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Coriolis interaction and variable reflection asymmetry in fission vibrational resonances

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A new analysis of the 720 keV gross resonance in the neutron-induced fission cross section of ^{230}Th has been made. While confirming some of the main conclusions of earlier analyses (such as presence of a double-parity rotational band, suggesting an underlying vibrational mode in a tertiary well of the fission barrier), the new analysis explores additional features of the vibrational model, as well as employing new fission product angular distribution data and treating already available data in a different way. The new model and least squares analysis indicate strong evidence for Coriolis coupling of the $K=\frac{1}{2}$ single-particle state and its coupled rotational motion with a $K=\frac{3}{2}$ single-particle state and rotational band, probably lying at 930 keV neutron energy. There is also strong evidence that the opposite parity band in the 720 keV gross resonance, which is brought down by the reflection asymmetry in the tertiary potential energy well, is attenuated by a factor of about 3–5 by the fact that reflection asymmetry is not a property expected of the potential energy surface at the inner barrier containing the well.

I. INTRODUCTION

The occurrence of gross resonances in the fission cross sections of many of the lower-charge actinide nuclides is a striking phenomenon.[1] These resonances, which are much larger in their energy scale than the dense fine-structure resonances found in slow neutron fission cross sections, are generally attributed to the complexity of the potential energy of the nucleus as a function of its deformation. Originally, it was supposed[1] that they were due to simple vibrational modes quasibound in the secondary well of a double-humped fission barrier as theoretically calculated[2] for a broad range of actinide nuclei. The developing theory suggested, however, that even for thorium and neighboring nuclides the secondary well would remain so deep[3,4] that such simple vibrational modes would be damped and hence spread over denser and more complex states associated with the secondary well deformation. However, refined calculations[4] of the potential energy surface indicated that for these particular nuclides a shallow tertiary well could exist in the fission barrier and this could be responsible for a virtually undamped vibrational mode causing the gross resonance phenomenon.

The information that they can thus possibly give on the behavior of nuclear potential energy as a function of extended deformation has led to a great deal of analysis of these vibrational resonances (as the gross fission resonances are commonly called), but the states responsible for such resonances are not merely vibrational in character, in the sense of oscillating in the direction of the principal path towards fission (elongation), but have other intrinsic and collective properties associated with them. One of the collective properties is rotation of the elongated nucleus, and identification of the members of the rotational band built on the vibrational state can yield the moment of inertia, giving a measure of the mean elongation of the nucleus in its quasistationary state at some significant point of the fission path. The effective moment of inertia can, in turn, be modified by the intrinsic particle properties of the responsible state. For neutron-induced fission of an even target nucleus such as ^{230}Th the intrinsic state could be a single-particle neutron Nilsson orbital; the properties of this are clearly strongly influenced by the mean state of deformation, especially in its energy, both absolute and relative to other single-particle states coupled to the same vibrational mode. Another collective property that can be of great significance is reflection asymmetric vibration; calculations of the potential energy surface indicate that a tertiary well in the fission barrier would, in fact, be split in the reflection asymmetry direction,[4,5] giving rise to nearly degenerate states of opposite reflection quantum number and, hence, parity.

Thus, a vibrational resonance in a fission cross section may be due to a quite complex superposition of states, some of which may appear as separate peaks if conditions are right. Otherwise, the determination of the properties of the complex of states will depend on measurement of other properties, such as the angular distribution of fission products, which is controlled by the angular momentum of the members of the rotational bands. This was the basis for the first attempts [for ^{230}Th(n,f)] to elucidate the complexity of structure[1,6] the assumption being in this original work that the vibrational mode was associated with a secondary well, with no reflection asymmetric near-degeneracy, and was coupled to one single-particle state with a superposed rotational band. Later, very high energy resolution measurements revealed some substructure in the gross resonance of the ^{230}Th cross section.[7] This was later analyzed[11,12] in conjunction with new angular distribution data,[8–10] on the assumption that two rotational bands existed, built on near-degenerate states of opposite parity due to reflection asymmetry of the tertiary well.

While these later analyses have on the whole been very successful and appear to confirm the general validity of the tertiary well concept, some unsatisfactory features remain. These generally surround the angular distribution data, which are generally less satisfactorily fitted by the models than are the cross-section data. Partly, this has been due to the doubtful quality of much of the data, discrepancies between data sets from various laboratories having occurred, and also inconsistencies within sets, but partly also because the analyses have not allowed sufficiently for deviations from an ideal model, in which the collective motion towards fission is considered to be completely decoupled from intrinsic nucleonic motion and other collective degrees of freedom. Models in which single-particle nucleon motion is affected by the collective motion[13,14] suggest considerable modification of the fission strengths of the vibrational resonances and some mixing of the wave functions of the intrinsic wave functions associated with them, thus affecting in marked degree the channel structure of their fission widths.

In the present paper we consider two variants on previous model fitting, such as would arise from modification of intrinsic and other degrees of freedom by their coupling to the vibrational mode. The first of these is the occurrence of Coriolis coupling between single-particle states with different quantum number, K, for projection of angular momentum along the axis of elongation; this will have a modifying effect on the angular distribution of fission products. The second variant is an effect on the relative values of coupling and fission widths of the opposite parity components of the gross resonance. This is expected to arise because the potential energy hill between the twin reflection asymmetric routes to fission is calculated to be much lower at the barrier deformation than at the tertiary well, and hence significant near degeneracy of the opposite parity states at fixed barrier elongation will not occur.

II. BASIC EXPRESSIONS AND DATA-FITTING PROCEDURES

The fission cross section of an even target nucleus can be written in the Hauser-Feshbach[15] form

$$\sigma_{\rm nf}(E)=\sum_{J,\pi}\sigma_{\rm nf}(E,J^{\pi}) = \pi\lambda\!\!\!^{-2}\sum_{J,\pi} g_J T_{{\rm n}(J^{\pi})}T_{{\rm f}(J^{\pi})}/T_{(J^{\pi})}\;, \tag{1}$$

where $\lambda\!\!\!^{-}$ is the de Broglie wavelength of the neutron (divided by 2π), g_J is the statistical spin weight for the unpolarized interacting system having total angular momentum J, $T_{{\rm n}(J^{\pi})}$ is the transmission coefficient for the entrance neutron channel with orbital angular momentum l and hence parity $\pi=(-1)^l$, $T_{{\rm f}(J^{\pi})}$ is the fission transmission coefficient for the compound nucleus in total angular momentum and parity state J,π, and $T_{(J^{\pi})}$ is the total transmission coefficient of this state. This last quantity is the sum of the neutron and radiative transmission coefficients for all open channels (this sum being denoted by $T_{p(J^{\pi})}$) and the fission transmission coefficient.

When an intermediate "vibrational" state is controlling the transmission through the fission barrier, the transmission coefficient has the intermediate resonance form

$$T_{{\rm f}(J^{\pi})}=\frac{\Gamma_{v(c,J^{\pi})}\Gamma_{v({\rm f},J^{\pi})}}{(E-E_{v(J^{\pi})})^2+\frac{1}{4}\Gamma^2_{v(J^{\pi})}}\;, \tag{2}$$

where $E_{v(J^{\pi})}$ is the resonance energy of a vibrational state with total angular momentum and parity J^{π}, and $\Gamma_{v(c,J^{\pi})},\Gamma_{v({\rm f},J^{\pi})}$ are the coupling and fission widths, respectively; these sum to give the total vibrational resonance width, Γ_v. The coupling width governs the strength of coupling of the vibrational mode to the fine structure states of the compound nucleus, these being the states excited in the first stage by absorption of the projectile neutron. The coupling and fission width are deemed to be governed in strength in the main by penetration, respectively, through the inner (A) and outer (B) potential barriers containing the well responsible for the vibrational mode; they are usually assumed to have the Hill-Wheeler form[16] of dependence on excitation energy,

$$\frac{2\pi\Gamma_{v(c,J^{\pi})}}{D_{v(J^{\pi})}}\approx\{1+\exp[2\pi(\mathcal{V}_A-E)/\hbar\omega_A]\}^{-1}\;, \tag{3a}$$

$$\frac{2\pi\Gamma_{v({\rm f},J^{\pi})}}{D_{v(J^{\pi})}}\approx\{1+\exp[2\pi(\mathcal{V}_B-E)/\hbar\omega_B]\}^{-1}\;, \tag{3b}$$

where $D_{v(J^{\pi})}$ is the spacing of vibrational modes, $\mathcal{V}_A,\mathcal{V}_B$ are the potential heights of the inner and outer barriers, and $\hbar\omega_A,\hbar\omega_B$ parametrize the barrier penetrability characteristics. At the simplest level it might be assumed that D_v, $\Gamma_{v(c)}$, and $\Gamma_{v({\rm f})}$ are independent of angular momentum and parity, for a given vibrational mode, but at a more detailed level it is clear that $\mathcal{V}_A,\mathcal{V}_B,\hbar\omega_A,\hbar\omega_B$ in Eq. (3), and hence the vibrational coupling and fission widths, can depend on J^{π} because of the rotational energy and the mass reflection vibrational energy, and their variation with changing deformation through the barrier peaks.

The fission cross section of Eq. (1) will reflect the Lorentzian form of the fission transmission coefficient of Eq. (2), but the width of each angular momentum and parity component will be greater than Γ_v ($\Gamma\equiv\Gamma_{v(c)}+\Gamma_{v({\rm f})}$); it is, in fact,

$$G_{v(J^{\pi})}=(\Gamma_{v(J^{\pi})}+\Gamma_{v(c,J^{\pi})}\Gamma_{v({\rm f},J^{\pi})}/T_{p(J^{\pi})})^{1/2}\;. \tag{4}$$

Experimental resolution of a simple rectangular form of width $2R$ can now be taken into account very easily:

$$\sigma_{\rm nf,expt}(E,J^{\pi})\equiv\sigma_{\rm CN}(J^{\pi})\frac{\Gamma_{v(c,J^{\pi})}\Gamma_{v({\rm f},J^{\pi})}}{T_p}\frac{1}{2G_{v(J^{\pi})}R}\left|\arctan\left[\frac{E+R}{G_{v(J^{\pi})}}\right]+\arctan\left[\frac{E-R}{G_{v(J^{\pi})}}\right]\right|\;, \tag{5}$$

where

$$\sigma_{\rm CN}(J^\pi)=\pi\lambda\!\!\!^{-2} g_J T_{n(J^\pi)} \,. \tag{6}$$

By means of Eq. (5), a moderate degree of experimental resolution can thus be included analytically in the fitting procedures. The total resolution-broadened fission cross section is

$$\sigma_{\rm nf,expt}(E)=\sum_{J^\pi}\sigma_{\rm nf,expt}(E,J^\pi) \,. \tag{7}$$

Equations (5) and (6) can be further developed to describe the functional dependence of the cross section on angle of emission θ of the fission products with respect to the projectile beam axis. In the simplest case it is assumed that the fissioning nucleus is in a pure K quantum number state as it passes through the outer barrier. The angular distribution functions are related to the symmetric top wave functions $d^J_{MK}(\theta)$, so we have

$$\frac{\partial\sigma_{\rm nf,expt}(\theta)}{\partial\Omega}=\sum_{J,\pi,M}\sigma_{\rm nf,expt}(E,J^\pi)W_{JKM}(\theta) \,, \tag{8}$$

$$W_{JKM}(\theta)\propto |\, d^J_{MK}(\theta)\,|^2+|\, d^J_{M-K}\,|^2 \,, \tag{9a}$$

$$d^J_{MK}(\theta)=\sum_n \frac{(-)^n[(J+K)!(J-K)!(J+M)!(J-M)!]^{1/2}}{(J-M-n)!(J+K-n)!n!(n+M-K)!} \times\cos^{2J+K-M-2n}(\tfrac{1}{2}\theta)\sin^{2n+M-K}(\tfrac{1}{2}\theta) \,, \tag{9b}$$

where M is the projection of J on the projectile beam axis. For an even target nucleus the angular behavior of the function $\sum_M W_{JKM}(\theta)$ is illustrated graphically for low values of K and low to moderate values of J in Ref. 17.

The data that are available on the ^{230}Th neutron-induced fission reaction comprise the fission cross section (integrated over solid angle) measured with high energy resolution,[7] the relative angular variations of the fission cross section measured with somewhat coarser energy resolution, and two measurements of the cross section at specific angles (100° and 125°) measured with the same (nominally high) energy resolution.[10] Of the many measurements on relative angular variation of the fission product yields, we have chosen the latest very careful measurement[18] as the most definitive, and have used the $\sigma(20°)/\sigma(70°)$ ratios in the neutron energy range 690–750 keV for the fitting procedure. To these and the cross sections referenced above, we have used a straightforward least squares fitting procedure to attempt to deduce a range of parameters governing the vibrational mode properties of Eq. (2). The remaining parameters required for Eqs. (1)–(3) and hence Eqs. (7) and (8) are the entrance neutron channel transmission coefficients $T_{n(J^\pi)}$ and total particle and radiative transmission coefficients $T_{p(J^\pi)}$. The radiative transmission coefficient part of the latter is small and can be estimated with good accuracy from radiation width data and simple models.[13] The neutron transmission coefficients can be calculated by extrapolation from observed values of neutron strength functions, or from optical model calculations[19] with parameters fitted to a wide range of elastic and inelastic neutron scattering data at MeV energies. The two approaches yield similar values of $T_{p(J^\pi)}$ and $\sigma_{\rm CN}(J^\pi)$ within 20%. Over the range of energy considered in the fitting process, the chosen set of transmission coefficient parameters has been taken to be independent of energy.

The intermediate state responsible for the vibrational resonance in the cross section is assumed to have the character of a single-particle neutron state with K quantum number $\frac{1}{2}$ (this is established by the forward peaked coupled angular distribution of the fission products) and parity π_s to a rotational state to give total angular momentum J. The vibrational state energies of Eq. (2) are then related by the rotational energy displacements, giving

$$E_{v(J^{\pi_s})}=E_{v(K)}+\frac{\hbar^2}{2\mathcal{J}}\,|\,J(J+1)-K(K+1)+\delta_{K,1/2}a(-1)^{J+1/2}(J+\tfrac{1}{2})\,| \,, \tag{10}$$

where $\mathcal{J}$ is the effective moment of inertia of the rotational band and a is the decoupling parameter introduced by the Coriolis force for $K=\frac{1}{2}$ states (see, e.g., Ref. 20). If this intermediate state were based on a vibrational mode in a secondary well with no mass reflection asymmetry, this postulated band should be sufficient to give an adequate fit to the cross-section data in the absence of any perturbation effects from another single-particle state. In fact, as has been demonstrated by Blons *et al.*,[11] such a postulate is inadequate to fit the detailed data on ^{230}Th(n,f) now available.

The detailed theoretical calculations of the potential energy surface across the broad outer peak of the double-humped fission barrier of thorium nuclides indicate that the shallow tertiary well appearing in this peak is itself in the mass asymmetry degree of freedom, and a potential hill of several MeV lies between the two opposite metastable pear shapes, as illustrated in Fig. 1. The mode of mass symmetry vibration from one shape to its reflection

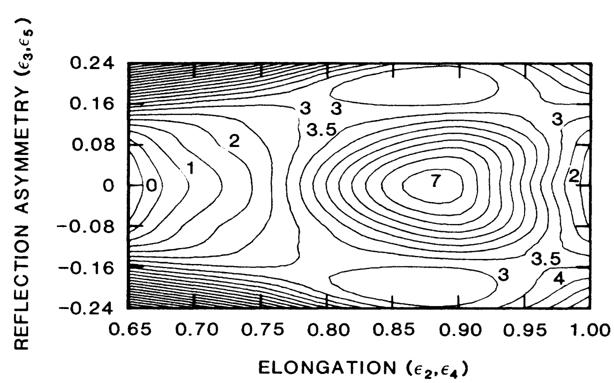

FIG. 1. The reflection asymmetric second fission barrier and tertiary wells as calculated by Åberg *et al.* (Ref. 5). The deformation parameters are related to the coefficients of the Legendre polynomial expansion of the stretched coordinates of the deformed harmonic oscillator system (Ref. 23), ϵ_2,ϵ_4 being principally a measure of elongation with some necking, and ϵ_3,ϵ_5 describing reflection asymmetry (pear shapes). The secondary well is at the extreme left of the diagram.

must then be considered in the wave function of the intermediate state. The zero-point vibration in this mode is assumed already to be coupled into the basic rotational band of Eq. (2), but the one-phonon state will be almost degenerate with the zero-phonon state, because of the high potential hill. This second state has opposite parity, $\pi_r = -\pi_s$, from that of the first, and the Coriolis decoupling parameter has opposite sign.[20,12] Hence, the second rotational band has energies

$$E_{v(J^{\pi_r})} = E_{v(K)} + \Delta E_v + \frac{\hbar^2}{2\mathcal{J}} \,|\, J(J+1) - K(K+1) - \delta_{K,1/2}\, a(-1)^{J+1/2}(J+\tfrac{1}{2}) \,| \ . \quad (11)$$

The fission and coupling widths of the two intermediate states are assumed to be the same in first approximation.

With these assumptions on the structure of the intermediate state, we can fit the data to determine the quantities $\Gamma_{v(c)}$, $\Gamma_{v(f)}$, $E_{v(K)}$, $\hbar^2/2\mathcal{J}$, a, and ΔE_v. In doing this we have to take account of possible systematic errors or discrepancies among the data. The chief of these are likely to be overall normalization errors between the total fission cross-section data of Blons *et al.*[7] and the angle-specific fission cross sections of Veeser and Muir.[10] Blons *et al.*[11] avoided this question by fitting the ratio of the Veeser and Muir data at the two angles, i.e., $\sigma_{nf}(125°)/\sigma_{nf}(100°)$. We have not done this because (a) it could lead to loss of useful information, and (b) it could introduce additional errors if the energy scales of the two angle measurements are not in exact agreement with each other or that of the total fission cross section. We have therefore introduced as additional free parameters normalization constants, C_{125} and C_{100} (to multiply into the calculated cross section for each angular measurement of Ref. 10), and two energy shifts, ϵ_{125} and ϵ_{100}, which are deducted from the energy E in calculating the respective angular cross sections. (The Veeser-Muir data have already been shifted down by 6 keV, as suggested by Blons *et al.*[11] in order to align them approximately with the energy scale of the data of Ref. 7.)

A typical fit based on a typical set of parameters recommended by Blons *et al.*[11] is shown in Fig. 2. The basic parameters $E_{v(K)}$, $\hbar^2/2\mathcal{J}$, and a have been held fixed at

$$E_{v(K)} = 714.9 \text{ keV}, \quad \hbar^2/2\mathcal{J} = 1.75 \text{ keV} ,$$

$$a = 0.3, \quad \Delta E_v = -9.48 \text{ keV} .$$

The fission and coupling widths were allowed to vary and their "best" values were found to be

$$\Gamma_{v(f)} = 10.9 \text{ keV}, \quad \Gamma_{v(c)} = 0.51 \text{ keV}$$

(or vice versa). The neutron transmission coefficients were the optical model values of Ref. 19. The fits are not nearly as good as those found by Blons *et al.*[11] We make four comments. First, we have used a strict Lorentzian formula, Eq. (2), for the fission transmission with only a modest amount of barrier penetration variation, Eq. (3) with $\hbar\omega_A = 1$ MeV. Blons *et al.* used a transmission coefficient calculated for a specific shape of barrier of smoothly joining parabolas parametrized by well-depth,

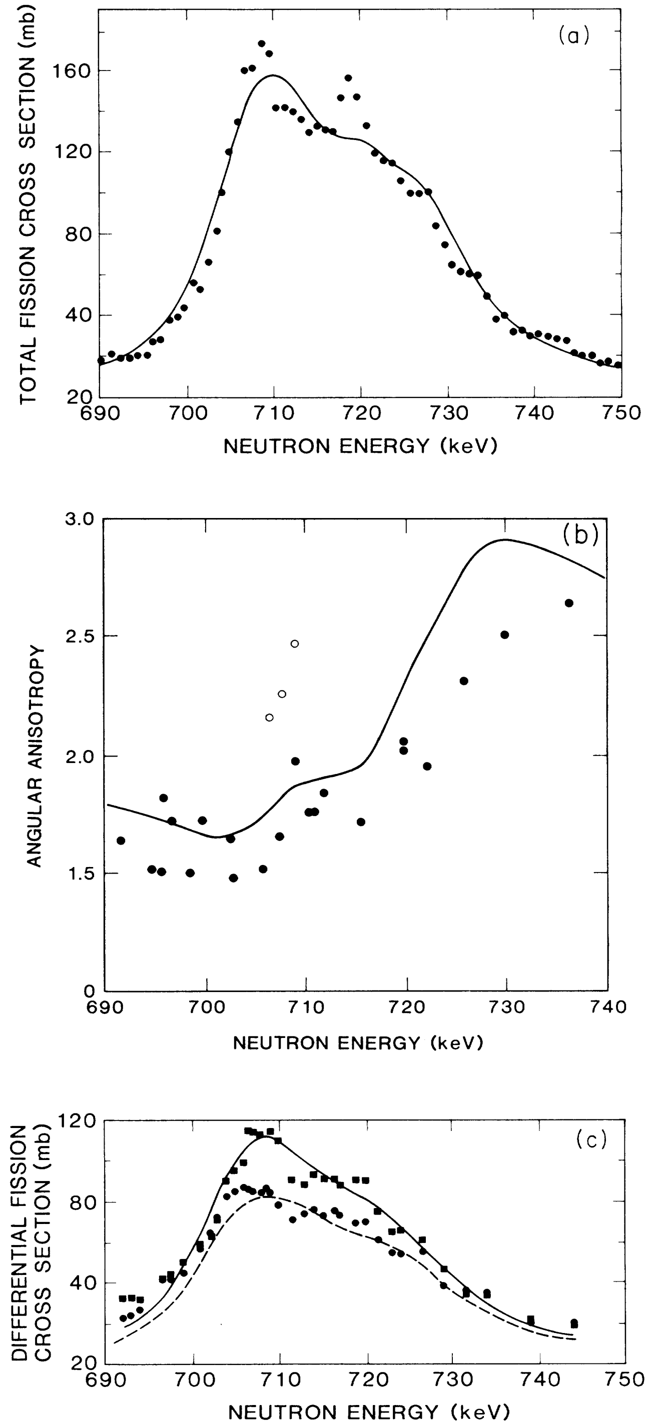

FIG. 2. Simple fit to ^{230}Th(n,f) data, based on the model of near-degenerate opposite parity rotational bands with rotational parameters postulated by Blons *et al.* (Ref. 11). Panel (a) is the fit to the cross-section data of Ref. 7. Panel (b) is the fit to the ratio of fission product angular differential cross sections at 20° and 70° from Ref. 17. Panel (c) shows the fits to fission product angular differential cross sections of Ref. 10 at 125° (solid squares and curve) and 100° (open circles and dashed curve). The normalization constants C_{125}, C_{100} were taken equal at a value of 0.715, as were the energy shifts $\epsilon_{125}, \epsilon_{100}$ at 1.6 keV. The open circles in (b) are data of doubtful validity.

phonon energy, barrier heights, and penetrability characteristics. One of the last, $\hbar\omega_A$, is determined as 0.2 MeV, which seems to be an extremely small value from a physical point of view. Such a small value will cause much greater variation of the barrier penetration factor with changing energy and effects considerable variation in the resonance widths of the rotational states. Second, Blons *et al.* rely considerably on the ratio of the 125° and 100° cross-section data of Muir and Veeser. Our comparison [Fig. 2(c)] is with the fission cross-section data themselves, and indicates a possible relative energy shift of the two sets, which would invalidate the ratio fit. Third, Blons *et al.* fit $\sigma_{\rm nf}(0°)/\sigma_{\rm nf}(90°)$ ratio data from the measurements of Refs. 8 and 9. Our comparison [Fig. 2(b)] is with the more recent $\sigma_{\rm nf}(20°)/\sigma_{\rm nf}(70°)$ data of James *et al.*,[18] which has a quite different behavior. Finally, Blons *et al.* allow modest differences ($\sim 20\%$) between the moments of inertia of the two rotational bands, a departure from the ideal model.

III. MODIFICATION OF THE INTERMEDIATE STATE BY CORIOLIS COUPLING

In general, the intermediate state responsible for a vibrational resonance effect in the fission cross section will be describable as pure transmission through a barrier containing a potential well. The variation of intrinsic state properties (single-particle, rotational, mass-asymmetry vibration) with elongation will give rise to mixing of both intrinsic state properties and phonons as the nucleus oscillates in shape in the fission path. The effect of such mixing on intrinsic states of specifically single-particle character was studied in Ref. 14, in which it was noted that both fission and coupling widths could depart significantly from their expected "pure transmission" values, and furthermore that Coriolis interaction could allow significant mixing into the fission width of channel components with different K quantum number from that of the principal component of the intermediate state.

In this paper, we treat this Coriolis effect as simple perturbation. The Coriolis coupling matrix element between two intermediate levels of the same total angular momentum J but axis projection K' differing by one unit is (see, e.g., Ref. 20)

$$\mathcal{V}_{\rm cor} = \langle J, K+1 \,|\, -(\hbar^2/2\mathcal{J})(2\mathbf{I}\cdot\mathbf{J}) \,|\, J,K \rangle = -\,|(J-K)(J+K+1)|^{1/2} A_K \;, \tag{12a}$$

where

$$A_K = (\hbar^2/2\mathcal{J}_u)\langle K+1 \,|\, j_1 \,|\, K\rangle \;, \tag{12b}$$

j_1 being the single-particle angular momentum operator perpendicular to the cylindrical symmetry axis of the system and $\mathcal{J}_u$ the unperturbed value of the moment of inertia (see Sec. V). If the state of different K' is far enough removed in energy from the intermediate state under study, the admixture of the intruder state in intensity terms is

$$I(J) \approx \frac{(J-K)(J+K+1)A_K^2}{(E_{v(K)} - E_{v'(K+1)})^2} = (J-K)(J+K+1)B_c \;, \tag{13a}$$

where

$$B_c = A_K^2/(E_{v(K)} - E_{v'(K+1)})^2 \;. \tag{13b}$$

The fission width of the intermediate state becomes therefore

$$\begin{aligned}\Gamma_{v({\rm f},J)} &\approx [1 - B_c(J-K)(J+K+1)]\Gamma_{v({\rm f},J,K)} + B_c(J-K)(J+K+1)\Gamma_{v'({\rm f},J,K+1)} \\ &= \Gamma_{v({\rm f},J,K)} - B_c[J(J+1) - K(K+1)](\Gamma_{v({\rm f},J,K)} - \Gamma_{v'({\rm f},J,K+1)}) \;;\end{aligned} \tag{14}$$

similarly,

$$\Gamma_{v(c,J)} \approx \Gamma_{v(c,J,K)} - B_c[J(J+1) - K(K+1)](\Gamma_{v(c,J,K)} - \Gamma_{v'(c,J,K+1)}) \;. \tag{15}$$

The cross section for specific direction of emission of fission products becomes

$$\begin{aligned}\frac{\partial\sigma_{\rm nf,expt}(\theta)}{\partial\Omega} = \sum_{J,\pi} \sigma_{\rm nf,expt}(E,J^\pi)\{&[1 - B_c(J-K)(J+K+1)]\mathcal{W}_{J,K}(\theta)\Gamma_{v({\rm f},J,K)}/\Gamma_{v({\rm f},J)} \\ &+ B_c(J-K)(J+K+1)\Gamma_{v'({\rm f},J,K+1)}\mathcal{W}_{J,K+1}(\theta)/\Gamma_{v({\rm f},J)}\} \;.\end{aligned} \tag{16}$$

A fit to the data that takes account of Coriolis coupling through the use of Eqs. (14)–(16) is shown in Fig. 3. This is based on the parameters derived from the starting point of Blons *et al.* as shown in Fig. 2. The fission cross-section curves are not shown in Fig. 3 because they are barely distinguishable from those shown in Fig. 2. If it is assumed that the widths of the "contaminating" $K+1$ rotational band are the same as those of the K band, a "best fit" value for $B_c = 0.015$ is found. It can be seen that this gives a better overall agreement with the $\sigma(20°)/\sigma(70°)$ data [Fig. 3(a)], but there is little visible improvement to the $\sigma(100°)$ and $\sigma(125°)$ data. The normalization constant for the latter has been raised by 5%.

The introduction of Coriolis coupling could also be applicable to a single-parity rotational band fit based on the concept of the secondary well as the source of the vibrational resonance. Indeed, the steady reduction of strength of the rotational band members with increasing J found necessary by James *et al.*[6] to fit the data using the

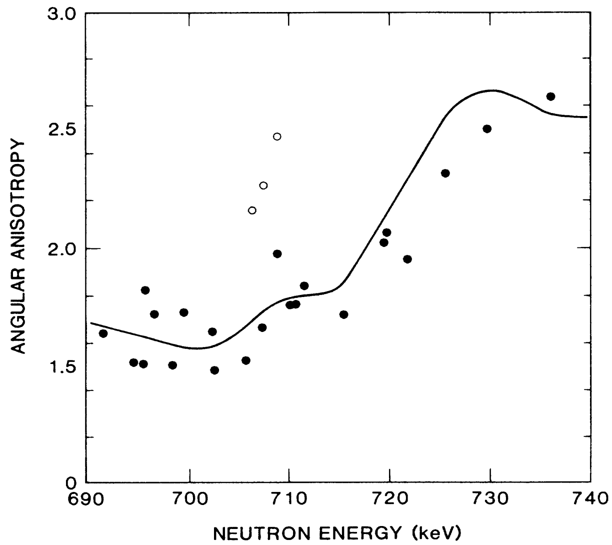

FIG. 3. Same as Fig. 2, but with admixture of the $K'=K+1$ band due to Coriolis coupling. Only the $\sigma(20°)/\sigma(70°)$ data and fit are shown.

single band hypothesis could be attributed to Coriolis coupling. An attempted fit for an even-parity band is shown in Fig. 4. The parameters of the fitted curves are

$$E_{v(K)}=709.2 \text{ keV} ,$$

$$\hbar^2/2\mathcal{J}=1.41 \text{ keV} ,$$

$$a=-0.9 ,$$

$$\left.\begin{array}{l}\Gamma_{v(f)}=14.7 \text{ keV} \\ \\ \Gamma_{v(c)}=1.47 \text{ keV}\end{array}\right\} \text{(or vice versa)} ,$$

$$C_{100}=C_{125}=0.76 ,$$

$$\epsilon_{100}=-3.2 \text{ keV} ,$$

$$\epsilon_{125}=-1.9 \text{ keV} ,$$

$$B_c=0.044 ,$$

$$(\Gamma_{v(f,K)}-\Gamma_{v'(f,K+1)})/\Gamma_{v(f,K)}$$
$$=(\Gamma_{v(c,K)}-\Gamma_{v'(c,K+1)})/\Gamma_{v(c,K)}=0.9 \text{ (assumed)} .$$

Attempts to fit the data with an odd-parity band were much poorer.

IV. REDUCED FISSION STRENGTH OF THE ONE-PHONON MASS ASYMMETRY VIBRATION STATE

Study of the potential energy surface (Fig. 1) for the fissioning nucleus in the thorium region shows that while the tertiary wells of opposite reflection asymmetric shape are separated by a high potential hill, giving rise to near degeneracy of opposite parity bands associated with a sin-

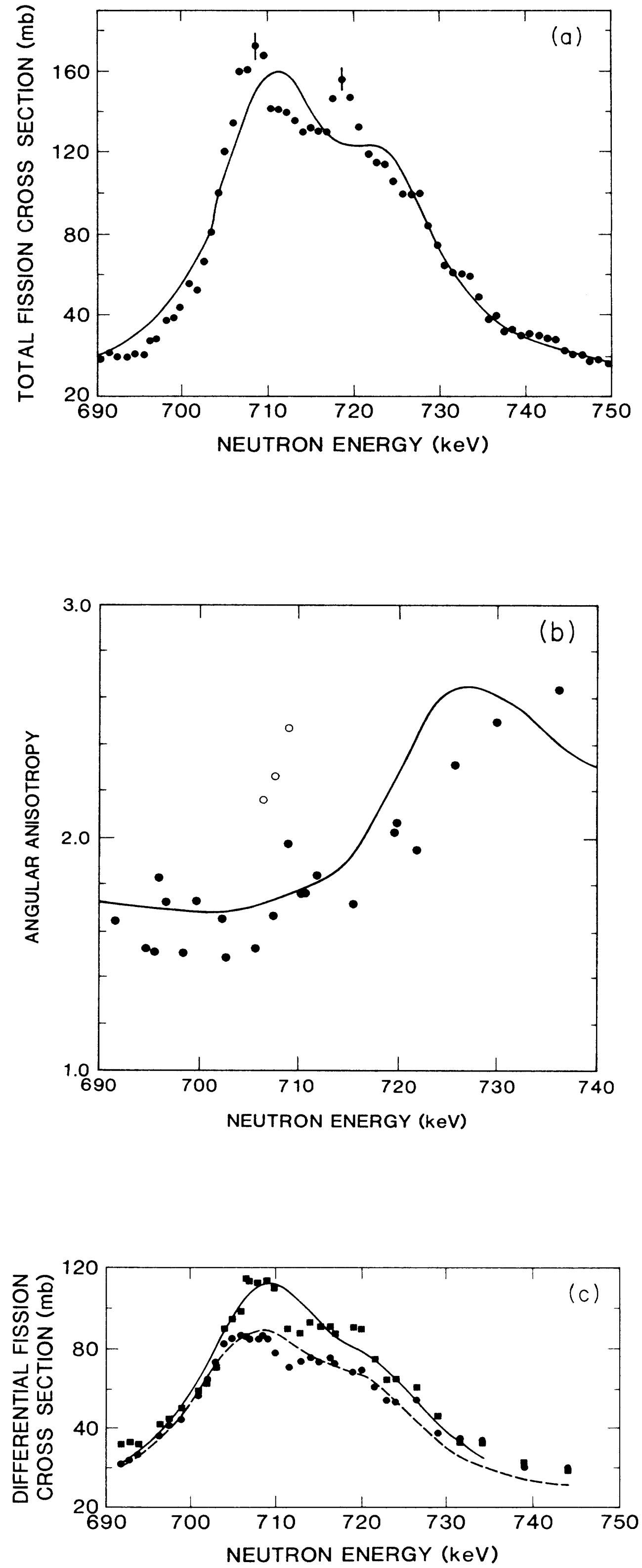

FIG. 4. Attempted fit to $^{230}Th(n,f)$ data with a single parity band allowing for Coriolis interaction.

TABLE I. Parameters for fits with reduced degeneracy for reflection asymmetry vibration and Coriolis interaction. Curves are those of Fig. 6.

	E_v (keV)	$\hbar^2/2\mathcal{J}$ (keV)	a	ΔE_v (keV)	$\Gamma_{v^+(f)}$ (keV)	$\Gamma_{v^+(c)}$ (keV)	$\Gamma_{v^-(f)}$ (keV)	$\Gamma_{v^-(c)}$ (keV)	B_c
Fit I (solid curve)	706.8±0.2	1.14±0.07	−0.70±0.14	14.9±0.4	8.4±0.3	0.81±0.03	10.5±0.8	0.24±0.02	0.019±0.002
Fit II (dashed curve)	706.8	1.16	−0.67	14.0	8.1	0.81	12.6	0.36	0.046

gle vibrational and single-particle state, at the inner barrier into these wells the mass asymmetry feature is very weak. At fixed inner barrier deformation, therefore, the degeneracy between the two opposite parity bands will be removed and there could be considerable differences in their amplitudes of vibrational motion at the inner barrier, and hence in their coupling widths through this barrier. Similar behavior, but probably much less marked, could occur at the outer barrier, thus affecting the fission width.

In our final set of studies of models based on the tertiary well concept of the vibrational mode, we have therefore allowed for differing coupling and fission widths for the opposite parity rotational bands.

We have first attempted a fit with no Coriolis interaction. The results are shown in Fig. 5. In Fig. 5(a) the fit to the total fission cross section is obviously very good. The $\sigma(20°)/\sigma(70°)$ calculated ratio [Fig. 5(b)] is rather too high over much of the energy range, but is generally a little better than that of Fig. 2(a) (equally weighted opposite parity bands). The fit to the $\sigma(100°)$ and $\sigma(125°)$ data also appear qualitatively as good as those of Fig. 2(c). The parameters for the fits of Fig. 5 are

$$E_v = 706.8 \text{ keV}, \quad \hbar^2/2\mathcal{J} = 1.134 \text{ KeV},$$

$$a = -0.664, \quad \Delta E_v = 14.36 \text{ keV},$$

$$\Gamma_{v^+(f)} = 8.0 \text{ keV}, \quad \Gamma_{v^+(c)} = 0.75 \text{ keV},$$

$$\Gamma_{v^-(f)} = 11.1 \text{ keV}, \quad \Gamma_{v^-(c)} = 0.28 \text{ keV},$$

$$C_{100} = 0.78, \quad C_{125} = 0.76,$$

$$\epsilon_{100} = -3.3 \text{ keV}, \quad \epsilon_{125} = -2.7 \text{ keV}.$$

There is a factor of 3 between the magnitudes of the coupling widths of even and odd parity bands. The smaller factor of difference (in an opposite sense) between the fission widths is probably not statistically significant.

Even better fits are obtainable if Coriolis interaction is included. Figure 6 shows some examples. In Figs. 6(a) and 6(b) the dashed curves are based on the assumption that

$$(\Gamma_{v(f,K)} - \Gamma_{v'(f,K+1)})/\Gamma_{v(f,K)} = 0.9,$$

$$\Gamma_{v(c,K)} = \Gamma_{v'(c,K+1)},$$

while the solid curves are based on

$$(\Gamma_{v(c,K)} - \Gamma_{v'(c,K+1)})/\Gamma_{v(c,K)} = 0.9,$$

$$\Gamma_{v(f,K)} = \Gamma_{v'(f,K+1)}.$$

The curves of Fig. 6(c) show only the result of the first assumption. The main parameters for the two fits are shown in Table I.

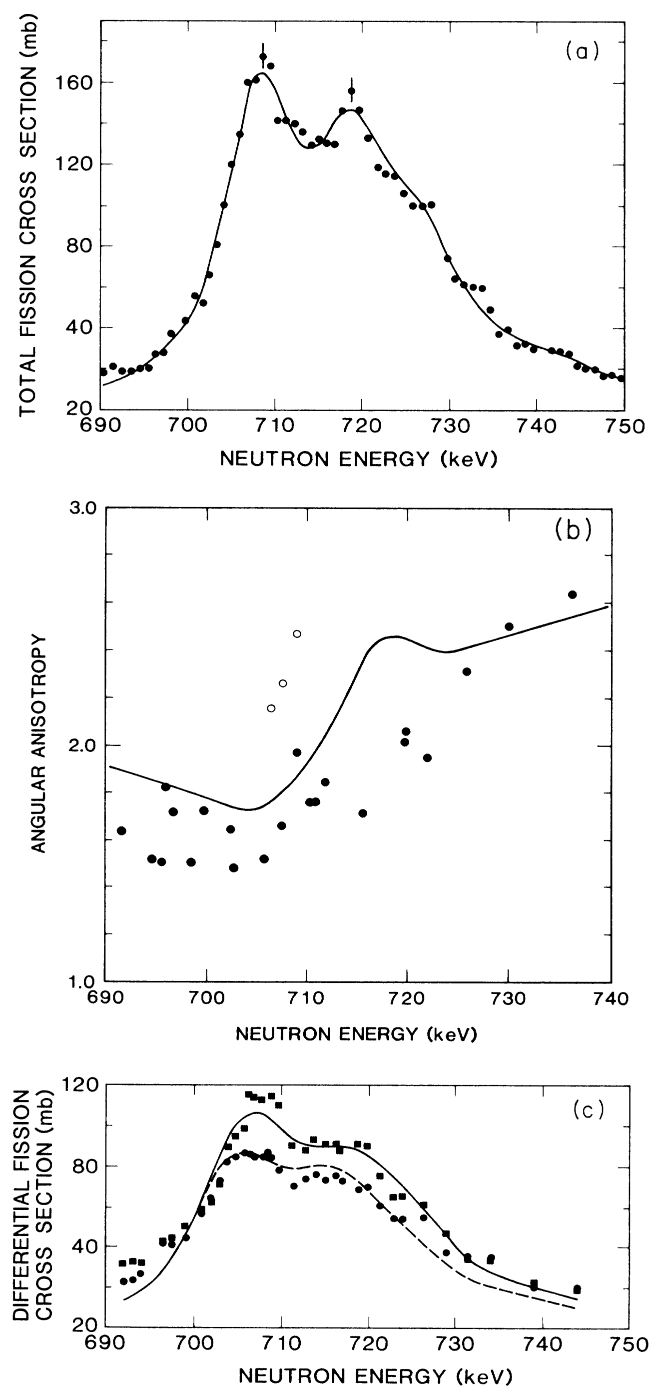

FIG. 5. Fit to ^{230}Th(n,f) data with different coupling and fission widths for the opposite parity bands. No Coriolis interaction is included.

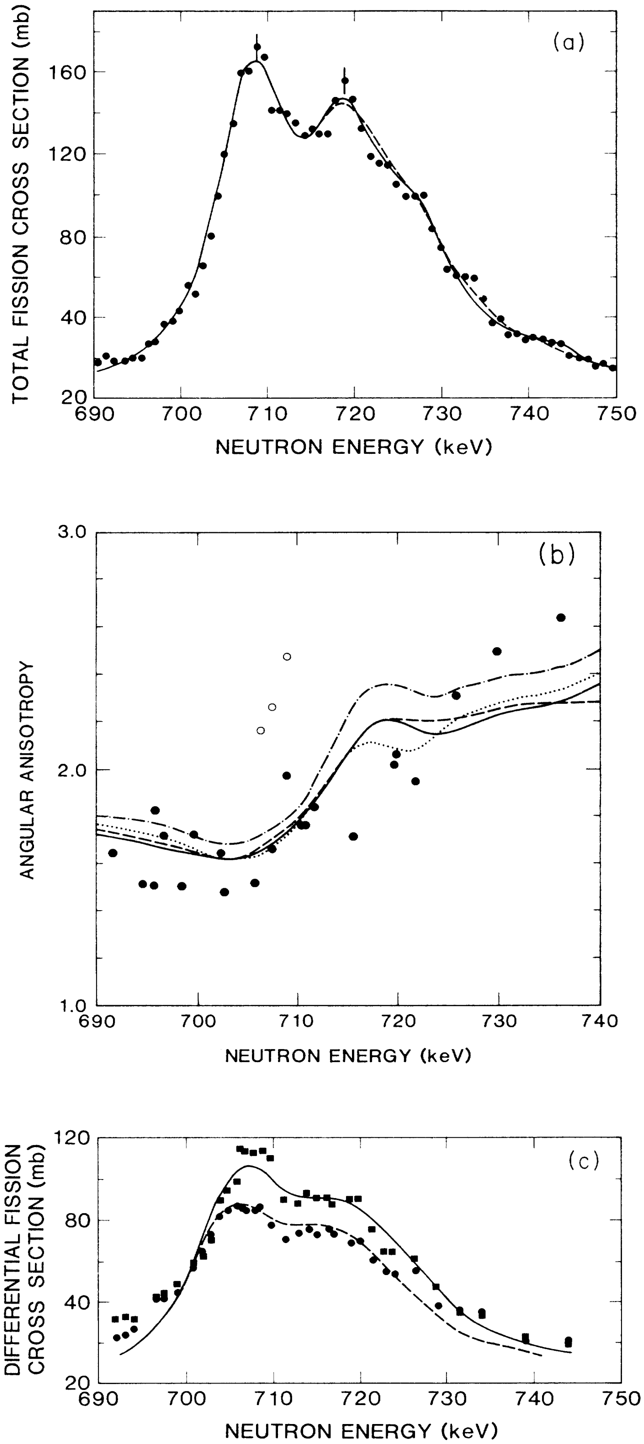

FIG. 6. Generalized fits to ^{230}Th(n,f) data. Coriolis interaction is included in addition to the possibility of breakdown of reflection asymmetry degeneracy at the barriers. Parameters are shown in Table I. The normalization constants for the curves of (c) are $C_{100}=0.764$, $C_{125}=0.736$, and the energy shifts are $\epsilon_{100}=-3.1$ keV, $\epsilon_{125}=-2.1$ keV. The dotted-dashed curve of (b) is for the first case of Table I, but with B_c reduced to 0.007, while the dotted curve is also for the reduced value of B_c but with $(\Gamma_{v(f,K)}-\Gamma_{v'(f,K+1)})/\Gamma_{v(f,K)}=-2.2$ and $\Gamma_{v(c,K)}=\Gamma_{(c,KN)}$.

V. DISCUSSION AND CONCLUSION

We have shown that greatly improved fits to what we regard as the soundest of available cross-section data on the ^{230}Th neutron-induced fission reaction in the region of the 250 keV vibrational resonance can be obtained if allowance is made for Coriolis interaction with nearby vibrational states with $K=\frac{3}{2}$, and for reduced strength of the non-normal parity band in the reflection asymmetric tertiary well model. Indeed, Coriolis interaction alone allows greatly improved fits for a single parity rotational band; such fits are almost as good as those based on a two-band model of Blons *et al.*, which in our procedure has turned out to be disappointingly poor, possibly because we allow no mechanism that produces varying resonance widths among the different spin members of a given band. However, even though our single parity model is improved by Coriolis interaction, we feel that it is not nearly good enough to allow renewed credence that the secondary well in the fission barrier could be the source of the vibrational resonance.

Of course, the generalized two-band models offer a great many more degrees of freedom for fitting, and therefore we would naturally expect much improved fits. Nevertheless, the extra parameters have a sound physical basis, and we have retained certain constraints such as equal moment of inertia, equal magnitude for the $K=\frac{1}{2}$ decoupling parameter, and equal strength for the Coriolis interaction of the two bands. It is clearly important to establish that the values determined for the new degrees of freedom have a reasonable physical magnitude.

To take the Coriolis interaction first, we start by pointing out that there is clear evidence for a $K\geq\frac{3}{2}$ state[6] in the fission cross section at neutron energy $E_n\approx950$ keV, about 230 keV higher than the $K=\frac{1}{2}$ resonance under study. Its coupling and fission width, at its resonance energy, are somewhat larger than those of the $K=\frac{1}{2}$ state, but at the energy of the latter they will be similar or smaller. Without *a priori* knowledge of the Nilsson orbitals involved, a precise estimate of the Coriolis coupling matrix element cannot be given, but if we accept estimates for the actinides at their normal (ground state) deformation,[22] we would expect a value of the order of 7.5 for the quantity $\langle K+1\,|\,j_1\,|\,K\rangle$ of Eq. (12b). Using $\hbar^2/2\mathcal{J}_u\approx2.5$ keV as a reasonable estimate for the unperturbed value of the rotational constant for a nucleus at the outer barrier deformation, we then obtain $B_c\approx0.007$ as an estimate for the order of magnitude we expect for this quantity, to be compared with fitted values ranging from about 0.019 to 0.04 for the two-band models. We note that a value of $B_c\approx0.007$ would give a better fit to the fission product angular distribution data in the region $E_n\approx720-740$ keV, although it would raise it slightly above the data at lower energies. In fact, if B_c is held at 0.007 in the fitting process, and the fission width difference of the K and K' states is allowed to vary, just as acceptable an angular distribution is obtained; this is also shown in Fig. 6(b), the K' fission width being found to be about 3 times greater than that of the $K+\frac{1}{2}$ state. The value of $B_c=0.044$ determined for the single band model (Fig. 4) is clearly untenable.

The effective moment of inertia $\mathcal{J}$ and decoupling parameter a for the rotational band are also modified from the unperturbed value $\mathcal{J}_u$ by the Coriolis interaction with the $K=\frac{3}{2}$ band. The relative shift in energy of the members of the $K=\frac{1}{2}$ band implies that

$$E_J=E_K+\frac{\hbar^2}{2\mathcal{J}_u}[J(J+1)-K(K+1)+a_u(-)^{J+1/2}(J+\tfrac{1}{2})]+(J-K)(J+K+1)\langle K\,|\,j_1\,|\,K+1\rangle^2\left[\frac{\hbar^2}{2\mathcal{J}_u}\right]^2\frac{1}{E_K-E_{K+1}}\,, \tag{17}$$

giving

$$\frac{\hbar^2}{2\mathcal{J}}=\frac{\hbar^2}{2\mathcal{J}_u}\left[1+\langle K\,|\,j_1\,|\,K+1\rangle^2\left[\frac{\hbar^2}{2\mathcal{J}_u}\right]\frac{1}{E_K-E_{K+1}}\right]\,, \tag{18}$$

$$a=a_u/[1+\langle K\,|\,j_1\,|\,K+1\rangle^2(\hbar^2/2\mathcal{J}_u)/(E_K-E_{K+1})]\;. \tag{19}$$

A value of $B_c\approx 0.015$ with $\hbar^2/2\mathcal{J}=1.15$ keV from the data analysis would imply that $\hbar^2/2\mathcal{J}_u\approx 5$ keV, a value that is much too high; again, a value of $B_c\approx 0.007$ would appear to be much more acceptable physically. This would lead to a value of $a\approx -0.27$.

The determination of the values of $\mathcal{J}_u$ and a_u with some degree of confidence is important for our knowledge of nuclear structure at extreme deformations. Clearly, we need to know the strength of the Coriolis interaction with some accuracy, and we do not believe we have yet attained the degree of accuracy required. The elucidation of this quantity rests principally on the fission product angular distribution data, and the published values of these in the literature now show a great deal of scatter and systematic deviation from one set to another. Among other problems, there is a persistent tendancy for a narrow group of high points in the forward to sideways angle ratios to occur. Although these are generally on the low energy side of the vibrational resonance, there is no precise agreement on their location among the various data sets. Their existence badly requires confirmation or otherwise. If the effect is real, it will be difficult to explain within the present class of models. But the main problem remains the deviation of the data at energies above the resonance. These are the energies at which the Coriolis interaction will show its greatest effect.

To judge the physical significance of the reduced strength of the non-normal parity band, we have made a model calculation to judge this effect. This is an extension of the particle-vibration coupling model described in Ref. 14. In addition to the dependence of single-particle energy on deformation, we have included an energy term to represent the energy of the first reflection-asymmetric vibration mode. This energy is described by the sum of a quadratic and cubic term in elongation deformation about the tertiary minimum; these have coefficients related so that the energy of the reflection-asymmetry mode is positive at the inner barrier and remains roughly zero from the center of the tertiary well to the outer barrier. We find that if the energy of the reflection-asymmetry mode is of the order of 2–3 meV at the inner barrier, there is significant reduction in the coupling width, by a factor of 3–4, while the increase in eigenvalue of the fully mixed state is only of the order of 30 keV. This is qualitatively consistent with the results of our analysis of the ^{230}Th fission data, and leads us to conclude that the reduced strength of the non-normal parity band is a real physical effect.

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