Reduced alpha transfer rates in a schematic model

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(Received 27 January 1987)

The reduced alpha transfer rates are studied microscopically with a schematic model. Results for ground state to ground state alpha transfer reactions are given.

The occurrence of α-clustering inside nuclei has been suggested by various experimental facts. Among them is the α-transfer in direct nuclear reactions.[1]

Recently the reduced α-transfer rates have been studied within the framework of the interacting boson model IBM.[2] While in this paper we try to investigate the α-transfer microscopically with a schematic model in order to clarify the main points of the problem.

We consider a two-level model with spin and isospin independent pairing forces. The two levels having different parities represent, respectively, the levels of neighboring major shells. The short-range pairing forces come from the high multipolarity components in the multipole expansion of the two-body interactions and can therefore be classified into two categories: (1) pairing force between levels of same parity arising from even-l multipole components and (2) pairing force between levels of different parities arising from odd-l multipole components. The first kind of pairing force is mainly responsible for the pairing and hence α-correlation in nuclei, while the second kind is significant to the α-clustering in heavy nuclei.

The model Hamiltonian is as follows:

$$H=H_0(+)+H_0(-)+H_1(+,-)\ , \tag{1}$$

where

$$H_0(\pm)=\pm\epsilon A(\pm)-2\lambda_0\left[\sum_\alpha B^\dagger_\alpha(\sigma,\pm)B_\alpha(\sigma,\pm)+\sum_\mu B^\dagger_\mu(\tau,\pm)B_\mu(\tau,\pm)\right]\ , \tag{2a}$$

indicate, respectively, the individual Hamiltonians of the upper and lower levels, and

$$H_1(+,-)=-2\lambda_1\left[\sum_\alpha B^\dagger_\alpha(\sigma,+)B_\alpha(\sigma,-)+\sum_\mu B^\dagger_\mu(\tau,+)B_\mu(\tau,-)\right]+\text{H.c.} \tag{2b}$$

represents the pairing interaction between the two levels. In the above expressions,

$$A(\pm)=\sum_{mm_sm_t}a^\dagger_{mm_sm_t}(\pm)a_{mm_sm_t}(\pm)\ , \tag{3a}$$

$$B^\dagger_\alpha(\sigma,\pm)=\frac{1}{\sqrt{2}}[a^\dagger(\pm)a^\dagger(\pm)]^{L=0,\ S=1,\ T=0}_{M=0,M_S=\alpha,M_T=0}\ ,$$

$$B_\alpha(\sigma,\pm)=[B^\dagger_\alpha(\sigma,\pm)]^\dagger\ , \tag{3b}$$

$$B^\dagger_\mu(\tau,\pm)=\frac{1}{\sqrt{2}}[a^\dagger(\pm)a^\dagger(\pm)]^{L=0,\ S=0,\ T=1}_{M=0,M_s=0,M_T=\mu}\ ,$$

$$B_\mu(\tau,\pm)=[B^\dagger_\mu(\tau,\pm)]^\dagger\ , \tag{3c}$$

represent the nucleon number and the creation and annihilation of nucleon pairs with $L=0$, $T=0$, $S=1$, $M_s=\alpha$ and $L=0$, $S=0$, $T=1$, $M_T=\mu$, respectively. It is assumed that $2\epsilon>2\lambda_0>2\lambda$ and $H_1(+,-)$ can be treated as a perturbation term.

In case the proton and neutron Fermi levels lie in the same major shell, the ground states and low-lying excited states have all protons and neutrons situated on the lower level in zero-order approximation. For even-even systems with seniority $v=0$, they can be generally written as[3]

$$|\Psi^{(0)}\rangle=\sum_{\eta_\beta=0}^{n}C^{(n)}_{\eta_\beta}|\eta_\beta\rangle\ , \tag{4a}$$

$$|\eta_\beta\rangle=[C^\dagger(-)]^{\eta_\alpha}[D^\dagger(-)]^{\eta_\beta}(S_+)^{S+M_s}(T_+)^{T+M_T}\times[B^\dagger_{-1}(\sigma,-)]^S[B^\dagger_{-1}(\tau,-)]^T|0\rangle\ , \tag{4b}$$

where

$$C^\dagger(\pm)=\tfrac{1}{2}\{[B^\dagger(\sigma,\pm)B^\dagger(\sigma,\pm)]^{00}-[B^\dagger(\tau,\pm)B^\dagger(\tau,\pm)]^{00}\}\ , \tag{5a}$$

$$D^\dagger(\pm)=\tfrac{1}{2}\{[B^\dagger(\sigma,\pm)B^\dagger(\sigma,\pm)]^{00}+[B^\dagger(\tau,\pm)B^\dagger(\tau,\pm)]^{00)}\}\ , \tag{5b}$$

$$S_+=-\sqrt{2l(+)+1}[a^\dagger(+)\tilde{a}(+)]^{10}_{10}-\sqrt{2l(-)+1}[a^\dagger(-)\tilde{a}(-)]^{10}_{10}\ , \tag{5c}$$

$$T_+=-\sqrt{2l(+)+1}[a^\dagger(+)\tilde{a}(+)]^{01}_{01}-\sqrt{2l(-)+1}[a^\dagger(-)\tilde{a}(-)]^{01}_{01}\ , \tag{5d}$$

$$4(\eta_\alpha+\eta_\beta)+2(T+S)=Z+N\ . \tag{5e}$$

$C^\dagger(C)$ is just the creation (annihilation) operator of the α-cluster. Set $|\eta_\beta\rangle$, together with another set $|\overline{\eta}_\beta\rangle$, form a biorthonormal set. It is easier to solve the problem with such a biorthonormal set. The eigenenergies found are[3]

$$E^{(0)}=2\left[-\epsilon-\lambda_0-\frac{3\lambda_0}{2l(-)+1}\right]\frac{Z+N}{2}+\frac{\lambda_0}{2l(-)+1}\left[\frac{Z+N}{2}\right]^2+\frac{\lambda_0}{2l(-)+1}(T+S+2n)(T+S+2n+4)\ , \qquad (6a)$$

$$T+S+2n=\left|\frac{Z-N}{2}\right|+\left[0,2,4,\cdots\left[\frac{2+N}{2},2[2l(-)+1]-\frac{Z+N}{2}\right]_<\right]\ , \qquad (6b)$$

which is a function of $T+S+2n$, a quantum number of the Casimir operator of the SU(4) group. Nuclear states with the same value of $T+S+2n$ constitute a band in the gauge space and ground states of nuclei with the same value of $|Z-N|/2$ constitute a band with the least possible value of $T+S+2n$.

The reduced α-transfer rates are defined as

$$\begin{aligned}B(Z-2,N-2\rightarrow Z,N)&=|\langle\Psi(2-2,N-2)|C(+)+C(-)|\Psi(Z,N)\rangle|^2\\&=B(ZN\rightarrow Z-2,N-2)=|\langle\Psi(Z,N)|C^\dagger(+)+C^\dagger(-)|\Psi(Z-2,N-2)\rangle|^2\ .\end{aligned} \qquad (7)$$

We see from Eqs. (4) and (5) the selection rule for α transfer is $\Delta(T+S+2n)=0$. α-transfer just occurs between states within the same band. The reduced α-transfer rates between ground states of nuclei with same $|Z-N|/2$ are found in zero-order approximation as

$$B_0(Z-2,N-2\rightarrow Z,N)=B_0(Z,N\rightarrow Z-2,N-2)=\begin{cases}\frac{1}{12}Z(N+4)\left[1-\frac{Z-6}{2(2l(-)+1)}\right]\left[1-\frac{N-2}{2(2l(-)+1)}\right], & N\geq Z\ ,\\ \frac{1}{12}N(Z+4)\left[1-\frac{N-6}{2(2l(-)+1)}\right]\left[1-\frac{Z-2}{2(2l(-)+1)}\right], & N<Z\ .\end{cases} \qquad (8)$$

The pairing force between levels of different parities will induce cross shell excitations of correlated nucleon pairs. But the corrections due to such an effect are very small in our case.

In Fig. 1, curves representing the variation of the reduced α-transfer rate versus the proton number for ground states of nuclei with constant value of $N_\pi+N_{\tilde{\nu}}=2[2l(-)+1]-(N-Z)$ are given. These states constitute a band in the gauge space with $T+S+2n=|Z-N|/2$. The reduced α-transfer rate attains its maximum value when the combined effect of the proton and neutron pairing is largest.

Recently, Frank[2] obtained results similar to those shown in Fig. 1 within the framework of the IBM. It is assumed in the IBM that (1) the ground states of nuclei with constant value of $N_\pi+N_{\tilde{\nu}}$ form an F multiplet and (2) the α transfer is directly related to the transformation of π and $\tilde{\nu}$ pairs. Now we see clearly from discussions given above the implications of these two assumptions.

In cases where the proton and neutron Fermi levels lie in neighboring major shells, the zero-order solutions for even-even systems with seniority $v=0$ can be obtained by generalizing Eq. (4) to two levels. α transfer can only occur between states within the same band characterized by quantum numbers for each of these two levels. The ground states can be expressed as

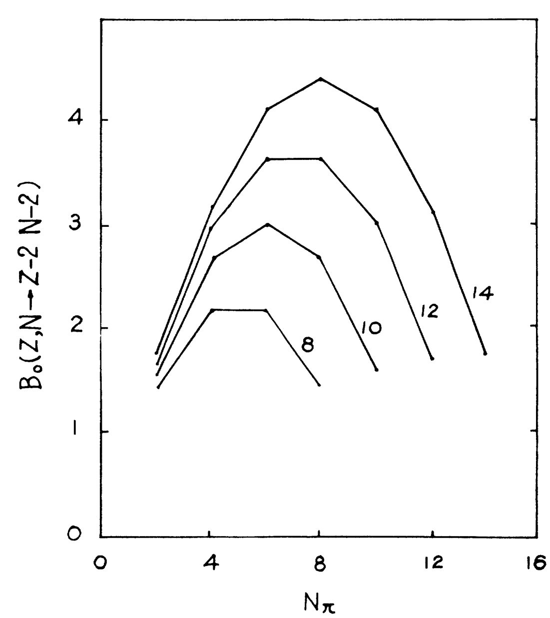

FIG. 1. Reduced α-transfer rates $B(Z,N\rightarrow Z-2,N-2)$ between ground states of nuclei with same value of $N_\pi+[4l(-)+2-N_\nu]$ as indicated in the figure, $N_\pi,N_\nu<4l(-)+2$. $\epsilon=3.5$ MeV, $\lambda_0=1.0$ MeV, $\lambda_1=0.5$ MeV, $l(-)=5$, $l(+)=6$.

$$|\Psi_0^{(0)}\rangle=[B_{+1}^{\dagger}(\tau,+)]^{N_\nu/2}[B_{-1}(\tau,-)]^{N_{\tilde{\pi}}/2}|\phi_0\rangle\ , \quad (9)$$

where $|\phi_0\rangle$ is the reference state with the lower level fully occupied and the upper level fully unoccupied, N_ν is the number of neutrons in the upper level and $N_{\tilde{\pi}}$ the number of proton holes in the lower level. Different from the previous case, ground states of nuclei with same $N_\nu+N_{\tilde{\pi}}=|Z-N|$ but different N_ν and $N_{\tilde{\pi}}$ belong to different bands in the gauge space. α transfer is forbidden in the zero-order approximation. It is necessary to consider further the first-order perturbation.

With the help of the eigensolutions of $H_0(+)+H_0(-)$ obtained as in the previous case, we can readily carry out the perturbation. The correction to the energy of the system from the perturbation is very small. Hence

$$E_0\approx E_0^{(0)}\ . \quad (10)$$

Indeed, the analysis on binding energies of nuclei did not show any clear evidence of α correlation in heavy nuclei.[4]

The correction to the reduced α-transfer rate is very significant since the zero-order value vanishes itself. We have

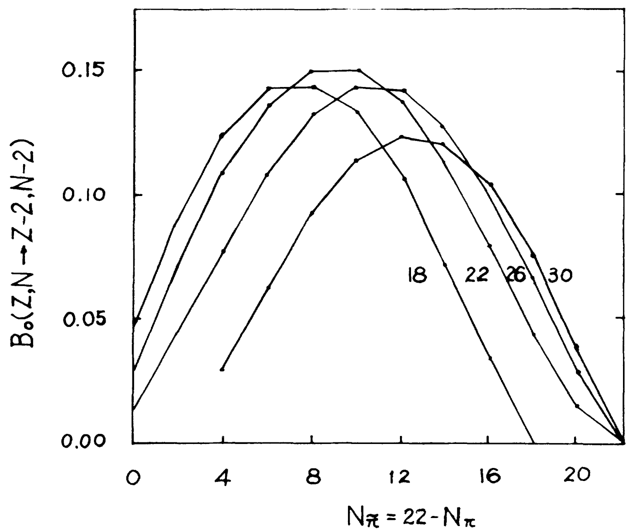

FIG. 2. Reduced α-transfer rates $B(Z,N\rightarrow Z-2,N-2)$ between ground states of nuclei with same value of $N_\nu-N_\pi$ as indicated in the figure, $N_\pi<4l(-)+2$, $N_\nu>4l(-)+2$. $\epsilon=3.5$ MeV, $\lambda_0=1.0$ MeV, $\lambda_1=0.5$ MeV, $l(-)=5$, $l(+)=6$.

$$\begin{aligned}B_0(Z,N\rightarrow Z-2,N-2)&=B_0(Z-2,N-2\rightarrow Z,N)\\&=|C_\alpha(Z,N)\langle\Psi_\alpha^{(0)}(Z,N)|C^\dagger(+)|\Psi_0^{(0)}(Z-2,N-2)\rangle\\&\quad+\langle\Psi_0^{(0)}(Z,N)|C^\dagger(-)|\Psi_{\tilde{\alpha}}^{(0)}(Z-2,N-2)\rangle C_{\tilde{\alpha}}(Z-2,N-2)|^2\ ,\end{aligned} \quad (11)$$

where $|\Psi_\alpha^{(0)}\rangle$ and $|\Psi_{\tilde{\alpha}}^{(0)}\rangle$ are zero-order solutions of $H_0(+)+H_0(-)$ with one α cluster in the upper level and one α-cluster hole in the lower level, respectively, c_α and $c_{\tilde{\alpha}}$ are the corresponding coefficients. The results for ground states of nuclei are as follows:

$$\begin{aligned}B_0(Z,N\rightarrow Z-2,N-2)&=B_0(Z-2,N-2\rightarrow Z,N)\\&=(N_{\tilde{\pi}}+2)\left[1-\frac{N_{\tilde{\pi}}}{2[2l(-)+1]}\right]N_\nu\left[1-\frac{N_\nu-2}{2[2l(+)+1]}\right]\\&\quad\times\frac{1}{3}\lambda_1^2\left[\frac{1}{\Delta E_\alpha(N_{\tilde{\pi}},N_\nu)}\left[1+\frac{2}{2l(+)+1}\right]+\frac{1}{\Delta E_{\tilde{\alpha}}(N_{\tilde{\pi}}+2,N_\nu-2)}\left[1+\frac{2}{2l(-)+1}\right]\right]\ ,\end{aligned} \quad (12)$$

where

$$\Delta E_\alpha(N_{\tilde{\pi}},N_\nu)=4(\epsilon-\lambda_0)-\frac{8\lambda_0}{2l(+)+1}+\frac{12\lambda_0}{2l(-)+1}+\frac{2\lambda_0N_{\tilde{\pi}}}{2l(-)+1}\ , \quad (13a)$$

$$\Delta E_{\tilde{\alpha}}(N_{\tilde{\pi}},N_\nu)=4(\epsilon-\lambda_0)+\frac{4\lambda_0}{2l(-)+1}+\frac{2\lambda_0N_\nu}{2l(+)+1}\ . \quad (13b)$$

The α-transfer in heavy nuclei is directly related to the cross shell excitation of correlated nucleon pairs and hence increases with the increase of λ_1 and decrease of ϵ. The α correlation is based on the existence of proton and neutron pairs and hence increases with λ_0. These can be clearly seen from Eqs. (12) and (13). The reduced α-transfer rates depend also on the value of N_ν and $n_{\tilde{\pi}}$. As shown in Fig. 2, the reduced α-transfer rate between states within the band attains its maximum value when the combined effect of proton and neutron pairing is largest. However, the absolute magnitude is much smaller than that of the previous case.

It should be noted that consideration of merely one kind of odd-l multipole force is unable to give the proper amount of α correlation and the coherent superposition of the odd-l multipole components represented by the pairing force between levels of different parities is requisite for the α clustering in heavy nuclei.

[1]K. Bethge, Ann. Rev. Nucl. Sci. **20**, 255 (1970). F. D. Becchetti, J. Jänecke, J. Overway, J. D. Cossairt, and R. L. Spross, Phys. Rev. C **19**, 1775 (1979).
[2]A. Frank, Phys. Rev. C **34**, 351 (1986).
[3]Ren Zhong-zhou and Xu Gong-ou, Phys. Energ. Fort. Phys. Nucl. (China) (to be published).
[4]A. S. Jensen, P. G. Hansen, and B. Jonson, Nucl. Phys. A **431**, 394 (1984).