Sum rule for two-particle excitation processes in heavy-ion reactions

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A measure of the strength available for the excitation of collective pair transfer channels in heavy-ion reactions is introduced. This combines direct pair pickup and stripping processes, and is expressed in the form of an energy-weighted sum rule. Distributions of strength and of coupling matrix elements in the case of ^{208}Pb are presented.

Formalisms for describing the excitation of two-particle transfer modes are normally presented in a way in which the structural aspects of the problem are closely linked with the reaction mechanism which is used to probe them. Microscopic calculations, which keep track of the transferred nucleons, invariably bring the structure of the projectile into the picture. Thus, it is often difficult to trace properties which should be attributed to only one of the reaction partners, such as enhanced transfer probabilities resulting from residual interactions. Difficulties of this sort have been encountered in previous attempts to introduce sum rules for two-particle transfer reactions.[1,2]

An alternative to this traditional approach can be obtained by assuming that a description in terms of elementary modes of excitation of the target alone is meaningful. To this end one may use generalized degrees of freedom which do not respect particle-number conservation. These have been exploited[3] to characterize the special relation of a nuclear system with those neighbors in the mass table whose particle numbers differ by ± 2, ± 4, etc. We emphasize that a study along these lines centers on the isolated target nucleus. Consequently, the focus is placed on quantities which are not affected by particular range of Q values, separation distances, overlaps, etc., which follow from a given choice of projectile and bombarding energy. The adoption of this point of view eases the implementation of a simple picture to describe the reaction mechanism. Defining a collective coordinate for transfer and modeling the local pair transition density, macroscopic form factors can be introduced.[4]

Within this context, a natural step to take is to exploit the generalized one body character of the fields to define a measure of the total excitation strength. This is a familiar procedure for residual interactions which generate density fluctuations.[3] In the particle transfer case and for a monopole field of the form

$$F=\sum f(r_k)Y_{00}(\theta_k,\phi_k) ,$$

the relevant operators are

$$\hat{F}^{\dagger}=\sum \langle \alpha \mid F \mid \alpha \rangle [a_{\alpha}^{\dagger}a_{\alpha}^{\dagger}]_0$$

and its Hermitian conjugate. Here, α stands for $n_\alpha, l_\alpha, j_\alpha$ and we have assumed that the interaction does not connect states with different values of n. The square brackets indicate coupling to total angular momentum zero.

To establish a point of contact with the case of inelastic excitations, we introduce the Hermitian combination

$$\hat{F}_H=(\hat{F}+\hat{F}^{\dagger})/2 ,$$

which conserves the number of particles only as an average. An energy-weighted sum rule can then be obtained through the expectation value

$$\begin{aligned} S&=2\langle 0 \mid [\hat{F}_H,[\hat{H},\hat{F}_H]] \mid 0\rangle \\ &=\sum (E_{n+}-E_0)\mid \langle n_+ \mid \hat{F}^{\dagger} \mid 0\rangle \mid^2 \\ &\quad +\sum (E_{n-}-E_0)\mid \langle n_- \mid \hat{F} \mid 0\rangle \mid^2 , \end{aligned}$$

where $E_{n\pm}$ are energies in the $(A\pm 2)$-particles systems and E_0 is the energy of the reference state in the A-particle system. While the sum on the right-hand side of this equation combines transition matrix elements connecting systems with different numbers of particles, we evaluate the expectation value on the left-hand side as usually done for an ordinary (number conserving) Hermitian one-body field. Thus we get[3]

$$S=(\hbar^2/2\pi M)\mathcal{N}\langle (df/dr)^2\rangle ,$$

where the angular brackets represent the average per particle in the ground state of the system with $\mathcal{N}$ particles of mass M.

The excitation energies $(E_{n\pm}-E_0)$ in the absence of correlations correspond to the use of an independent-particle Hamiltonian of the form $\hat{H}=\sum \epsilon_k a_k^{\dagger}a_k$. The single-particle energies are extracted from the relative binding energies of the odd systems, i.e.,

$$(E_{n\pm}-E_0)_U=\mp 2[B_n(A\pm 1)-B_0(A)] .$$

The correlated excitation energies are instead identified with the states of the $A\pm 2$ systems through the analogous expressions

$$(E_{n\pm}-E_0)_C = \mp[B_n(A\pm2)-B_0(A)] \ .$$

With these standard prescriptions the energies corresponding to the bound states of the $A+2$ system are negative. Sometimes, however, an alternative definition of the energies is introduced by subtracting from the absolute scale a term which is linear in the number of particles.[3] Choosing the origin of the single particle levels in the middle between the occupied and unoccupied shells, all two-particle energies become positive. The estimation of the sum rule is not affected by the substitution $\hat{H}\rightarrow\hat{H}-\epsilon_F\hat{N}$, insofar as we take the expectation value of the number-conserving version of the field F. Consistent with this step, the value of S should be rather insensitive to the convention adopted to define the energy weighting factors.

More than the sum rule in itself, one is often interested in the distribution of strength which makes up its total value. This function depends on the particular form of the operator $\hat{F}$ since the relevant matrix elements are given by

$$\langle n \mid \hat{F} \mid 0\rangle = \int f(r)\delta\rho_n(r)dr \ .$$

As we see from the expression above, however, the field $f(r)$ is common to all terms and thus the shape of the distribution actually reflects characteristics of the transition densities $\delta\rho_n(r)$ corresponding to the different states. The choice of the radial dependence $f(r)$ of the operator $\hat{F}$ can then be inspired by the features of the transition densities one wishes to emphasize. Since the applications in mind correspond to the construction of transfer form factors whose scale is determined by the magnitude of the transition densities in the surface region, in what follows we use $f=[d\rho(r)/dr]/\rho_0$, where ρ is the density function with Woods-Saxon shape and conventional parameters.[5] In Fig. 1 we display the sum rule S as a function of A for both protons and neutrons.

As an illustration of the use of these concepts, we analyze neutron pair excitations in ^{208}Pb. For this purpose

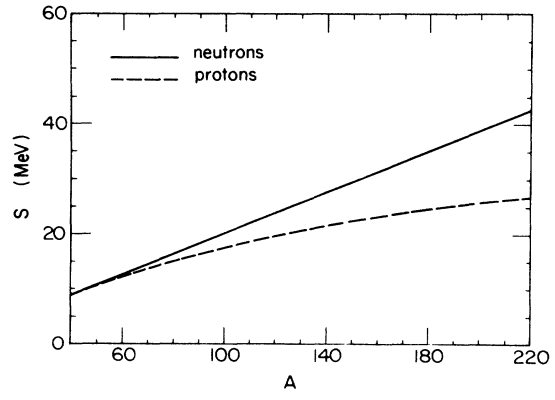

FIG. 1. Sum rule S (in MeV) as a function of the mass number A for protons (dashed line) and neutrons (solid line). The field used is of the form $[1+\exp(r-1.1A^{1/3})]^{-1}$ and the division between proton and neutron contributions was achieved by assuming $Z/A=[0.5-0.106(A-40)/170]$, which is geared to give $Z=N$ for $A=40$ and $Z=(\frac{82}{126})N$ for $A=210$.

we carried out a microscopic calculation within the random-phase-approximation (RPA) formalism using a residual interaction of the form $\hat{V}=-G\hat{F}^\dagger\hat{F}$ and single-particle levels generated with the parametrization of Ref. 6. The energy of the levels close to the Fermi surface were taken from experiment. The radial matrix elements for a pure configuration were constructed as

$$\left\langle 0 \left| \frac{[a_\alpha a_\alpha]_0}{\sqrt{2}}\hat{F}^0 \right| 0 \right\rangle = \left[\int R_\alpha(r)^2 f(r) r^2 dr \right] \times Z_\alpha \frac{\sqrt{2J_\alpha+1}}{\sqrt{2\pi}} \ ,$$

where $R_\alpha(r)$ is the single-particle radial wave function. Diagonal matrix elements like the one listed above are known to retain unrealistic high values within a discrete basis which extends high into the continuum. This may present a convergence problem even for a surface-peaked function like $f(r)$. The contributions of the high-energy configurations are cut down by the attenuation factor Z_α, taken from Ref. 7. The strength of the residual interaction is adjusted to reproduce the empirically observed energy of the ground state of ^{210}Pb. The energy for the ground state of ^{206}Pb is then predicted at -14.16 MeV, to be compared with the experimental value of -14.10 MeV.

The sum rule displayed in Fig. 1 is estimated in a model independent way. Therefore, the accumulated strength should not be dependent on the magnitude of the coupling.[8] The values accumulated for the uncorrelated and correlated strengths are in our case $S_U=38.6$ MeV and $S_C=38.3$ MeV, respectively. The calculations were made including 12 major oscillator shells. By adding or subtracting a shell, the results obtained were found to be stable within 12%. Note that the quantities S_U and S_C, calculated in this space, are thus close to the value of the neutron sum rule shown in Fig. 1, namely $\sim$40 MeV.

In Fig. 2 we display the distribution of uncorrelated and correlated energy weighted strength. A redistribution of strength is induced by the residual interaction mostly around the ground states of the $A\pm2$ nuclei. The rest of the strength remains rather unchanged.

To eliminate the distortion introduced by the energy weighting, we show in the lower part of Fig. 2 the distribution of the matrix elements $|\langle n \mid \hat{F} \mid 0\rangle|^2$. The histogram displays prominently the twin peaks corresponding to the ground states of ^{206}Pb and ^{210}Pb. Note that the magnitude of the transition matrix elements reveals the underlying symmetry between the two states. This result lends support to the macroscopic interpretation of the two levels as components of a single excitation mode. The combination of addition and removal modes is indeed used in the introduction of a macroscopic variable—the number of particles—to describe collective pair transfer.

Besides the ground state transitions, additional coupling strength is found at higher excitation energies in both the $A-2$ and $A+2$ systems. This aspect may be related to the high-lying resonances discussed in Ref. 9. The structures obtained in our calculations do not seem to be of collective character, and their coupling strengths are, at most, about a third as large as compared to the ground

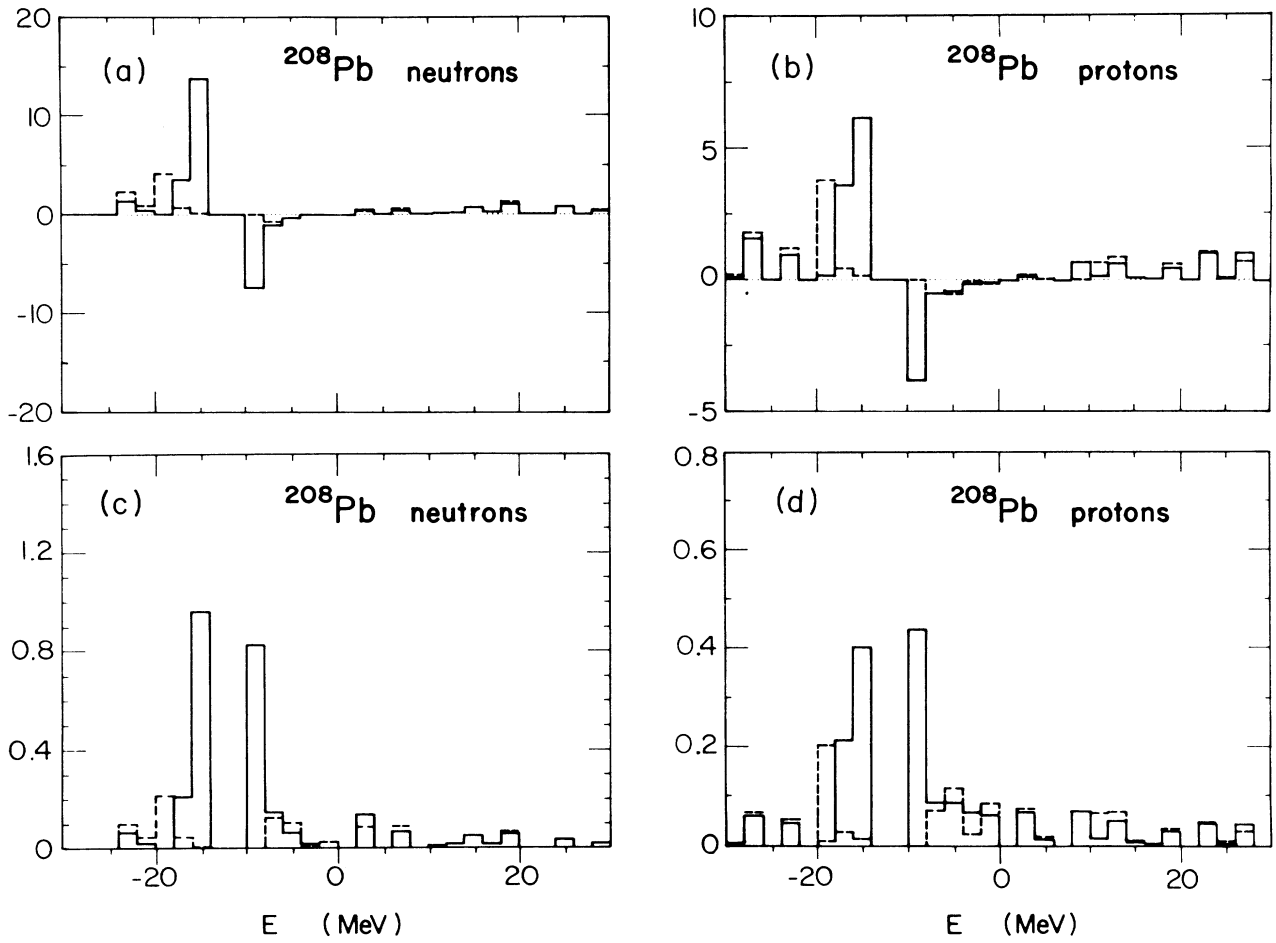

FIG. 2. (a) Distribution of the energy-weighted strength as a function of energy E (both in MeV), for neutron pair excitations in ^{208}Pb. The ground state energies are $E(^{206}\text{Pb}) = -14.11$ MeV and $E(^{210}\text{Pb}) = -9.12$ MeV. The dashed and solid lines correspond to the uncorrelated and correlated contributions, respectively. The areas of both cures are $S_U \cong S_C \cong 38$ MeV. (b) Same as (a), but for protons. The ground state energies are $E(^{206}\text{Hg}) = -15.38$ MeV and $E(^{210}\text{Po}) = -8.78$ MeV. The areas of both curves are $S_U \sim S_C \sim 29$ MeV. (c) Distribution of matrix elements $\langle n \mid \hat{F} \mid 0 \rangle^2$ as a function of energy E (in MeV) for the same case as shown in (a) above. Dashed and solid curves correspond to the uncorrelated and correlated matrix elements, respectively. (d) Same as (c), but for protons.

state transitions. Nevertheless, for a convenient reaction Q value and selected bombarding energies they may effectively compete with the more collective ground state transitions.[9]

As a further check we have investigated the proton pair transitions in ^{208}Pb. The calculations were carried out as described above, adjusting the coupling constant G in this case to fit the ground state energy of ^{210}Po. The results of the RPA calculation were again found to be satisfactory, since the calculated ground state energy of ^{206}Hg turned out to be -15.39 MeV, as compared with the empirical value -15.38 MeV. The uncorrelated and correlated strength content in the proton configurations were $S_U = 29.1$ MeV and $S_C = 29.0$ MeV, respectively. These quantities, calculated within 11 shells, correspond closely to the value of ~ 25 MeV extracted from the proton curve in Fig. 1. We have also repeated the calculations using different origins for the single-particle energies. The distributions of energy-weighted strength are, of course, modified, but the total area of the curves is not significantly affected. The quantities of physical interest [i.e., the distribution of matrix elements displayed in Figs. 2(c) and 2(d)] are, on the other hand, independent of the energy scale.

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