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Test of the fermion dynamical symmetry model microscopy in the *sd* shell

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The recently formulated fermion dynamical symmetry model treats low-lying collective levels as states classified in a pseudo-orbit pseudo-spin (k-i) basis having either $k=1$ and zero i seniority, or $i=\frac{3}{2}$ and zero k seniority. The validity of this suggestion, which has not previously been subjected to a microscopic examination, is determined for even-even nuclei in the *sd* shell, for which the model is phenomenologically successful, by comparing these states with the eigenfunctions of a realistic Hamiltonian. Most low-lying levels are almost orthogonal to the fermion dynamical symmetry model zero seniority subspaces.

I. INTRODUCTION

A long-standing goal of nuclear structure physics has been to find a shell model truncation scheme capable of describing collective motion; this is often approached by invoking symmetries, the discovery of (approximately) conserved quantum numbers being of interest in its own right.[1] The concept of symmetry is combined with that of S-D pairing, suggested by the success of the interacting boson model (IBM) (Ref. 2), in the monopole and quadrupole pairing model,[3] originally intended as a schematic model of nuclear structure, which can be applied to systems of particles occupying a set of angular momentum levels $\{j\}$ which allows either of the pseudo-orbit pseudo-spin (k-i) decompositions $[k=1]\times\{i\}$ or $\{k\}\times[i=\frac{3}{2}]$. For either choice, the number of particles not in particular S or D pairs appears as a seniority label, and an interesting phenomenology is obtained on the assumption that low-lying levels belong to the subspaces with zero seniority. Subsequently, it has been noted that the normal parity levels of each major shell can indeed be decomposed according to one of the above schemes, giving rise to the fermion dynamical symmetry model (FDSM),[4] and a suggestion has been made that the various symmetries may actually be realized in nature.[5] Some successful phenomenological studies have been made where low-lying levels (below high-spin band crossings) are modeled as states with zero seniority,[6,7] but the extent to which this *ansatz* is microscopically valid remains to be investigated.

A detailed microscopic study, involving, for instance, a comparison of wave functions, clearly requires the solutions of a more general model. The only nuclei with spectra displaying rotational collectivity for which a complete shell model calculation is available at present are those in the *sd* shell; here an interaction has been derived from Kuo-Brown matrix elements by fitting to energy levels throughout the shell, and gives wave functions which allow good reproduction of moments and transition rates.[8] The FDSM can indeed be applied to the *sd* shell, whose j values $\{\frac{5}{2},\frac{1}{2},\frac{3}{2}\}$ can be decomposed as ($k=1$, $i=\frac{3}{2}$), and a proton-neutron version of the model suitable for isospin invariant systems, as required in this region, has been formulated.[9–11] (The lighter Te, Xe, and Ba isotopes also have protons and neutrons filling the same major shell, and might require an explicitly isospin invariant formalism.) Moreover, the phenomenology associated with the i seniority zero subspace in the SU(3) limits has been investigated and found to satisfactorily reflect trends in the spectra of even-even *sd* shell nuclei;[10] for instance, this scheme provides candidates for both low-lying $K=2$ bands of ^{22}Ne, in contrast to the L-S SU(3) classification,[12] while still explaining naturally the single such band in ^{24}Mg. We now proceed to examine the extent to which this success of the zero seniority phenomenology is actually due to dominance of such states in low-lying levels.

Although an effective interaction which reproduces a subset of the original energies can always be constructed for any truncated space, its simplicity, and the applicability of perturbative renormalization in its derivation, will clearly depend on the extent to which the chosen states are important in the full solution. Moreover, our understanding of the original problem is certainly enhanced by the knowledge of a realistic truncation. Thus a direct evaluation of the realism of a truncation is desirable, whether as an end in itself or as providing some indication of the complexity to be expected in a renormalization procedure. Indeed, the validity of other symmetry or truncation schemes in use has been investigated on the basis of model and realistic state overlaps:[1] for instance, SU(3),[12–14] pseudo-SU(3),[15,16] j-shell seniority,[17] the OAI *SD* pair construction,[18–20] and the shell model approximation itself.[21–23]

It should also be stressed that the results to be presented here imply no definite conclusion for heavier nuclei, this being simply a test in one of the many regions to which the formalism can be applied; however it is at the very least an examination of the argument that phenomenology can be used to support a microscopic truncation. Other collective models have been successfully applied to the *sd* shell, for instance, the geometrical model,[24] SP(3,R),[23,25,26] and the IBM.[27,28] Moreover, the isospin invariant FDSM itself is phenomenologically successful,[10] justifying an examination of the microscopy. The isospin invariant FDSM deals with $T=0$ pairs in addition to the $T=1$ SD pairs of the identical particle version. Nevertheless, the properties of both are due entirely to taking either $k=1$ and i seniority $v_i=0$, or $i=\frac{3}{2}$ and k seniority $v_k=0$, and it is precisely these prescriptions which are to be tested in the present work. In the same way the IBM concept of *sd* bosons for identical particles has been extended to systems of protons and neutrons (IBM-2 and IBM-4).[2,27]

Other regions of the nuclear chart should be investigated, where possible, to obtain a more global picture of the validity of the FDSM. The relevance of the seniority zero subspace to heavier rotational nuclei could be estimated from the degree of convergence of $v=0$ and $v=0,2$, etc., calculations using some realistic Hamiltonian, the required technology now being available.[29] It might also be instructive to compare the SU(3) wave functions of the FDSM with those of the $\tilde{L}$-$\tilde{S}$ pseudo-orbit pseudo-spin scheme which has also had considerable phenomenological success,[30] in the regions where both models can be applied. In addition, since the FDSM is also potentially capable of describing vibrational motion, full calculations for near semimagic nuclei could be used in the same spirit as in the present work to evaluate its relevance in this regime.

The isospin invariant FDSM is briefly discussed in Sec. II. In Sec. III the collectivity of the zero seniority subspace for the *sd* shell is investigated in a fully microscopic way by comparing the corresponding states to the eigenfunctions of a realistic Hamiltonian. Section IV contains some discussion of the results.

II. THE ISOSPIN INVARIANT FDSM

The k-active FDSM (Refs. 3 and 4) can be applied to systems of particles in a set of angular momentum levels $\{j\}$ which may be decomposed into pseudo-orbit and pseudo-spin factors as $[k=1]\times i$:

$$j=|i-1|,i,i+1 \ . \tag{1}$$

Similarly, the i-active version[3,4] requires that $\{j\}$ be decomposed as $k\times[i=\frac{3}{2}]$:

$$j=|k-\tfrac{3}{2}|,|k-\tfrac{1}{2}|,k+\tfrac{1}{2},k+\tfrac{3}{2} \ . \tag{2}$$

The formalism is readily extended to incorporate many i or k values, respectively, given that successive i (k) values differ by at least 3 (4) units (that is, no j value occurs more than once). In the interest of clarity, a single i (k) value is used throughout this section, the generalization of any expression to the multi-i (k) cases being straightforward.

A. Group structure

The group chains appropriate to these decompositions for identical particles are discussed in the literature.[3] Correspondingly, for protons and neutrons in the same orbits, creation operators with sharp projections m_k and m_i are

$$\mathbf{a}^{\dagger(t=1/2\,k\,i)}_{m_t m_k m_i}=\mathbf{a}^{\dagger(t=1/2\,j)}_{m_t m}\langle k m_k i m_i \,|\, jm\rangle \ , \tag{3}$$

a summation over j and m being implied. The SO$(24i+12)$ and SO$(32k+16)$ groups generated by all number conserving and pair creation and annihilation operators can be reduced to[9]

$$\mathrm{SO}(24i+12)\supset \mathrm{Sp}(12)\times \mathrm{Sp}(2i+1) \ , \tag{4}$$

$$\mathrm{Sp}(12)\supset \mathrm{SU}(6)\supset \mathrm{Sp}(6)\supset \mathrm{SO}(3)\times \mathrm{SU}^T(2), \tag{5a}$$
$$\mathrm{Sp}(12)\supset \mathrm{SU}(6)\supset [\mathrm{SU}(3)\supset \mathrm{SO}(3)]\times \mathrm{SU}^T(2), \tag{5b}$$
$$\mathrm{Sp}(12)\supset [\mathrm{SO}(6)\supset \mathrm{SU}(3)\supset \mathrm{SO}(3)]\times \mathrm{SU}^T(2), \tag{5c}$$
$$\mathrm{Sp}(12)\supset \mathrm{SO}(3)\times[\mathrm{Sp}(4)\supset \mathrm{SU}^T(2)], \tag{5d}$$

$$\mathrm{Sp}(2i+1)\supset \mathrm{SU}^I(2) \ , \tag{6}$$

and

$$\mathrm{SO}(32k+16)\supset \mathrm{SO}(16)\times \mathrm{SO}(2k+1) \ , \tag{7}$$

$$\mathrm{SO}(16)\supset \mathrm{SU}(8)\supset \mathrm{SO}(8)\supset [\mathrm{Sp}^I(4)\supset \mathrm{SU}^I(2)]\times \mathrm{SU}^T(2), \tag{8a}$$
$$\mathrm{SO}(16)\supset \mathrm{SU}(8)\supset [\mathrm{SU}(4)\supset \mathrm{Sp}^I(4)\supset \mathrm{SU}^I(2)]\times \mathrm{SU}^T(2), \tag{8b}$$
$$\mathrm{SO}(16)\supset [\mathrm{Sp}(8)\supset \mathrm{SU}(4)\supset \mathrm{Sp}^I(4)\supset \mathrm{SU}^I(2)]\times \mathrm{SU}^T(2), \tag{8c}$$
$$\mathrm{SO}(16)\supset [\mathrm{Sp}^I(4)\supset \mathrm{SU}^I(2)]\times[\mathrm{Sp}(4)\supset \mathrm{SU}^T(2)], \tag{8d}$$

$$\mathrm{SO}(2k+1)\supset \mathrm{SO}(3) \ , \tag{9}$$

the total angular momentum J in both cases being obtained by coupling the pseudo-orbital SO(3) and pseudo-spin $\mathrm{SU}^I(2)$ labels K and I. The U(1) group generated by the number operator has been omitted from Eqs. (5) and (8). The groups appearing in Eqs. (4)–(9), and some aspects of the representation theory, have been described in Refs. 9–11, each chain having associated with it characteristic spectra. For instance, the SU(3) group in Eqs. 5(b) and 5(c) describes transformations within the three-dimensional ($k=1$) pseudo-orbit space, and has representations which contain angular momenta corresponding to sets of truncated rotational bands.[13] It is therefore potentially capable of describing, at least phenomenologically, rotational nuclei. SU(4) and $Sp^I(4)$ [Eqs. (8)] are homomorphic to SO(6) and SO(5),[3,9] which have been used to describe γ-unstable spectra in the IBM,[2] the latter notation often being used in the literature for this reason. The SO(6) and SO(5) labels ($\mathbf{a}$) and (τ) are related to those of SU(4) and Sp(4) [see Eq. (17)] by

$$(a_1a_2a_3)=(\tfrac{1}{2}\alpha+\beta+\tfrac{1}{2}\gamma\ \ \tfrac{1}{2}\alpha+\tfrac{1}{2}\gamma\ \ \tfrac{1}{2}\alpha-\tfrac{1}{2}\gamma)\ , \tag{10}$$

$$(\tau_1\tau_2)=(\tfrac{1}{2}\sigma_1+\tfrac{1}{2}\sigma_2\ \ \tfrac{1}{2}\sigma_1-\tfrac{1}{2}\sigma_2)\ . \tag{11}$$

It is sometimes useful to consider the classification schemes completely equivalent to those above [Eqs. (4)–(9)] whose groups are generated by number conserving operators only;[9–11] see for example Eq. (20).

It is possible to generalize Eq. (3) by the introduction of a j-dependent real phase while maintaining the same group structure, there being four independent choices for a three-level system: $(+++)$, $(++-)$, $(+-+)$, and $(-++)$. The assignment $(+++)$ for each of the $j=\frac{5}{2}$, $\frac{1}{2}$, and $\frac{3}{2}$ orbits turns out to give the highest overlaps between realistic and zero seniority states; those obtained using the other choices will be described briefly in Sec. III B.

States classified by any of these group chains span the full shell model space, and so represent valid classification schemes; the suggestion which has yet to be justified is that the corresponding symmetries are approximately realized, and further that low-lying levels are dominated by states which transform either as $\langle 0^{i+1/2}\rangle$ under $Sp(2i+1)$, or as (0^k) with respect to $SO(2k+1)$, as applicable.

B. Pair states

The ($T=1$, $K=0,2$ and $T=0$, $K=1$) zero i-seniority pairs, and ($T=1$, $I=0,2$ and $T=0$, $I=1,3$) zero k-seniority pairs, are particular combinations of the j-j coupled states[3,9] [Eqs. (12) and (13), respectively, a summation over j and j' is implied, each pair being counted only once],

$$\mathbf{A}^{\dagger(TK0)}=(-)^{j'+K-i}\left[\frac{2(2j+1)(2j'+1)}{(2i+1)(1+\delta_{jj'})}\right]^{1/2}\begin{Bmatrix} j & j' & K\\ 1 & 1 & i\end{Bmatrix}\mathbf{A}^{\dagger(TK)}(jj')\ , \tag{12}$$

$$\mathbf{A}^{\dagger(T0I)}=(-)^{j+I+k-1/2}\left[\frac{2(2j+1)(2j'+1)}{(2k+1)(1+\delta_{jj'})}\right]^{1/2}\begin{Bmatrix} j & j' & I\\ \frac{3}{2} & \frac{3}{2} & k\end{Bmatrix}\mathbf{A}^{\dagger(TI)}(jj')\ , \tag{13}$$

where

$$\mathbf{A}^{\dagger(TKI)}=\sqrt{1/2}(\mathbf{a}^{\dagger(1/2\,k\,i)}\mathbf{a}^{\dagger(1/2\,k\,i)})^{(TKI)} \tag{14}$$

and

$$\mathbf{A}^{\dagger(TJ)}(jj')=\sqrt{1/(1+\delta_{jj'})}(\mathbf{a}^{\dagger(1/2\,j)}\mathbf{a}^{\dagger(1/2\,j')})^{(TJ)}\ . \tag{15}$$

Taking i-k coupling [as opposed to the order k-i, Eq. (3)] results in an additional relative phase $(-)^{j-j'}$ in Eqs. (12) and (13), thus corresponding to the phase assignment $(++-)$ (Sec. II A). States with good isospin could of course be constructed using only the $T=1$ S and D pairs, but these do not allow a closed algebra to be constructed; the resulting pair space is clearly a subspace of that considered here.

C. Isospin invariant FDSM in the *sd* shell

The group chains for the general case can be carried over directly, taking $i=\frac{3}{2}$ in Eqs. (4) and (6) and $k=1$ in Eqs. (7) and (9). Since it is primarily the zero seniority dominance itself which is to be tested in the present work, and not the goodness of any further symmetries, such as SU(3) or SU(4), although this is also of interest, it will be sufficient in the following to consider states classified according to any one of the chains in Eqs. (5), and any one of the chains in Eq. (8); we choose Eqs. 5(b) and 8(b):

$$\begin{aligned}SO(48)\supset\{&\underset{\langle\Sigma\rangle}{Sp(12)}\supset\underset{[f]}{U(6)}\supset\underset{T}{SU^T(2)}\\ &\times[\underset{(\lambda\mu)}{SU(3)}\supset\underset{K}{SO(3)}]\}\\ &\times[\underset{\langle\sigma\rangle}{Sp(4)}\supset\underset{I}{SU^I(2)}]\ ,\end{aligned} \tag{16}$$

$$\begin{aligned}SO(48)\supset\underset{K}{SO(3)}\times\{&\underset{\langle P\rangle}{SO(16)}\supset\underset{[g]}{U(8)}\supset\underset{T}{SU^T(2)}\\ &\times[\underset{(\alpha\beta\gamma)}{SU(4)}\supset\underset{\langle\sigma\rangle}{Sp(4)}\supset\underset{I}{SU^I(2)}]\}\ ,\end{aligned} \tag{17}$$

where the representations of the quasispin and seniority groups are related by[9]

$$\Sigma_q=2-(\tilde{\sigma})_{7-q} \tag{18}$$

and

$$P_q=\tfrac{3}{2}-(\tilde{\rho})_{9-q}\ . \tag{19}$$

ρ here refers to the unmodified[31] SO(3) label. The former is implemented by assuming that low-lying levels transform as $\langle 0\,0\rangle$ under Sp(4), the latter by considering (0) of SO(3). A classification scheme completely equivalent to those of Eqs. (15) and (16) whose subgroups are generated solely by number conserving operators is

TABLE I. K-I classification for two sd shell nucleons [Eq. (20)].

$SU^T(2)$ T	$SU(3)$ $(\lambda\mu)$	$SO(3)$ K	$U(4)$ $[\tilde{f}]$	$Sp(4)$ $\langle\sigma_1\sigma_2\rangle$	$SU^I(2)$ I	$SU^J(2)$ J
1	(2 0)	0	[1 1]	⟨0 0⟩	0	0
		2				2
				⟨1 1⟩	2	2
						0,1,2,3,4
	(0 1)	1	[2]	⟨2 0⟩	1,3	$0,1,2^2,3,4$
0	(2 0)	0	[2]	⟨2 0⟩	1,3	1,3
		2				$1^2,2^2,3^2,4,5$
	(0 1)	1	[1 1]	⟨0 0⟩	0	1
				⟨1 1⟩	2	1,2,3

$$SO(48)\supset \underset{[1^n]}{U(24)}\supset\{\underset{[f]}{U(6)}\supset \underset{T}{SU^T(2)}\times[\underset{(\lambda\mu)}{SU(3)}\supset \underset{K}{SO(3)}]\}\times[\underset{[\tilde{f}]}{U(4)}\supset \underset{\langle\sigma\rangle}{Sp(4)}\supset \underset{I}{SU^I(2)}] , \tag{20}$$

where

$$(\alpha\beta\gamma)=(\tilde{f}_1-\tilde{f}_2\ \tilde{f}_2-\tilde{f}_3\ \tilde{f}_3-\tilde{f}_4) , \tag{21}$$

$$[\mathbf{g}]=(3^{1/3(n-\lambda-2\mu)}2^{\mu}1^{\lambda}) ; \tag{22}$$

the corresponding classification for two sd shell nucleons is shown in Table I. There has, in fact, been a suggestion[4] that a truncation scheme based on the $SU(4)\times SU(3)$ decomposition [Eq. (20)] might alternatively be relevant to the sd shell; this possibility is being investigated,[32] but lacks the phenomenological rationale of a similarity to the IBM and is not considered further in this work.

Specializing the expressions for the zero seniority pairs [Eqs. (12) and (13)] in j-j coupling we have

$$\mathbf{A}^{\dagger(TK0)}=(-)^{j'+K+1/2}\sqrt{(2j+1)(2j'+1)/2(1+\delta_{jj'})}\begin{Bmatrix} j & j' & K \\ 1 & 1 & \frac{3}{2}\end{Bmatrix}\mathbf{A}^{\dagger(TK)}(jj') , \tag{23}$$

$$\mathbf{A}^{\dagger(T0I)}=(-)^{j+I+1/2}\sqrt{2(2j+1)(2j'+1)/3(1+\delta_{jj'})}\begin{Bmatrix} j & j' & I \\ \frac{3}{2} & \frac{3}{2} & 1\end{Bmatrix}\mathbf{A}^{\dagger(TI)}(jj') , \tag{24}$$

the coefficients of each configuration (jj') for the various states being given in Table II.

III. COMPARISON OF REALISTIC AND $v=0$ WAVE FUNCTIONS

Because the angular momenta $\{j\}$ of the sd shell $(j=\frac{5}{2},\frac{1}{2},\frac{3}{2})$ can be decomposed into pseudo-orbit and pseudo-spin factors as $k=1$ $i=\frac{3}{2}$, both k- and i-active versions of the isospin invariant FDSM can in principle be used.[9] The classification of zero seniority states in the SU(3) limits of the k-active scheme has been found to be capable of reflecting qualitative trends in the spectra of even-even sd shell nuclei,[10] and the SU(4) representations occurring for the i-active version have also been determined;[11] we now proceed to find the extent to which the zero seniority subspaces actually exhaust low-lying eigenfunctions arising from a full sd shell calculation.

TABLE II. Coefficients (represented by the phase and squared magnitude) of each configuration (jj') for the FDSM seniority zero pairs in the sd shell given k-i coupling [Eqs. (23) and (24)], the order i-k yielding phase differences of $(-)^{j-j'}$. The k-active ($v_i=0$) pairs have $I=0$ and $J=K$; the i-active ($v_k=0$) pairs have $K=0$ and $J=I$.

			(jj')					
T	K	I	$(\frac{5}{2}\ \frac{5}{2})$	$(\frac{5}{2}\ \frac{1}{2})$	$(\frac{5}{2}\ \frac{3}{2})$	$(\frac{1}{2}\ \frac{1}{2})$	$(\frac{1}{2}\ \frac{3}{2})$	$(\frac{3}{2}\ \frac{3}{2})$
1	0	0	$\frac{3}{6}$			$\frac{1}{6}$		$\frac{2}{6}$
1	2	0	$\frac{21}{150}$	$\frac{45}{150}$	$\frac{63}{150}$		$\frac{5}{150}$	$\frac{16}{150}$
1	0	2	$\frac{21}{75}$	$\frac{5}{75}$	$-\frac{28}{75}$		$\frac{20}{75}$	$\frac{1}{75}$
0	1	0	$\frac{63}{180}$		$\frac{54}{180}$	$-\frac{5}{180}$	$-\frac{50}{180}$	$\frac{8}{180}$
0	0	1	$\frac{567}{1350}$		$-\frac{216}{1350}$	$\frac{125}{1350}$	$\frac{200}{1350}$	$\frac{242}{1350}$
0	0	3	$\frac{9}{75}$	$\frac{25}{75}$	$-\frac{32}{75}$			$-\frac{9}{75}$

TABLE III. Dimensions of the total and zero-seniority spaces for the *sd* shell systems considered in Sec. III B.

n	T	J	Total	$v_i=0$	$v_k=0$
4	0	0	21	3	4
		2	56	3	5
		4	44	1	3
6	1	0	148	6	11
		1	351	5	10
		2	525	11	24
		3	537	4	15
		4	502	5	17
		6	255	1	6
8	0	0	325	10	21
		1	779	3	16
		2	1206	15	44
		3	1304	5	29
		4	1311	10	39
		6	835	3	19
		8	329	1	6

A. Realistic *sd* shell wave functions

Collective effects in the *sd* shell are well established, nuclei such as ^{20}Ne, ^{22}Ne, and ^{24}Mg displaying rotational-like spectra and enhanced electric quadrupole moments and transitions.[24] Shell model calculations using an untruncated $(d_{5/2}s_{1/2}d_{3/2})$ space[8] reproduce properties such as these with quantitative accuracy so that the corresponding effective interaction, obtained by small adjustments of the Kuo-Brown matrix elements, is known with much more certainty than in heavier nuclei. Moreover, from these observations, it can also be surmised that the corresponding wave functions do indeed represent the "reality" of *sd* shell nuclear structure, or at least its projection on the *sd* valence space. For a truly realistic description the mixing of *sd* states with particle-hole excitations would have to be considered; this can be neglected here since the shell model approximation is also used in the FDSM, and moreover such effects are calculated to be small for these nuclei.[8,21–23]

TABLE IV. Squared overlaps, expressed as percentages, of the ^{20}Ne realistic wave functions arising from the fitted interaction, and *i*-seniority zero states classified using $[\tilde{f}](\lambda\mu)K=J$ [Eqs. (16) and (20)].

		[1111]	[22]	
K_f^ξ	J	(02)	(40)	(02)
0_1	0	0.7	58.4	1.1
	2	0.1	39.7	0.1
	4		9.3	
0_2	0	3.1	0.2	2.3
	2	1.8	0.0	0.5
0_3	0	6.6	0.2	14.5
	2	1.7	0.0	6.7

B. Results

States classified by the group chains of Eqs. (16) and (17) can be explicitly constructed by using the Oak Ridge-Rochester shell model code to diagonalize Hamiltonians having the corresponding symmetries. The dimensions of the full and seniority zero spaces for the systems to be considered are shown in Table III. Squared overlaps of the (k-active) FDSM zero *i*-seniority states[10] and realistic eigenfunctions are shown in Tables IV–VI

TABLE V. Squared overlaps of the ^{22}Ne realistic wave functions and *i*-seniority zero states. See Table IV.

		[2211]							[33]		
K_f^ξ	J	(60)	(41)	$(22)^2$	(30)	$(11)^2$	(00)		(22)	(11)	(00)
0_1	0	32.7		2.4			0.1		13.2		0.1
	2	27.0	0.0	0.6		0.0			4.8	0.0	
	4	11.7	0.0	0.0					0.3		
	6	3.2									
2_1	2	0.9	4.8	0.8		0.0			4.1	0.0	
	3		0.9	0.2	0.0				1.5		
	4	5.7	3.1	0.0				0.1	0.1		
2_2	2	0.0	3.4	0.7		0.0			12.7	0.1	0.0
	3		3.5	0.1	0.0				4.0		
	4	0.0	0.6	0.1					0.8		
1_1	1		7.7		0.1	0.0				0.0	
	2	1.8	1.0	0.5		0.2			3.3	0.1	
	3		0.2	0.0	0.1				3.3		
0_2	0	0.4		9.1			0.2		5.3		0.1
	2	0.5	1.2	2.5		0.1			3.9	0.0	
0_3	0	0.0		2.0			0.0		10.0		0.0
	2	1.5	1.9	2.4		0.0			4.5	0.0	

TABLE VI. Squared overlaps of the ^{24}Mg realistic wave functions and i-seniority zero states. See Table IV.

		[2222]				[3311]					[44]	
K_i^ξ	J	(80)	(42)	(04)	(20)	(42)	(04)	$(31)^2$	(12)	$(20)^2$	(04)	(20)
0_1	0	22.8	0.2	0.1	0.0	13.0	0.1			0.1	4.4	0.1
	2	21.8	0.1	0.0	0.0	8.9	0.0	0.0	0.0	0.0	0.7	0.0
	4	16.6	0.0	0.0		4.3	0.0	0.0			0.1	
	6	8.0	0.0	0.0		0.8						
	8	3.0										
2_1	2	0.0	0.0	0.0	0.0	7.6	0.0	0.0	0.0	0.0	4.3	0.0
	3		0.0			4.8		0.0	0.0			
	4	0.1	0.0	0.0		3.0	0.0	0.0			0.3	
0_2	0	1.8	1.0	0.1	0.0	3.4	1.8			0.1	9.3	0.1
	2	0.1	0.3	0.0	0.0	0.8	0.3	0.6	0.0	0.0	0.8	0.0
	4	0.3	0.0	0.0		0.2	0.0	0.0			0.1	
1_1	1							1.2	0.0			
	2	0.2	0.8	0.0	0.0	0.4	0.8	0.2	0.0	0.0	1.6	0.0
	3		0.1			0.0		0.2	0.0			

for (n,T)=(4,0), (6,1), and (8,0) respectively, these being the quantum numbers associated with the low-lying levels of the rotational-like nuclei ^{20}Ne, ^{22}Ne, and ^{24}Mg. Tables VII–IX show the overlaps corresponding to the (i-active) zero k-seniority states[11] for the same nuclei, where in the interest of simplicity only the total occupancies for all such states are shown for ^{22}Ne and ^{24}Mg, the SU(4) classification being both more complicated than that obtained using SU(3) and of less similarity with the observed sd shell spectra.[11] (The fitted levels can be arranged into bands labeled by the projection K^ξ of the angular momentum onto an intrinsic axis,[12] K_i^ξ denoting those in the ith such band.) These results correspond to the $(++++)$ phase assignment (Sec. II A), those obtained for the other choices are described briefly below.

The overlaps for each nucleus are almost without exception very low; in addition there is considerable fluctuation between levels. For the k-active scheme, all levels other than the lower members of the ground bands have very small overlaps with the zero seniority states, the summed occupancies being typically less than 10%. Even the ground band is poorly described—the entire zero seniority subspace exhausts only 60% of the $J=0$ wave function for ^{20}Ne, 50% for ^{22}Ne, and 40% for ^{24}Mg, a more important failing, however, being that the overlaps decrease significantly as the angular momentum is increased, falling through 10% by only $J=4$. Further, the dominant $v_i=0$ SU(3) representations in the excited bands of both ^{22}Ne and ^{24}Mg are not the optimal phenomenological candidates[10]; for instance, the two low-lying $K_i^\xi=2$ bands in ^{22}Ne were associated with the two [2211] (22) representations, however these have occupancies less than 1%, the main contributions to the (small) total coming from [2211] (41) and the single [33] (22). The $k=1$ $v_i=0$ states do not approximate the low-lying rotational levels in this region; overlaps summed over a similar number of states classified according to the L-S SU(3) scheme give considerably larger values for all low-lying levels, and remain reasonably constant throughout each band.[1,12–14] For the i-active version similar results are obtained; the $J=0$ levels have typically only 20%–40% occupancy of zero k-seniority states, this figure falling dramatically

TABLE VII. Squared overlaps, expressed as percentages, of the ^{20}Ne realistic wave functions arising from the fitted interaction and k-seniority zero states classified using $[\tilde{g}]$ $(\alpha\beta\gamma)$ $\langle\sigma\rangle$ $I=J$ [Eqs. (17) and (20)–(22)].

				[22]				[4]	
		(400)		(020)		(000)		(020)	
K_i^ξ	J	⟨40⟩	⟨00⟩	⟨11⟩	⟨22⟩	⟨00⟩	⟨00⟩	⟨11⟩	⟨22⟩
0_1	0	0.8	1.1			0.7	58.4		
	2	0.1		0.2	0.1			26.9	4.0
	4	0.1			0.0				5.5
0_2	0	2.1	2.3			3.1	0.2		
	2	1.5		1.0	1.6			1.7	0.5
0_3	0	20.7	14.5			6.6	0.2		
	2	19.9		4.5	5.5			2.0	0.0

TABLE VIII. Squared overlaps, expressed as percentages, of the ^{22}Ne realistic wave functions arising from the fitted interaction, and the k-seniority zero subspace [Eqs. (17) and (20)—(22)].

	J					
K_i^{ξ}	0	1	2	3	4	6
0_1	49.4		16.1		3.2	0.9
2_1			21.4	2.1	3.1	
2_2			7.1	6.0	3.3	
1_1		6.3	10.6	2.3		
0_2	17.2		4.2			
0_3	15.9		13.7			

with increase in angular momentum. It is, however, interesting to note that the largest contributions to these (small) overlaps are due to the states with IBM-1 like SU(4) labels $(0\mu 0)$ [Eq. (10)].

The phase ambiguities (Sec. II A) have been investigated by comparing the corresponding pairs to the $n=2$ states most favored by the realistic Hamiltonian. For example, with the assignments $(+++)$, $(++-)$, $(+-+)$, and $(-++)$ the k-active D pairs account for 44%, 21%, 0.2%, and 0.7%, respectively, of the lowest $J=2$ state, the i-active pairs 39%, 27%, 1.3%, and 20%. It is seen that the best results in both cases are given by the assignment used above, and that the choices not corresponding to unmodified phases for either k-i or i-k coupling [$(+-+)$ and $(-++)$] give the lowest overlaps, these being virtually zero in the phenomenologically successful k-active scheme. The choice of i-k coupling [equivalent to the assignment $(++-)$], which gives only slightly smaller overlaps for the pairs, yields similar but smaller overall occupancies for many particles. For instance, the i- (k-) seniority zero states then exhaust 5.5%, 6.3%, and 2.6% (13.3%, 8.9%, and 17.2%) of the three lowest $J=2$ levels for (n, T)=(6,1) [^{22}Ne]. In the *sd* shell, realistic wave functions of most low-lying levels are almost orthogonal to the both the i- and k-seniority zero subspaces.

IV. DISCUSSION AND CONCLUSION

The validity of the FDSM zero seniority shell model truncations, in the sense of the degree to which such states exhaust low-lying wave functions, has been examined for the first time, in the only region where full shell model calculations for rotational nuclei exist at present, the *sd* shell. Here it is seen that neither such truncation is realistic—the majority of low-lying levels are almost orthogonal to both the $k=1$ $v_i=0$ and $i=\frac{3}{2}$ $v_k=0$ subspaces, and no wave functions are well described. Thus the zero seniority basis is, in this instance, not even a good starting point for a perturbation renormalization treatment, and moreover provides no means of understanding low-lying levels. It is however possible that these truncation prescriptions are reasonable in other regions of the nuclear chart, although it should be noted that in larger shells the seniority zero subspace will exhaust a considerably smaller proportion ($\simeq 10^{-10}$) of the full space than that typical (10^{-1}—10^{-2}) for the nuclei described here (Table III).

There is a considerable weight of opinion that S and D pairs, no matter how they are chosen, are alone incapable of an accurate description of even the lowest rotational states,[21,30] and it has recently been argued[31] that the lowest $K^{\xi}=0$ excitations that can be constructed from the *SD* pairs optimized for the ground state are substantially higher in energy than those obtained by introducing a second S pair, implying that models which rely on single S and D pairs, such as OAI and the FDSM, are omitting essential physics. However, even the advocates of this truncation treat the determination of the optimum structures for the Hamiltonian being considered as a crucial step.[20,32] For instance, in the OAI-like procedures, the coefficient of each (jj') configuration in the "collective" S and D pairs is obtained dynamically,[20,32] whereas these are determined purely kinematically by 6-j symbols via coupling either the pseudo-orbit or pseudo-spin angular momenta to zero in the FDSM (Refs. 3 and 5) [Eqs. (12) and (13)]. Although the S pairs of both the k- and i-active versions are just the zero generalized seniority states given by equal weighting for each $(jm, j-m)$ pair,[3] which have been used as shell model analogues of the IBM s boson,[19] it is not obvious that the D pairs [Eqs. (12) and (13)] are as realistic; these have been contrasted with the eigenstates of the surface-delta interaction,[37] and it might be of interest to compare with those determined using, say, variational procedures and a realistic nuclear Hamiltonian,[35,36] or maximization of the quadrupole matrix element with the S pair.[19] For the *sd* shell, the dominant component of both D pairs is antisymmetric in the orbital space,[38] whereas the energetically favored states are orbitally symmetric. Thus the FDSM might give a realistic truncation for systems where its kinematics and the nuclear dynamics coincide; as explained in Sec. I, calculations to investigate this possibility for heavier nuclei are feasible.

This work also serves to illustrate the general conclusion that a successful phenomenology does not by itself indicate that the important degrees of freedom have been isolated. The success of the isospin invariant FDSM in predicting the two quasidegenerate $K^{\xi}=2$ bands in ^{22}Ne (Ref. 10), though intriguing, is now seen to be fortuitous because these bands have less than 1% occupancy of the corresponding FDSM candidates ([2211] (22), Table V). In the same way, the success of the FDSM phenomenology in heavier nuclei[6–7] does not, by itself, imply that the corresponding pairs describe the low-lying levels. It is interesting to note in this context the Pseudonium calculations[39] where an unrealistic truncation allowed results comparable to those of an exact solution for nearly all ob-

TABLE IX. Squared overlaps, expressed as percentages, of the ^{24}Mg realistic wave functions arising from the fitted interaction, and the k-seniority zero subspace [Eqs. (17) and (20)—(22)].

	J						
K_i^{ξ}	0	1	2	3	4	6	8
0_1	42.3		9.1		5.4	1.4	0.2
2_1			4.3	2.1	1.6		
0_2	18.9		7.6		3.2		
1_1		1.7	19.5	1.9			

servables after a free fitting of the corresponding parameters.

It should be reiterated that results such as these do not preclude the construction of effective interactions suitable for the zero seniority subspaces, indicating however that the necessary renormalization procedures would be complex. Here, on a more fundamental level, the suggestion that the FDSM symmetries are approximately realized in low-lying nuclear levels has itself been tested; this is not supported by the evidence from the *sd* shell.

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