# Microscopic theory of the $^8Be(\alpha,\gamma)^{12}C$ reaction in a three-cluster model

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The generator-coordinate method is applied to the study of the three-$\alpha$-particle system. The choice of the basis functions allows us to calculate the $\alpha+^8$Be phase shifts. We perform a comparison between the full three-cluster approach and two of its approximations. Distortion effects are shown to be important in the spectroscopic properties of $^{12}$C. We present the $^8Be(\alpha,\gamma)^{12}C$ $S$ factor derived in the three-$\alpha$-particle model. At low temperatures, the reaction rate is found to be slightly different from earlier studies.

## I. INTRODUCTION

The triple-$\alpha$ reaction, which leads to the production of $^{12}$C nuclei, is expected to occur in two steps.[1] The first is the formation of $^8$Be from two $\alpha$ particles. The $^8$Be nucleus is unstable with respect to $\alpha$ decay, but in some stellar sites a quantity of $^8$Be exists in thermal equilibrium with the $\alpha$ particles. The second step involves the $\alpha$ capture by $^8$Be. The latter reaction depends crucially on the properties of the $0_2^+$ excited state in $^{12}$C (see Ref. 1). The triple-$\alpha$ reaction has recently been reconsidered by Nomoto *et al.*,[2] and by Langanke *et al.*[3] in nonmicroscopic models. However, microscopic descriptions of capture reactions may lead to rather different results.[4] We have already presented the $^4He(\alpha,\gamma)^8Be$ cross section obtained in a microscopic framework[5] with an exact account of the unstable nature of the $^8$Be ground state. In this paper we investigate microscopically the second step of the triple-$\alpha$ reaction, the $^8Be(\alpha,\gamma)^{12}C$ capture reaction. We assume that the $^8$Be ground-state wave function can be approximated by a square-integrable wave function.

Microscopic calculations give a satisfactory description of the bound-state and scattering properties of two-cluster systems.[6] The generator-coordinate method (GCM) has recently been applied to the determination of electromagnetic properties of such systems.[7] Until now, mainly single-channel approaches have been investigated. The single-channel model is not reliable when one of the colliding nuclei is easily deformable.[8] In this case distortion effects have to be considered in the calculation. These effects can be taken into account by introducing configurations involving either physical excited states [9] or pseudostates of the compressible nucleus.[8] Both techniques improve the quality of the wave functions, but in different ways. The physical configurations introduce a distortion in the wave function of the *unified* nucleus only, but allow one to study inelastic reactions. In the pseudostate approach, the wave functions of the unified nucleus are distorted through the distortion of the *clusters*.

In this paper we want to study the $\alpha+^8$Be system. Since the ground state of the $^8$Be nucleus is a short-lived nuclear molecule formed of two $\alpha$ particles, a one-center description of this nucleus is expected to be poor. A way of solving this problem is to describe the $\alpha+^8$Be system by a three-$\alpha$-particle configuration. This leads to the introduction of two generator coordinates, one internal to the $^8$Be nucleus and the other associated with the $\alpha+^8$Be relative motion.

The spectroscopic properties of $^{12}$C have already been studied by Kamimura,[10] in a fully microscopic three-cluster framework. However, the choice of the antisymmetric basis functions employed by Kamimura is not appropriate for the investigation of the $\alpha+^8$Be collision. Here we perform the calculation with a different basis of wave functions. This investigation takes correct account of the two-center structure of $^8$Be, but in an approximation where its wave function is square integrable. In order to evaluate the importance of distortion effects, we compare three different approaches. The first is a multichannel study based on the full three-cluster model, while the second consists of a single-channel calculation in which the $^8$Be nucleus is described in a two-center model. Our investigation is completed by a conventional two-cluster calculation of the $\alpha+^8$Be system with the ground-state SU(3) wave function for $^8$Be. Since the $^8$Be nucleus is a particularly deformable nucleus, the $\alpha+^8$Be system offers a favorable case for such a comparison. It will allow us to evaluate the influence of the $^8$Be description in the study of the $\alpha+^8$Be system.

In Sec. II we present the three-cluster model. Application to the $^{12}$C nucleus is presented in Sec. III, and the $^8Be(\alpha,\gamma)^{12}C$ reaction is investigated in Sec. IV. Concluding remarks are given in Sec. V.

## II. THE THREE-CLUSTER MODEL

Let us consider translation-invariant $\alpha$-particle wave functions $\phi_i$ (where $i$ is the cluster index), constructed in the harmonic oscillator model with parameter $b$. A basis function of the 12-nucleon system reads

$$\Phi(\mathbf{R}_1,\mathbf{R}_2)=\phi_{\text{c.m.}}\mathcal{A}\phi_1\phi_2\phi_3\Gamma(b_1,\boldsymbol{\rho},\mathbf{R}_1)\Gamma(b_2,\boldsymbol{\rho}',\mathbf{R}_2)\ , \qquad (1)$$

where $b_1=(\frac{3}{8})^{1/2}b$, $b_2=2^{-1/2}b$, $\mathcal{A}$ is the antisymmetrization projector, and $\mathbf{R}_1$ and $\mathbf{R}_2$ are the generator coordinates. The relative coordinates $\boldsymbol{\rho}$ and $\boldsymbol{\rho}'$ are defined by

 

$$\boldsymbol{\rho}=\frac{1}{4}\sum_{i=9}^{12}\mathbf{r}_i-\frac{1}{8}\sum_{i=1}^{8}\mathbf{r}_i\ ,$$
$$\boldsymbol{\rho}'=\frac{1}{4}\sum_{i=1}^{4}\mathbf{r}_i-\frac{1}{4}\sum_{i=5}^{8}\mathbf{r}_i\ , \tag{2}$$

where $\mathbf{r}_i$ is the coordinate of nucleon $i$. In (1), the function $\Gamma$ is given by

$$\Gamma(\beta,\boldsymbol{\rho},\mathbf{R})=(\pi\beta^2)^{-3/4}\exp[-(\boldsymbol{\rho}-\mathbf{R})^2/2\beta^2]\ , \tag{3}$$

and $\phi_{\text{c.m.}}$ is chosen as

$$\phi_{\text{c.m.}}=\Gamma((12)^{-1/2}b,\mathbf{R}_{\text{c.m.}},0)\ , \tag{4}$$

where $\mathbf{R}_{\text{c.m.}}$ is the center-of-mass coordinate. With definitions (3) and (4), the wave function (1) is readily written as a Slater determinant:

$$\Phi(\mathbf{R}_1,\mathbf{R}_2)=\mathscr{A}\Phi(-\tfrac{1}{3}\mathbf{R}_1+\tfrac{1}{2}\mathbf{R}_2)\Phi(-\tfrac{1}{3}\mathbf{R}_1-\tfrac{1}{2}\mathbf{R}_2)\Phi(\tfrac{2}{3}\mathbf{R}_1), \tag{5}$$

where $\Phi(\mathbf{S})$ represents a Slater determinant composed of four $0s$ particles centered at $\mathbf{S}$. This property allows one to use simple formulas for the calculation of matrix elements.[11]

The GCM basis functions are obtained by projecting out $\Phi$ on the total angular momentum $J$. Moroever, in order to study the $\alpha+^8$Be reaction, it is useful to introduce the orbital momentum $L$ of the $^8$Be nucleus and the relative orbital momentum $l$. The basis functions read

$$\Phi_{lL}^{JM\pi}(R_1,R_2)=\int[Y_l(\Omega_1)\otimes Y_L(\Omega_2)]_M^J\Phi(\mathbf{R}_1,\mathbf{R}_2)d\Omega_1 d\Omega_2\ , \tag{6}$$

where $\Omega_1$ and $\Omega_2$ denote the directions of $\mathbf{R}_1$ and $\mathbf{R}_2$, respectively, and where $\pi=(-1)^{l+L}$.

For an irreducible tensor operator $O_\mu^\lambda$ of rank $\lambda$, the reduced matrix elements are given by

$$\langle\Phi_{lL}^{J\pi}(R_1,R_2)||O^\lambda||\Phi_{l'L'}^{J'\pi'}(R_1',R_2')\rangle=8\pi^2(-1)^{J'}[(2J+1)(2\lambda+1)]^{-1/2}$$
$$\times\sum_\mu\int[[Y_l(0,0)\otimes Y_L(\Omega_2)]^J\otimes[Y_{l'}(\theta_1',0)\otimes Y_{L'}(\Omega_2')]^{J'}]_\mu^{\lambda*}$$
$$\times\langle\Phi(\mathbf{R}_1,\mathbf{R}_2)\,|\,O_\mu^\lambda\,|\,\Phi(\mathbf{R}_1',\mathbf{R}_2')\rangle\sin\theta_1' d\theta_1' d\Omega_2 d\Omega_2'\ , \tag{7}$$

where we have chosen $\mathbf{R}_1$ along the $z$ axis, and $\mathbf{R}_1'$ forms an angle $\theta_1'$ with $z$, in the $x$-$z$ plane. The matrix elements involve five-dimensional integrals. With the choice of basis functions (1), the integrals have to be evaluated numerically. On the contrary, the choice of Kamimura[10] allows an analytical evaluation of the matrix elements, but is not appropriate for the study of reactions.

The total wave function of the system reads

$$\Psi^{JM\pi}=\sum_{lL}\int f_{lL}^{J\pi}(R_1,R_2)\Phi_{lL}^{JM\pi}(R_1,R_2)dR_1 dR_2\ , \tag{8}$$

where $f_{lL}^{J\pi}$ is the generator function, and where the sum over $l$ and $L$ is restricted by the condition $\pi=(-1)^{l+L}$. In practice, the integrals in (8) are discretized over a finite set of generator-coordinate values, which leads to

$$\Psi^{JM\pi}\simeq\sum_{lL}\sum_{mn}f_{lL}^{J\pi}(R_{1m},R_{2n})\Phi_{lL}^{JM\pi}(R_{1m},R_{2n})\ . \tag{9}$$

For $^{12}$C bound-state wave functions, the coefficients $f_{lL}^{J\pi}$ are deduced from a variational principle. For $\alpha+^8$Be scattering wave functions, the coefficients are calculated with the microscopic $R$-matrix method (Ref. 7 and references herein).

Equation (9) exhibits the three-cluster nature of the wave function. However, this form is not practical for the study of reactions. Therefore, it is useful to decompose (9) on a basis emphasizing the $\alpha+^8$Be structure. Let us consider the $N$ variational solutions of the two-cluster equation ($N$ is the number of values of $R_{2n}$),

$$H\phi_{12}^{LM\omega}=E^{L\omega}\phi_{12}^{LM\omega}\quad(\omega=0,\ldots,N-1)\ . \tag{10}$$

The two-center trial function $\phi_{12}^{LM\omega}$ is given by

$$\phi_{12}^{LM\omega}=\sum_{n=1}^{N}g^{L\omega}(R_{2n})\int Y_L^M(\Omega_2)$$
$$\times\mathscr{A}\Phi(-\tfrac{1}{2}\mathbf{R}_{2n})\Phi(\tfrac{1}{2}\mathbf{R}_{2n})d\Omega_2 \tag{11}$$
$$=4\pi\sum_{n=1}^{N}g^{L\omega}(R_{2n})\mathscr{A}\phi_1\phi_2\Gamma_L(b_2,\rho',R_{2n})Y_L^M(\Omega_{\rho'})\ . \tag{12}$$

The coefficients $g^{L\omega}(R_{2n})$ are determined by solving (10) and by normalizing $\phi_{12}^{LM\omega}$. The function $\Gamma_L$ is defined as

$$\Gamma_L(\beta,\rho,R)=(\pi\beta^2)^{-3/4}\exp[-(\rho^2+R^2)/2\beta^2]i_L(\rho R/\beta^2)\ ,$$

where $i_L$ is a spherical Hankel function. Let us point out that, for each $L$ value, only the $\omega=0$ wave function has a physical meaning. It represents, in a bound-state approximation, the lowest $^8$Be state in the partial wave $L$. In the $\alpha+\alpha$ case, eigenvalues with $\omega>0$ cannot be related to a physical $^8$Be state. In accord with the definitions of the Introduction, the three-cluster model mixes physical states ($\omega=0$) with pseudostates ($\omega\neq0$).

In order to introduce the internal wave functions of $^8$Be in the 12-nucleon wave function (9), we define

$$\tilde{\Phi}_{lL\omega}^{JM\pi}(R_{1m})=\phi_{\text{c.m.}}\mathscr{A}\phi_3[\phi_{12}^{L\omega}\otimes Y_l(\Omega_\rho)]_M^J\Gamma_l(b_1,\rho,R_{1m})\ , \tag{13}$$

and the total wave function (9) reads

$$\Psi^{JM\pi}\simeq\sum_{lL\omega}\sum_m F_{lL\omega}^{J\pi}(R_{1m})\tilde{\Phi}_{lL\omega}^{JM\pi}(R_{1m})\ . \tag{14}$$

The coefficients $F_{lL\omega}^{J\pi}(R_{1m})$ represent a new generator

TABLE I. Bound-state and resonance energies. The energies are expressed with respect to the $\alpha+{}^8$Be threshold.

| | Calculations | | | |
|---|---|---|---|---|
| $J^\pi$ | 1 | 2 | 3 | Expt.[a] |
| $0_1^+$ | −7.01 | −5.65 | −11.07 | −7.36 |
| $0_2^+$ | 0.29 | 0.84 | 0.27 | 0.29 |
| $2_1^+$ | −3.73 | −0.22 | −2.02 | −2.93 |
| $2_2^+$ | 1.90 | 2.24 | 3.07 | 3.80 |
| $4_1^+$ | 5.20 | 5.40 | 5.00 | 6.72 |
| $1_1^-$ | 5.13 | 6.06 | | 3.48 |
| $3_1^-$ | 0.78 | 1.37 | 1.69 | 2.28 |

[a]Reference 13.

function, defined in the basis (13). Therefore, the three-cluster wave functions can be expressed as a linear combination of two-center wave functions. The three-cluster problem is replaced by a multichannel two-cluster problem. The scattering and bound-state wave functions (14) are therefore determined according to the method of Ref. 7.

## III. THREE-CLUSTER STUDY OF $^{12}$C

### A. Conditions of the calculation

The calculations are performed with the interaction $V_2$ (Ref. 12) and an oscillator parameter $b=1.36$ fm. The full three-cluster calculation (referred to as calculation 1) is carried out with a Majorana parameter $m=0.5955$. This choice reproduces the experimental energy of the $0_2^+$ state. The $R_{1m}$ values are located from 1.2 to 10.2 fm, with a step of 1 fm. This set of generator coordinates contains larger values than in usual two-center studies. Such large $R_{1m}$ values are necessary to take account of the important size of the $^8$Be nucleus. The $R_{2n}$ values, relative to $^8$Be, are 2.8, 3.8, and 4.8 fm. This choice allows one to simulate the compressibility of $^8$Be with reasonable computation times. The energy of the ground state is found to be 0.36 MeV above the $\alpha+\alpha$ threshold, which differs from a more accurate value by a few tenths of a MeV only. Hence, the description of $^8$Be is expected to be reliable. For $L=0$, the energies of the pseudostates $\omega=1$ and 2 are located near 13 and 30 MeV, respectively. In order to reduce the computation time, we restrict our study to $L=0$ in (9). With this assumption, Kamimura[10] obtains a valuable description of the $^{12}$C nucleus.

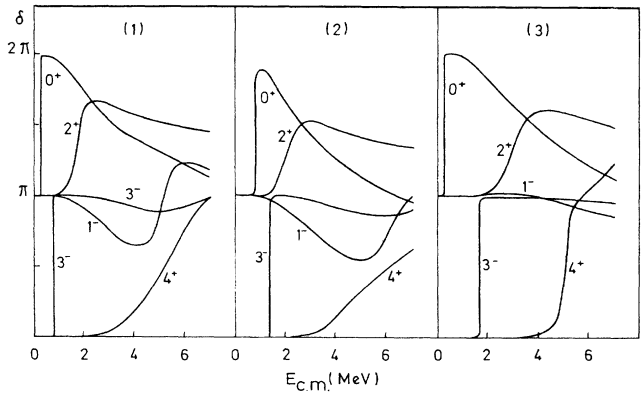


FIG. 1. $\alpha+{}^8$Be phase shifts calculated in the three models. 1, Multichannel three-cluster model; 2, single-channel three-cluster model; and 3, two-cluster model.

The calculation denoted calculation 2 is performed under the same conditions, but the terms with $\omega\neq 0$ in the wave function (14) are dropped. This procedure will allow us to estimate the importance of the role of the pseudostates in the three-cluster wave functions. A third calculation, denoted calculation 3, is performed by describing the $^8$Be nucleus in the one-center $LS$ coupling scheme. The $^8$Be ground state is then the usual $L=S=0$ component of the (4,0) representation of SU(3). Notice that this case corresponds to the limit of a three-cluster calculation with the generator coordinate $R_2$ tending to zero. This approach corresponds to a usual two-center calculation. The $m$ value is chosen as 0.642 in order to reproduce the $0_2^+$ energy of $^{12}$C. The comparison of calculations 1 and 3 is expected to give valuable information on the influence of the $^8$Be description in the capture process. In the following, energies are expressed in MeV and lengths in fm.

### B. Bound-state and resonance properties

In Table I we give the bound-state and resonance energies of the $^{12}$C nucleus. The energy locations and widths of the resonances are extracted from the $\alpha+{}^8$Be elastic phase shifts displayed in Fig. 1, for the three approaches. The energies of calculation 1 are in excellent agreement with Kamimura's. Notice that he uses slightly different values of the oscillator and Majorana parameters. The rotational constant of the ground-state band is better reproduced with the present parameter choice. The phase shifts obtained in calculation 1 suggest identification of the broad $2^+$ and $1^-$ resonances to the experimental $2_2^+$ and $1_1^-$ states. The energy difference between the $1_1^-$ and $3_1^-$ states is overestimated in our model.

As expected from the variational principle, the energies deduced from calculation 2 are systematically higher than in 1. This effect is not really a problem, since it could be compensated by a slight decrease of the Majorana parameter. The phase shifts presented in Fig. 1 are qualitatively equivalent.

In calculation 3 the energy difference between states of the same partial wave is too large. For the sake of completeness, let us add that the binding energy of the $^{12}$C ground state is found to be $-19.6$ MeV, with the Majorana parameter $m=0.5955$. This effect arises from the fact that, because of the choice of identical oscillator parameters for $\alpha$ and $^8$Be, the internal energy of the $^8$Be nucleus is not minimum. Hence the binding energy of $^{12}$C, calculated with respect to the $\alpha+{}^8$Be threshold, is overestimated. This problem is also encountered in the $\alpha+{}^{14}$C system,[9] where the ground-state band is too deeply bound. The phase shifts presented in Fig. 1 show that the $1_1^-$ resonance is not reproduced by a simple two-cluster model. The $0^+$, $2^+$, and $3^-$ phase shifts are not very different from one calculation to another. The $4^+$ phase shift exhibits a narrower resonance in the two-center approach.

We display in Table II the reduced $\alpha$ widths, calculated

TABLE II. Reduced $\alpha$ widths and quadrupole moments of $^{12}C$ states. The reduced $\alpha$ widths are calculated at 7.0 fm. The experimental quadrupole moment for the $2_1^+$ state is $6\pm3$ $e$ fm$^2$. 1–3 refer to calculations 1–3 in the text.

| | $\theta_\alpha^2$ (%) | | | | $Q$ ($e$ fm$^2$) | | |
|---|---|---|---|---|---|---|---|
| $J^\pi$ | 1 | 2 | 3 | Expt.[a] | 1 | 2 | 3 |
| $0_1^+$ | 0.1 | 0.4 | 0.06 | | | | |
| $0_2^+$ | 39.8 | 37.8 | 14.9 | 37.3 | | | |
| $2_1^+$ | 0.1 | 2.4 | 1.5 | | 5.7 | 3.0 | −8.2 |
| $2_2^+$ | 38.2 | 37.1 | 27.7 | 15.0 | −37.6 | −39.0 | −17.4 |
| $4_1^+$ | 31.0 | 31.2 | 11.4 | 7.1 | −63.6 | −67.0 | −10.9 |
| $1_1^-$ | 6.6 | 25.6 | | 10.3 | −7.0 | −11.3 | |
| $3_1^-$ | 4.3 | 6.8 | 1.8 | 5.2 | −15.0 | −16.1 | −10.7 |

[a]Reference 13.

at 7 fm, and the quadrupole moments of low-lying $^{12}C$ states. Table II shows significant differences between the three approaches. Except for the $0_1^+$ state, whose small $\theta_\alpha^2$ value obtained in calculation 3 is due to an overly large binding energy, the bound states have a larger reduced width in calculations 2 and 3. This means that the molecular character of these states is enhanced and often overestimated in single-channel models.[9] On the other hand, the resonant states present closer $\theta_\alpha^2$ values in the three approaches. The agreement between theory and experiment is good. Fortuitously, the reduced width of the $2_2^+$ and $4_1^+$ states is better in calculation 3 than in the others.

The calculation of $\theta_\alpha^2$ requires the knowledge of the wave functions at the channel radius only. On the other hand, the quadrupole moments allow one to test the wave functions over a larger spatial domain. A striking feature is the disagreement between calculations 1 or 2 and calculation 3 for the ground-state band. The sign of the quadrupole moment of the $2_1^+$ state is even incorrect in calculation 3. The results of approximation 3 are characteristic of the dinuclear rotator model, where the quadrupole moments are obtained from an intrinsic quadrupole moment $Q_0$ by

$$Q=-Q_0J/(2J+3)\ .$$

The quadrupole moments of the $2_1^+$ and $4_1^+$ states correspond nearly to $Q_0=30$ $e$ fm$^2$. This result confirms the conclusion drawn from the $\theta_\alpha^2$ values: In calculation 3 the ground-state band has too large a molecular component. For the negative-parity states, the agreement between the three models is relatively good.

### C. $E2$ transitions

The $E2$ reduced transition probabilities calculated in a bound-state approximation[7] are presented in Table III. Our three-cluster calculation fairly reproduces the available experimental data. It also agrees with the microscopic results of Kamimura.[10] The results of calculations 2 and 3 are systematically higher than those of 1, except for the $2_2^+\rightarrow0_1^+$ transition, whose radiation width is probably inaccurate in the bound-state approximation. This effect arises from too important a clustering when the three-alpha structure of $^{12}C$ is not properly taken into account. It is particularly striking for the $0_2^+\rightarrow2_1^+$ transition; since this transition probability plays an important role in the $^8Be(\alpha,\gamma)^{12}C$ cross section, the present comparison shows that a three-cluster model is required for the study of this radiative capture reaction.

The importance of the pseudostates is clearly visible from Table III, by comparing the columns for calculations 1 and 2. It confirms the result obtained by Tang and co-workers,[8] who studied the elastic scattering of different systems composed of $s$ shell nuclei. The conclusion drawn by these authors is that the importance of the distortion is correlated with the compressibility of the clusters involved in the system. The influence of the distortion effects has been investigated by Tang and co-workers on the elastic phase shifts only. Our study shows that, in spite of the fact that these effects do not qualitatively modify the $\alpha+^8Be$ phase shifts, they drastically affect the electromagnetic properties in the $^{12}C$ nucleus. Therefore, radiative-capture calculations involving soft nuclei should be considered with special care.

TABLE III. $E2$ reduced transition probabilities.

| | | $B(E2)$ ($e^2$ fm$^4$) | | | | |
|---|---|---|---|---|---|---|
| | | Calculations | | | | |
| $J_i^{\pi_i}$ | $J_f^{\pi_f}$ | 1 | 2 | 3 | Ref. 10 | Expt.[a] |
| $2_1^+$ | $0_1^+$ | 10.3 | 20.4 | 15.9 | 9.3 | 7.7 |
| $2_2^+$ | $0_1^+$ | 4.1 | 3.0 | 0.8 | | |
| $0_2^+$ | $2_1^+$ | 8.6 | 108 | 98.6 | 5.6 | 13.4 |
| $4_1^+$ | $2_1^+$ | 3.9 | 36.5 | 29.2 | | |

[a]Reference 13.

## IV. THREE-CLUSTER STUDY OF THE $^8$Be$(\alpha,\gamma)^{12}$C REACTION

### A. Capture cross sections

We present in Fig. 2 the astrophysical $S$ factors for the $^8$Be$(\alpha,\gamma)^{12}$C reaction towards the $0_1^+$ and $2_1^+$ states of $^{12}$C. Since the electromagnetic properties of the $^{12}$C states are not well reproduced by calculations 2 and 3, we only display the results deduced from the exact three-cluster model. We use an effective charge $\delta e = 0.12e$ in order to reproduce the correct experimental value of the $B(E2, 0_2^+ \rightarrow 2_1^+)$ which, as we shall see, is an important ingredient of the calculation. Moreover, the experimental energies of the bound $0_1^+$ and $2_1^+$ states are employed in the calculation of the transition energies. The astrophysical $S$ factor relative to the ground state presents a broad peak near 1.75 MeV, corresponding to the $2_2^+$ resonance. Over the energy range considered in Fig. 2, it can be parametrized with good accuracy by the simple Breit-Wigner-type expression

$$S_0 \simeq \frac{5.49}{(E-1.75)^2+(0.28)^2} \text{ keV b} . \quad (15)$$

Notice that the $B(E2, 2_1^+ \rightarrow 0_1^+)$ value is overestimated in our model. Therefore, in order to take account of the experimental value, we recommend replacement of the numerator of $S_0$ by 2.67 keV b. This value will be used in the following.

The capture cross section towards the $2_1^+$ state is dominated at astrophysical energies by the contribution of the $0_2^+$ resonance. A small bump appears near 1.75 MeV, due to the $2_2^+$ resonance. The astrophysical $S$ factor towards the $2_1^+$ state is well parametrized below 1 MeV by

$$S_2 \simeq \frac{0.29}{(E-0.29)^2+(4.72\times10^{-6})^2} \text{ keV b} . \quad (16)$$

Notice that in this case the usual expansion

$$S_2(E) \simeq S_2(0) + ES_2'(0) + \tfrac{1}{2}E^2 S_2''(0)$$

is not accurate for astrophysical purposes. The comparison of $S_2$ and $S_0$ shows that the contribution of the ground-state transition is not negligible as $E$ tends towards zero.

What do the capture cross sections become in the approximate calculations 2 and 3? Since the cross section is mainly dominated by the contribution of the $0_2^+$ resonance, we can estimate the $S$ factor from Tables II and III. The main ingredients necessary to determine the cross section at low energies are the $\theta_\alpha^2(0_2^+)$ and $B(E2, 0_2^+ \rightarrow 2_1^+)$ values. Since the reduced widths are similar in the three models, the differences arise from the $E2$ transition probability. Table III shows that this value is strongly overestimated in calculations 2 and 3. Therefore, the radiative capture cross sections calculated in the approximate models are expected to be unrealistically enhanced by about a factor of 10.

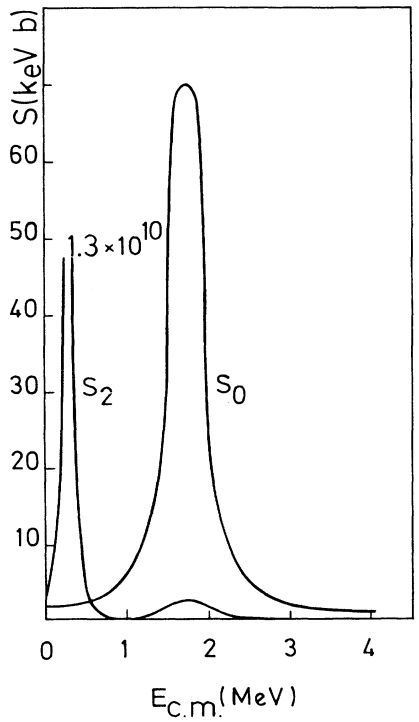


FIG. 2. $^8$Be$(\alpha,\gamma)^{12}$C astrophysical $S$ factors for transitions towards the $0_1^+$ and $2_1^+$ states of $^{12}$C.

### B. Reaction rates

The reaction rates[1] at low temperature $T$, towards the ground and first excited states of $^{12}$C, are given in Table IV. They are obtained by a numerical integration of the capture cross section multiplied by the Boltzmann distribution. We compare our results with those of Nomoto *et al.*[2] and of Langanke *et al.*[3] These authors evaluate the reaction rates from a Breit-Wigner formula with energy-dependent widths[2] plus a direct contribution.[3] Our reaction rate is slightly higher than that of Ref. 2, where the authors neglect the capture towards the ground state. On the contrary, the GCM values are smaller than the results of Ref. 3. This is probably due to an inaccurate estimation of the direct capture rate in Ref. 3. Indeed, since the $S$ factor presents a marked resonant structure, it is not possible to separate the direct and resonant capture rates with accuracy. Hence the direct reaction rate presented in Ref. 3 is probably overestimated.

The low-temperature reaction rates presented in Table IV can be reproduced by using the off-resonance approximation[1]

$$\langle \sigma v \rangle = \frac{4}{kT}\left[\frac{2E_0}{3\mu}\right]^{1/2} S(E_0)\exp\left[-3\frac{E_0}{kT}\right] , \quad (17)$$

where $E_0$ is the Gamow-peak energy,

$$E_0 = [(\mu c^2/2)^{1/2}\pi Z_1 Z_2 e^2 kT/\hbar c]^{2/3} , \quad (18)$$

and $S(E)$ is given by (15) and (16). In (17) and (18), $\mu$ is the reduced mass of the system and $Z_1 e$ and $Z_2 e$ are the charges of the colliding nuclei. In this way, the reaction rates of Table IV are reproduced within 20%. At higher

TABLE IV. $\alpha+^8Be$ reaction rates (in $cm^3 s^{-1}$). The rates are given in powers of 10, where the power is denoted in square brackets.

| $T$ ($10^7$ K) | $0_1^+$ | $2_1^+$ | Total | Ref. 2 | Ref. 3 |
|---|---|---|---|---|---|
| 1 | 8.36[−64] | 4.50[−63] | 5.34[−63] | 3.65[−63] | 9.42[−63] |
| 2 | 3.45[−54] | 2.19[−53] | 2.55[−53] | 1.73[−53] | 4.30[−53] |
| 3 | 1.59[−49] | 1.15[−48] | 1.31[−48] | 9.06[−49] | 2.25[−48] |
| 4 | 1.38[−46] | 1.14[−45] | 1.28[−45] | 8.98[−46] | 2.04[−45] |
| 5 | 1.71[−44] | 1.61[−43] | 1.78[−43] | 1.27[−43] | 2.73[−43] |
| 6 | 6.71[−43] | 7.23[−42] | 7.90[−42] | 5.73[−42] | 1.18[−41] |
| 7 | 1.26[−41] | 1.96[−40] | 2.09[−40] | 1.26[−40] | 2.45[−40] |

temperatures, the usual resonant rate[1-3] due to the $0_2^+$ state should be added to the approximation (17).

## V. CONCLUSION

In this paper we compare the properties of the $\alpha+^8Be$ system obtained in three microscopic approaches. The first one is a three-cluster model in which the distortion of the $^8Be$ nucleus in the collision is realistically included. The second approach freezes the two-cluster $^8Be$ wave function by dropping the pseudoconfigurations. The third model involves a $^8Be$ cluster in its ground-state SU(3) configuration. The spectrum of $^{12}C$ and the $\alpha+^8Be$ phase shifts are not qualitatively modified in the approximate approaches, provided the interaction is readjusted in the third model. However, in calculation 3 the energy difference between two states of the same partial wave is obviously overestimated. The reduced $\alpha$ widths of the $0_1^+$ and $2_1^+$ states, and the $E2$ transition probabilities involving the ground-state band are enhanced in approximate models. Hence radiative-capture calculations of the $^8Be(\alpha,\gamma)^{12}C$ reaction require a full three-cluster model.

Our results extend—to a heavier system—the conclusions of Tang and co-workers,[8] who show the importance of distortion effects in systematic studies of systems involving a soft cluster. Introducing distortion seems to be a general necessity for clusters with high compressibility. Other types of distortion could be considered. It would be worthwhile to compare the relative importance of distortion effects introduced by physical configurations or pseudoconfigurations.

Our microscopic three-cluster model brings information about the $^8Be(\alpha,\gamma)^{12}C$ capture reaction. With respect to the results of Nomoto *et al.*[2] and of Langanke *et al.*,[3] who make use of less sophisticated models, we find intermediate results for the reaction rate. Moreover, we propose a simple parametrization fitting accurately the reaction rate at low temperatures.

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