Inelastic electron scattering charge form factor of ^{4}He

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(Received 16 October 1986)

In this work we calculate the inelastic electron scattering charge form factor of ^{4}He using several N-N interactions. We employ the hyperspherical harmonics expansion method and include the contribution of harmonics above the minimal subset ($L_{\min}$). We find that the inelastic electron scattering form factor obtained by us is in excellent agreement with the experimental observations of Walcher. However, the inelastic form factor is insensitive to the N-N interaction. Further, we find that the minimal subset approximation is inadequate for form factor calculations when the momentum transfer is large. We also calculate the elastic electron scattering form factor and find the usual disagreement between theory and experiment. A possible explanation for this discrepancy is given.

I. INTRODUCTION

The electromagnetic properties of nuclear systems are a unique laboratory for testing various models of nuclear structure. Although there is a large body of high accuracy experimental data on the elastic electron scattering charge form factor (chff) of ^{3}H, ^{3}He, and ^{4}He, the corresponding information for inelastic electron scattering chff [^{4}He(e,e')^{4}He*] is quite scanty. Moreover, the results of different experimental groups are in serious disagreement with each other. In the present work we have tried to resolve this lacunae by calculating the inelastic electron scattering chff (abbreviated to inelastic chff) to a higher degree of accuracy using various N-N interactions.

It is worth noting that Furutani[1] has calculated the inelastic chff of ^{4}He using the generator coordinate method (GCM) technique. The model used in that work envisaged the excited state (e.s.) as a $(3+1)$ state, i.e., a cluster of three nucleons with the fourth nucleon loosely bound to it; this amounts to a single particle excitation. However, as discussed towards the end of Sec. II, the 0^+ e.s. has to be collective excitation. Thus, it is not surprising that the results of Furutani are in poor agreement with both the experimental groups.

The 0^+ e.s. of ^{4}He is quite loosely bound with a binding energy (BE) of 8.2 MeV compared to the ground state (g.s.) BE of 28.4 MeV. Therefore we expect the Coulomb interaction to play an important role in the determination of the wave function (WF) of the e.s. It is well known that long range potentials (like the Coulomb repulsion) are difficult to incorporate in a Faddeev equation formalism since too many partial waves have to be taken into account. A coordinate space approach is more suitable for this problem. Thus, for this work, we have used the hyperspherical harmonics (HH) expansion method. In this technique the few body WF is expanded in the complete orthonormal basis of HH, which are harmonic polynomials in $3N$-dimensional space. The few body Schrödinger equation then reduces to an infinite set of coupled, one dimensional differential equations. For practical purposes the expansion of the WF is truncated to a finite number of HH and the resulting finite system of coupled differential equations is solved numerically.

A general feature of the HH expansion technique is that the convergence of the expansion of the WF for a bound state is guaranteed. Also, the largest contribution to the WF comes from the first term ($L_{\min}$) of the expansion. Very often the chff of few nucleon systems are calculated in the $L_{\min}$ approximation[2,3] (i.e., only the first term of the WF is retained in the calculation). However, for the 0^+ e.s. the convergence of the expansion is quite slow. Thus, in order to ensure accuracy of the calculations it is desirable to go beyond the $L_{\min}$ approximation.

With the above discussion in view, we have undertaken the present work with two objectives: (1) to calculate the inelastic electron scattering chff of ^{4}He using the HH technique, and (2) to go beyond the $L_{\min}$ approximation. In the next section we give a brief overview of the HH technique. In Sec. III we derive expressions for chff of ^{4}He in hyperspherical coordinates. The results are shown and discussed in the last section.

II. THE HH EXPANSION METHOD

In this section we describe, very briefly, the HH expansion technique. Greater details may be found in the excellent review by Fabre de la Ripelle.[4]

The conventional Jacobi vectors, ξ_i, are used to construct translationally invariant WF's for four identical particles:

$$\begin{aligned}\xi_1&=(\mathbf{r}_2-\mathbf{r}_1)\ ,\\ \xi_2&=[\mathbf{r}_3-\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)]\sqrt{4/3}\ ,\\ \xi_3&=[\mathbf{r}_4-\tfrac{1}{3}(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3)]\sqrt{3/2}\ .\end{aligned}\tag{1}$$

We then transform to the hyperspherical coordinates defined by (i) the six polar angles (w_i) of ξ_i, and (ii) the hyperradius ρ and the angles φ_2 and φ_3 expressed in terms of the magnitudes of ξ_i:

$$\begin{aligned}\xi_1&=|\,\xi_1\,|=\rho\sin\varphi_2\sin\varphi_3\ ,\\ \xi_2&=|\,\xi_2\,|=\rho\cos\varphi_2\sin\varphi_3\ ,\\ \xi_3&=|\,\xi_3\,|=\rho\cos\varphi_3\ .\end{aligned}\tag{2}$$

In these new coordinates the volume element is given by

$$d\tau=\rho^8 d\rho \sin^5\varphi_3 \cos^2\varphi_3 d\varphi_3 \sin^2\varphi_2 \cos^2\varphi_2 d\varphi_2 \prod_{i=1}^{3} dw_i \ , \tag{3}$$

and the nine dimensional Laplace operator is

$$\sum_{i=1}^{3} \nabla_{\xi_i}^2 = \frac{\partial^2}{\partial\rho^2}+\frac{8}{\rho}\frac{\partial}{\partial\rho}+\frac{L^2(\Omega)}{\rho^2} \ , \tag{4}$$

where (Ω) denotes the set of eight hyperangles. The HH, $Y_{(L)}(\Omega)$, are defined as the eigenvectors of the grand orbital angular momentum operator $L^2(\Omega)$:

$$L^2(\Omega)Y_{(L)}(\Omega)=-L(L+7)Y_{(L)}(\Omega) \ , \tag{5}$$

where (L) denotes the eight quantum numbers required to specify a HH completely. Explicitly, we have

$$Y_{(L)}(\Omega)=\prod_{i=1}^{3} Y_{m_i}^{l_i}(w_i) C_2(L_2,l_1,l_2)(\sin\varphi_2)^{l_2}(\cos\varphi_2)^{l_1} P_{(L_2-l_1-l_2)/2}^{(l_2+1/2,\, l_1+1/2)}(\cos 2\varphi_2)$$
$$\times C_3(L_3,L_2,l_3)(\sin\varphi_3)^{L_2}(\cos\varphi_3)^{l_3} P_{(L_3-L_2-l_3)/2}^{(L_2+2,\, l_3+1/2)}(\cos 2\varphi_3) \ , \tag{6}$$

subject to the conditions

$$L_3=L, \quad (L_2-l_1-l_2)/2\geq 0, \text{ integer}$$

and

$$(L_3-l_3-L_2)/2\geq 0, \text{ integer} \ .$$

The normalization factors C_2 and C_3 are

$$C_3(L_3,L_2,l_3)=\left\{\frac{(2L_2+4)[(L_2-l_1-l_2)/2]!\Gamma[1+(L_2+l_1+l_2)/2]}{\Gamma[(L_2+l_1-l_3+3)/2]\Gamma[(L_2+l_2-l_1+3)/2]}\right\}^{1/2},$$

$$C_2(L_2,l_1,l_2)=\left\{\frac{(2L_3+7)[(L_3-L_2-l_3)/2]!\Gamma[(L_3+L_2+l_3+\frac{7}{2})/2]}{(2+(L_3+L_2-l_3)/2]!\Gamma[(L_3-L_2+l_3+3)/2]}\right\}^{1/2}.$$

The HH obey the orthonormality condition

$$\int Y_{(L)}^*(\Omega)Y_{(L')}(\Omega)d\Omega=\delta_{(L),(L')} \ , \tag{7}$$

where the δ function implies that the HH are separately orthonormal in each of the eight dimensions. The HH form a complete orthonormal set and can be employed to expand any arbitrary function $f(\rho,\Omega)$. In particular, we can expand the space WF and the total potential of a few body system in the HH.

The S state WF of ^{4}He can be written as

$$\psi=\sum_L U_L(\rho)\rho^{-4}S_L(\Omega)A(s,t) \ . \tag{8}$$

$A(s,t)$ is a spin-isospin WF, totally antisymmetric with respect to exchange of nucleons. $S_L(\Omega)$ is a linear combination of HH of order $2L$, totally symmetric with respect to exchange of nucleons:

$$S_L(\Omega)=D_L\sum_{(2L)} Y_{(2L)}(\Omega)\sum_{i,j>i} Y_{(2L)}^{0,0}(\varphi^{ij}) \ . \tag{9}$$

The $Y_{(2L)}^{0,0}(\varphi^{ij})$ are geometrical coefficients,[4] independent of all extraneous factors like the potential. D_L are normalization constants such that

$$\int S_L^*(\Omega)S_{L'}(\Omega)d\Omega=\delta_{L,L'} \ ,$$

which gives

$$D_L=\left\{\sum_{(2L)}\left[\sum_{i,j>i} Y_{(2L)}^{0,0}(\varphi^{ij})\right]^2\right\}^{-1/2} . \tag{10}$$

Introducing (8) in the four-nucleon Schrödinger equation, multiplying from the left by $A^*(s,t)S_{L'}^*(\Omega)$, summing over the spin-isospin variables, and integrating over (Ω), we get a coupled set of differential equations in the hyperradial functions $U_L(\rho)$,

$$-U_L''(\rho)+\frac{(2L+3)(2L+4)}{\rho^2}U_L(\rho)-\frac{mE}{\hbar^2}U_L(\rho)$$
$$=-\sum_{L'} V_L^{L'}(\rho)U_{L'}(\rho) \ , \tag{11}$$

where

$$V_L^{L'}(\rho)=\frac{m}{\hbar^2}\int S_{L'}^*(\Omega)V(\rho,\Omega)S_L(\Omega)d\Omega \ .$$

$V(\rho,\Omega)$ is the total potential in the system:

$$V(\rho,\Omega)=\sum_{i,j>i} V_{\text{N-N}}(i,j)+\tfrac{1}{4}[1+t_3(i)][1+t_3(j)]\frac{e^2}{r_{ij}} \ .$$

Some details of the procedure for calculation of the potential matrix elements, $V_L^{L'}(\rho)$, are given in our earlier work on the photodisintegration of ^{4}He.[5] The system of coupled differential equations can be solved if the expansion over WF, (8), is truncated to a finite number of terms. A special technique of iterative improvement of adiabatic approximation[6] has been employed in the present work. The details will be published elsewhere.[7] In brief, we calculate the hyperradial functions $U_L(\rho)$ as

$$U_L(\rho)=-I_{2L+7/2}(\epsilon\rho)\sqrt{\rho}\int\sqrt{\rho'}K_{2L+7/2}(\epsilon\rho')[W_0(\rho')-(2L+3)(2L+4)/\rho'^2]U_L(\rho')d\rho'$$
$$-K_{2L+7/2}(\epsilon\rho)\sqrt{\rho}\int\sqrt{\rho'}I_{2L+7/2}(\epsilon\rho')[W_0(\rho')-(2L+3)(2L+4)/\rho'^2]U_L(\rho')d\rho' , \quad (12)$$

where $I_n(x)$, $K_n(x)$ are the modified spherical bessel functions, $W_0(\rho)$ is the zeroth eigenpotential, and $U_L(\rho')$ in the integrands, are the hyperradial functions calculated using the uncoupled adiabatic approximation and $\epsilon[=(-mE/\hbar^2)^{1/2}]$ is the corresponding energy eigenvalue.[6]

For the case of ^{4}He, both the g.s. and e.s. have $J^p=0^+$. Thus, their hyperspherical expansions are similar and only their expansion coefficients [i.e., the hyperradial functions $U_L(\rho)$] are different. In this sense the e.s. is a pure hyperradial excitation of the g.s. On the other hand, from the definition of the hyperspherical coordinates we see that the hyperradius is a collective variable. Therefore we conclude that the 0^+ e.s. of ^{4}He is a collective excitation of the g.s. It is probably due to this reason that the results of Furutani[1] did not agree with experiment.

III. ELECTRON SCATTERING CHFF IN THE HH FORMALISMS

In this section we derive hyperspherical expressions for the electron scattering chff of ^{4}He. The electron scattering form factor of a transition from state A to B is given by

$$F(q)_{B\text{-}A}=\left\langle\psi_B\left|\sum_{j=1}^{4}\tfrac{1}{4}[1+t_3(j)]e^{i\mathbf{q}\cdot(\mathbf{r}_j-\mathbf{R})}\right|\psi_A\right\rangle f_p(q) . \quad (13)$$

The WF ψ_A,ψ_B are given by expansions of the form (8). Summing over the spin-isospin variables and noting that the space part of the WF is symmetric with respect to exchange of identical particles, we can write

$$F_{B\text{-}A}(q)=\int\left[\sum_L U_L^B(\rho)S_L^*(\Omega)\right]e^{i\mathbf{q}\cdot\boldsymbol{\xi}_3\sqrt{3/8}}$$
$$\times\left[\sum_L U_L^A(\rho)S_L(\Omega)\right]\rho^{-8}d\tau . \quad (14)$$

It should be noted that the above form is valid for both the elastic and inelastic electron scattering chff's. The expansion of the plane wave in HH is given by[8]

$$e^{i\mathbf{q}\cdot\boldsymbol{\xi}_3\sqrt{3/8}}=\frac{(2\pi)^{9/2}}{x^{7/2}}\sum_{\chi=0}\sum_{(\chi)}i^\chi Y_{(\chi)}(\Omega)$$
$$\times Y_{(\chi)}(\Omega_q)J_{\chi+7/2}(x) , \quad (15)$$

where $x=q\rho(\frac{3}{8})^{1/2}$ and

$$Y_{(\chi)}(\Omega_q)=\left[\prod_{i=1}^{3}Y_{\mu_i}^{\lambda_i}(\omega_q)\right]C_2(\chi_2\lambda_1\lambda_2)(\sin\alpha_2)^{\lambda_2}(\cos\alpha_2)^{\lambda_1}P_{(\chi_2-\lambda_1-\lambda_2)/2}^{(\lambda_2+1/2,\,\lambda_1+1/2)}(\cos2\alpha_2)$$
$$\times C_3(\chi_3\chi_2\lambda_3)(\sin\alpha_3)^{\chi_2}(\cos\alpha_3)^{\lambda_3}P_{(\chi_3-\chi_2-\lambda_3)/2}^{(\chi_2+2,\,\lambda_3+1/2)}(\cos2\alpha_3) ,$$

with the restrictions $\chi_3=\chi$, $(\chi_2-\lambda_1-\lambda_2)/2\geq0$, integer, and $(\chi_3-\chi_2-\lambda_3)/2\geq0$, integer. The (ω_q) are the polar angles of the momentum transfer $\mathbf{q}$. The angles α_2 and α_3 are "kinematic rotation" angles characterizing $\boldsymbol{\xi}_3$; for the hyperspherical coordinates defined earlier (2), $\alpha_3=0$. Therefore, for $Y_{(\chi)}(\Omega_q)$ to be different from zero we require $\chi_2=0$. However, due to restrictions on the quantum numbers we also require $\lambda_1=\lambda_2=0$. Moreover, since the system is spherically symmetric we can assume $\mathbf{q}$ to be along the z axis. Thus $\mu_3=0$ and the expansion of the plane wave reduces to

$$e^{i\mathbf{q}\cdot\boldsymbol{\xi}_3\sqrt{3/8}}=2\frac{2^{9/2}}{x^{7/2}}\pi^4\sum_\chi i^\chi J_{\chi+7/2}(x)\sum_{(\chi)}Y_{(\chi)}(\Omega)Y_0^{\lambda_3}(w_q)C_3(\chi,0,\lambda_3)P_{(\chi-\lambda_3)/2}^{(2,\,\lambda_3+1/2)}(1)\,\delta_{\chi_2,0}\delta_{\lambda_1,0}\delta_{\lambda_2,0}\delta_{\mu_1,0}\delta_{\mu_2,0}\delta_{m_3,0} . \quad (16)$$

The above expansion of the plane wave has to be introduced in the expression for chff (13). Noting that the $L=0$ component of the WF is the dominant term in the expansion of the WF, we retain only those terms in (13) which have $L=0$ in either ψ_A or ψ_B:

$$F(q)_{B\text{-}A}=\left[\int U_0^B(\rho)S_0^*(\Omega)e^{i\mathbf{q}\cdot\boldsymbol{\xi}_3\sqrt{3/8}}U_0^A(\rho)S_0(\Omega)\rho^{-8}d\tau+\sum_{L=2}\int U_0^B(\rho)S_0^*(\Omega)e^{i\mathbf{q}\cdot\boldsymbol{\xi}_3\sqrt{3/8}}U_L^A(\rho)S_L(\Omega)\rho^{-8}d\tau\right.$$
$$\left.+\sum_{L=2}\int U_L^B(\rho)S_L^*(\Omega)e^{i\mathbf{q}\cdot\boldsymbol{\xi}_3\sqrt{3/8}}U_0^A(\rho)S_0(\Omega)\rho^{-8}d\tau\right]f_p(q) . \quad (17)$$

In the $L_{\min}$ approximation only the first term of the above expression is retained in the calculation of $F(q)$. Introducing (9) and (15) in the above expression, noting that $S_0(\Omega)$ is a constant and the HH are orthonormal in each dimension, we get

TABLE I. Norms of the "partial waves" together with the rms radius and BE calculated using the Volkov and $S1$ potentials (with Coulomb interaction).

Potential	Volkov		$S1$	
L	g.s.	e.s.	g.s.	e.s.
0	98.6436	90.0160	96.7955	91.4529
2	0.9593	7.3389	1.3979	5.9038
3	0.2233	1.6738	0.7527	1.5834
4	0.1063	0.5698	0.5095	0.5605
5	0.0366	0.1213	0.2151	0.1347
6	0.0280	0.0652	0.1778	0.1003
7	0.0068	0.0244	0.0718	0.0617
8	0.0034	0.0203	0.0450	0.0727
9	0.0013	0.0144	0.0219	0.0648
10	0.0006	0.0121	0.0127	0.0649
BE (MeV)	29.769	5.579	30.833	2.585
rms radius (fm)	1.48	3.66	1.43	3.77

$$F_{B\text{-}A}(q)=\left\{105\int U_0^A(\rho)U_0^B(\rho)j_3(x)x^{-3}d\rho + \sum_{L=2}^{L_{\max}} D'_L \int d\rho[U_0^A(\rho)U_L^B(\rho)+U_L^A(\rho)U_0^B(\rho)]x^{-3}j_{2L+3}(x)\sum_{l_3} G(L,l_3)\right\} f_p(q)\ , \tag{18}$$

with $D'_L=(-1)^L 16(\frac{105}{8})^{1/2}\pi^{3/2}D_L$,

$$G(L,l_3)=\left[\sum_{i,j>i} Y^{0,0}_{(2L,2l_3)}(\varphi^{ij})\right] Y_0^{2l_3}(0)C_3(2L,0,2l_3)P^{(2,\,2l_3+1/2)}_{(L-l_3)}(1)\ .$$

$Y^{0,0}_{(2L,2l_3)}(\varphi^{ij})$ are the geometric coefficients defined earlier with all quantum numbers, except for L and l_3, equal to zero. In the above expression L takes even values only (i.e., $L=2,3,4,\ldots,L_{\max}$) and l_3 varies from 0 to L.

Using the above expression we can calculate the chff of ^{4}He easily. Only the integrations over the hyperradius need to be performed. In practical calculations we found that the integrands were quite smooth and an ordinary Simpson's rule works efficiently.

IV. RESULTS AND DISCUSSION

The system of coupled differential equations is truncated to a finite ($L_{\max}$) set in practical calculations. This amounts to truncating the solution space and the corresponding eigenvalue is an upper limit to the exact eigenvalue. The presence of the centrifugal barrier $[\sim(2L+3)(2L+4)/\rho^2]$ ensures convergence of the WF. We increase $L_{\max}$ until the eigenvalue converges. In the present work we found that both the g.s. and the 0^+ e.s. of ^{4}He converge when we retain the first ten equations (i.e., $L_{\max}=10$). In Table I we give the norms of the "partial waves" together with the root-mean-square radius and the BE calculated using the Volkov and $S1$ potentials

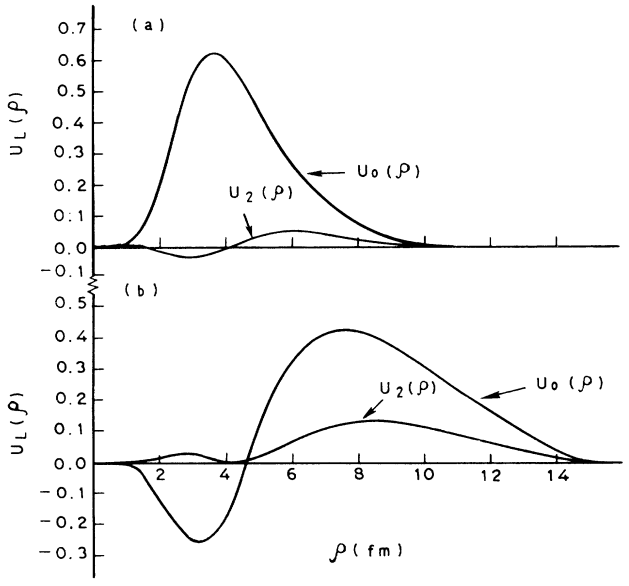

FIG. 1 (a) The hyperradial functions $U_0(\rho)$ and $U_2(\rho)$ of the g.s. using the Volkov potential. (b) The hyperradial functions $U_0(\rho)$ and $U_2(\rho)$ of the e.s. using the Volkov potential.

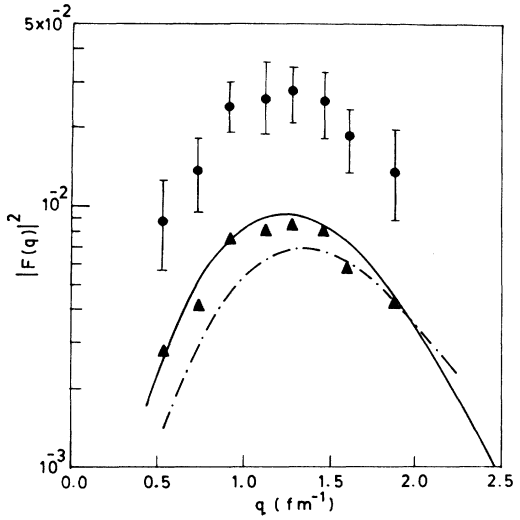

FIG. 2. The inelastic chff of ^{4}He obtained using the Volkov (solid line) and $S1$ (dashed-dotted) line potentials. The experimental data are due to Walcher (Ref. 11) (triangles) and Frosch (Ref. 12) (dots).

(with Coulomb interaction). An inspection of the table reveals that the $L=0$ component of the WF is the most important term. This justifies the approximation used in the calculation of $F(q)$.

In Figs. 1(a) and Fig. 1(b) we plot the functions $U_0(\rho)$ and $U_2(\rho)$ of the g.s. and e.s., respectively obtained using the Volkov potential. Remarkable features are that $U_0(\rho)$ of the e.s. has a node and both $U_0(\rho)$ and $U_2(\rho)$ decay very slowly compared to their g.s. counterparts. Thus the e.s. has a large rms radius (cf Table I) and this state is sensitive to the Coulomb interaction. Similar hyperradial functions are obtained with other potentials.

The BE of the four nucleon system has been investigated using the same N-N potentials by Ballot *et al.*[9] However, they have integrated only the first seven equations and ignore the Coulomb interaction. Thus, we obtain a slightly different set of norms though the trend is similar. Moreover, they have not calculated the chff of ^{4}He. Ballot *et al.* have also obtained a node in $U_0(\rho)$ of the e.s. This result can be understood on the basis of the lemmas due to Levinger,[10] which state that for local noncentral forces in multidimensional space we have that (i) the lowest partial wave of the g.s. is nodeless, and (ii) the lowest partial wave of the first e.s. has at least one node. It should be noted that, in general, forces which are central in three-dimensional space become noncentral in $3N$-dimensional space ($N>1$), resulting in the coupling terms $V_L^{L'}(\rho)$ in (11).

The electron scattering chff is calculated using (18). In Fig. 2 below we plot the inelastic chff of ^{4}He calculated

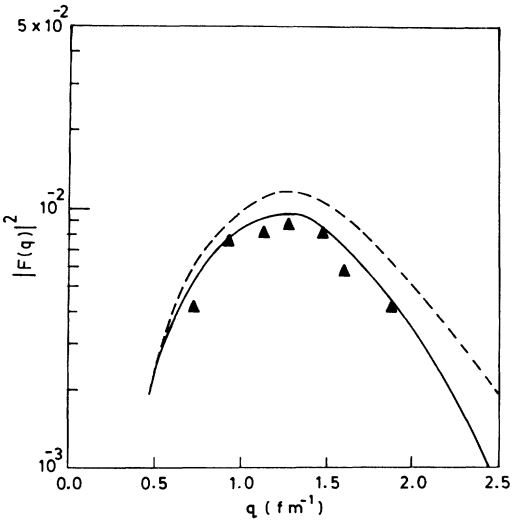

FIG. 3. The inelastic chff of ^{4}He obtained using the $L_{\min}$ approximation (dashed line) and the full expression (solid line).

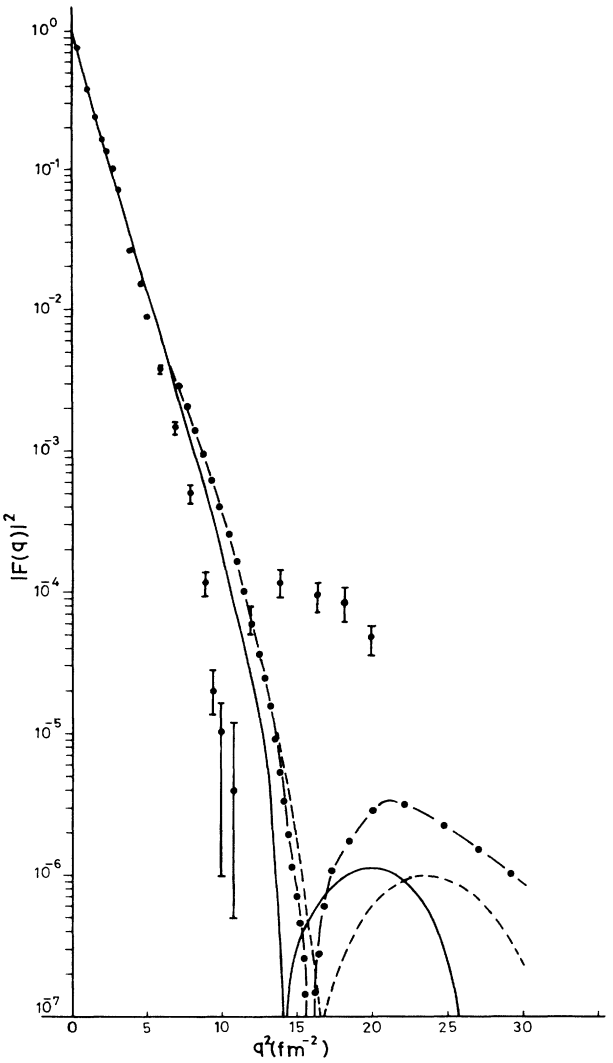

FIG. 4. The elastic chff of ^{4}He obtained using the Volkov potential with the $L_{\min}$ approximation (dashed line) and the full expression (solid line). We compare our results with those obtained using the realistic Urbana ϑ_{14} potential (plus three nucleon interaction) (Ref. 15).

using the Volkov and $S1$ potentials. These two potentials are central but vary in the strength of the soft core. Remarkably, the inelastic chff's obtained agree with each other. The calculations were repeated for a number of other potentials and similar inelastic chff's were obtained in each case. This feature clearly exhibits the insensitivity of inelastic chff's to the explicit form of the N-N interaction. Thus the ^{4}He(e,e')^{4}He* reaction is unlikely to reveal any information regarding the details of nuclear potential. Moreover, there is close agreement between the theoretical values and the experimental observations of Walcher[11] (marked with triangles). The other set of experimental results (marked with dots) is due to Frosch.[12] Both experimental groups studied the reaction ^{4}He(e,e')^{4}He* and obtained similar cross sections, though Walcher's experiment had better resolution. Frosch used an asymmetric curve, based on R-matrix analysis,[13] to fit the resonance due to the 0^+ e.s. On the other hand, Walcher used a symmetric Lorentzian to fit the same resonance. The inelastic chff was then extracted by the usual process and are reproduced in Fig. 2. The discrepancy between the two sets is obvious. On the basis of our calculations using various N-N potentials, we get the impression that the inelastic chff obtained by Walcher is more reliable. However, the final test rests with the experimentalists.

A detailed analysis of the chff shows that the $L_{\min}$ approximation is quite good for low momentum transfers. However, the approximation becomes poor with increasing momentum transfer and for $q \gtrsim 2.5$ fm^{-1} the discrepancy is alarming. In Fig. 3 we compare the inelastic chff obtained using the $L_{\min}$ approximation [i.e., the first term in (18)] and the actual chff calculated using the Volkov potential.

It is well known that for large momentum transfer the chff is sensitive to the details of the WF. From Fig. 1(b) we see that $U_2(\rho)$ gives an appreciable contribution to the asymptotic part of the WF. Thus the $L \geq 2$ components of the WF become significant for increasing momentum transfer.

For the sake of completeness we have also calculated the elastic chff of ^{4}He. In this case both ψ_B and ψ_A [cf. (12)] refer to the WF of the g.s. The results are shown in Fig. 4. In this case we see that the theoretical estimates with the Volkov potential (solid line) are in very poor agreement with the experimental observations.[14] The first minima are too far and the secondary maxima are too small ($\sim 5\times 10^{-6}$). Our results are in agreement with those of other group; in the figure we compare our values with those obtained using the Urbana ϑ_{14} (Ref. 15) potential. In Fig. 4 the dashed line indicates the elastic chff of ^{4}He obtained with the Volkov potential in the $L_{\min}$ approximation. As in the earlier case of inelastic chff, we see that this approximation becomes poorer with increasing momentum transfer. A similar situation exists for the elastic chff of trinucleon systems. However, recent calculations of chff's of ^{3}H and ^{3}He (Ref. 16) indicate that the discrepancy can be resolved if the quark substructure of the nucleons is incorporated accurately in the few nucleon WF. Coon *et al.*[17] have indicated a procedure for obtaining the quark cluster probability in ^{4}He. It is worth investigating the elastic chff of ^{4}He with the underlying quark dynamics taken into account. Since the nucleons are well separated in the 0^+ e.s. of ^{4}He, the six, nine, and twelve quark clustering probability will be very small. Thus quark dynamics will have a small effect on the inelastic chff of ^{4}He.

With the results of this work we can conclude the following.

(i) The inelastic electron scattering chff's of ^{4}He, calculated using various spin independent N-N potentials, agree closely with the experimental observations of Walcher.

(ii) It is important to retain the $L > L_{\min}$ components of the WF for $q \gtrsim 1$ fm^{-1}.

(iii) The underlying quark dynamics should be included in the calculation of elastic chff.

The authors are grateful to Council of Scientific and Industrial Research, India, and the Department of Atomic Energy, India, for financial support.

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