PHYSICAL REVIEW C VOLUME 36, NUMBER 2 AUGUST 1987

Analysis of primary electric dipole gamma rays from slow-neutron capture by Ca isotopes

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(Received 5 January 1987)

We have analyzed the available data on the primary $E1$ transitions in the thermal-neutron capture by even isotopes of calcium with the aim of determining the relative importance of direct and compound-nucleus mechanisms of capture in these nuclides. In some of these cases, the thermal-neutron scattering length, as well as the spectroscopic factors of the final states, is known, and an absolute estimate of the direct capture cross sections can be made. In these cases (^{40}Ca, ^{44}Ca) the estimates made from a specialized optical model (which reproduces the scattering length precisely) and a global optical model with a valency contribution from a local level are in agreement and are also generally close in absolute value to the experimental cross sections. The detailed deviations can be accounted for as a compound-nucleus contribution to the transitions, which fluctuates, but has an overall magnitude in agreement with Cameron's semiempirical relationship for this mechanism. In both nuclides the compound-nucleus contribution to the capture can plausibly be attributed to a bound local level. For ^{48}Ca there is some indirect evidence for the value of the thermal-neutron scattering length, and the use of this value in the calculations of direct capture leads to agreement with the data. There is no evidence for an important compound-nucleus contribution in this magic number nucleus. In ^{42}Ca and ^{46}Ca the thermal-neutron scattering length can be adjusted to "best fit" the calculation of direct capture from the optical model to the experimental data. After this adjustment the detailed agreement between theory and data is very good, indicating the major predominance of the direct mechanism in capture at thermal-neutron energies by these nuclides.

I. INTRODUCTION

A direct mechanism for the slow-neutron-capture reaction of many nuclides was first recognized and systematically explained almost three decades ago.[1,2] Since then the stock of available data, especially absolute cross sections for individual electric dipole ($E1$) gamma transitions, has increased enormously. Some of these data have been analyzed[3] using a simple approximate formula (the channel-capture formula with some emendations) from Ref. 1, and shown to be reasonably reproduced. It has also long been recognized that more accurate quantitative estimates of direct capture than those of the channel-capture formula can be obtained from an optical-model formulation of the direct-capture mechanism. Recently, definitive analyses of two important sets of slow-neutron-capture gamma-ray data have been published using this optical-model approach. One of these comprises an extensive set of data, mostly recently measured, on all the stable sulfur isotopes.[4] In Ref. 4 the theory of slow-neutron direct capture was fully recapitulated, and the basic method of analysis was formulated. These details are therefore to be found in that paper and are not repeated here. The second publication[5] comprises the analysis of fairly long-standing, but reliable, data on the light compound nuclides ^{10}Be, ^{13}C, and ^{14}C. For nuclides that have a single-particle s state close to the neutron separation energy, complications appear in the use of the relatively straightforward method of analysis adopted in Ref. 1. Hence, more sophisticated variants of that method were adopted in Ref. 5. In the analyses of these two very different sets of nuclides it was shown that the bulk of the primary $E1$ gamma-ray cross sections could be reproduced quantitatively by the direct-capture theory, while individual discrepancies could be explained as due to the contribution to the transition matrix element of a fluctuating compound-nucleus component, the magnitude of which is in agreement with long-established, semiempirical, global estimates for the compound-nucleus radiation widths.

Several other sets of slow-neutron-capture gamma-ray data[6] could yet be profitably analyzed using the optical-model formulation of direct capture. Such sets are indicated by the qualitative signature of a strong correlation between the (d,p) stripping strength and the primary capture-gamma-ray cross section to a sequence of com-

mon final states. One set of nuclides for which this correlation is particularly strong and the data are particularly complete is the even calcium (target) nuclides bridging two major shell closures: ^{40}Ca, ^{42}Ca, ^{44}Ca, ^{46}Ca, and ^{48}Ca. The aim of this paper is to establish the degree to which the cross sections of these isotopes can be described quantitatively by the direct-capture theory and to check whether departures between the theory and the data can be attributed to the statistical compound-nucleus hypothesis.

II. DIRECT-CAPTURE CROSS SECTIONS

The earliest quantitative attempts to analyze thermal-neutron-capture cross-section data relying on the hypothesis of a direct mechanism were those of Lane and Lynn.[1] In that work a very simple formula was established for "pure" direct capture, namely "hard-sphere" capture, in which the scattering wave function of the neutron was assumed to have a node at the radius of the nucleus; the nucleus, in turn, was assumed to be representable as a hard sphere with negligible internal penetration of the neutron wave function permitted. This simple concept could be easily generalized to account for actual cases of neutron scattering, for which the thermal-neutron scattering length is generally different, sometimes strongly so, from the nuclear potential radius; this generalization is known as "channel" capture, because any contribution from the internal component of the neutron-nucleus wave function is ignored. The channel-capture formula—in which the required parameters are the nuclear potential radius, the thermal-neutron scattering length (for each allowed spin state of the neutron-target system), the binding energy of the final state populated by the primary capture gamma ray, and the spectroscopic (single-particle) factor of the final state—was shown to be successful in explaining semi-quantitatively a considerable volume of capture data extant at that time.[1] Since then this formula has been shown by Mughabghab,[3,7–11] and others[12–22] to be approximately valid for many new and more precise data that have been published in the ensuing two decades.

In spite of this success, the channel-capture formula of Ref. 1 can only be regarded as an approximation to a more precise estimate of the cross section obtained from a model of direct capture based on single-particle motion in a nuclear optical-potential well. The diffuseness of the well and the possible effects of the contributions from the internal region must be taken into account. For this reason, when systematic analysis of a large body of accurate neutron capture gamma-ray data on four sulfur isotopes was required, it was considered important to further develop the optical model (or intermediate coupling) concept originally formulated by Lane and Lynn[1] and later discussed by Cugnon and Mahaux.[23] The full account of this treatment can be found in Ref. 4.

In the sulfur work,[4] the optical-model computations were based on a global set of optical-model parameters. This set was capable of reproducing not only the broad trends of neutron strength-function data over the range of light- to medium-weight nuclides but also the general trend of the p-wave single-particle binding energy. Because the model was a global one, it could not reproduce the scattering lengths and the binding energies of all final states in the individual sulfur isotopes. Therefore, a device was used in which the ratio of the computed optical-model direct-capture cross section to the channel-capture cross section (calculated using the computed optical-model potential scattering length and p-wave single-particle binding energy as parameters) was studied over a range of variation of the optical-model parameters. This ratio, denoted by the symbol $C_{\rm opt}$, was found to be very slowly varying (although often considerably different from unity) for the range of optical-model parameters and observables expected to be met in the sulfur isotopes. The final theoretical estimate of the capture cross section of a given transition was then computed from the channel-capture formula using the experimentally known values of scattering length and transition (binding) energy and multiplying this computed value by $C_{\rm opt}$ from an optical model close to the original global one but now approximately reproducing the scattering length and binding energy.

This approach was also attempted in a subsequent study[5] of the thermal-neutron capture by ^{9}Be, ^{12}C, and ^{13}C. In those cases, however, it was found that $C_{\rm opt}$ was often extremely sensitive to small changes in optical-model parameters. To be certain of obtaining a sound estimate of the direct-capture cross section, it was found necessary to vary certain parameters until both the potential scattering length and the final p-state binding energy precisely reproduced the experimental data on thermal-neutron scattering length and transition energy. This parameter set was dubbed a "specialized optical model" for the case in question. The difficulty found with this approach was that certain parameters in the set were being pushed to values that were at the limits of physical acceptability.

This difficulty could be avoided by also using the concept of "valence" capture by the compound resonance states of the neutron-nucleus system. Valence capture is closely related to the direct-capture model inasmuch as its source is the capture amplitude due to the projection of the single-particle s-wave state out of the compound-nucleus wave function (this amplitude connects through the electric dipole operator to the p-wave single-particle content of the final state f). The average radiation width for valence capture [Eqs. (35) and (36) of Ref. 4] arises from the imaginary part of the capture amplitude.

In this approach to the analysis, a global optical model is used to give both the amplitude $\sigma^{1/2}_{\rm pot,\gamma}$ of the direct potential capture and the average valence radiation width $\overline{\Gamma}_{\lambda\gamma({\rm val})}$ as well as the potential scattering length $a_{\rm pot}$. This last quantity will, in general, differ from the thermal-neutron scattering length a_J of the nucleus (in given spin state J) under study. The difference between $a_{\rm pot}$ and a_J gives a measure of the influ-

ence on the scattering due to nearby compound-nucleus levels. This effect is quantified as the local-level contribution $\mathcal{R}_J^{\text{loc}}$ to the $\mathcal{R}$ function[24,25] underlying the neutron scattering:

$$\mathcal{R}_J^{\text{loc}} = \sum_{\lambda(\text{local})} \gamma_{\lambda(n)}^2/(E_\lambda - E) = (a_{\text{pot}} - a_J)/R \ , \quad (1)$$

where R is the potential radius, $\gamma_{\lambda(n)}^2$ the reduced widths, E_λ the eigenvalues of the local levels λ (of spin J), and E the neutron energy for the capture observations (usually thermal neutron energy). The quantity $\mathcal{R}_J^{\text{loc}}$ is directly proportional to the amplitude of the valence capture from the local levels, and the direct-capture cross section is given by (Ref. 5)

$$\sigma_\gamma \approx \sum_J g_J \left| \sqrt{4\pi} R \mathcal{R}_J^{\text{loc}} \left(\frac{\bar{\Gamma}_{\lambda\gamma(\text{val})}}{\Gamma_{\lambda n}} \right)^{1/2} + \sigma_{\text{pot},\gamma}^{1/2} \right|^2 , \quad (2)$$

where g_J is the spin-weighting factor in the initial scattering state and $\Gamma_{\lambda n}$ is the neutron width.

The real part of the global optical potential (with spin-orbit term added) will not, in general, reproduce the actual final-state binding energy. Hence, $\sigma_{\text{pot},\gamma}^{1/2}$ can be computed from the channel-capture formula, substituting therein the actual binding energy and the potential scattering length from the global optical potential and multiplying by C_{opt}, which will normally be quite stable against variation of the optical-model parameters, provided these have not been forced into an unstable regime as described in Ref. 5. Alternatively, the real potential well for the final p state can be adjusted quite separately from the global optical potential that describes the general s-wave neutron-scattering properties, so that the final-state binding energy is reproduced and the resulting value of $\sigma_{\text{pot},\gamma}^{1/2}$ is then used directly in Eq. (2). This approach also has the advantage of obviating the need for making any gamma-ray energy (E_γ) correction to the valence radiation width; the reduced valence radiation strength is

$$s_{\lambda,\text{val}} = \frac{10^{-4}}{E_\gamma^3 \ (\text{in MeV})} \left(\frac{\bar{\Gamma}_n^0}{D_J} \right)^{-1} \times \left(\frac{A}{2Z} \right)^2 \frac{\Gamma_{\gamma,\text{val}}}{D_J} \ , \quad (3)$$

where $\Gamma_n^0 = \Gamma_n/\sqrt{E}$ (E in eV) is the reduced neutron width, D_J the average spacing of levels of spin J at the initial state energy, A the mass number, and Z the charge number of the nucleus. The reduced valence strength is roughly proportional to E_γ^{-1} (see Fig. 12 of Ref. 4).

We have commonly used the above method of a global optical model augmented with local valence contribution in our treatment of capture by the even calcium isotopes. In addition we have used a specialized optical model treatment in which the s-wave optical potential and p-wave real potential are varied independently (but within physically realistic limits) to reproduce the values of the scattering lengths (where known) and the final-state binding energies. The results of both these approaches follow.

III. EXPERIMENTAL DATA ON Ca ISOTOPES

There are five stable isotopes of Ca with even mass numbers. The γ rays from thermal-neutron capture in ^{40}Ca, ^{42}Ca, and ^{44}Ca have been studied by Arnell *et al.*[26] at Stockholm and by Gruppelaar *et al.*[27–29] at Petten. In each isotope, the bulk of the capture cross section[30] is due to a *few* primary $E1$ transitions—six transitions in ^{41}Ca account for 82% of the cross section, eight in ^{43}Ca for 93%, ten in ^{45}Ca for 98%, four in ^{47}Ca for 96%, and two in ^{49}Ca for 100%. The overall agreement between the Stockholm and Petten data sets, particulary concerning the intensity values for these strong transitions, is good. The ^{46}Ca(n,γ) reaction has been studied by Cranston *et al.*[31] at Livermore and the ^{48}Ca(n,γ) reaction by Arnell *et al.*[26] at Stockholm, both with thermal neutrons. The appropriate data from the above studies have been presented in the various tables below; it is with the analysis of the partial cross sections for the strong $E1$ transitions that this paper is primarily concerned.

For our analysis, we need to know the thermal-neutron scattering length $a_{1/2^+}$. Unfortunately, scattering lengths have been measured for only two Ca isotopes; $a_{1/2^+} = 4.9 \pm 0.2$ fm for ^{40}Ca, and $a_{1/2^+} = 1.8 \pm 0.1$ fm for ^{44}Ca. Both measurements were made by Shull and Wollan[32] more than three decades ago at Oak Ridge using Bragg diffraction of monochromatic neutrons scattered on polycrystalline samples. In the case of ^{48}Ca, it is possible to infer an approximate scattering length of $+1.95$ fm or -1.12 fm from an analysis[33] of new total neutron cross section measurements made at the Oak Ridge Electron Linear Accelerator (ORELA). The newer measurements were made with much thicker samples, and the inferred scattering length should be more reliable than the value 2.2 ± 0.8 fm reported from earlier ORELA measurements.[34] The scattering lengths are not known for the remaining isotopes, ^{42}Ca and ^{46}Ca.

We have relied on the existing compilations[35–38] for the energies (E_f), spins (J_f), and parities (π) of the final states in the various Ca isotopes. The only other quantity needed for our analysis is the (d,p) spectroscopic strength $(2J_f+1)S$ for each final state and the corresponding data base is fortunately satisfactory. For ^{40}Ca, ^{42}Ca, and ^{44}Ca, differential (d,p) cross sections have been measured by Belote *et al.*[39–41] at MIT, Brown *et al.*[42] at Aldermaston, and Schär *et al.*[43] at Basel and analyzed with distorted-wave Born approximation

(DWBA) codes. The respective spectroscopic strengths, for the most part, are in good agreement and, therefore, can be averaged. In a similar manner, we have averaged the ^{46}Ca(d,p) strengths deduced by Bjerregaard *et al.*[44] at Copenhagen and Belote *et al.*[45] at MIT. Finally, in the case of the ^{48}Ca(d,p) reaction we have averaged the results of Metz *et al.*[46] at Yale and Abegg *et al.*[47] at Wisconsin, which are mutually consistent. We stress here that there is an overall uncertainty of $\sim$ 20% generally associated with absolute (d,p) strengths derived from comparisons of experimental results with DWBA calculations. This uncertainty will propagate and indeed dominate the uncertainties in our calculated values.

IV. CAPTURE CROSS SECTIONS OF Ca ISOTOPES

A. General Remarks

For our analysis, we have based our choice of a global potential on the work of Moldauer.[48] His choice of optical-model parameters gave an especially favorable fit to slow-neutron strength-function data of near-spherical medium-mass nuclides from $A = 40$ to 140 and, therefore, seems especially appropriate to use in our work. However, the surface-peaked imaginary component of his potential was centered 0.5 fm outside the potential radius. We have chosen to center this term at the potential radius, and have adjusted its magnitude so that the strength-function behavior calculated by Moldauer[48] in the range $A = 40$ to 50 is approximately reproduced. Our (modified Moldauer) global optical potential is

$$\mathcal{U}(r) = \mathcal{V}(r) + i\mathcal{W}_s(r) , \quad (4)$$

where

$$\mathcal{V}(r) = \mathcal{V}_0/\{1 + \exp[(r - R)/d]\} , \quad (5)$$

and

$$\mathcal{W}_s(r) = Z_0 \exp[-(r - R)^2/b^2] , \quad (6)$$

with potential radius $R = 1.16A^{1/3} + 0.6$ fm, real well depth $\mathcal{V}_0 = -46$ MeV, imaginary surface-peak potential $Z_0 = -15$ MeV, surface diffuseness parameter $d = 0.62$ fm, and imaginary surface spreading width $b = 0.7$ fm.

To obtain the wave function of the p-wave single-particle component of the final state, we have employed a real Eckart potential of the form

$$\mathcal{V}_p(r) = \mathcal{V}_1/\{1 + \exp[(r - R)/d]\} + (\ell\cdot\sigma)K_s\mathcal{V}_1 \exp[(r - R)/d]/[rd\{1 + \exp[(r - R)/d]\}^2] , \quad (7)$$

where ℓ is the single-particle orbital angular momentum, σ the neutron spin, and K_s the spin-orbit coupling constant; this last quantity has been set numerically at $K_s = 0.00435$. The magnitude of the p-wave well depth was adjusted, independently of the optical-model parameters, to reproduce the binding energy of the final state of every individual transition studied.

B. ^{40}Ca(n,γ)^{41}Ca

The direct-capture cross sections expected for the primary gamma rays (listed in Table I) in the ^{40}Ca(n,γ) reaction were first calculated using the global optical model plus local valence [G+V] method. The global optical model for ^{40}Ca + n gives a neutron strength-function value $(\Gamma_n^0/D) = 1.87 \times 10^{-4}$ and a potential scattering length $a_{pot} = 2.54$ fm. From the latter value and the known thermal-neutron scattering length of ^{40}Ca we deduce $R\mathcal{R}^{loc}_{1/2^+} = -2.36$ fm to substitute into Eq. (2). The results for the calculated capture cross sections are listed in Table I, column 7.

We then calculated the cross sections for the same transitions using the specialized optical model [S] procedure. We adjusted the real well depth of the modified Moldauer optical model to a value that resulted in a_{pot} being identical to the thermal-neutron scattering length. For ^{40}Ca + n this depth was found to be $\mathcal{V}_0 = -52.4$ MeV. The resulting cross sections are listed in column 8 of Table I. The difference between these and the value calculated by the other procedure [G+V] is less than 6%, thus strongly supporting our confidence in the methods of calculation.

The difference between the theoretical values and the measured capture cross sections (given in column 9 of Table I) is in nearly every case considerably less than the measured cross section, showing that direct capture is the principal mechanism governing the $E1$ transitions following thermal-neutron capture by ^{40}Ca. The differences, where significant, between experimental and calculated cross sections may possibly be attributed to a compound-nucleus component in the capture amplitude from the tails of nearby resonance states. The term "compound-nucleus radiative amplitude" is used here as a generic term for mechanisms involving more general features of the wave functions than the simple projections of neutron motion in the field of the unexcited core of the target. Because of the overall complexity of the wave functions of the states responsible for resonances in neutron cross sections, it is expected that a statistical view of compound-nucleus radiative transitions will be a valid one; in particular, unlike the direct amplitude, the

magnitude and sign of the compound-nucleus component will be quite uncorrelated with the simple single-particle character of the initial and final states as indicated by their elastic and deuteron stripping strengths, respectively. The data on the partial radiative widths of the neutron resonances of a large number of very heavy nuclides clearly exhibit this lack of correlation with resonance neutron width and final state spectroscopic factor, and also exhibit the characteristic statistical feature of belonging to a Porter-Thomas distribution. It follows that a hypothesized compound-nucleus component of the radiative amplitude will be uncorrelated with the direct amplitude, and we may write

$$\sigma_\gamma(\mathrm{X}) = (\sigma_{\mathrm{dir},\gamma}^{1/2} \pm \sigma_{\mathrm{CN},\gamma}^{1/2})^2 , \tag{8}$$

from which we may deduce an estimate of the compound-nucleus contribution. In doing this we have assumed that such contribution is small compared with the direct amplitude, as indicated by the relative closeness of the direct capture estimate and the measured cross sections. The resulting values (all assumed significant, i.e., theoretical and experimental uncertainties are ignored) are presented in column 10 of Table I.

The hypothetical compound-nucleus cross sections thus deduced can be compared with models of possible compound-nucleus mechanisms (such as Brink's model[51] of the damped giant resonance built on each final state) or with semiempirical rules for statistical (i.e., compound-nucleus) model estimates of neutron radiation widths. The best-tested of these is that of Cameron,[49] who gives for the mean partial radiation widths of electric dipole transitions:

$$\Gamma_{J,\gamma,\mathrm{CN}} = 0.33 \times 10^{-9} E_\gamma^3 A^{2/3} D_J . \tag{9}$$

To compare our numbers with this estimate we first divide the entries in column 10 by E_γ^3 and average the result:

$$\langle \sigma_{\gamma,\mathrm{CN}}/E_\gamma^3 \rangle = 0.027 \text{ mb MeV}^{-3} . \tag{10}$$

We then assume that the compound-nucleus capture at thermal-neutron energy is due to a single level; the Breit-Wigner formula gives us

$$\sigma_{\gamma,\mathrm{CN}} \approx \frac{\pi}{k^2} \frac{\Gamma_{\lambda n}\Gamma_{\lambda\gamma,\mathrm{CN}}}{E_\lambda^2} . \tag{11}$$

Factoring the neutron width into $\Gamma_{\lambda n} = 2kR\gamma_{\lambda(n)}^2$ (k being the neutron wave number) we then obtain

$$\frac{\langle \Gamma_{\lambda\gamma,\mathrm{CN}}/E_\gamma^3 \rangle}{E_\lambda} \approx \frac{k \langle \sigma_{\gamma,\mathrm{CN}}/E_\gamma^3 \rangle}{2\pi R \mathcal{R}_{1/2^+}^{\mathrm{loc}}} , \tag{12}$$

in which we have further assumed that $\mathcal{R}^{\mathrm{loc}}$ [defined in Eq. (1), and equal in this case to $-2.36/R$] is due only to the single local level dominating the compound-nucleus capture. Equations (12) and (10) lead to the estimate

$$\frac{\langle \Gamma_{\gamma,\mathrm{CN}}/E_\gamma^3 \rangle}{E_\lambda} \approx -6.4 \times 10^{-9} \text{ MeV}^{-3} . \tag{13}$$

This estimate is to be compared with the Cameron formula [Eq. (9)] for ^{41}Ca:

$$\frac{\langle \Gamma_{\gamma,\mathrm{CN}}/E_\gamma^3 \rangle}{D_{1/2^+}} = 3.8 \times 10^{-9} \text{ MeV}^{-3} , \tag{14}$$

TABLE I. Direct capture cross sections for primary $E1$ transitions in the ^{40}Ca(n,γ) reaction. Columns 1, 2, and 3 give the energy, J^π value, and the (d,p) spectroscopic strength of the final state. Column 4 is the primary transition energy (essentially the binding energy of the final state). The entries in columns 5 and 6 are for idealized final states, i.e., they have not been multiplied by the spectroscopic factor and spin-coupling factor $\mathcal{W}_J$ described in Ref. 4. Column 7 is the calculated cross section (including the last-mentioned factors) using the global plus valence [G+V] procedure. Column 8 contains the calculated cross sections from the specialized optical-model procedure [S]. The experimentally determined cross sections are given in column 9. Finally, column 10 gives the minimum hypothesized compound-nucleus cross sections deduced from the differences between column 7 and column 9.

E_f[a] (MeV)	J_f^π[a]	(d,p)[b] $(2J_f+1)S$	E_γ (MeV)	$(\Gamma_{\gamma,\mathrm{val}}/DE_\gamma^3)$ $\times 10^7$ (MeV^{-3})	$\sigma_{\mathrm{pot},\gamma}$ (mb)	$\sigma_{\mathrm{dir},\gamma}$[G+V] (mb)	$\sigma_{\mathrm{dir},\gamma}$[S] (mb)	σ_γ[X][c] (mb)	$\sigma_{\mathrm{CN},\gamma}$ (mb)
1.943	$3/2^-$	2.53	6.421	0.615	1534	160	158	167 ± 25	0.07
2.462	$3/2^-$	0.90	5.901	0.690	1400	53	53	31 ± 5	2.9
3.614	$1/2^-$	0.22	4.750	0.755	920	10	10	9 ± 2	0.03
3.944	$1/2^-$	1.09	4.419	0.835	854	47	47	86 ± 13	5.8
4.603	$3/2^-$	0.15	3.760	1.228	836	7	7	13 ± 3	0.9
4.753	$1/2^-$	0.35	3.611	1.094	687	13	13	30 ± 5	3.5

[a]Reference 35.

[b]The (d,p) strengths given in Ref. 39 were first renormalized, as recommended in Ref. 35, and then averaged with the values given in Refs. 42 and 43. In Ref. 43, we used their (I) value in Table 2.

[c]From σ_γ = 410 ± 20 mb (Ref. 30) and the average of branchings given in Refs. 26 and 27. The 4.9% uncertainty in the σ_γ value is not included.

indicating that $E_\lambda \approx -0.6D_{1/2^+}$ and $\gamma^2_{\lambda(n)} \approx 0.09D_{1/2^+}$ —an entirely reasonable result supporting the starting hypothesis.

C. ^{42}Ca(n,γ)^{43}Ca

In this case (see Sect. III) the value of the thermal-neutron scattering length is unknown. We have used the global optical model plus valence procedure, varying the scattering length $a_{1/2^+}$ and hence $\mathcal{R}^{\rm loc}_J$ to find a "best fit" between the calculated cross sections and the experimental data. We found that the optimum match was obtained for $a_{1/2^+} = 3.4$ fm. The comparison between theory and experiment is shown in Table II.

There is extremely close agreement between the calculated and experimental cross sections, and this agreement implies again that the direct-capture model is the predominant thermal-neutron-capture mechanism of $E1$ transitions. Of course, the matching patterns of columns 7 and 8 in Table II are no stronger an indication of direct capture than the well known correlation plots of (d,p) final-state stripping factors with partial capture cross sections, except that the agreement in magnitude between experiment and theory, when a physically reasonable estimate of the scattering length is employed in the latter, reinforces the belief in the correctness of the detailed model.

It should be stressed, however, that the close agreement shown in Table II is not to be taken as giving a quantitative estimate for the scattering length.[50] Quite apart from possible systematic uncertainties in experimental data, such as the spectroscopic factors of the final state, leading to distortions of the inferred value of this length, there could be deviations of a stochastic nature due to compound-nucleus components in the capture amplitude (the inherently random nature of these is illustrated in the last column of Table I). Indeed, the difference between the "best-fit" scattering length $a_{1/2^+} = 3.4$ fm, and the global-potential scattering length $a_{\rm pot} = 2.25$ fm indicates a local-level contribution, at thermal-neutron energy, characterized by $R\mathcal{R}^{\rm loc}_J \approx -1.15$ fm, similar in magnitude to that of ^{40}Ca + n. Perhaps in this case, however, the local level (or levels) is much stronger, and proportionately more strongly bound, thus reducing considerably the magnitude of the compound-nucleus amplitude.

D. ^{44}Ca(n,γ)^{45}Ca

We can proceed with the analysis of the ^{44}Ca(n,γ) data along the same lines we took in the ^{40}Ca case. The global optical model gives a neutron strength-function value $(\bar{\Gamma}^0_n/D) = 3.87 \times 10^{-4}$ and a potential scattering length $a_{\rm pot} = 2.19$ fm. The latter value implies that $R\mathcal{R}^{\rm loc}_{1/2^+} = 0.39$ fm. The results of our calculations using the global optical model plus valence procedure are given in Table III. In the alternative procedure, the thermal-neutron scattering length of ^{44}Ca, $a_{1/2^+} = 1.8$ fm, can be reproduced by a specialized optical model in which $\mathcal{V}_0 = -46$ MeV, $d = 0.66$ fm. The calculations from the global + valence [G+V] and specialized [S] procedures agree to within 5%.

There is overall agreement between the theoretical and observed values of the cross section, but there are some individual discrepancies of up to a factor of ~ 2. If these discrepancies are attributed to a compound-nucleus term, we find that

$$\langle \sigma_{\gamma,\rm CN}/E^3_\gamma \rangle \approx 0.05 \text{ mb MeV}^{-3} . \quad (15)$$

We notice, however, that the overall trend of the discrepancies is such that the theoretical values are higher than the experimental ones. For the strongest transitions this discrepancy is about 30% of the experimental value. This trend suggests that there could be a systematic error in the spectroscopic factors of the final states. Indeed, the values given in Ref. 42 are, on the

TABLE II. Direct capture cross sections (calculated for $a_{1/2^+} = 3.4$ fm) for primary $E1$ transitions in the ^{42}Ca(n,γ) reaction. The column headings are explained in Table I.

E_f[a] (MeV)	J^π_f[a]	(d,p)[b] $(2J_f+1)S$	E_γ (MeV)	$(\Gamma_{\gamma,\rm val}/DE^3_\gamma) \times 10^7$ (MeV^{-3})	$\sigma_{\rm pot,\gamma}$ (mb)	$\sigma_{\rm dir,\gamma}$[G+V] (mb)	σ_γ[X][c] (mb)
0.593	$3/2^-$	0.30	7.340	0.685	95	56	48 ± 8
2.046	$3/2^-$	2.73	5.886	0.931	668	407	393 ± 60
2.611	$1/2^-$	0.27	5.322	0.860	49	30	37 ± 6
2.878	$1/2^-$	0.19	5.055	0.928	33	20	18 ± 3
2.943	$3/2^-$	0.20	4.990	1.157	41	28	28 ± 4
3.286	$3/2^-$	0.13	4.647	1.266	30	19	21 ± 5
3.572	$3/2^-$	0.16	4.360	1.371	33	21	25 ± 4
4.207	$1/2^-$	0.85	3.726	1.406	88	57	65 ± 10

[a]Reference 35.
[b]The (d,p) strengths listed are averages of those given in Refs. 40, 42, and 43. In Ref. 43, we used their (I) value in Table 3.
[c]From $\sigma_\gamma = 680 \pm 70$ mb (Ref. 30) and the average of branchings given in Refs. 26 and 29. Before averaging, we multiplied the intensities of the primary γ rays in Ref. 29 by 1.14 such that the sum of the intensities of the primary γ rays is 100 photons per 100 captures. The 10.3% uncertainty in the σ_γ value is not included in calculating the uncertainties.

average, ~ 15% smaller than the values given in column 3 of Table III. If the $(2J_f+1)S$ values are uniformly reduced by 25%, this has the effect of reducing the deduced compound-nucleus magnitude to

$$\langle \sigma_{\gamma,\mathrm{CN}}/E_\gamma^3 \rangle \approx 0.017 \text{ mb MeV}^{-3} . \quad (16)$$

We can take this and the previous value as encompassing the range of the possible compound-nucleus effect. The corresponding compound-nucleus radiation width is in the range

$$\frac{0.9\times10^{-9}}{R\mathcal{R}^{\mathrm{loc}}_{1/2^+}} \lesssim \frac{\langle \Gamma_{\gamma,\mathrm{CN}}/E_\gamma^3 \rangle}{E_\lambda} \lesssim \frac{2.8\times10^{-9}}{R\mathcal{R}^{\mathrm{loc}}_{1/2^+}} \quad (17)$$

if the compound-nucleus capture and $\mathcal{R}^{\mathrm{loc}}_{1/2^+}$ are due to the same single level.

This last assumption is, in fact, unlikely. The value of $R\mathcal{R}^{\mathrm{loc}}_{1/2^+} = 0.39$ fm implied by the difference between the global optical-potential scattering length and the observed thermal-neutron scattering length is small enough in magnitude that it may not be dominated by a single close level. Indeed, the known resonance[30] at 51.6 keV in the ^{44}Ca cross section contributes 0.41 fm to $R\mathcal{R}^{\mathrm{loc}}_{1/2^+}$ but probably only contributes 0.003 mb MeV^{-3} to the average strength $\langle \sigma_{\gamma,\mathrm{CN}}/E_\gamma^3 \rangle$ of the compound-nucleus transitions. The remainder could be due to a very weak level bound by an energy much less than the average level spacing.

E. ^{46}Ca(n,γ)^{47}Ca

The thermal-neutron scattering length of ^{46}Ca is not known. We have, therefore, treated this case in the same way that we treated ^{42}Ca(n,γ), seeking a "best-fit" value for the scattering length. The calculations for the resulting "best-fit" value of $a_{1/2^+} = 3.0$ fm are shown in Table IV. Again, the detailed quantitative agreement between theory and experiment for a physically reasonable value of the scattering length is very good and supports the proposition that the optical model of direct capture is the predominant $E1$ capture mechanism for this nuclide. Nevertheless, we again stress that the value of $a_{1/2^+}$ quoted above cannot be taken as a determination of this quantity; even a minor contribution to the capture amplitude from the compound-nucleus mechanism could give a net overall distortion of the pattern of transition strengths.

F. ^{48}Ca(n,γ)^{49}Ca

A direct measurement of the thermal-neutron scattering length of ^{48}Ca has not been made. However, the total neutron cross section of a thick sample of ^{48}Ca has been measured recently at ORELA from 30 keV to 4 MeV, and an $\mathcal{R}$-matrix analysis of these data up to 500 keV has been done by Johnson.[33] The parameters fitted in this analysis are the $\mathcal{R}$-matrix reduced neutron widths, the energies of the resonances observed within the measured interval, and energy-dependent quantities $\mathcal{R}_J^{\mathrm{ext}}(E)$. In this case, $\mathcal{R}^{\mathrm{ext}}$ was found to be adequately represented by a simple linear function of energy to give the effect of optical-model scattering plus the effect of local levels outside the measured range. Only one s-wave resonance was found; this is at 450 keV, and its reduced width $\gamma^2_{\lambda(n)}$ is 7.03 keV. This resonance makes negligible contribution to the $\mathcal{R}$ function at zero energy. The main contribution to the $\mathcal{R}$ function comes from the $\mathcal{R}_{1/2^+}$ function. Johnson[33] finds two possible linear functions for $\mathcal{R}^{\mathrm{ext}}_{1/2^+}$ that give acceptable fits to the total cross section data. On extrapolation to zero energy, one fit

TABLE III. Direct capture cross sections for primary $E1$ transitions in the ^{44}Ca(n,γ) reaction. The column headings are explained in Table I.

E_f[a] (MeV)	J_f^π[a]	(d,p)[b] $(2J_f+1)S$	E_γ (MeV)	$(\Gamma_{\gamma,\mathrm{val}}/DE_\gamma^3)$ × 10^6 (MeV^{-3})	$\sigma_{\mathrm{pot},\gamma}$ (mb)	$\sigma_{\mathrm{dir},\gamma}$[G+V] (mb)	$\sigma_{\mathrm{dir},\gamma}$[S] (mb)	σ_γ[X][c] (mb)	$\sigma_{\mathrm{CN},\gamma}$ (mb)
1.435	$3/2^-$	0.43	5.980	0.123	1474	122	127	95 ± 10	1.7
1.900	$3/2^-$	2.35	5.515	0.137	1338	604	631	460 ± 46	9.8
2.249	$1/2^-$	0.35	5.166	0.122	1037	70	74	85 ± 10	0.7
2.842	$3/2^-$	0.40	4.573	0.174	1064	81	85	35 ± 5	9.5
3.241	$3/2^-$	0.13	4.173	0.194	950	24	25	21 ± 4	0.1
3.418	$1/2^-$	0.68	3.996	0.173	775	100	105	95 ± 10	0.1
3.783	$3/2^-$	0.11	3.632	0.229	798	17	17	8 ± 3	1.7
	or								
	$1/2^-$			0.195	693	14	15		0.8
3.838	$(1/2)^-$	0.24	3.577	0.199	681	31	33	14 ± 3	3.3
4.616	$1/2^-$	0.40	2.799	0.270	508	38	40	31 ± 5	0.4
5.000	$(1/2)^-$	0.47	2.415	0.321	424	37	39	16 ± 4	4.3

[a]Reference 36.
[b]The (d,p) strengths listed are averages of those given in Refs. 41, 42, and 43. In Ref. 43, we used their (I) values in Table 4.
[c]From $\sigma_\gamma = 880 \pm 50$ mb (Ref. 30) and the average of branchings given in Refs. 26 and 28. The 5.7% uncertainty in the σ_γ value is not included.

TABLE IV. Direct capture cross sections (calculated for $a_{1/2^+} = 3.0$ fm) for primary $E1$ transitions in the ^{46}Ca(n,γ) reaction. The column headings are explained in Table I.

E_f[a] (MeV)	J_f^π[a]	(d,p)[b] $(2J_f+1)S$	E_γ (MeV)	$(\Gamma_{\gamma,\mathrm{val}}/DE_\gamma^3)$ $\times 10^6$ (MeV^{-3})	$\sigma_{\mathrm{pot},\gamma}$ (mb)	$\sigma_{\mathrm{dir},\gamma}$[G+V] (mb)	σ_γ[X][c] (mb)
2.014	$3/2^-$	3.60	5.262	0.186	685	520	548 ± 59
2.875	$3/2^-$[d]	0.51	4.401	0.232	75	60	77 ± 8
4.058	$(1/2)^-$[e]	1.10	3.218	0.290	101	79	70 ± 7
4.809	$3/2^-$ or $1/2^-$	0.28	2.467	0.449	21	16	13 ± 2
				0.397	18	15	

[a]Reference 37 except as noted.
[b]The (d,p) strengths are averages of those given in Refs. 44 and 45.
[c]From $\sigma_\gamma = 740 \pm 70$ mb (Ref. 30) and branchings (Ref. 31). The 9.5% uncertainty in the σ_γ value is not included.
[d]From ^{48}Ca (d,t) measurements by M. E. Williams-Norton and R. Abegg, Nucl. Phys. **291**, 429 (1977).
[e]Based on the dip in cross section at back angles (Ref. 45).

(for the chosen 7.5 fm boundary radius) gives $\mathcal{R}^{\mathrm{ext}}_{1/2^+}(0) = 0.74$, and hence $a_{1/2^+} = 1.95$ fm, while the other gives $\mathcal{R}^{\mathrm{ext}}_{1/2^+}(0) = 1.15$, and hence $a_{1/2^+} = -1.12$ fm. We have employed both of these possible scattering lengths in turn in calculating theoretical values of the direct-capture cross sections of the two observed $E1$ gamma rays in the ^{48}Ca(n,γ) reaction.

The global optical model yields a strength function value $(\bar{\Gamma}^0_n/D) = 6.92 \times 10^{-4}$ at thermal-neutron energy and a potential scattering length $a_{\mathrm{pot}} = 3.38$ fm. With the assumption $a_{1/2^+} = 1.95$ fm, we obtain $R\mathcal{R}_{1/2^+} = 1.43$ fm. The calculated cross sections are given in Table V. The specialized optical-model procedure ($\mathcal{V}_0 = -44$ MeV, $d = 0.67$ fm) yields cross sections that differ by less than 3%. There seems to be reasonable agreement between theory and experiment with $a_{1/2^+} = 1.95$ fm. With the assumption $a_{1/2^+} = -1.12$ fm, we obtain $R\mathcal{R}^{\mathrm{loc}}_{1/2^+} = 4.50$ fm. The calculated capture cross sections using this value of $R\mathcal{R}^{\mathrm{loc}}_{1/2^+}$, are also given in Table V. In this case the two cross sections are greatly overestimated by the theory.

Although the general magnitudes of the observed cross sections support the predominance of the direct-capture mechanism in the ^{48}Ca case, a direct measurement of the ^{48}Ca scattering length and improved measurements of the ^{48}Ca(n,γ) cross sections and the (d,p) spectroscopic factors are required before a fully quantitative assessment can be made.

V. CONCLUSIONS

The calcium isotopes, being fairly light and dominated by the proximity of closed shells of both neutrons and protons, should be good candidates for direct processes to occur. This is especially so for low-energy neutron capture because the unfilled single neutron $2p_{3/2}$ and $2p_{1/2}$ states should be found not far above the ground state, and the $3s_{1/2}$ single neutron state should lie not far above zero binding energy.

Our analysis shows that for all the even calcium isotopes, from ^{40}Ca to ^{48}Ca inclusive, the direct mechanism that can be described as a single $3s_{1/2}$ neutron orbiting in the potential field of the target and falling to a bound $2p_{3/2}$ or $2p_{1/2}$ single-neutron state is the dominant one. Because we have used careful numerical calculations of the direct-capture cross sections based on realistic optical-model potentials, we have been able to study the deviations of theory from experiment. In those cases for which sufficiently full and accurate data are available [including thermal-neutron scattering lengths and (d,p) spectroscopic factors], we have shown that these devia-

TABLE V. Direct capture cross sections (calculated for $a_{1/2^+} = +1.95$ and -1.12 fm) for primary $E1$ transitions in the ^{48}Ca(n,γ) reaction. The column headings are explained in Table I.

E_f[a] (MeV)	J_f^π[a]	(d,p)[b] $(2J_f+1)S$	E_γ (MeV)	$(\Gamma_{\gamma,\mathrm{val}}/DE_\gamma^3)$ $\times 10^6$ (MeV^{-3})	$\sigma_{\mathrm{pot},\gamma}$ (mb)	$a_{1/2^+} = +1.95$ fm $\sigma_{\mathrm{dir},\gamma}$[G+V] (mb)	σ_γ[X][c] (mb)	$a_{1/2^+} = -1.12$ fm $\sigma_{\mathrm{dir},\gamma}$[G+V] (mb)
0.0	$3/2^-$	3.56	5.147	0.221	677	703	818 ± 110	1632
2.023	$1/2^-$	2.06	3.123	0.345	340	188	272 ± 40	398

[a]Reference 38.
[b]The (d,p) strengths are an average of those given in Refs. 46 and 47.
[c]From $\sigma_\gamma = 1090 \pm 140$ mb (Ref. 30) and branchings (Ref. 26). The 12.8% uncertainty in the σ_γ value is not included.

tions can reasonably be attributed to compound-nucleus admixtures to the transition amplitudes, and there is no substantial systematic deviation which would suggest the presence of a significant mechanism, such as semi-direct capture, also correlated to the single-particle properties of the capturing state.

In none of these cases, however, is an observed unbound resonance the obvious candidate for contributing the compound-nucleus admixtures and, hence, the interpretation of compound-nucleus interference cannot be fortified. In two cases, a bound s-wave level would have to be stipulated to fully explain the capture data. In the case of ^{40}Ca(n,γ)^{41}Ca, that bound level appears likely to be a very typical one lying some tens of keV below the neutron binding energy. In the case of the ^{42}Ca(n,γ)^{43}Ca reaction, the level could be much stronger (in terms of its reduced neutron width) and more strongly bound. In contrast, the ^{44}Ca(n,γ)^{45}Ca reaction is probably significantly affected by a very weak level bound by only about 100 eV, while in the ^{48}Ca(n,γ)^{49}Ca reaction (which has a very large s-wave level spacing) there is no evidence as yet for any compound-nucleus effects, which, for their elucidation, would require more accurate values of spectroscopic factors of the final states and very careful observations of the energy dependence of the potential scattering lengths extending to some keV in neutron energy.

ACKNOWLEDGMENTS

The current research was sponsored by the U.S. Department of Energy under Contract W-7405-eng-36 with the University of California (Los Alamos) and DE-AC05-84OR21400 with Martin Marietta Energy Systems, Inc. (Oak Ridge). Both J. E. L. and S. K. are grateful to the Oak Ridge National Laboratory for hospitality during part of this work. The authors thank J. A. Harvey and C. H. Johnson for making available their new ^{48}Ca results.

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