$\Delta(1232)$ contribution to three-nucleon force

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A Δ-mediated effective three-nucleon potential which takes into proper account of the Δ-propagation effects is derived. A reliable estimate of the Δ contribution to three-body force effects can be obtained only if this representation of the three-nucleon potential is used.

The calculation of the effects of three-nucleon force in the bound three-nucleon system has recently become quite sophisticated and reliable.[1-8] It appears to be a good time to reexamine more critically the approximations used in deriving these three-nucleon potentials (3NP's). In this Brief Report we would like to do this for the type of three-nucleon force which arises from the excitation of a bound nucleon to a virtual Δ isobar.

Two approaches have been used to treat the effects of Δ-mediated three-nucleon force. The traditional approach is to obtain an effective three-nucleon potential for Δ to be used in pure nucleonic Hilbert space.[9-11] The second approach[1] is to enlarge the Hilbert space by including explicitly the Δ's degrees of freedom in coupled channels. It was found[12] in the resulting coupled-channel calculations that proper inclusion of the propagation of Δ substantially reduces the Δ effects. However, static approximation, i.e., the kinetic energy in the propagation of Δ is neglected, is widely used in the effective operator treatment. Most of the recent "state of the art" Faddeev calculations[2-8] of the three-body force effects are performed in pure nucleonic Hilbert space. Thus we would like to expound the point of including the propagation of Δ in the effective potential approach so that the Δ's contribution to the 3NP would be properly evaluated.

It is instructive to see first how the static approximation is commonly made in the derivation of the Δ-mediated 3NP. The excitation of a negative energy Δ in the intermediate states gives a negligible contribution to the 3NP and we need to consider only the process of exciting a nucleon into positive energy Δ states as shown in Fig. 1. With the following effective Lagrangian as used in Refs. 9 and 11 (i.e., $Z=-\frac{1}{2}$ in the notation of Ref. 11),

$$L_{\pi NN}=\left[\frac{f_{\pi NN}}{m_\pi}\right]\bar{N}\gamma_\mu\gamma_5 N\partial^\mu\pi \ , \tag{1}$$

$$L_{\pi N\Delta}=\left[\frac{f_{\pi N\Delta}}{m_\pi}\right]\bar{\Delta}^\mu g_{\mu\nu}N\partial^\nu\pi+\text{H.c.} \ , \tag{2}$$

one can easily write the T matrix for the process depicted in Fig. 1 as

$$T^{(\Delta,+)}\sim\left[\frac{f_{\pi NN}}{m_\pi}\right]^2[\bar{u}(p_2')\not{q}'\gamma_5 u(p_2)]\frac{1}{q'^2-m_\pi^2}\left[\left[\frac{f_{\pi N\Delta}}{m_\pi}\right]^2\left[\frac{M_\Delta}{E_\Delta}\right]\bar{u}(p_3')\frac{q'^\mu\Lambda_{\mu\nu}(P_\Delta)q^\nu}{E_\Delta-p_{30}-q_0}u(p_3)\right]$$
$$\times\frac{1}{q^2-m_\pi^2}[\bar{u}(p_1')\not{q}\gamma_5 u(p_1)] \ , \tag{3}$$

where π, N, and Δ denote, respectively, the pion, nucleon, and delta fields, whose masses are m_π, m_N, and M_Δ. $q=p_1-p_1'$ and $q=p_2'-p_2$. The intermediate positive energy is now on mass shell and $P_\Delta=(E_\Delta,\mathbf{P}_\Delta)$, $E_\Delta=(M_\Delta^2+\mathbf{P}_\Delta^2)^{1/2}$. $f_{\pi NN}$ and $f_{\pi N\Delta}$ are the πNN and $\pi N\Delta$ coupling constants. The projection operator $\Lambda_{\mu\nu}(P_\Delta)$ is

$$\Lambda_{\mu\nu}(P_\Delta)=\frac{\not{P}_\Delta+M_\Delta}{2M_\Delta}\left[g_{\mu\nu}-\tfrac{1}{3}\gamma_\mu\gamma_\nu-\frac{2P_{\Delta\mu}P_{\Delta\nu}}{3M_\Delta^2}+\frac{P_{\Delta\mu}\gamma_\nu-P_{\Delta\nu}\gamma_\mu}{3M_\Delta}\right] . \tag{4}$$

We have omitted the isospin indices. To obtain the effective 3NP, a nonrelativistic reduction is made on Eq. (3) and terms of order $(\mathbf{p}^2/m_N^2)$ are neglected. The nonrelativistic reduction of the numerator of Eq. (3) is straightforward and no ambiguity arises. For the energy denominator of the Δ propagator in Eq. (3), the nonrelativistic reduction keeping terms up to $\mathbf{p}^2$, gives

$$E_\Delta-p_{30}-q_0=(M_\Delta-m_N)+\frac{\mathbf{P}_\Delta^2}{2M_\Delta}-\frac{\mathbf{p}_3^2}{2m_N}+\frac{1}{2m_N}(\mathbf{p}_1'^2-\mathbf{p}_1^2)$$
$$=(M_\Delta-m_N)+\left\{\left[\frac{\mathbf{p}_1'^2}{2m_N}+\frac{1}{2M_\Delta}(\mathbf{p}_3+\mathbf{p}_1-\mathbf{p}_1-\mathbf{p}_1')^2+\frac{\mathbf{p}_2^2}{2m_N}\right]-\left[\frac{\mathbf{p}_1^2}{2m_N}+\frac{\mathbf{p}_3^2}{2m_N}+\frac{\mathbf{p}_2^2}{2m_N}\right]\right\} . \tag{5}$$

The prescriptions for approximating Eq. (5) as adopted in Refs. 9 and 11 are identical; namely, throwing away the kinetic energy difference terms inside the curly bracket in the second line of Eq. (5). This then leads to the following familiar form of the Δ-mediated 3NP in momentum space:

$$T^{(\Delta,+)} \sim \left[\frac{f_{\pi NN}}{m_\pi}\right]^2 (\sigma_2\cdot\mathbf{q}')\tau_\beta^{(2)} \frac{1}{\mathbf{q}'^2+m_\pi^2} \left\{ \left[\frac{f_{\pi N\Delta}}{m_\pi}\right]^2 \frac{[-\frac{2}{3}\mathbf{q}\cdot\mathbf{q}'+\frac{1}{3}\sigma_3\cdot(\mathbf{q}'\times\mathbf{q})]}{M_\Delta-m_N} (\tfrac{2}{3}\delta_{\alpha\beta}^{(3)}-\tfrac{1}{6}[\tau_\beta^{(3)},\tau_\alpha^{(3)}]) \right\} \frac{1}{\mathbf{q}^2+m_\pi^2}(\sigma_1\cdot\mathbf{q})\tau_\alpha^{(1)} , \tag{6}$$

and all permutation terms in (123).

The isospin structures are now exhibited explicitly in Eq. (6). α and β denote the isospin indices of the pion exchanged between nucleon pairs (13) and (32), respectively. The static approximation described above cannot be very good, because in Eq. (5) $M_\Delta - m_N \simeq 2m_\pi$ and the kinetic energy difference term is at least of the order (p/m_N) as compared to $M_\Delta - m_N$ for typical values of nucleon momentum $p \simeq m_\pi$. This is why large reduction of the Δ effects is found in the coupled-channels calculations of Ref. 12, when the kinetic energy difference terms are properly kept. The terms inside the curly brackets on the right-hand side of Eq. (5) represent the nonrelativistic reduction of the relevant πN scattering amplitude. In the derivation of the Tucson-Melbourne potential,[10] only terms up to the order $(\mathbf{q}^2/m_N^2)$ are kept in the expansion of the πN scattering amplitude. However, keeping the kinetic energy terms in the Δ propagator amounts to retaining terms of higher orders in the expansion of the πN scattering amplitude. Thus, even though Δ contributions are not dealt with explicitly in Ref. 10, the effects of Δ propagation are not included in the Tucson-Melbourne potential.

Straightforward inclusion of the kinetic energy terms in the Δ propagator leads to the following 3NP in momentum space:[13]

$$T^{(\Delta,+)} \sim \left[\frac{f_{\pi NN}}{m_\pi}\right]^2 (\sigma_2\cdot\mathbf{q}')\tau_\beta^{(2)} \frac{1}{\mathbf{q}'^2+m_\pi^2} \left\{ \left[\frac{f_{\pi N\Delta}}{m_\pi}\right]^2 \frac{[-\frac{2}{3}\mathbf{q}\cdot\mathbf{q}'+\frac{1}{3}\sigma_3\cdot(\mathbf{q}'\times\mathbf{q})]}{(M_\Delta-m_N)+K_\Delta-K_i} \times [\tfrac{2}{3}\delta_{\alpha\beta}^{(3)}-\tfrac{1}{6}\tau_\beta^{(3)}\tau_\alpha^{(3)}] \frac{1}{\mathbf{q}^2+m_\pi^2} \right\} (\sigma_1\cdot\mathbf{q})\tau_\alpha^{(1)} + \text{all permutation terms in } (123) , \tag{7}$$

where

$$K_\Delta = \frac{\mathbf{p}_1'^2}{2m_N} + \frac{(\mathbf{p}_3+\mathbf{p}_1-\mathbf{p}_1')^2}{2M_\Delta}$$

and

$$K_i = \frac{1}{2m_N}(\mathbf{p}_1^2+\mathbf{p}_3^2) .$$

However, Eq. (7) is not suitable for bound state calculation because pole singularity will be encountered. To overcome this problem, we proceed as follows.

For a system of nucleons and deltas, the Hamiltonian takes the form

$$H = \sum_{i=1}^{A} \{T_i(N)+[T_i(\Delta)+(M_\Delta-m_N)]\} + V_{NN} + V_\Delta , \tag{8}$$

where the T_i refer to the kinetic energy operators, with the $T_i(\Delta)$ having the mass difference between the Δ and the nucleon added. V_{NN} is the sum of the potentials between NN. $V_{N\Delta}$ stands for the sum of the potentials between NΔ and ΔΔ pairs and transition potentials for NN↔NΔ, NΔ↔ΔΔ, and NN↔ΔΔ. The Schrödinger equation for such a system can be written as

$$H\Psi = E\Psi . \tag{9}$$

Let P be the projection operator onto the pure nucleonic space and $Q=1-P$. Then the equation satisfied by the pure nucleonic component of the total wave function Ψ is

$$\left[\sum_{i=1}^{A} T_i(N) + \sum_{i<j} V_{NN}(ij) + V_{\text{eff}}\right] P\Psi = EP\Psi , \tag{10}$$

where

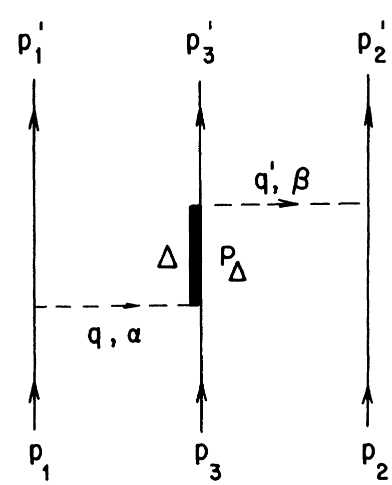

FIG. 1. The two-pion–exchange three-nucleon force which arises from the excitation of a nucleon into the Δ isobar. In nuclei the dominant contribution comes from positive energy Δ excitation.

$$V_{\rm eff}=PV_{\Delta}Q\frac{1}{E-QH_0Q-QV_{\Delta}Q}QV_{\Delta}P\ , \tag{11}$$

and

$$H_0=\sum_{i=1}^{A}\{T_i({\rm N})+[T_i(\Delta)+(M_{\Delta}-m_{\rm N})]\}\ .$$

The effective interaction $V_{\rm eff}$ is, in general, a many-body interaction (i.e., two-, three-, etc. nucleon force). The two-body force component of the $V_{\rm eff}$, when combined with $V_{\rm NN}$, is supposed to give a realistic description of the two-nucleon system. $V_{\rm eff}$ also contains a three-body force component which corresponds to that depicted in Fig. 1. This is given by

$$V_3^{(\Delta)}=PV_{{\rm N}\Delta\to{\rm NN}}(\pi)Q\frac{1}{E-QH_0Q}QV_{{\rm NN}\to{\rm N}\Delta}(\pi)P\ , \tag{12}$$

where $V_{{\rm NN}\leftrightarrow{\rm N}\Delta}(\pi)$ is the NN$\leftrightarrow$NΔ transition potential due to one pion exchange. It has the form

$$\langle \mathbf{p}'\mid V_{{\rm NN}\to{\rm N}\Delta}(\pi)\mid \mathbf{p}\rangle =-\frac{1}{(2\pi)^3}\left[\frac{f_{\pi{\rm NN}}f_{\pi{\rm N}\Delta}}{m_\pi^2}\right]\tau^{(1)}\cdot\mathbf{T}^{(3)}_{\Delta{\rm N}}\frac{(\sigma_2\cdot\mathbf{q})(\mathbf{S}^{(3)}_{\Delta{\rm N}}\cdot\mathbf{q})}{\mathbf{q}^2+m_\pi^2}\ , \tag{13}$$

where $\mathbf{q}=\mathbf{p}'-\mathbf{p}$ is the three-momentum of the pion exchanged; $\mathbf{S}_{\Delta{\rm N}}$ and $\mathbf{T}_{\Delta{\rm N}}$ are the spin and isospin transition operators transforming a nucleon into a Δ. This leads to the following representation of the Δ-mediated three-nucleon potential $V_3^{(\Delta)}$ in momentum space:

$$\langle \mathbf{p}_1',\mathbf{p}_2',\mathbf{p}_3'\mid V_3^{(\Delta)}\mid \mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3\rangle=\frac{1}{(2\pi)^6}\left[\frac{f_{\pi{\rm NN}}}{m_\pi}\right]^2\left[\frac{f_{\pi{\rm N}\Delta}}{m_\pi}\right]^2(\sigma_2\cdot\mathbf{q}')(\sigma_1\cdot\mathbf{q})(\tfrac{2}{3}\tau^{(1)}\cdot\tau^{(2)}-\tfrac{1}{6}[\tau^{(2)}\cdot\tau^{(3)},\tau^{(3)}\cdot\tau^{(1)}])$$
$$\times\frac{1}{\mathbf{q}'^2+m_\pi^2}\frac{1}{\mathbf{q}^2+m_\pi^2}\frac{\left[\frac{2}{3}(\mathbf{q}\cdot\mathbf{q}')-\frac{i}{3}\sigma_3\cdot(\mathbf{q}'\times\mathbf{q})\right]}{E-(M_\Delta+2m_{\rm N})-\frac{\mathbf{p}_{13}^2}{2\mu_{13}}-\frac{\mathbf{q}_2^2}{2\mu_{13,2}}}+\text{all permutation terms in (123)}\ , \tag{14}$$

where

$$\mu_{13}=\frac{m_{\rm N}M_\Delta}{m_{\rm N}+M_\Delta}\ ,$$
$$\mu_{13,2}=\frac{(m_{\rm N}+M_\Delta)m_{\rm N}}{2m_{\rm N}+M_\Delta}\ ,$$
$$\mathbf{p}_{13}=\frac{M_\Delta\mathbf{p}_1'-m_{\rm N}\mathbf{P}_\Delta}{m_{\rm N}+M_\Delta}\ ,$$
$$\mathbf{q}_2=\frac{(m_{\rm N}+M_\Delta)\mathbf{p}_2-m_{\rm N}(\mathbf{p}_1'+\mathbf{P}_\Delta)}{2m_{\rm N}+M_\Delta}\ , \tag{15}$$

with $\mathbf{P}_\Delta=\mathbf{p}_1+\mathbf{p}_3-\mathbf{p}_1'$. The energy E is understood not to include the kinetic energy of the c.m. motion. For scattering problems, Eqs. (7) and (14) are equivalent. In a bound state, like a triton, the energy denominator in Eq. (14) is reduced to

$$-E_B-(M_\Delta-m_{\rm N})-\frac{\mathbf{p}_{13}^2}{2\mu_{13}}-\frac{\mathbf{q}_2^2}{2\mu_{13,2}}$$

and will not give rise to any singularity. E_B is the triton binding energy.

In the coupled-channels calculations of Refs. 1 and 12, the πNΔ coupling constant is taken to be $f_{\pi{\rm N}\Delta}^2/4\pi=0.35$, which is determined[14] from the experimental value of the Δ width. A value of $f_{\pi{\rm N}\Delta}^2/4\pi=0.27$ is used in the Brazil 3NP model,[11] which is obtained[15] from a fit to a wide range of low energy experimental data. Since the pions exchanged between nucleons in nuclei are mostly of about zero energy, we feel that the latter value of $f_{\pi{\rm N}\Delta}$ is to be preferred. In a static approximation, this value of $f_{\pi{\rm N}\Delta}$ leads to a 3NP with the following strength parameter, $b_\Delta=-1.49m_\pi^{-3}$ and $d_\Delta=-0.373m_\pi^{-3}$ (in the notation of Ref. 10). The contribution of this Δ-mediated 3NP to the triton binding energy E_B, in a first-order perturbation approximation, can be readily obtained from Table III of Ref. 6 by simple scaling. For the Reid soft-core potential, it gives 0.67 MeV extra binding with the 18-channel Faddeev solution if a dipole pionic form factor with cutoff momentum $\Lambda=800$ MeV is employed. The 3NP arising from other processes gives a net contribution of 0.22 MeV to E_B. Experience from coupled-channels calculations indicates that if the proper form of the Δ-mediated 3NP of Eq. (14) is used, the contribution of 0.67 MeV will be reduced by almost a factor of 2.

In summary, we have derived a Δ-mediated effective three-nucleon potential which takes into account the effects of Δ-propagation effects. Reliable estimate of the Δ's contribution to three-body force effects can be obtained only if this representation of the 3NP is used.

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