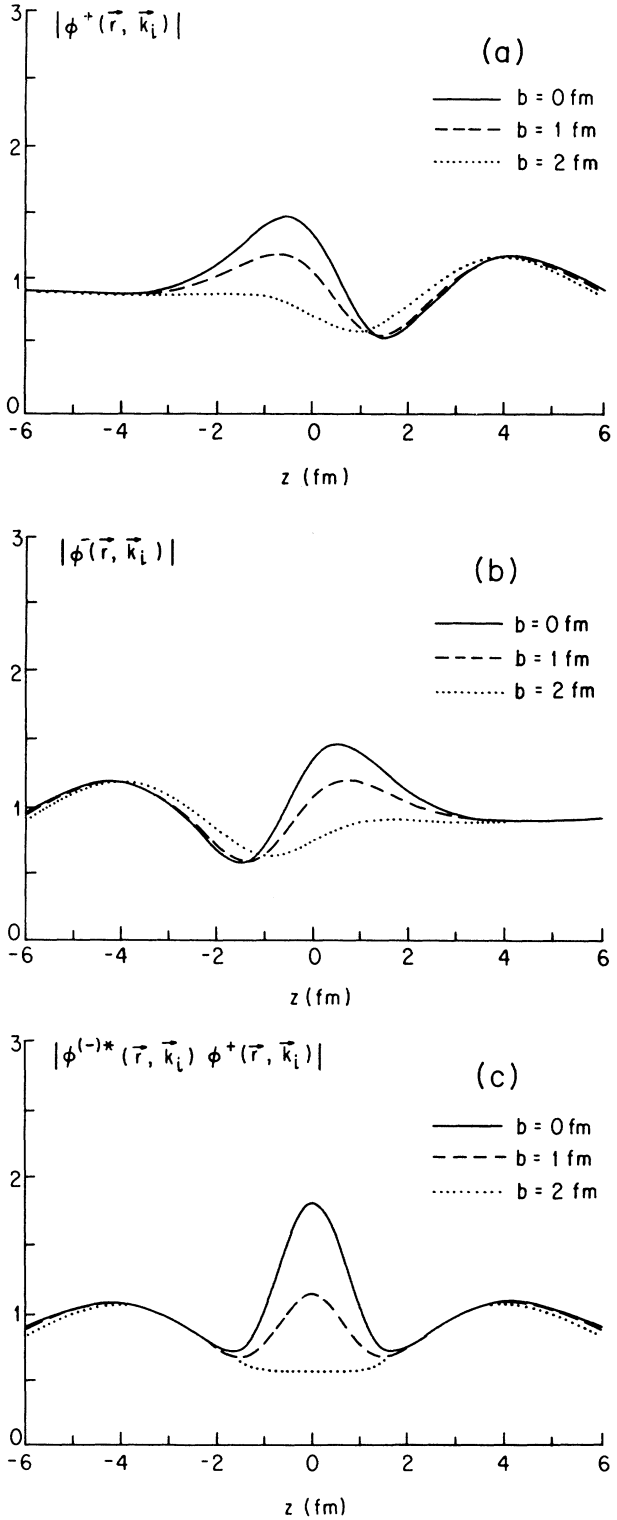

FIG. 9. (a) $|\phi^{(+)}(\mathbf{r},\mathbf{k}_i)|$, (b) $\phi^{(-)}(\mathbf{r},\mathbf{k}_i)|$, and (c) $|\phi^{(-)*}(\mathbf{r},\mathbf{k}_i)\phi^{(+)}(\mathbf{r},\mathbf{k}_i)|$ for three impact parameters b as functions of z (component of $\mathbf{r}$ along the normal to the impact parameter plane). The pion incoming $\phi^{(+)}(\mathbf{r},\mathbf{k}_i)$ and the outgoing $\phi^{(-)}(\mathbf{r},\mathbf{k}_i)$ wave functions result from the Klein-Gordon equation which describes the elastic scattering of positive pions from ^{14}C at 50 MeV.

modifications of about 10–30% in the single-charge-exchange cross section.

To estimate the corresponding corrections for the double-charge-exchange cross section, we have applied analogous correction factors to each of the charge-exchange amplitude F_1 and F_2 in the integrand of Eq. (3.2). This procedure amounts to inserting the factor $R(q_1,q_2)=\tilde{S}(q_1)\tilde{S}(q_2)/S(q_1)S(q_2)$, with the function $\tilde{S}$ defined by Eq. (4.1). This factor $R(q_1,q_2)$ into the integrand and carrying out the integration numerically. It implicitly neglects the off-shell dependence of the correction factors and so may only represent a crude approximation. For the double-charge-exchange reaction, however, much as for the single-charge-exchange reaction, our procedure indicates that the overall corrections to the plane wave analysis are fairly small. A calculation, for example, based on the parameters for $V(r)$ noted earlier leads to the double-charge-exchange angular distribution shown in Fig. 2. For our other solutions the deviations from the plane wave result amounts to only about 10%. If the absorbtion and scattering corrections are small, we should emphasize, it is not because the pion wave functions correspond closely to plane waves. That can be seen for example, in the graph of the modulus of $\phi^{(+)}(\mathbf{r},\mathbf{k}_i)$, shown in Fig. 9(a), as a function of position in the nucleus for three values of impact parameter b. The wave function has a modulus considerably larger than unity within the forward hemisphere of the nucleus. Its modulus then drops to values well below the unity in the shadowed rear hemisphere of the nucleus. The wave function $\phi^{(-)}(\mathbf{r},\mathbf{k}_i)$, since it describes the escape of pions from the nucleus rather than their entry, exhibits the complementary variation shown in Fig. 9(b). In the product of $\phi^{(+)}(\mathbf{r},\mathbf{k}_i)$ and $\phi^{(-)*}(\mathbf{r},\mathbf{k}_i)$, which occurs in the expressions for the forward charge-exchange amplitudes, the variations of the two wave functions compensate each other to a considerable degree. The modulus of the product of $\phi^{(+)}(\mathbf{r},\mathbf{k}_i)$ and $\phi^{(-)*}(\mathbf{r},\mathbf{k}_i)$, presented in Fig. 9(c) is seen to differ appreciably from the unit value, one would have for plane waves only in a central region of the nucleus of radius of about 1 fm. But, this wave function product only occurs in the single-charge-exchange reaction amplitude of Eq. (2.1) multiplied by the single particle density of the valence nucleons given by Eq. (2.8). The latter density, which is characteristic of the p-shell vanishes at the nuclear center, $r=0$, and thus give zero weight to the larger deviations from plane wave behavior indicated in Fig. 9(c). The net effect then of all these compensations is a result for the single-charge-exchange amplitude not to be different from that of the plane wave impulse approximation. The double-charge-exchange amplitude, involving only products of two such amplitudes also seems to undergo only the smallest corrections.

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[1]I. Navon, M. J. Leitch, D. A. Bryman, T. Numano, P. Schlatter, G. Azuelos, R. Poutissou, R. A. Burnham, M. Hasinoff, J. M. Poutissou, J. A. McDonald, J. E. Spuller, C. K. Hargrove, H. Mes, M. Blecher, K. Gotow, M. Moinester, and H. W. Baer, Phys. Rev. Lett. **52**, 105 (1984).

[2]M. J. Leitch, E. Piasetzky, H. W. Baer, J. D. Bowman, R. L. Burman, B. J. Dropesky, P. A. M. Gram, F. Irom, D. Roberts, G. A. Rebka, J. N. Knudson, J. R. Comfort, V. A. Pinnack, D. H. Wright, and S. A. Wood, Phys. Rev. Lett. **54**, 1482 (1985).

[3]G. A. Miller, Phys. Rev. Lett. **53**, 2008 (1984).

[4]J. L. Ullmann, P. W. F. Alons, J. J. Krashauser, J. H. Mitchell, R. J. Peterson, R. A. Rinstinen, J. N. Knudson, J. R. Comfort, H. W. Baer, J. D. Bowman, D. H. Fitzgerald, F. Irom, M. J. Leitch, and E. Piasetzky, Phys. Rev. C **33**, 2092 (1986).

[5]R. Arndt, computer program (unpublished).

[6]G. Brown, B. Jennings, and V. Rostokin, Phys. Rep. **50C**, 229 (1979).

[7]G. Fäldt, Nucl. Phys. **B10**, 597 (1967).

[8]W. Gibbs and R. Siegel, Phys. Rev. C **33**, 1407 (1986).

[9]J. Eisenberg and D. Koltun, *Theory of Meson Interactions with Nuclei* (Wiley, New York, 1980).

[10]M. Abramowitz and L. A. Stegun, *Handbook of Mathematical Functions* (National Bureau of Standards, Washington, D.C., 1970).

[11]S. Cohen and D. Kurath, Nucl. Phys. **73**, 1 (1965).

[12]H. J. Rose, O. Häusser, and E. K. Warburton, Rev. Mod. Phys. **40**, 591 (1968).

[13]W. Gibbs, R. Kaufmann, and R. Siegel, in Proceedings of the Los Alamos Workshop on Double Charge Exchange, 1985 (unpublished).