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Singularities in π-N scattering amplitude in nuclear matter

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The possibility of π-N bound states in nuclear matter is examined. It turns out that a very special many-body effect allows states which have no counterpart in the two-body system. In fact, around a logarithmic singularity, generated by the Pauli blocking, an attractive interaction—however small—creates bound states whose origin greatly resembles that of the Cooper bound state of two electrons in a superconductor or the Suhl-Abrikosov resonance in the Kondo effect. Coupling with other reaction channels gives the bound states a finite lifetime.

I. INTRODUCTION

The unexpected coherent production of pions in heavy ion reactions reported in Ref. 1 raises the question that something is amiss in our current understanding of the pion dynamics in nuclei. Along this line, a careful reanalysis has been undertaken of the Pauli blocking (PB) which is normally expected to have only a slight effect on pion dynamics.[2] This is effectively true on the average; but for the total momentum of the pion-nucleon pair near zero ($P \approx 0$) some singularities are present.[3–5] These singularities have been considered very accidental due to a simplifying assumption of a sharp form of the nucleon momentum distribution, and few explanations have been proposed to justify their suppression: In Refs. 3 and 4 the authors supposed a removal of the singularities by coupling with other reaction channels; in Ref. 5 a smeared finite temperature nucleon distribution is used to calculate the averages over the nucleon distribution (i.e., pion self-energy), taking the limit for T going to zero at the end.

It is the aim of this work to shed some light on these anomalies and show that they originate from a genuine many-body effect which has very striking analogies to Cooper condensation[6] and the Kondo effect.[7] Once the display of a very general effect has been recognized, the ineffectiveness of the proposed suppressions will be shown.

It is well known that an attractive effective interaction, however small, greatly modifies the properties of the ground state of a Fermi liquid, and generates superconductivity in some conductors. The drastic variation in the conducibility properties of a metal below a critical temperature is a consequence of a peculiar many-body effect; in fact, as Cooper showed,[6] an attractive interaction between two fermions in the presence of other fermions in the ground states can generate bound states for any value—however small—of the coupling constant. On the contrary, in the free two-particle system, bound states are present only if the coupling constant exceeds some minimum value. The source of this difference is the presence of a logarithmic singularity in the two-particle Green's function in the medium at the Fermi energy which, in the presence of an attractive interaction, gives poles below the Fermi energy; the position of these poles has a typical nonanalytic relation to the coupling constant.

A similar many-body effect is present even in the π-N system in nuclear matter. In this case, the logarithmic singularity is generated by the sharp momentum distribution of the nucleons; it is natural to expect polar singularities in the partial waves with an attractive π-N interaction. There are some differences with respect to the case of two electrons; here, in fact, the PB is effective only on the nucleons. This configuration bears greater resemblance more to a magnetic impurity atom interacting with the conduction electrons of a metal:[7] The PB is effective only on the electrons. The electron-impurity scattering amplitude shows a resonance, the Suhl-Abrikosov (SA) resonance,[8] around the Fermi surface in the partial wave attractively coupled to the impurity. As in Cooper condensation, the SA resonance originates from a sharp end point distribution of intermediate states (the density of conduction electrons). Also, it has a very important role in the explanation of many properties of some conductors, in particular in explaining the minimum in resistivity versus the temperature of dilute magnetic alloys (the Kondo effect[7]).

II. POLAR SINGULARITY

To display in detail the generation of the polar singularity for the pion-nucleon propagator in nuclear matter, we shall use an extension of the Low equation that allows a derivation very similar to that given in Ref. 9 for the Cooper bound states. At any rate, other approaches can provide the same results.[10]

The Low equation for the π-N effective scattering amplitude in nuclear matter has been derived in Refs. 5 and 11. At its lowest level of approximation, this approach differs from others in that the PB is present even in the Born terms. This feature is introduced by the request that the integral equation generate a set of Feynman diagrams with the PB in each nucleon line. Some motivations have been proposed[2] to justify the suppression of the PB from the Born terms. We prefer to use a more complex approach (linearization and coupled channels), which is safer from a fundamental viewpoint, in place of suppression. In fact, terms which could suppress the PB in the Born terms (with some simplifying assumption about the effective vertex function) are generated by a disconnected term of the two-particle–one-hole intermediate states; it is clearly better to consider all the effects of this set of intermediate states on the same footing. At any rate, in Ref. 5 a method of successive linearization around a main solution has been developed to solve the analytic problem introduced by the lack of separability of the Born terms due to PB or other more realistic dependence from the momenta of the incoming particles. This method generates the complete solution at any desired degree of approximation.

For the present task, we can skip many of the complications of the complete equation; in fact, for $P\approx 0$, the s waves in the momenta of the PB factors suffice. If the medium corrections are suppressed in the crossing terms (the rationale of this is given below), we can limit ourselves to considering the partial waves of the Chew and Low model, although it will be evident that other partial waves could develop the singularities described here. For nucleons so heavy to take on momentum but no energy, the projected amplitudes have the following equations:

$$F^{\alpha}(\omega,P,q,q')=\frac{\lambda_\alpha}{\omega}[1-n_{00}(P,q,q')]-\frac{9}{8}\frac{\lambda_\alpha}{\omega}[n(P)-n_{00}(P,q,q')]\delta_{\alpha,11}$$
$$+\frac{1}{\pi}\int_{m\pi}^{\infty}d\omega_n\frac{F^{\alpha}(\omega_n,P,q,q_n)[1-n_0(P,q_n)]F^{\alpha *}(\omega_n,P,q',q_n)}{\omega_n-\omega-i\epsilon}q_n^3v^2(q_n)+\text{CT}\ . \tag{1}$$

The functions $n_0(P,q),n_{00}(P,q,q')$ are the angular average of the PB function, P is the total momentum of the π-N pair, and only a parametric P dependence survives. The PB in the B terms is effective at low pion momenta (excluding the P_{11} partial wave, which requires special treatment). However, since the low pion momenta are cut away by the PB in the integral term, important contributions are expected only from the high energy integrations controlled by the cutoff $v(q)$. This fact immediately suggests the splitting

$$F^{\alpha}(\omega,P,q,q')=H^{\alpha}(\omega,P)+E^{\alpha}(\omega,P,q,q')\ , \tag{2}$$

where $E^{\alpha}(\omega,P,q,q')$ embodies all the complications given by the nonseparability of the B terms, and must be solved as in Ref. 5. $H^{\alpha}(\omega,P)$ accounts for all the contributions given by the high energy integrations, and is defined by the integral equation ($\alpha\neq 11$)

$$H^{\alpha}(\omega,P)=\frac{\lambda_\alpha}{\omega}+\frac{1}{\pi}\int_{m_\pi}^{\infty}d\omega_n q_n^3v^2(q_n)\left[\frac{|H^{\alpha}(\omega_n,P)|^2[1-n_0(P,q_n)]}{\omega_n-\omega-i\epsilon}+\text{CT}\right]\ . \tag{3}$$

Here the PB is present only in the singular integration and is the only source of logarithmic diverging terms. The crossing terms (CT) contain nonsingular integrations ($\omega>0$) and depend on the shifted total momentum $|\mathbf{P}-\mathbf{q}-\mathbf{q}'|$. Therefore, excluding special values of $\mathbf{q}$ and $\mathbf{q}'$, the terms are insensitive to the singularities generated by the sharp integration edge of the direct term. For special values of $\mathbf{q},\mathbf{q}'$, they develop a Cooper-type singularity of their own at negative ω. For the positive values of ω which are of interest to us, the effect of this singularity is fairly negligible and, whenever possible, the free values for the crossing terms are used.

In these assumptions, an N/D (Ref. 12) solution of Eq. (3) normalized at the nucleon pole to give the residuum λ_α has the form

$$H^{\alpha}(\omega,P)=\frac{N_\alpha(\omega,P)}{D_\alpha(\omega,P)}=N_\alpha(\omega,P)\left[1-\frac{\omega}{\pi}\int_{m_\pi}^{\infty}d\omega_n q_n^3v^2(q_n)\frac{N_\alpha(\omega_n,P)[1-n_0(P,q_n)]}{\omega_n(\omega_n-\omega-i\epsilon)}\right]^{-1}\ ,$$

which can be recast in the form

$$H^{\alpha}(\omega,P)=\left[\frac{D_\alpha^0(\omega,P)}{N_\alpha(\omega,P)}+\frac{\omega}{\pi}\frac{1}{N_\alpha(\omega,P)}\int_{m_\pi}^{\infty}d\omega_n q_n^3v^2(a_n)\frac{N_\alpha(\omega_n,P)n_0(P,a_n)}{\omega_n(\omega_n-\omega-i\epsilon)}\right]^{-1}\ ,$$
$$D_\alpha^0(\omega,P)=1-\frac{\omega}{\pi}\int_{m_\pi}^{\infty}\frac{N_\alpha(\omega_n,P)q_n^3v^2(a_n)}{\omega_n(\omega_n-\omega-i\epsilon)}d\omega_n\ . \tag{4}$$

If one neglects the PB corrections in $N_\alpha(\omega,P)$ and uses its free expression, Eq. (4) assumes a more transparent form which shows a remarkable independence from the dynamics assumed for the free π-N scattering amplitude:

$$H^{\alpha}(\omega,P)=\left[\frac{1}{h_\alpha(\omega)}+\frac{\omega}{\pi}\frac{1}{N_\alpha(\omega,P)}\int_{m_\pi}^{\infty}\frac{N_\alpha(\omega_n,P)n_0(P,q_n)}{\omega_n(\omega_n-\omega-i\epsilon)}q_n^3v^2(q_n)d\omega_n\right]^{-1}\ . \tag{4'}$$

A symmetric nuclear matter, whose Fermi momentum is much lower than the cutoff point given by $v(q)$, is supposed; $h_\alpha(\omega)$ is the free π-N scattering amplitude. Let us now examine the $P=0$ region; here for $\omega<\omega_F$ [$\omega_F=(k_F^2+m_\pi^2)^{1/2}$] the imaginary part of $1/h_\alpha(\omega)$ is suppressed by the imaginary part of the integral; the discontinuity in the singular integration gives a logarithmic singularity.[13] The N/D solution can have poles not explicitly considered in (3); they are given by

$$\mathrm{Re}\left[\frac{1}{h_\alpha(\omega)}\right]+\frac{q^3}{\pi}\ln\left[\frac{\omega_F-\omega}{\omega-m_\pi}\right]+\frac{\omega}{\pi}\frac{1}{N_\alpha(\omega,P)}\int_{m_\pi}^{\infty}\left[\frac{q_n^3N_\alpha(\omega_n,P)}{(\omega_n-\omega)\omega_n}-\frac{q^3N_\alpha(\omega,P)}{\omega(\omega_n-\omega)}\right]d\omega_n\ . \tag{5}$$

Equation (5) has solutions for any attractive interaction $[\mathrm{Re}h_\alpha(\omega)>0]$, however small, and, like superconductivity, it has the typical expression for the low coupling $[h_\alpha(\omega)\approx\lambda_\alpha/\omega]$

$$\omega_B^\alpha\approx\omega_F-(\omega_F-m_\pi)e^{-(\pi\omega_F/\lambda_\alpha q_F^3)}\ ,$$
$$\omega_B^\alpha=\frac{\omega_F+m_\pi e^{-[\pi/\tan\delta_\alpha(\omega_F)]}}{1+e^{-[\pi/\tan\delta_\alpha(\omega_F)]}}\ , \tag{6}$$

with an essential singularity for $\lambda_\alpha\to 0$. The residuum at the pole has the sign of a bound state. Above ω_F, another zero of $\mathrm{Re}(H^\alpha(\omega,P))$ is present, but here the full free imaginary part is effective. Very similar results can be easily obtained from a Lippmann-Schwinger equation with a separable potential and the same density of intermediate states considered here. It is clear from (5) that a Cooper-type bound state can be generated only for $\omega_F<\omega_{\Delta^{++}}$ in the P_{33} partial wave; above the resonance, $\mathrm{Re}h_{33}(\omega)$ is lower than zero.

For a small increase in P or temperature T, the variation of $n_0(P,q)$ is rapid enough around ω_F to maintain the pole. The survival of the pole of a small finite increase of the temperature renders ineffective the method proposed in Ref. 5 to get a nondivergent pion self-energy in the hole-line approximation; in fact, a fictitious smearing of $N_0(P,q)$ with a finite temperature was supposed sufficient to get rid of the singularities.

The form of Eq. (5) implies the absence of logarithmic divergence in $E^\alpha(\omega,P,q,q')$. In fact, the linearized part has integrals of $H^\alpha(\omega,P)[1-n_0(P,q)]$, but $H^\alpha(\omega,P)$ is of the order $[\ln|\omega_F-\omega|]^{-1}$ around ω_F, and this gives at most singularity of the dilogarithmic type [a closer inspection of the first linearization using the equations given in Ref. 5 shows $\ln|\omega_F-\omega|^{-1}$ behavior even for $E^\alpha(\omega,P,q,q')$].

For small $P\ll(\omega_F-\omega_B)$ and $|\omega-\omega_B|\ll(\omega_F-\omega_B)$ the amplitude $H^\alpha(\omega,P)$ becomes

$$H^\alpha(\omega,P)=\frac{\omega_F-\omega_B^\alpha+\dfrac{Pq_F}{\omega_F}}{\dfrac{q_F^3}{\pi}\left[\omega_B^\alpha-\omega+\dfrac{Pq_F}{\omega_F}\right]}\ .$$

A similar set of singularities is encountered even in the P_{11} partial wave. Here, in fact, we have to consider the PB in the B terms. From (1) it is evident that the direct nucleon pole term is suppressed for $P<K_F$, and only the crossed attractive B term survives, contributing to the high energy integral up to the cutoff. While complete solution of (1) necessitates a calculation of the crossing terms, our task of showing the presence of a Cooper-type pole does not require careful estimation of the high energy integrals. All they have to do is give a positive contribution, which is easily checked from the signs of the crossing matrix; in addition to this, from the results of Mizutani *et al.*,[14] the P_{11} partial wave without the direct pole term should show the Roper resonance at an energy comparable to the $\Delta^{++}(1236)$ and be of similar width. Hence, the situation should greatly resemble that of the $\Delta^{++}(1236)$. In our approach without the crossing terms we can employ the low coupling result with $\lambda_\alpha=\frac{1}{3}f_\pi^2/\mu^2$. This picture can be changed by the insertion of more complex intermediate states, but one must keep in mind that even the real part of the free P_{11} partial wave becomes positive above a certain energy.

III. COUPLING WITH OTHER REACTION CHANNELS

The states of Eq. (5) are located at ω_B^α above the ground state, so the states below this energy will be populated by their decay, and the polar form of the π-N scattering amplitude breaks down. In general, for a not too strong coupling with other reaction channels one expects the transformation of a bound state in a narrow resonance: We can prove this is the case for our states. To see the modifications of the present picture introduced by the channel couplings, one must resort to an analytical approach because only in this way can one safely develop the calculation to the logarithmic accuracy required by the problem. A numerical approach is prone to losing logarithmic singularities; the absence of any trace of these states in the numerical calculation of Ref. 15 is just due to the impossibility of reaching a sufficient degree of accuracy with their method.

Without giving a detailed demonstration, let us say that it is not difficult to indicate the modifications introduced in Eq. (1) by the presence of other reaction channels; in addition to the standard one-nucleon and one-nucleon–one-pion excitations over the ground state as intermediate states, one must consider a more complex set of excited states. As in Eq. (1), the Born terms will be the states without integration over a current variable. The rescattering terms will be given by the intermediate states with at least a nucleon and a strong interacting boson (pion or something else) with the cut on the right-hand side of the real axis (unitarity cut); for each rescattering term there is a crossing term with a cut on the left-hand side of the real axis. Continuous channels given by intermediate states with more than two particles will be possible in a many-body system even at very low energy. To treat a problem so complex analytically, one must introduce some simplifying assumptions:

(a) the Born terms and the amplitudes are approximated with separable forms;

(b) no anomalous threshold is around the discontinuity we are studying;

(c) the continuous channels are approximated with a finite number, however large, of discrete channels.

Assumptions (b) and (c) are not critical and could be released with a generalization of the formalism; condition (a) is unavoidable, its effect can be mitigated accounting for the effects of the nonseparability with a many channel generalization of the approach given in Ref. 5, i.e., with successive linearizations around a main solution. But no important modification can originate from these corrections because all the logarithmic diverging terms are absorbed in the main solution, as explained above for the noncoupled case. For this only the structure of the main solution shall be examined. The set of coupled nonlinear singular equations can be solved as in Ref. 16. In the following equations the parametric P dependence, which is present in all terms, will be dropped from the notation. The channel in which we are exploring the effects of the coupling is 1,1:

$$\mathcal{H}_{ij}(\omega)=b_{ij}(\omega)+\frac{1}{\pi}\int_{m_\pi}^{\infty}\frac{\mathcal{H}_{ie}(\omega_n)\rho_e(\omega_n)\mathcal{H}^*_{ej}(\omega_n)}{\omega_n-\omega-i\epsilon}d\omega_n \;.$$

The solutions are

$$\mathcal{H}_{ij}(\omega)=N_{ie}(\omega)[D^{-1}(\omega)]_{ej} \;,$$

$$D_{ij}(\omega)=\delta_{ij}-\frac{\omega}{\pi}\int_{m_\pi}^{\infty}\frac{\rho_i(\omega_n)N_{ij}(\omega_n)d\omega_n}{\omega_n(\omega_n-\omega-i\epsilon)} \;,$$

$$N_{ij}(\omega)=b_{ij}(\omega)+\frac{1}{\pi}\int_{m_\pi}^{\infty}\frac{[\omega_n b_{ie}(\omega_n)-\omega b_{ie}(\omega)]}{\omega_n(\omega_n-\omega)}\rho_e(\omega_n)N_{ej}(\omega_n)d\omega_n \;.$$

The functions $N_{1,j}(\omega)$ are regular on the right-hand side cut. The terms $D_{1,j}(\omega)$ for $P=0$ have a jump in the integrand given by the PB on the nucleon momentum distribution; manipulating the solution as in Eq. (5), it is possible to evidence the free scattering amplitude yielding ($\omega<\omega_F$):

$$\mathcal{H}_{11}(\omega)=\frac{\left[1+\frac{\sum_{j>1}N_{1j}(\omega)A_{j1}(\omega)}{N_{11}(\omega)A_{11}(\omega)}\right]}{\mathrm{Re}\left[\frac{1}{h_\alpha(\omega)}\right]+\frac{q^3}{\pi}\ln\left[\frac{\omega_F-\omega}{\omega-m_\pi}\right]\left[1+\sum_{j\neq 1}\frac{N_{1j}(\omega)A_{j1}(\omega)}{N_{11}(\omega)A_{11}(\omega)}\right]+\frac{g_j(\omega)A_{j1}(\omega)}{N_{11}(\omega)A_{11}\omega}} \;, \tag{7}$$

$$A_{em}(\omega)=[D^{-1}(\omega)]_{em}\det\{D(\omega)\} \;.$$

$g_j(\omega)$ is the regular part of $D_{1,j}(\omega)$. The logarithmic term exactly factorizes the numerator of $\mathcal{H}_{11}(\omega)$, making the structure of $\mathcal{H}_{11}(\omega)$

$$\mathcal{H}_{11}(\omega)=\left[\alpha(\omega)+i\gamma(\omega)+\frac{q^3}{\pi}\ln\left[\frac{\omega_F-\omega}{\omega-m_\pi}\right]\right]^{-1} \;, \tag{8}$$

where $\alpha(\omega)$ and $\gamma(\omega)$ are real functions whose definition can be extracted from Eq. (7). For $P=0$ and $\omega<\omega_F$, the width $\gamma(\omega)$ receives contributions only from the medium correction (reflection broadening, absorption, etc.), the imaginary part of the free scattering amplitude being suppressed by the PB. To prove that the singularity of Eq. (8) is a resonance, one has to locate the zero of its denominator. The imaginary part $\gamma(\omega)$ is given by the discontinuity across the right-hand side cuts of the reaction channels, and it is negative. In fact, from Eq. (7), it is

$$\gamma(\omega)=\mathrm{Im}\left[\frac{D^0_{1j}(\omega)A_{j1}(\omega)}{N_{1j}(\omega)A_{j1}(\omega)}\right]+q^3 \;,$$

$$D^0_{1j}(\omega)=D_{1j}-\frac{1}{\pi}\int_{m_\pi}^{\omega_F}\frac{N_{1j}(\omega)q^3v^2(q_n)}{(\omega_m-\omega-i\epsilon)}d\omega_n \;,$$

$$\gamma(\omega)=\mathrm{Im}\left[\frac{1}{\mathcal{H}^0_{11}(\omega)}\right]+q^3 = -\sum_{j>1}\frac{|\mathcal{H}_{1j}(\omega)|^2\rho_j(\omega)}{|\mathcal{H}^0_{11}(\omega)|^2} \;.$$

$\mathcal{H}^0_{11}$ is an amplitude formally identical to $\mathcal{H}_{11}$ but with the integrals of D_{1j} extended down to the threshold m_π (D^0_{1j}); the phase of the logarithmic term is zero on the upper side of the right-hand side cut for $\omega<\omega_F$; so the point at which this phase equals $-\gamma(\omega)$ is located in the second Rieman sheet below the right-hand side cut. Hence, in a small range of energies around the zeros of $\mathrm{Re}(\mathcal{H}_{11}^{-1})$, $\mathcal{H}_{1,1}(\omega)$ assumes the form of a resonance which escapes the unitarity limit ($1/q^3$) for small inelasticity. It has a Breit-Wigner shape only very close to the position of the resonance, and the tails are logarithmically decreasing. The delicacy of the proper position of the resonance in the unphysical sheet is evident from Eq. (7). In fact, a small phase difference from the numerator of (7) and the factor in parentheses of the logarithmic term can scatter the singularity almost everywhere in the complex plane. This explains the enormous care required for a proper numerical calculation.

IV. CONCLUSIONS

The results of the present calculation force us to conclude that the PB can greatly affect the π-N scattering amplitude for low P by inducing a large fluctuation in its real and imaginary parts. Such effects are consequences of a genuine many-body effect which generates bound states around an end point discontinuity in the density of intermediate states whenever attractive interactions are present. This many-body effect is very similar to that which allows the Cooper bound states of two electrons in a superconductor (with a similar nonanalytic relationship of the energy shift from the coupling constant) or, better still, similar to the SA resonance.[8] The SA resonance explains the minimum in the resistivity versus the temperature observed in some alloys (the Kondo effect[7]). Around a magnetic impurity introduced in a Fermi liquid the discontinuity in the density of electron states generates a bound state very near to the Fermi surface; this state is turned to a resonance by the coupling with a particle-hole reaction channel. As in our states, the PB is effective only on one kind of particle, i.e., the conduction electrons, the impurity is assumed infinitely heavy. The main difference with respect to the pions in nuclear matter is that the fermions (the nucleons) are much heavier than the pions. Hence in the singular integration the energy of the nucleon is disregarded compared to the pion energy, and the PB factor is effective only for low P.

So, in nuclear matter, a branch of fermion-type excitations which have quantum numbers $(\frac{3}{2},\frac{3}{2})$ and perhaps $(\frac{1}{2},\frac{1}{2})$ appears possible. These are peculiar to a many-body system and have no correspondence in the two-body system. The effective interaction which makes these states possible is the strong short range π-N interaction. In general, their wave functions decrease at large distances as $1/r^2$, and, being weakly bound, they will have a very large $\langle r^2 \rangle$.[6] A small range of P values around $P=0$ are possible for these fermions. Channel couplings give these states a finite lifetime, turning them into resonances: Their positions are defined by a formally invariant expression [Eq. (5)] upon switching on the coupling. The imaginary parts are exclusively given by the coupling. An exact factorization in Eq. (7) is fundamental in demonstrating this property. No numerical approach can reach this accuracy and can even confuse the resonances in a uniform background (or place them in a wrong position in the complex plane).

As in superconductivity at a critical temperature, which is dependent upon density, the smearing of the nucleon distribution makes these states disappear. A similar disappearance is produced by an increase in P for $T=0$. The presence of poles in the π-N scattering amplitude will require an upgrading of the formalism used to calculate the properties of the pions in nuclear matter; for example, the pion self-energy, calculated in the hole-line approximation, is divergent at the energy of the poles.

It is beyond the aim of this work to indicate some possible experimental testing of these excitations (nuclear matter is a too conceptual matter for experimentation). Nevertheless, some properties of neutron stars can bear their signature. In finite nuclei, few assumptions of this approach (especially the sharpness of the nucleon momentum distribution) break down, and further work will be required for an extension (if any). However, it is worthwhile noting that the unusually long range of these states could have very unexpected effects on the pion production mechanism.

The results of this approach require very few explicit reference to the π-N system; for that they can be easily extended even to other many-fermion systems attractively coupled to a particle (boson or fermion) distinguishable from the background.

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[1]W. A. Zajc *et al.*, Phys. Rev. C **29**, 2173 (1984).

[2]G. E. Brown and W. Weise, Phys. Rep. **22c**, 280 (1975); C. B. Dover and R. H. Lemmer, Phys. Rev. C **14**, 2211 (1976); H. C. Chiang and Mikkel B. Johnson, *ibid.* **32**, 531 (1985).

[3]R. Cenni and G. Dillon, Nucl. Phys. **A333**, 413 (1980).

[4]E. J. Moniz and A. Sevgen, Phys. Rev. C **24**, 224 (1981).

[5]G. Landi, Nuovo Cimento **A71**, 409 (1982).

[6]L. N. Cooper, Phys. Rev. **104**, 1189 (1956).

[7]J. Kondo, *Solid State Physics* (Academic, New York, 1969), Vol. 23.

[8]H. Suhl, Phys. Rev. **138**, A515 (1965); **141**, 483 (1966); A. A. Abrikosov, Physics (N.Y.) **2**, 5 (1965). The first one is very interesting. Here the author, with a Chew and Low type of approach, deduces the equations for the scattering amplitude of a conduction electron form a magnetic impurity—his final equations have the familiar form of a system of nonlinear singular integral equations.

[9]A. L. Fetter and J. D. Walecka, *Quantum Theory of Many Particle Systems* (McGraw-Hill, New York, 1971).

[10]In fact, the first indication of the polar form of scattering amplitude is in Ref. 3 with an isobaric model. In Figs. 9 and 10 of C. B. Dover and R. H. Lemmer, Phys. Rev. C **7**, 2312 (1973), the real and imaginary parts of the amplitude reported in the plots show very bizarre oscillations and cusps right in the expected positions of these states. Very noticeably the positions of the oscillations and cusps move to higher energy with increasing densities. Equations (62) and (65) of Chiang and Johnson (Ref. 2) must display the poles herein described.

[11]G. Landi and F. Matera, Nuovo Cimento **A54**, 469 (1979).

[12]G. Chew and S. Mandelstam, Phys. Rev. **119**, 467 (1960).

[13]N. Muskhelishvili, *Singular Integral Equation* (Noordhoff, Groningen, 1963).

[14]T. Mizutani, C. Fayard, G. H. Lamot, and S. Nahabetian, Phys. Rev. C **24**, 2633 (1981).

[15]R. Cenni and G. Dillon, Nucl. Phys. **A392**, 438 (1983).

[16]A. S. Rinat and M. Stingl, Ann. Phys. (N.Y.) **65**, 141 (1971).