

### Beta and neutrino spectra in the decay of <sup>8</sup>B

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We have measured the upper range of the positron momentum spectrum from the decay of <sup>8</sup>B. This spectrum is compared to that predicted on the basis of various available measured alpha spectra from the breakup of the daughter <sup>8</sup>Be nucleus. Recoil order corrections are included in the comparison. After choosing the most consistent alpha spectrum we infer the <sup>8</sup>B neutrino spectrum which we integrate over the <sup>37</sup>Cl( $\nu_e, e^-$ )<sup>37</sup>Ar cross section. We obtain  $(1.07 \pm 0.02) \times 10^{-42}$  cm<sup>2</sup> for this average cross section where the error is an estimate of the rms (i.e.,  $1\sigma$ ) statistical uncertainty from our positron spectrum measurement. This value is appropriate for predicting the solar neutrino capture rate in <sup>37</sup>Cl detectors. Systematic errors in the determination of the integrated cross section are discussed.

The beta decays of <sup>8</sup>Li and <sup>8</sup>B proceed mainly to the broad first excited state of <sup>8</sup>Be, which quickly decays into two alpha particles (see Fig. 1). Thus, for example, the beta spectrum deviates from an allowed shape. This was in fact first demonstrated in 1950 by Hornyak and Lauritsen,<sup>2</sup> who measured the  $\beta$  spectrum in <sup>8</sup>Li decay. Since then, however, most detailed experimental studies of beta decay in this system have precisely measured the <sup>8</sup>Be breakup alpha spectra as opposed to the beta spectra. In principle, however, the beta (and neutrino) spectra can be derived from the alpha spectrum for the particular  $A = 8$  decay in question.

In this paper, we first discuss a formalism for deriving the beta spectrum from the alpha spectrum, including the

weak magnetism and second-forbidden terms. The available measured alpha spectra are discussed. (These alpha spectra are not entirely consistent with each other.) Next we describe our measurement of the beta momentum spectrum in the decay of <sup>8</sup>B, including a comparison to the expected  $\beta^+$  spectra for the various alpha spectra. Using this comparison to determine the most consistent alpha spectrum, we calculate the neutrino spectrum from <sup>8</sup>B decay and integrate it over the cross section for neutrino capture on <sup>37</sup>Cl. Implications for solar neutrino detection in <sup>37</sup>Cl detectors are discussed.

The differential beta momentum spectrum for an allowed beta decay of a nucleus of mass  $M_1$  to a nucleus of mass  $M_2$  (including excitation energy), evaluated to “recoil order”<sup>3</sup> can be expressed as follows:

$$\frac{dN}{dp}(E_0) = \text{const} \times p^2(E - E_0)^2 F(Z, E) \times R(E, E_0) c^2 S(E, E_0) \tag{1}$$

for an electron or positron of momentum  $p$ , where  $E^2 = p^2 + m_e^2$  ( $m_e$  is the electron mass),

$$E_0 = \Delta(1 + m_e^2/2M\Delta)/(1 + \Delta/2M)$$

$[\Delta = M_1 - M_2, M = (M_1 + M_2)/2]$ ,  $F(Z, E)$  is the “Fermi function” (for example, see Ref. 4),  $R(E, E_0)$  is the radiative correction,<sup>5</sup> and  $c^2 S(E, E_0)$  is the weak matrix element including terms to “recoil order.”

The radiative correction  $R(E, E_0)$  diverges at  $E = E_0$ , but is otherwise rather linear with slope  $dR/dE \approx -0.5\%/MeV$  independent of  $E_0$ . [Corrections to the Fermi function  $F(Z, E)$  proportional to  $\alpha Z$  are discussed in Ref. 3. These corrections will not be included in  $R(E, E_0)$ ; for  $Z = 4$ , they are much smaller than the effects we consider here.] For pure Gamow-Teller positron-emitting transitions, assuming time-reversal invariance and ignoring the  $q^2$  dependence of the form factors, we can express the weak matrix element in the form

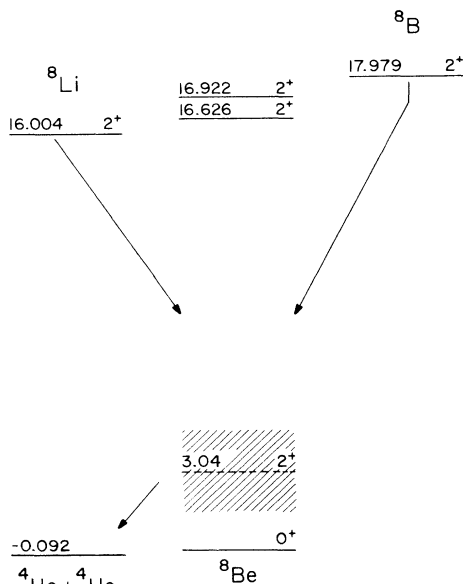


FIG. 1. Energy levels in the  $A = 8$  system. The data are taken from Ref. 1.

$$c^2 S(E, E_0) = c^2 \left\{ 1 - (2E_0/3m_N)(1/A + d/Ac - b/Ac) + (10E/3m_N)(1/A - \frac{2}{3}b/Ac) \right. \\ \left. - \frac{2}{3}(m_c^2/m_N E) [1/A + \frac{1}{2}d/Ac - b/Ac - (h/4A^2c)(E_0 - E)/m_N] \right\}, \quad (2)$$

where  $m_N$  is the nucleon mass,  $A$  is the mass number, and the form factors  $c$ ,  $b$ ,  $d$ , and  $h$  are explicitly defined in Ref. 3. In particular, the Gamow-Teller form factor  $c$  can be deduced from the experimental  $ft_{1/2}$ ,

$$c^2 \approx 2ft_{1/2}(0^+ \rightarrow 0^+)/ft_{1/2},$$

with  $ft_{1/2}(0^+ \rightarrow 0^+) = 3080$  sec (Ref. 6), and the weak magnetism form factor  $b$  is given by the conserved vector current (CVC) hypothesis and experiment by the relation

$$b \approx M(6\Gamma_{M_1}/\alpha E_\gamma^3)^{1/2},$$

where  $E_\gamma$  is the photon energy of the analog gamma transition whose width is  $\Gamma_{M_1}$ .

In  ${}^8\text{B}$  decay,  $M_2$  is not a constant, and so  $c$ ,  $b$ ,  $d$ , and  $h$  are all functions of  $\Delta$  or, equivalently, the excitation energy in  ${}^8\text{Be}$ . The expected beta spectrum is then derived by integrating over  $\Delta$ . The largest effect is from the  $\Delta$  dependence of  $c^2$  which we derive directly from the shape of the alpha spectrum and the known nuclear masses and lifetime.<sup>1</sup> The weak magnetism form factor has been measured as a function of  $\Delta$  in Ref. 7, where it was shown that  $b/Ac$  has an average value of about 7. However,  $b/Ac$  is not constant but varies by roughly 50% over the range of excitation energies considered.

Warburton<sup>8</sup> has pointed out that  $c^2$  is not the result of a single nuclear matrix element, but rather several interfering matrix elements to different final states in  ${}^8\text{Be}$ . Since we are concerned primarily with the shape of the resulting  $\beta$  spectrum, we treat  $c^2(\Delta)$  as a simple phenomenological quantity. However, this may affect the calculation of the form factor ratios  $b/Ac$  and  $d/Ac$  which are used in the calculation of  $S(E, E_0)$ . McKeown *et al.*<sup>9</sup> have precisely measured the  $\beta$ - $\alpha$  correlation in the beta decay of  ${}^8\text{Li}$  and  ${}^8\text{B}$  and show that using the results of Ref. 7 for  $b$  and obtaining  $c$  directly from the alpha spectrum, one finds that  $b/Ac$  (as a function of  $\Delta$ ) correctly describes their measurement. We use their procedure to determine  $b/Ac$ .

There are no published measurements of the "first class" current contributions to  $d$  and  $h$  for  $A = 8$ . Calculations using various shell model wave functions suggest<sup>10</sup> that  $d/Ac \approx 5$ . As for  $h$ , if we assume (see Ref. 3) that

$$M_{1y} = (16\pi/5)^{1/2} 1 \text{ fm}^2,$$

then we have

$$(h/4A^2c)(E_0/m_N) \approx 1.4,$$

which is of the same order as other terms multiplied by  $(m_c^2/m_N E)$  in Eq. (2). However, since  $(m_c^2/m_N E) \ll E_0/m_N$  for the energies we are considering, we are justified in neglecting all these terms.

Warburton<sup>8</sup> has analyzed beta decay in the  $A = 8$  system using the alpha data of Wilkinson and Alburger<sup>11</sup> which were originally used to search for second class

currents. Warburton tabulates the data after they have been corrected for energy loss effects in the target. We refer to Warburton's tabulation of the data as alpha spectrum A, and the raw data<sup>12</sup> as spectrum B. In addition, we use the alpha spectra data of Farmer and Class<sup>13</sup> (spectrum C) and of Clark *et al.*<sup>14</sup> (spectrum D).

Spectrum A does not differ significantly from the raw data except at alpha energies below  $\approx 2$  MeV. Here, the ratio of spectrum A to the raw data changes from  $\approx 0.5$  at 1 MeV to  $\approx 1.2$  at 2 MeV, then becoming unity at higher energies. We note that these changes in the low energy alpha spectrum most significantly affect the high energy beta spectrum (which is where our measurements are carried out) as well as the high energy neutrino spectrum (which is most important for solar neutrino  ${}^{37}\text{Cl}$  capture experiments).

The beta spectrometer used in our measurements has been described elsewhere.<sup>15</sup> Briefly, it is a flat field  $180^\circ$  spectrometer which operates with magnetic fields up to  $\approx 6$  kG. The device has a momentum acceptance  $\Delta p/p \approx 40\%$  and a solid angle acceptance  $\Delta\Omega/4\pi \approx 4 \times 10^{-5}$ . The spectrometer solid angle is defined by a thin plastic scintillator active collimator. For these measurements the active collimator was followed by a thin-window ionization chamber operated in coincidence with the focal-plane detector. The position along the focal plane is determined from resistive charge division. The source position is occupied by thin targets bombarded by beams from the Physics Division Dynamitron accelerator.

The spectrometer momentum was calibrated using the procedure outlined in Ref. 15 using the  $\beta^-$  decay of  ${}^{12}\text{B}$  produced via  ${}^{11}\text{B}(d, p)$  with  $E_d \approx 2$  MeV. Recoil order corrections<sup>5</sup> as well as branching ratios to excited states in  ${}^{12}\text{C}$  are included in the fit. The measured  ${}^{12}\text{B}$  spectrum is shown in Fig. 2(a) along with the fitted curve.

${}^8\text{B}$  was produced via  ${}^6\text{Li}({}^3\text{He}, n)$  at a beam energy  $E_{3\text{He}} \approx 4$  MeV. The experimental  $\beta^+$  spectrum is shown in Fig. 2(b), where it is compared to the prediction using the alpha spectrum A (see details below). For comparison we also show the prediction based on a sharp final state of nominal excitation energy in  ${}^8\text{Be}$ .

In Table I, we compare our measured  ${}^8\text{B}$   $\beta^+$  spectrum to calculations using the various alpha spectra described above. The calculated spectrum is obtained by integrating Eq. (1) over  $\Delta$  using 250 keV steps. Our procedure accounts for the  $\Delta$  dependence of  $R(E, E_0)$  and  $S(E, E_0)$ , including the measured<sup>7</sup> behavior of  $b/Ac$ . The term  $d/Ac$  is assumed to be constant and equal to 5 (unless otherwise noted in Table I). Only the first two terms in Eq. (2) are used to determine  $S(E, E_0)$ . The comparison is judged on the basis of  $\chi^2$ . The calculation is constrained to give the observed number of counts. There are 33 degrees of freedom.

As shown in Table I, the best agreement with our measured  $\beta^+$  spectrum is obtained using alpha spectrum A,

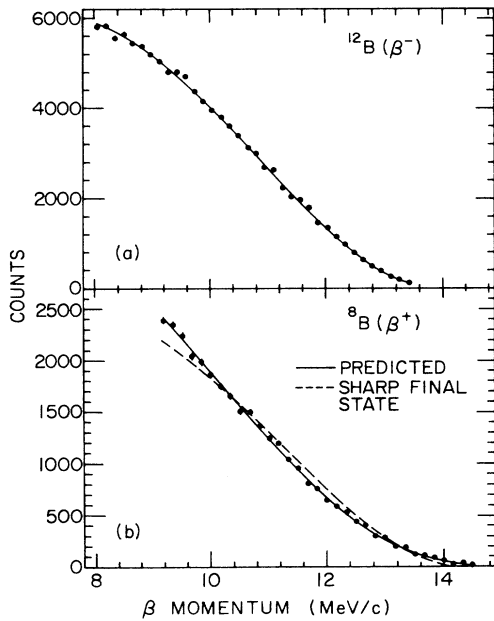


FIG. 2. Measured  $\beta$  momentum spectra. The error bars, where not shown, are smaller than the size of the points. (a) The  $\beta^-$  spectrum of  $^{12}\text{B}$  used to calibrate the spectrometer. A fit is performed to obtain the calibration parameter  $R_0$  (see Ref. 15). (b) The  $\beta^+$  spectrum of  $^8\text{B}$ . The solid line is the predicted spectrum which gives the best agreement (see Table I). The dashed line shows a normal allowed spectrum for a hypothetical sharp final state at  $E_x \approx 3$  MeV in  $^8\text{Be}$ .

when both recoil order terms and radiative corrections are included. The energy loss correction to the raw data significantly improves the agreement with experiment. Alpha spectra C and D only give mediocre agreement with experiment. *Consequently, we conclude that alpha spectrum A (tabulated in Ref. 8) is in best agreement with*

the  $^8\text{B}$  beta decay spectrum measurements. Hereafter, all results use alpha spectrum A, unless otherwise stated.

We now turn to the neutrino spectrum in  $^8\text{B}$  decay. This spectrum is particularly relevant to the detection of neutrinos from the sun since it extends to relatively high neutrino energies (for example, see Ref. 16). Indeed, it is the  $^8\text{B}$  neutrinos that account for most of the signal in the  $^{37}\text{Cl}$  experiment of Davis *et al.* which has consistently reported a measured flux that is much smaller than expected.<sup>17</sup> Present calculations of this neutrino spectrum use the  $^8\text{B}$  decay alpha spectrum, and it is important to determine which alpha spectrum best describes  $^8\text{B}$  decay.<sup>18,19</sup> Since the  $^{37}\text{Cl}(\nu, e)$  capture cross section increases dramatically for energies above  $\approx 8$  MeV,<sup>16</sup> the high energy portion of the neutrino energy spectrum, and consequently the low energy portion of the alpha spectrum, is particularly important.

To an adequate level of approximation, the neutrino spectrum may be calculated from the beta spectrum by making the substitution  $E = E_0 - E_\nu$  in Eq. (1), where  $E_\nu$  is the neutrino energy, and realizing that

$$dN/dE_\nu = (E/p)dN/dp.$$

However, the radiative correction function  $R(E, E_0)$  was calculated for the case where the charged lepton is observed. There is no *a priori* reason that the function would have exactly the same form for the neutrino spectrum.

Bahcall and Holstein<sup>19</sup> have also calculated the neutrino spectrum using a variety of alpha spectra. They, however, ignore the dominant radiative corrections. Their treatment of  $S(E, E_0)$  differs from ours in several other aspects: (1) The form of  $S(E, E_0)$  ignores the term with the form factor  $h$  as we do but includes other terms proportional to  $m_e^2/m_N E$ . Also, they include an energy-independent term proportional to  $\alpha Z$ . (2) They use

TABLE I. Results of calculations using different experimental alpha spectra.

| $^8\text{B}$<br>alpha spectrum | Description                                 | $\chi^2$<br>for comparison to data<br>(33 degrees of freedom) | Integrated $^{37}\text{Cl}(\nu, e)$<br>cross section<br>( $10^{-42}$ cm $^2$ ) |
|--------------------------------|---|---|--|
| A (Ref. 8)                     | Energy-loss-corrected spectrum from Ref. 11 |   |  |
|                                | No recoil order corrections                 | 42.8  | 1.016  |
|                                | Including $S(E, E_0)$ with $d/Ac = 5$       |   |  |
|                                | No radiative corrections                    | 32.3  | 1.047  |
|                                | With radiative corrections                  | 31.6  | 1.068  |
|                                | Including $S(E, E_0)$ with $d/Ac = 1.9$     |   |  |
| B                              | Raw data from Ref. 11                       |   |  |
|                                | No recoil order corrections                 | 157.9   | 1.063  |
|                                | Full recoil order corrections               | 91.6  | 1.117  |
| C (Ref. 13)                    | No recoil order corrections                 | 121.6   | 0.961  |
|                                | Full recoil order corrections               | 170.9   | 1.007  |
| D (Ref. 14)                    | No recoil order corrections                 | 40.3  | 0.994  |
|                                | Full recoil order corrections               | 56.4  | 1.043  |

TABLE II. Calculated  $^8\text{B}$  decay neutrino spectrum. [Note:  $dN/dE_\nu(E_\nu)$  normalized to unit area.]

| $E_\nu$ | $dN/dE_\nu$ | $E_\nu$ | $dN/dE_\nu$ | $E_\nu$ | $dN/dE_\nu$ | $E_\nu$ | $dN/dE_\nu$ |
|---------|-------------|---------|-------------|---------|-------------|---------|-------------|
| 0.2     | 0.000 615   | 4.2     | 0.103 546   | 8.2     | 0.117 293   | 12.2    | 0.025 553   |
| 0.4     | 0.002 351   | 4.4     | 0.108 032   | 8.4     | 0.113 966   | 12.4    | 0.021 671   |
| 0.6     | 0.005 035   | 4.6     | 0.112 163   | 8.6     | 0.110 341   | 12.6    | 0.018 049   |
| 0.8     | 0.008 507   | 4.8     | 0.115 923   | 8.8     | 0.106 444   | 12.8    | 0.014 723   |
| 1.0     | 0.012 627   | 5.0     | 0.119 297   | 9.0     | 0.102 300   | 13.0    | 0.011 712   |
| 1.2     | 0.017 298   | 5.2     | 0.122 275   | 9.2     | 0.097 933   | 13.2    | 0.009 029   |
| 1.4     | 0.022 441   | 5.4     | 0.124 855   | 9.4     | 0.093 392   | 13.4    | 0.006 790   |
| 1.6     | 0.027 955   | 5.6     | 0.127 010   | 9.6     | 0.088 673   | 13.6    | 0.004 889   |
| 1.8     | 0.033 759   | 5.8     | 0.128 738   | 9.8     | 0.083 811   | 13.8    | 0.003 342   |
| 2.0     | 0.039 765   | 6.0     | 0.130 035   | 10.0    | 0.078 833   | 14.0    | 0.002 141   |
| 2.2     | 0.045 917   | 6.2     | 0.130 905   | 10.2    | 0.073 770   | 14.2    | 0.001 272   |
| 2.4     | 0.052 153   | 6.4     | 0.131 357   | 10.4    | 0.068 683   | 14.4    | 0.000 778   |
| 2.6     | 0.058 407   | 6.6     | 0.131 379   | 10.6    | 0.063 560   | 14.6    | 0.000 440   |
| 2.8     | 0.064 627   | 6.8     | 0.130 979   | 10.8    | 0.058 444   | 14.8    | 0.000 234   |
| 3.0     | 0.070 763   | 7.0     | 0.130 165   | 11.0    | 0.053 365   | 15.0    | 0.000 117   |
| 3.2     | 0.076 773   | 7.2     | 0.128 948   | 11.2    | 0.048 354   | 15.2    | 0.000 054   |
| 3.4     | 0.082 618   | 7.4     | 0.127 354   | 11.4    | 0.043 482   | 15.4    | 0.000 027   |
| 3.6     | 0.088 249   | 7.6     | 0.125 373   | 11.6    | 0.038 730   | 15.6    | 0.000 011   |
| 3.8     | 0.093 633   | 7.8     | 0.123 024   | 11.8    | 0.034 138   | 15.8    | 0.000 004   |
| 4.0     | 0.098 741   | 8.0     | 0.120 324   | 12.0    | 0.029 736   | 16.0    | 0.000 001   |

$b/Ac=7.7$  and ignore any  $\Delta$  dependence. (3) A value of  $d/Ac=1.9$  is assumed. These differences contribute little to  $S(E,E_0)$  (see, for example, Table I), but nevertheless we find that if we ignore radiative corrections as they do, our result differs from theirs by  $\approx 10\%$  at  $E_\nu=12$  MeV, but the two calculations are consistent below  $E_\nu=10$  MeV. We believe that radiative corrections should be included and their effect may be significant relative to the contribution of the recoil order terms in the weak matrix element.<sup>20</sup> In lieu of a proper calculation of the radiative corrections, we apply the correction  $R(E,E_0)$  to the neutrino spectrum using  $E=E_0-E_\nu$ . The resulting neutrino spectrum is tabulated in Table II.

To calculate the  $^{37}\text{Cl}(\nu,e)$  cross section averaged over the  $^8\text{B}$  neutrino spectrum, we use the cross section as a function of neutrino energy as tabulated in Ref. 16. We then integrate the product of the cross section and the calculated neutrino spectrum over neutrino energy. Results for the various calculated neutrino spectra are listed in Table I. Having concluded that alpha spectrum A is the most appropriate, and with recoil order terms and radiative corrections included, we find a value of  $1.07 \times 10^{-42}$  cm<sup>2</sup> for the integrated cross section. It is difficult to assign a standard error to this cross section since our data do not cover the full positron momentum range. That is, a distortion of the positron spectrum below 9 MeV/c would not be detected but it would certainly affect the neutrino spectrum and average capture cross section. (Alpha spectrum A is of such high statistical quality that it does not contribute to the standard error.) Assuming that the different calculations in Table I represent reasonable distortions to the positron and neutrino spectra, we note that a difference in total  $\chi^2$  of one from the minimum value corresponds to a change of  $0.02 \times 10^{-42}$  cm<sup>2</sup> in the integrated cross section. We use this as an estimate of the standard error. Our result is slightly larger than the value

of  $1.03 \times 10^{-42}$  cm<sup>2</sup> obtained by Bahcall and Holstein.<sup>19</sup>

There are some additional uncertainties associated with our value for the integrated cross section. First, there are the uncertainties of radiative corrections to the neutrino spectrum. The effect of our assumed radiative correction is a 2% increase in the integrated cross section. Secondly, there are radiative corrections to the neutrino capture cross section itself which have been neglected. These have been included for the case of antineutrino capture on protons<sup>21</sup> and the effect is to increase the cross section by  $\approx 1\%$ . Thirdly, as indicated by Bahcall and Holstein,<sup>19</sup> nuclear and recoil order effects may increase the capture cross section by as much as 3%. These corrections suggest that the actual capture cross section may be as large as  $1.1 \times 10^{-42}$  cm<sup>2</sup>. These effects could increase the contribution of  $^8\text{B}$  neutrinos to the expected capture rate in  $^{37}\text{Cl}$  detectors by  $\approx 10\%$  over calculations that do not include recoil order and radiative corrections (see Ref. 19).

In conclusion, we have measured the  $\beta^+$  momentum spectrum from the decay of  $^8\text{B}$  for momenta greater than 9 MeV/c. This measurement has been compared to the predicted spectrum, with and without including recoil order corrections, based on a variety of previously measured alpha spectra. Best agreement is found by including recoil order corrections and using the alpha spectrum data with energy loss corrections.<sup>8</sup> We calculate the neutrino spectrum with radiative corrections and calculate the average capture cross section in  $^{37}\text{Cl}$ , finding a value of  $(1.07 \pm 0.02) \times 10^{-42}$  cm<sup>2</sup>. Radiative corrections to the cross section might increase this value by roughly 1%. Other corrections may increase the integrated cross section by an additional 3%.

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