Excitation of the Δ(3,3) resonance in compressed finite nuclei from a constrained mean-field method

Mahmoud A. Hasan* and Sigurd Köhler
Department of Physics, University of Arizona, Tucson, Arizona 85721

James P. Vary†
Kellogg Laboratory, California Institute of Technology, Pasadena, California 91125
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In this paper we present a nonrelativistic microscopic mean-field framework for finite nuclei in which the nucleus is described as a system of A baryons where each baryon is a dynamically determined superposition of a nucleon (N) and a Δ(3,3) resonance. The N-N interaction is a Brueckner G matrix based on the Reid soft-core interaction while the N-Δ transition potentials and Δ-Δ interactions are one-boson-exchange potentials. The ground-state properties of the nuclei ^{16}O and ^{40}Ca are determined within the spherical Hartree-Fock approximation in a no-core basis consisting of up to six major oscillator shells. A detailed study of the zero-temperature properties of these nuclei under compression and dilation is presented in order to gain insight into the nuclear equation of state. Under certain conditions, a transition to a mixture of nucleons and deltas is found in ^{40}Ca but not in ^{16}O. The transition is reminiscent of a first-order phase transition.

I. INTRODUCTION

Conventional nonrelativistic, microscopic nuclear-structure calculations are based on modeling the nucleus as a collection of nucleons which interact through a nucleon-nucleon potential. The nucleons are assumed to be elementary structureless particles. Thus no internal excitations of a single nucleon are considered in this case. With the advent of precision experiments at intermediate and high energies using electromagnetic and heavy ion beams, the contribution of baryon resonances to the structure of nuclei in their ground state and under compression is clearly a major theoretical question.

The possible importance of nucleon resonances in nuclear systems has been recognized for a long time.[1-3] Their effect has been investigated in the two-nucleon system, both in the bound[4] and in the scattering state.[5-10] The effect of nucleon resonances has been investigated also in infinite nuclear systems (nuclear matter[11] and neutron matter[12]). In all these investigations, it is the Δ(3,3) resonance that has been studied. This was seen to be the most important resonance for the understanding of the intermediate range of the nucleon-nucleon force because of the strong N-Δ transition strength and low mass of the delta relative to the nucleon. Thus the excitation of nucleons to deltas is the leading process to be considered when extending our models of the nucleus to incorporate the dynamics associated with the structure of the nucleons. Once the importance of this process is understood, we may consider processes whereby excitations to other baryon resonances are also allowed.

In this paper, we extend our model of nuclei to include the Δ(3,3) isobar with spin $s=\frac{3}{2}$, and isospin $\tau=\frac{3}{2}$ corresponding to the four charge states Δ^{++}, Δ^{+}, Δ^{0}, Δ^{-} each with a mass of 1236 MeV, and we neglect effects due to its finite width. We treat these isobars explicitly and on an equal footing with the nucleons with the goal of examining their nonperturbative effects in nuclear structure calculations. We are presenting a nonrelativistic microscope mean-field approach to finite nuclei, which includes nucleon and delta degrees of freedom. We select the nuclei ^{16}O and ^{40}Ca and calculate their ground-state properties at zero temperature within the spherical Hartree-Fock (SHF) approximation, neglecting the Coulomb force. Then with the constrained Hartree-Fock approximation (CSHF), we examine how the delta excitation is populated as a function of radial constraint at zero temperature. Future work will examine the role of finite temperature. A central goal of the present effort is to begin to explore the role of the delta resonance on the properties of finite nuclei under compression. Indeed, we present evidence indicating that a possible first-order phase transition can occur in nuclear matter since we find ^{40}Ca to undergo a discontinuous change in isobar occupation under compression. The present work extends a brief presentation given earlier.[13]

We organize this paper as follows. In Sec. II we develop an effective no-core Hamiltonian for our system. The two-body matrix elements are evaluated in a harmonic oscillator single-particle basis with j-j coupling to good total angular momentum J and good isospin T. In this section we also discuss the choice of model spaces for these no-core calculations. In Sec. III we introduce our Hartree-Fock (HF) method for the system of nucleons and Δ(3,3) particles. In Sec. IV, we discuss phenomenological adjustments of the matrix elements of the effective Hamiltonian. Finally, in Sec. V we display our constrained spherical Hartree-Fock (CSHF) results for ^{16}O and ^{40}Ca. Also, in Sec. V we discuss the conclusions that we draw from these results.

II. THE EFFECTIVE NO-CORE HAMILTONIAN, $H_{\rm eff}$

We consider a nuclear system of A baryons [nucleons: mass$=m$; spin $s=\frac{1}{2}$ isospin $\tau=\frac{1}{2}$ each, and $\Delta(3,3)$ baryons: mass$=M$; spin $s=\frac{3}{2}$; isospin $\tau=\frac{3}{2}$] at zero temperature and without the Coulomb interaction. We develop a dynamical treatment that accommodates transitions between nucleon and delta degrees of freedom. We introduce $\tau_{\rm op}^{1/2}$, $\tau_{\rm op}^{3/2}$ projection operators defined in the $\tau=\frac{1}{2}$ (nucleon) at $\tau=\frac{3}{2}$ (delta) spaces as

$$\tau_{\rm op}^{1/2}\,|\,\tau=\tfrac{1}{2}\rangle=|\,\tau=\tfrac{1}{2}\rangle,\quad \tau_{\rm op}^{1/2}\,|\,\tau=\tfrac{3}{2}\rangle=0$$
$$\tau_{\rm op}^{3/2}\,|\,\tau=\tfrac{1}{2}\rangle=0,\quad \tau_{\rm op}^{3/2}\,|\,\tau=\tfrac{3}{2}\rangle=|\,\tau=\tfrac{3}{2}\rangle \tag{1}$$

where

$$\tau_{\rm op}^{1/2}=\tfrac{5}{4}-\tfrac{1}{3}\tau^2,\quad \tau_{\rm op}^{3/2}=\tfrac{1}{3}\tau^2-\tfrac{1}{4}\ , \tag{2}$$

such that

$$\tau_{\rm op}^{1/2}+\tau_{\rm op}^{3/2}=1\ . \tag{3}$$

Our first approximation to the nuclear mass operator is

$$H'=\sum_{i=1}^{A}\left[\left[\frac{P_i^2}{2m}+m\right]\tau_{\rm op}^{1/2}+\left[\frac{P_i^2}{2M}+M\right]\tau_{\rm op}^{3/2}\right]+V^{BB'} \tag{4}$$

where $\mathbf{P}_i$ is the single particle momentum, $V^{BB'}$ is the two-baryon interaction operator given by

$$V^{BB'}=V^{\rm NN}+V^{\rm NN\leftrightarrow N\Delta}+V^{\rm N\Delta\leftrightarrow N\Delta}+V^{\rm N\Delta\leftrightarrow\Delta N}+V^{\rm N\Delta\leftrightarrow\Delta\Delta}+V^{\rm NN\leftrightarrow\Delta\Delta}+V^{\Delta\Delta\leftrightarrow\Delta\Delta}\ , \tag{5}$$

where $V^{\rm NN}$ is the nucleon-nucleon interaction operator, and $V^{B_1B_2\leftrightarrow B_1'B_2'}$ are the transition potentials.[14] The potential $V^{\rm NN}$ will be described in more detail below. For the transition potentials we adopt the one-boson exchange interactions of Gari, Niephaus, and Sommer.[14]

The intrinsic mass operator H is

$$H=H'-T_{\rm c.m.}\ , \tag{6}$$

where

$$T_{\rm c.m.}=\frac{P_{\rm c.m.}^2}{2\overline{m}},\quad \mathbf{P}_{\rm c.m.}=\sum_{i=1}^{A}\mathbf{P}_i \tag{7}$$

are the center-of-mass kinetic energy and momentum, respectively. $\overline{m}$ is the total mass of the nuclear system.

In general, $\overline{m}$ is state dependent. We use the following approximations. First, we approximate $\overline{m}=Am$ and therefore neglect binding energy effects in the kinetic energy operator. Then the intrinsic interaction Hamiltonian of the system is

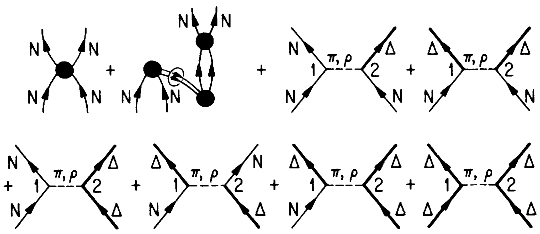

FIG. 1. Diagrammatic representation of $V_{\rm eff}^{BB'}$.

$$H=H_1(\text{one body})+H_2(\text{two body})\ , \tag{8}$$

where

$$H_1(\text{one body})=\left[\sum_{i=1}^{A}\frac{P_i^2}{2M}\left[\frac{m-M}{m}\right]+M-m\right]\tau_{\rm op}^{3/2} \tag{9}$$

and

$$H_2(\text{two body})=T_{\rm rel}(m)+V^{BB'}\ , \tag{10}$$

$$[T_{\rm rel}(m)]_{ij}=\frac{(\mathbf{P}_i-\mathbf{P}_j)^2}{2Am} \tag{11}$$

is the relative kinetic energy operator.

The term H_1(one body) of the Hamiltonian serves as a correction and gives a nonzero contribution when it acts on many-body states with Δ components. States with Δ components are said to comprise the Δ sector. H_1(one body) arises solely due to the mass difference between a nucleon and an isobar.

H_2(two body) is a two-body operator, composed of the relative kinetic energy operator $T_{\rm rel}(m)$ and the two-baryon interaction operator $V^{BB'}$.

We employ this Hamiltonian in nuclear Hartree-Fock calculations with nucleons and Δ orbitals. To this end, we replace the nucleon-nucleon interaction $V^{\rm NN}$ by an effective nucleon-nucleon interaction $V_{\rm eff}^{\rm NN}$. The effective nucleon-nucleon interaction we use is the sum of the Brueckner G matrix and the lowest-order folded diagram (second order in G) acting between pairs of nucleons in the no-core model space and calculated from the Reid soft-core potential.[15] The transition potential is treated in lowest order. For a more complete discussion of the evaluation of $V_{\rm eff}^{\rm NN}$, see Ref. 16.

Thus the two-body part of the effective Hamiltonian is then

$$H_2(\text{two body})=T_{\rm rel}(m)+V_{\rm eff}^{BB'}\ . \tag{12}$$

Figure 1 is a diagrammatic representation of $V_{\rm eff}^{BB'}$ which consists of the effective nucleon-nucleon interaction plus transition potentials.

We derived the following expression for the matrix elements of the two-body potential having good total angular momentum J and good total isospin T in a two-particle harmonic oscillator (HO) basis.

$$\langle n_1'l_1's_1'j_1'n_2'l_2's_2'j_2'(t_1't_2')JT \mid V \mid n_1l_1s_1j_1n_2l_2s_2j_2(t_1t_2)JT\rangle$$
$$=\sum_{n'l's'nls}\sum_{g}[D]\langle n'l's'(t_1't_2')gT \mid V \mid nls(t_1t_2)gT\rangle$$
$$\times\sum_{N\mathcal{L}}\left\{\sum_{L'}(-1)^{L'}\hat{\mathcal{L}}'[\hat{j}_1'\hat{j}_2'\hat{s}'\hat{g}]^{1/2}\begin{Bmatrix} l' & s' & g\\ J & \mathcal{L} & L'\end{Bmatrix}\begin{Bmatrix} l_1' & s_1' & j_1'\\ l_2' & s_2' & j_2'\\ L' & S' & J\end{Bmatrix}M_{L'}(n'l'N'\mathcal{L}',n_1'l_1'n_2'l_2')\right\}$$
$$\times\left\{\sum_{L}(-1)^{L}\hat{\mathcal{L}}[\hat{j}_1\hat{j}_2\hat{s}\hat{g}]^{1/2}\begin{Bmatrix} l & s & g\\ J & \mathcal{L} & L\end{Bmatrix}\begin{Bmatrix} l_1 & s_1 & j_1\\ l_2 & s_2 & j_2\\ L & S & J\end{Bmatrix}M_{L}(nlN\mathcal{L},n_1l_1n_2l_2)\right\}, \quad (13)$$

with

$$D=\frac{[1-(-1)^{l+s+T}][1-(-1)^{l'+s'+T}]}{\sqrt{2(1+\delta_{ab})2(1+\delta_{cd})}}\cdots . \quad (14)$$

Here, l, $\mathcal{L}$, and L stand for the relative, center-of-mass, and total orbital angular momentum, respectively; g and J stand for the relative and total angular momentum, respectively. n and N signify the relative and center-of-mass principal quantum number. s_i and s signify single-particle and total spins. The brackets $\{\ \}$,

$$\begin{Bmatrix} \\ \\ \end{Bmatrix},$$

and M_L represent the 6-J, 9-J, and Moshinsky coefficients extended to particles of unequal masses,[17] respectively. Finally, $(2i+1)$ is abbreviated by $\hat{i}$.

The evaluation of this expression has been accomplished in two major steps: First, we evaluate the relative center of mass (RCM) matrix elements, $\langle n'l's'(t_1't_2')gT \mid V \mid nls(t_1t_2)gT\rangle$. The upper limits on the quantum numbers n,n',l,l',s,s', and T are all taken to be 3. For g, the limit is 6, and changes of the orbital angular momentum $\Delta l=0,\pm 2$, and spin $\Delta s=0,\pm 2$ when we evaluate the tensor part of the potential. Under these limits on the quantum numbers, a total of 7505 nonvanishing RCM matrix elements are obtained. These RCM matrix elements serve as input for the second phase of our calculations, described below.

Second, we construct the two-body part of the effective Hamiltonian matrix in the harmonic oscillator single-particle basis. It is convenient to view the Hamiltonian matrix in its schematic form as

$$\langle H_2(\text{two body})\rangle=\left|\begin{array}{c|c} \text{pure N-N sector} & \text{N-}\Delta\text{ sector}\\ \langle V_{\text{eff}}^{\text{NN}}\rangle+\langle T_{\text{rel}}(m)\rangle & \langle V_{\text{transition}}^{\text{N}\Delta}\rangle\\ \hline \Delta\text{-N sector} & \text{pure }\Delta\text{-}\Delta\text{ sector}\\ \langle V_{\text{transition}}^{\Delta\text{N}}\rangle & \langle V_{\text{transition}}^{\Delta\Delta}\rangle+\langle T_{\text{rel}}(m)\rangle \end{array}\right|,$$

where H is decomposed into a pure N-N sector, an N-Δ sector, a Δ-N sector, and a pure Δ-Δ sector. The matrix elements are evaluated in two model spaces with $\hbar\omega=14$ MeV, and these spaces are referred to as the 3-space and the 5-space. In order to facilitate the calculations for this initial study, we select only those oscillator orbits that are allowed to mix to form the occupied orbits of ^{16}O and ^{40}Ca in the SHF approximation. We are not concerned at this stage with solving for the self-consistent, unoccupied orbits.

We refer to our no-core oscillator model spaces by the maximum of the value $2n+l$ of the orbitals included in the basis. Thus, the first four oscillator shells comprise the "3-space." With the eliminations just discussed the nucleon orbitals for ^{16}O and ^{40}Ca are

$$0s_{1/2},\ 0p_{1/2}, 0p_{3/2},\ 1s_{1/2},\ 0d_{3/2},\ 0d_{5/2},\ 1p_{1/2},\ 1p_{3/2}\ .$$

For the deltas, we keep those orbits which can mix with the retained nucleon orbits under our symmetry assumptions. These are

$$0p_{1/2},\ 0p_{3/2},\ 0d_{3/2},\ 0d_{5/2}\ .$$

There are thus a total of 12 single-particle orbitals in the 3-space. In the 5-space we use 13 nucleon orbitals (the eight-orbitals in 3-space plus $2s_{1/2}$, $1d_{3/2}$, $1d_{5/2}$, $2p_{1/2}$, $2p_{3/2}$) but only the same four delta orbitals as in 3-space so there are a total of 17 single-particle orbitals. We view these spaces as minimal choices to begin to search for qualitative effects such as the macroscopic occupation of the delta orbits. We are willing to make phenomenological adjustments to the effective Hamiltonian to reproduce appropriate ground-state properties in the SHF approximation; this is discussed further in Sec. IV. In future efforts, we will substantially expand the basis spaces.

III. HARTREE-FOCK EQUATIONS FOR A SYSTEM OF NUCLEONS AND Δ(3,3) PARTICLES

In a typical treatment of a system of A nucleons, the Hamiltonian H is assumed to be the sum of kinetic energies and two-body interactions,

$$H=\sum_i t(i)+\sum_{i<j} V(ij) , \tag{15}$$

which can be rewritten by adding and subtracting a single-particle potential as

$$H=\sum_i [t(i)+U(i)]+\sum_{i<j} V(ij)-\sum_i U(i) \tag{16}$$

$$=H_0+H_1 , \tag{17}$$

where H_0 is the single-particle Hamiltonian and H_1 is the residual interaction.

In conventional Hartree-Fock (HF) theory for nucleons only, the two-body interaction is replaced by the effective two-body interaction, and a variational treatment in the space of single Slater determinants yields equations for the mean-field Hamiltonian H_0.

We extend the pure nucleon HF approximation to a system of nucleons that can undergo transitions to isobars and derive expressions for the HF equations and the HF energy.

Let $|\alpha\rangle$, $|\beta\rangle$, $|\gamma\rangle$ represent the HF single-particle states determined as self-consistent solutions of the nucleon-delta HF problem; let λ,μ stand for any of these labels. The HF single-particle state $|\lambda\rangle$ is expanded in a harmonic oscillator basis

$$|\lambda\rangle=\sum_{\rm N} C_{\rm N}^{\lambda}\,|N\rangle+\sum_D C_D^{\lambda}\,|D\rangle . \tag{18}$$

The first sum in Eq. (18) extends over nucleon states while the second sum extends over delta states. N,D,λ are generalized single-particle-state labels, e.g.,

$$\begin{aligned} \mathrm{N}&=n_{\rm N}l_{\rm N}s_{\rm N}j_{\rm N}m_{j_{\rm N}}\tau_{\rm N}m_{\tau_{\rm N}}=\bar{N}m_{j_{\rm N}}m_{\tau_{\rm N}} ,\\ D&=n_Dl_Ds_Dj_Dm_{j_D}\tau_Dm_{\tau_D}=\bar{D}m_{j_D}m_{\tau_D} ,\\ \lambda&=n_{\lambda}l_{\lambda}J_{\lambda}m_{j_{\lambda}}m_{\tau_{\lambda}}=\bar{\lambda}m_{j_{\lambda}}m_{\tau_{\lambda}} . \end{aligned} \tag{19}$$

The $C_{\rm N}^{\lambda}$ and C_D^{λ} are the expansion coefficients of $|\lambda\rangle$ in the nucleon and delta states, respectively, and satisfy the equation

$$\sum_{\rm N} |C_{\rm N}^{\lambda}|^2+\sum_D |C_D^{\lambda}|^2=1 . \tag{20}$$

If we relabel the delta states and the coefficients, we can write Eq. (18) as

$$|\lambda\rangle=\sum_B C_B^{\lambda}\,|B\rangle . \tag{21}$$

The single sum now extends over both nucleon and delta states, and all states are referred to simply as baryon states. The coefficients C_B^{λ} are now defined as

$$\begin{aligned} C_B^{\lambda}&=C_{\rm N}^{\lambda} \text{ if } |B\rangle=|{\rm N}\rangle ,\\ C_B^{\lambda}&=C_D^{\lambda} \text{ if } |B\rangle=|D\rangle , \end{aligned} \tag{22}$$

and

$$\sum_B |C_B^{\lambda}|^2=1 .$$

The HF energy of the system in terms of HF single-particle states is

$$E_{\rm HF}=\sum_{\lambda}\langle\lambda|H_1|\lambda\rangle+\tfrac{1}{2}\sum_{\lambda\mu}\langle\lambda\mu|H_2|\lambda\mu\rangle_A \tag{23}$$

or

$$\begin{aligned} E_{\rm HF}=&\sum_{\lambda}\sum_{BB'}(C_B^{\lambda})^*(C_{B'}^{\lambda})\langle B|H_1|B'\rangle\\ &+\tfrac{1}{2}\sum_{\lambda\mu}\sum_{BB'B_1B_2}(C_B^{\lambda})^*(C_{B'}^{\lambda})(C_{B_1}^{\mu})^*(C_{B_2}^{\mu})\\ &\times\langle BB_1|H_2|B'B_2\rangle_A . \end{aligned} \tag{24}$$

The variational principle reads

$$\frac{\partial}{\partial C^*}\left[E_{\rm HF}-\sum_{\lambda}\left\{\sum_B(C_B^{\lambda})^*(C_B^{\lambda})-1\right\}\epsilon_{\lambda}\right]=0 , \tag{25}$$

where C is one member of the C_B^{λ} and ϵ_{λ} are the single-particle energies. We define

$$h\,|\lambda\rangle=\epsilon_{\lambda}|\lambda\rangle , \tag{26}$$

$$\langle B|h|\lambda\rangle=\epsilon_{\lambda}\langle B|\lambda\rangle , \tag{27}$$

or

$$\sum_{B'}C_{B'}^{\lambda}\langle B|h|B'\rangle=\epsilon_{\lambda}C_B^{\lambda} , \tag{28}$$

and, using Eqs. (24) and (25), we find

$$\langle B|h|B'\rangle=\langle B|H_1|B'\rangle+\sum_{\mu}\sum_{B_2B_1}(C_{B_1}^{\mu})^*(C_{B_2}^{\mu})\langle BB_1|H_2|B'B_2\rangle_A , \tag{29}$$

where h is the HF single-particle Hamiltonian. Using the relation

$$|ij\rangle=R_{\bar{i}\bar{j}}\sum_{JM}\sum_{TM_T}C_{m_im_jM}^{j_ij_jJ}C_{m_{\tau_i}m_{\tau_j}M_T}^{\tau_i\tau_jT}\,|(\bar{i}\,\bar{j})JMTM_T\rangle, \tag{30}$$

where

$$R_{\bar{i}\bar{j}}=\sqrt{(1+\delta_{\bar{i}\bar{j}})}=\sqrt{1+\delta_{n_in_j}\delta_{l_il_j}\delta_{j_ij_j}\delta_{s_is_j}\delta_{\tau_i\tau_j}}\ , \quad (31)$$

and imposing spherical symmetry ($\delta_{l_Bl_\lambda}$), time-reversal symmetry ($\delta_{m_Bm_\lambda}$), isospin projection symmetry ($\delta_{m_{\tau_B}m_{\tau_\lambda}}$), and total angular momentum symmetry ($\delta_{j_Bj_\lambda}$), the C_B^λ are defined as

$$C_B^\lambda=\delta_{j_Bj_\lambda}\delta_{l_Bl_\lambda}\delta_{m_Bm_\lambda}\delta_{m_{\tau_B}\mu_{\tau_\lambda}}C_{\bar{B}}^{\bar{\lambda}}(m_{\tau_B})\ . \quad (32)$$

Then Eq. (29) can be written as

$$\langle B\,|\,h\,|\,B'\rangle=\langle B\,|\,H_1\,|\,B'\rangle+\sum_{\bar{\mu}}\sum_{\bar{B}_1\bar{B}_2}\sum_{m_{\tau_{B_1}}m_{\tau_{B_2}}}\delta_{j_{B_1}j_\mu}\delta_{j_{B_2}j_\mu}\delta_{l_{B_1}l_\mu}\delta_{l_{B_2}l_\mu}\times\delta_{m_{\tau_{B_1}}m_{\tau_{B_2}}}\delta_{j_Bj_{B'}}\delta_{m_Bm_{B'}}R_{\bar{B}\bar{B}_1}R_{\bar{B}'\bar{B}_2}[C_{\bar{B}_1}^{\bar{\mu}}(m_{\tau_{B_1}})]^*[C_{\bar{B}_2}^{\bar{\mu}}(m_{\tau_{B_2}})]\times\sum_{JTM_T}\frac{2J+1}{2j_B+1}C_{m_{\tau_B}m_{\tau_{B_1}}M_T}^{\tau_B\tau_{B_1}T}C_{m_{\tau_{B'}}m_{\tau_{B_2}}M_T}^{\tau_{B'}\tau_{B_2}T}\langle\bar{B}\bar{B}_1\,|\,H_2\,|\,\bar{B}'\bar{B}_2\rangle_{JT}\ . \quad (33)$$

In the HF we restrict the particles to have good isospin projections $\pm\frac{1}{2}$. That is, $m_\tau=+\frac{1}{2}$ for protons and Δ^+ particles which are allowed to mix, while $m_\tau=-\frac{1}{2}$ for neutrons and Δ^0 particles which are allowed to mix. The following components of the HF Hamiltonian must then be considered:

(a) $\langle B(\mathrm{n})\,|\,h\,|\,B'(\mathrm{n})\rangle$, (b) $\langle B(\Delta^0)\,|\,h\,|\,B'(\Delta^0)\rangle$,
(c) $\langle B(\mathrm{n})\,|\,h\,|\,B'(\Delta^0)\rangle$, (d) $\langle B(\Delta^0)\,|\,h\,|\,B'(\mathrm{n})\rangle$,
(e) $\langle B(\mathrm{p})\,|\,h\,|\,B'(\mathrm{p})\rangle$, (f) $\langle B(\Delta^+)\,|\,h\,|\,B'(\Delta^+)\rangle$,
(g) $\langle B(\mathrm{p})\,|\,h\,|\,B'(\Delta^+)\rangle$, (h) $\langle B(\Delta^+)\,|\,h\,|\,B'(\mathrm{p})\rangle$.

Matrix elements in (c) and (d) are related by hermiticity, as are matrix elements in (g) and (h). We employed Eq. (33) to find expressions for the eight cases above. These are given in Appendix A. We also derived a detailed expression for the HF energy of the system. The following expression represents the most convenient form for the HF energy upon which the numerical evaluations are based:

$$E_{\mathrm{HF}}=\frac{1}{2}\sum_{\bar{B}\bar{B}'}(2j_B+1)\delta_{j_Bj_{B'}}\delta_{l_Bl_{B'}}\{\rho_{\bar{B}\bar{B}'}^{\mathrm{nn}}\langle B(\mathrm{n})\,|\,h\,|\,B'(\mathrm{n})\rangle+\rho_{\bar{B}\bar{B}'}^{\mathrm{n}\Delta^0}\langle B(\mathrm{n})\,|\,h\,|\,B'(\Delta^0)\rangle+\rho_{\bar{B}\bar{B}'}^{\Delta^0\mathrm{n}}\langle B(\Delta^0)\,|\,h\,|\,B'(\mathrm{n})\rangle+\rho_{\bar{B}\bar{B}'}^{\Delta^0\Delta^0}\langle B(\Delta^0)\,|\,h\,|\,B'(\Delta^0)\rangle+\rho_{\bar{B}\bar{B}'}^{\mathrm{pp}}\langle B(\mathrm{p})\,|\,h\,|\,B'(\mathrm{p})\rangle+\rho_{\bar{B}\bar{B}'}^{\mathrm{p}\Delta^+}\langle B(\mathrm{p})\,|\,h\,|\,B'(\Delta^+)\rangle+\rho_{\bar{B}\bar{B}'}^{\Delta^+\mathrm{p}}\langle B(\Delta^+)\,|\,h\,|\,B'(\mathrm{p})\rangle+\rho_{\bar{B}\bar{B}'}^{\Delta^+\Delta^+}\langle B(\Delta^+)\,|\,h\,|\,B'(\Delta^+)\rangle\}+\frac{1}{2}\sum_{\bar{B}\bar{B}'}(2j_B+1)\delta_{j_Bj_{B'}}\delta_{l_Bl_{B'}}\{\rho_{\bar{B}\bar{B}'}^{\Delta^0\Delta^0}\langle B(\Delta^0)\,|\,H_1\,|\,B'(\Delta^0)\rangle+\rho_{\bar{B}\bar{B}'}^{\Delta^+\Delta^+}\langle B(\Delta^+)\,|\,H_1\,|\,B'(\Delta^+)\rangle\}\ , \quad (34)$$

where the HF energy is expressed in terms of HF single-particle Hamiltonian matrix elements, single-particle densities, and single-particle transition densities, defined in Appendix A.

IV. SCALING RULES AND PHENOMENOLOGICAL ADJUSTMENTS

In Sec. II the two-body matrix elements of the effective Hamiltonian were evaluated using a harmonic oscillator basis with $\hbar\omega=14$ MeV. In order to simplify the application of the effective Hamiltonian to as wide a range of nuclei as possible, we follow the scaling rule of Ref. 16. That is, if we signify a matrix element of an operator, e.g., H_{eff} by $\langle H_{\mathrm{eff}}\rangle$ implying it was calculated in an oscillator basis with $\hbar\omega$, the matrix elements of H'_{eff} in a basis with $\hbar\omega'$ are approximately given by

$$\langle H'_{\mathrm{eff}}\rangle=\frac{\hbar\omega'}{\hbar\omega}\langle H_{\mathrm{eff}}\rangle\ . \quad (35)$$

Therefore, we introduce the factor $\hbar\omega'/\hbar\omega$ to scale the matrix element of H_{eff} when we use them in the 3- or 5-space for $^{16}\mathrm{O}$ and $^{40}\mathrm{Ca}$. The origins and physical significance of this linear dependence on $\hbar\omega$ for both the kinetic and effective potentials are discussed in Ref. 16

We also introduce overall factors λ_1,λ_2 for the kinetic energy and the effective nucleon-nucleon interaction matrix elements, respectively. Our motives behind this can

TABLE I. Factors $\hbar\omega'$, λ_1, λ_2, used in this work for ^{16}O and ^{40}Ca in the 3- and 5-spaces.

Nucleus	Model space	$\hbar\omega'$ (MeV)	λ_1	λ_2
^{16}O	3	9.47	0.95	1.18
	5	8.65	0.977	1.35
^{40}Ca	3	10.08	0.99	1.11
	5	7.87	0.985	1.29

be understood as follows. We anticipate that our spherical Hartree-Fock (SHF) results for a system of nucleons would be similar to those of the Brueckner-Hartree-Fock (BHF) approximation.[18] Small differences may be ascribed to different choices of the Pauli operator. Indeed, the standard deficiencies were found[16] in the solution of ^{16}O and ^{40}Ca in the SHF approximation, i.e., too little binding but approximately correct rms radii. Our philosophy is to adjust the matrix elements of $H_{\rm eff}$ in the nucleon-nucleon sector in order to achieve agreement with measured ground-state properties in the SHF approximation before proceeding to see the effect of including the delta degrees of freedom in the Hamiltonian.

Therefore, we adjust λ_1, λ_2, and $\hbar\omega'$ simultaneously to achieve the desired rms radius and binding energy for a given nucleus within SHF for each of the model spaces. The values of λ_1, λ_2, and $\hbar\omega'$ we have used in this work are similar to those of Ref. 16 and are listed in Table I. The small differences in these parameter values from those of Ref. 16 are due to a higher quality fit to the ground state (g.s.) properties of ^{16}O and ^{40}Ca in the SHF approximation with the Coulomb interaction included.

We note that the value of λ_1 is less than unity. This is because the kinetic energy operator ($T_{\rm rel}$) is a positive definite operator, and if it is normalized by itself into a finite model space, this will reduce its magnitude. We use values of λ_2 greater than unity in order to compensate for the lack of sufficient binding observed even with BHF. The value of λ_2 increases as the model space increases. This trend arises because, as the model space increases, the renormalization procedure used to obtain $V_{\rm eff}^{\rm NN}$ produces an effective interaction with weaker attraction. As the model space becomes very large, the effective interaction approaches the bare interaction whose oscillator matrix elements are large but positive. Thus, our whole procedure of remaining within the HF approximation must break down eventually with increasing model spaces.

V. RESULTS

We developed a computer code for radially constrained spherical Hartree-Fock (CSHF) calculations based on Eqs. (A2)–(A9) and (34). We calculate the following quantities: The Hartree-Fock energy (EHF), the rms radius, the number of delta particles in the occupied orbitals, the single-particle energies, and the occupation probability of the single-particle orbitals. We found the 3-space to be inadequate to significantly explore the consequences of a radial constraint. Thus we report here the (CSHF) results for ^{16}O and ^{40}Ca in the 5-space only. The single-particle orbitals we use in the 5-space are those which are used to evaluate the two-body matrix elements in Sec. II.

We display our results for ^{16}O and ^{40}Ca in Fig. 2 and Fig. 3, respectively. The equilibrium EHF equals −138.5 MeV (−414.5 MeV), and the rms radius equals 2.61 fm (3.38 fm) for ^{16}O (^{40}Ca). The measured binding energy and the rms radius are −128 MeV (−342 MeV) and 2.74 fm (3.48 fm) for ^{16}O (^{40}Ca). The difference between our calculated results and the measured quantities can be attributed to our neglect of the Coulomb interaction, which gives a repulsive energy of 14 MeV (74 MeV) for ^{16}O (^{40}Ca). The number of Δ particles in the occupied orbitals for all values of rms radii obtained in ^{16}O and ^{40}Ca within this limited model space is equal to zero. These results are consistent with results obtained for ^{4}He with a different set of transition potentials.[19] The ^{4}He calculations did not explore the effect of a radial constraint, but did explore the effects of finite temperature. No mixing of deltas was observed in the ^{4}He calculations.

We note that the transition potentials we use in this work are evaluated in Ref. 14 with a one-boson exchange-potential model. The model has numerous parameters: masses, coupling constants, and regularization

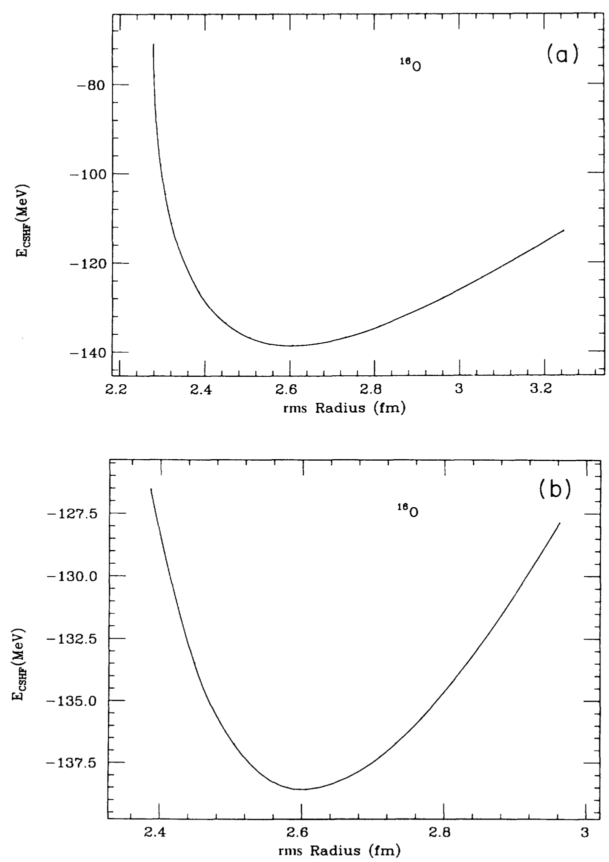

FIG. 2. Constrained spherical Hartree-Fock (CSHF) energy of $H_{\rm eff}$ defined in the text for ^{16}O in the 5-space vs rms radius. Coulomb effects are neglected. The transition potentials are those of Ref. 14. (a) Upper graph shows all the points we calculated. (b) Lower graph shows the portion of (a) in the immediate vicinity of the minimum.

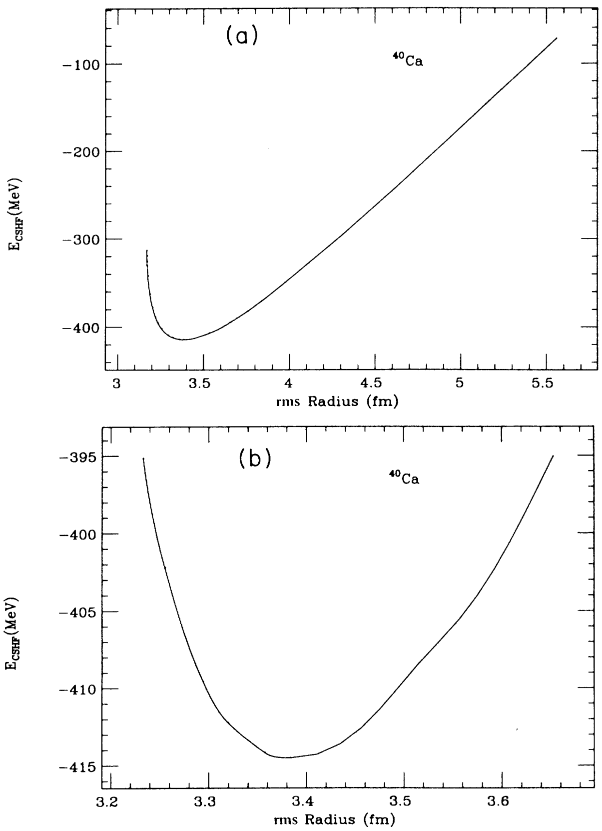

FIG. 3. Constrained spherical Hartree-Fock (CSHF) energy of H_{eff} for ^{40}Ca in 5-space vs rms radius. The transition potentials are those of Ref. 14. Coulomb effects are neglected. (a) Upper graph shows all points we calculated. (b) Lower graph shows the portion in the immediate vicinity of the equilibrium radius.

parameters. In principle, these parameters are constrained by the coupled channel fits to nucleon-nucleon scattering data. As in all such models, there are uncertainties. We remark that one uncertainty could be possibility of multiple parameter sets yielding equivalent fits to the nucleon-nucleon data. With this in mind, we sense some freedom, and we will now explore that freedom by simply adjusting the overall strength of the transition potentials. By doing this we hope to isolate the character of the transition from nucleon matter to delta matter in finite nuclei.

We have therefore increased the strength of the transition potentials in H_{eff} by multiplying the matrix elements of the transition potentials by factors of 5 to 25 in steps of 5, in a search for the transition to appreciable delta orbital occupation. We repeated the CSHF calculations with these various strengths. We found that the number of the deltas in the occupied orbitals remains zero for ^{16}O. The CSHF solutions are the same as the results obtained in the pure nucleon sector.

For ^{40}Ca we found Δ mixing first with compression when we increased the strength of the transition potentials by a factor of 5.0. The results with this strength are the same for a wide range of radii around the equilibrium radius as they are with strength unity. In the range $r=(3.16-3.97)$ fm, the results behave very smoothly and approximate the shape of a parabola. A portion of this parabola is shown in Fig. 4. However, for a small interval of the nuclear radius (see Fig. 4) around 3.09 fm, we find mixing of nucleons and deltas. In that region of r, a substantial change of the CSHF solutions occurs.

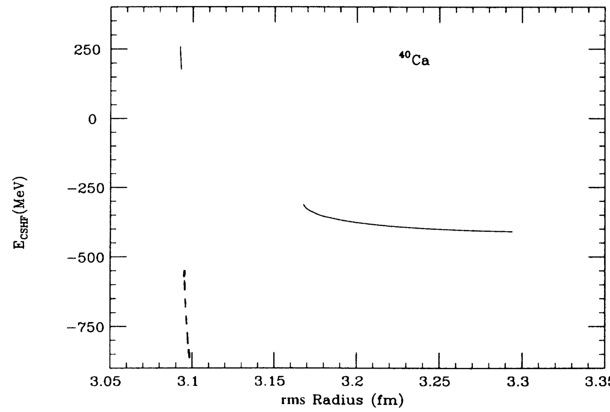

FIG. 4. Constrained spherical Hartree-Fock (CSHF) energy of H_{eff} for ^{40}Ca in 5-space vs rms radius for the strength of transition potentials increased by a factor of 5 (solid curve) and by a factor of 10 (dashed curve). The solid and dashed curves coincide for the nucleon sector which occurs only for rms radii greater than 3.15 fm and is only depicted up to 3.3 fm. The Δ sector occurs only for the rms less than 3.1 fm. Coulomb effects are neglected.

If we examine the region where Δ mixing occurs, we find several prominent features of the results.

(1) As the constraint parameter λ_3 (which is the coefficient of r^2 in the CSHF Hamiltonian) is changed slowly to increasing negative values (nucleus compressed), there is an abrupt change in the character of the CSHF solution.

(2) The nature of this change is characterized by a sudden discontinuous change in the number of delta particles N_Δ in the occupied orbitals. N_Δ changes suddenly from 0.0 to 4.1 at the critical value of $\lambda_3=-62$ MeV fm^{-2} (density increased by $\sim 30\%$).

(3) Accompanying this change, both the rms radius of the solution and the CSHF energy change discontinuously. In addition, in the range where mixing occurs, the binding energy is very sensitive to a small variation in the rms radius.

(4) The solutions of the CSHF equations take a dramatically increasing number of iterations to converge as we approach the critical value of λ_3.

All the above features are reminiscent of a first-order phase transition. However, we cannot completely rule out this possibility that the transition is second order in character. To explore this issue further we show as a dashed curve in Fig. 4 the solutions with Δ mixing when the strength of the transition potentials has been increased by a factor of 10. Note that in these same calculations the curve for the nucleons only phase is again obtained. That the Δ phase is obtained only under compression and is now lower in energy than the nucleon phase reflects the order of magnitude increase in the strength of the transition potentials and the use of constrained Hartree Fock. In order to have a Δ-mixing phase as the lowest Hartree Fock solution in ^{40}Ca without constraint we found the transition potential had to be increased by about a factor of 25 over the values given in Ref. 14.

To our knowledge this is the first observation of this transition in any calculations reported in the literature for finite nuclei. To further elucidate this transition it is necessary to go to larger nuclei and larger model spaces. Based on the trends from ^{16}O to ^{40}Ca we may speculate that the Δ phase would then be found with weaker strengths of the transition potentials when the nuclei are compressed. It is also possible that with larger model spaces where greater compression is achievable for these nuclei, this same transitions in ^{40}Ca will be observed with weaker transition potential strength.

ACKNOWLEDGMENTS

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APPENDIX

In this Appendix we use Eq. (33) of the text to find expressions for the HF single-particle Hamiltonian matrix element. Here we show the major steps of deriving an expression for the first case, i.e.,

$$\langle B(\mathrm{n})\,|\,h\,|\,B'(\mathrm{n})\rangle \;.$$

In Eq. (33), if we perform the sums over $m_{\tau_{B_1}}$, $m_{\tau_{B_2}}$, and over M_T, then we get

$$
\begin{aligned}
\langle B(\mathrm{n})\,|\,h\,|\,B'(\mathrm{n})\rangle = {} & \sum_{\bar\mu}\sum_{\bar B_1 \bar B_2} \delta_{j_{B_1} j_\mu}\delta_{j_{B_2} j_\mu}\delta_{l_{B_1} l_\mu}\delta_{l_{B_2} l_\mu}\sum_{j}\delta_{j_B j_{B'}}\delta_{m_B m_{B'}}\frac{2J+1}{2j_B+1}R_{\bar B \bar B_1}R_{\bar B' \bar B_2} \\
& \times \Bigg\{ [C^{\bar\mu}_{\bar B_1}(-\tfrac12)]^*[C^{\bar\mu}_{\bar B_2}(-\tfrac12)] \\
& \times \Bigg[\left[C^{\frac12\;\frac12\;\;1}_{-\frac12\;-\frac12\;-1} \right]^2 \langle \bar B(\tfrac12)\bar B_1(\tfrac12)|||H_2|||\bar B'(\tfrac12)\bar B_2(\tfrac12)\rangle_{JT=1} \\
& + \left[C^{\frac12\;\frac12\;\;T}_{-\frac12\;-\frac12\;1} \right]^2 \langle \bar B(\tfrac12)\bar B_1(\tfrac32)|||H_2|||\bar B'(\tfrac12)\bar B_2(\tfrac32)\rangle_{JT=1,2} \\
& + \left[C^{\frac12\;\frac12\;\;1}_{-\frac12\;-\frac12\;-1} C^{\frac12\;\frac32\;\;1}_{-\frac12\;-\frac12\;-1} \right] \langle \bar B(\tfrac12)\bar B_1(\tfrac12)|||H_2|||\bar B'(\tfrac12)\bar B_2(\tfrac32)\rangle_{JT=1} \\
& + \left[C^{\frac12\;\frac32\;\;1}_{-\frac12\;-\frac12\;-1} C^{\frac12\;\frac12\;\;1}_{-\frac12\;-\frac12\;-1} \right] \langle \bar B(\tfrac12)\bar B_1(\tfrac32)|||H_2|||\bar B'(\tfrac12)\bar B_2(\tfrac12)\rangle_{JT=1} \Bigg] \\
& + [C^{\bar\mu}_{\bar B_1}(\tfrac12)]^*[C^{\bar\mu}_{\bar B_2}(\tfrac12)] \\
& \times \Bigg[\left[C^{\frac12\;\frac12\;T}_{-\frac12\;\frac12\;0} \right]^2 \langle \bar B(\tfrac12)\bar B_1(\tfrac12)|||H_2|||\bar B'(\tfrac12)\bar B_2(\tfrac12)\rangle_{JT=0,1} \\
& + \left[C^{\frac12\;\frac32\;T}_{-\frac12\;\frac12\;0} \right]^2 \langle \bar B(\tfrac12)\bar B_1(\tfrac32)|||H_2|||\bar B'(\tfrac12)\bar B_2(\tfrac32)\rangle_{JT=1,2} \\
& + \left[C^{\frac12\;\frac12\;1}_{-\frac12\;\frac12\;0} C^{\frac12\;\frac32\;1}_{-\frac12\;\frac12\;0} \right] \langle \bar B(\tfrac12)\bar B_1(\tfrac12)|||H_2|||\bar B'(\tfrac12)\bar B_2(\tfrac32)\rangle_{JT=1} \\
& + \left[C^{\frac12\;\frac32\;1}_{-\frac12\;\frac12\;0} C^{\frac12\;\frac12\;1}_{-\frac12\;\frac12\;0} \right] \langle \bar B(\tfrac12)\bar B_1(\tfrac32)|||H_2|||\bar B'(\tfrac12)\bar B_2(\tfrac12)\rangle_{JT=1} \Bigg] \Bigg\} \;. \qquad \text{(A1)}
\end{aligned}
$$

Now we define the following one-body density matrices:

$$\rho^{\rm nn}_{\bar B_1\bar B_2}=\sum_{\bar\mu} C^{\bar\mu\,*}_{\bar B_1(\tau_{B_1}=1/2)}(-\tfrac12)C^{\bar\mu}_{\bar B_2(\tau_{B_2}=1/2)}(-\tfrac12),\quad \rho^{\rm pp}_{\bar B_1\bar B_2}=\sum_{\bar\mu} C^{\bar\mu\,*}_{\bar B_1(\tau_{B_1}=1/2)}(\tfrac12)C^{\bar\mu}_{\bar B_2(\tau_{B_2}=1/2)}(\tfrac12)\,,$$

$$\rho^{\Delta^0\Delta^0}_{\bar B_1\bar B_2}=\sum_{\bar\mu} C^{\bar\mu\,*}_{\bar B_1(\tau_{B_1}=3/2)}(-\tfrac12)C^{\bar\mu}_{\bar B_2(\tau_{B_2}=3/2)}(-\tfrac12),\quad \rho^{\Delta^+\Delta^+}_{\bar B_1\bar B_2}=\sum_{\bar\mu} C^{\bar\mu\,*}_{\bar B_1(\tau_{B_1}=3/2)}(\tfrac12)C^{\bar\mu}_{\bar B_2(\tau_{B_2}=3/2)}(\tfrac12)\,,$$

$$\rho^{\rm n\Delta^0}_{\bar B_1\bar B_2}=\sum_{\bar\mu} C^{\bar\mu\,*}_{\bar B_1(\tau_{B_1}=1/2)}(-\tfrac12)C^{\bar\mu}_{\bar B_2(\tau_{B_2}=3/2)}(-\tfrac12),\quad \rho^{\Delta^0{\rm n}}_{\bar B_1\bar B_2}=\sum_{\bar\mu} C^{\bar\mu\,*}_{\bar B_1(\tau_{B_1}=3/2)}(-\tfrac12)C^{\bar\mu}_{\bar B_2(\tau_{B_2}=1/2)}(-\tfrac12)\,,$$

$$\rho^{\rm p\Delta^+}_{\bar B_1\bar B_2}=\sum_{\bar\mu} C^{\bar\mu\,*}_{\bar B_1(\tau_{B_1}=1/2)}(\tfrac12)C^{\bar\mu}_{\bar B_2(\tau_{B_2}=3/2)}(\tfrac12),\quad \rho^{\Delta^+{\rm p}}_{\bar B_1\bar B_2}=\sum_{\bar\mu} C^{\bar\mu\,*}_{\bar B_1(\tau_{B_1}=3/2)}(\tfrac12)C^{\bar\mu}_{\bar B_2(\tau_{\bar B_2}=1/2)}(\tfrac12)\,.$$

Equation (33) can be rewritten as

$$\begin{aligned}\langle B({\rm n})\,|\,h\,|\,B'({\rm n})\rangle=&\sum_{\bar B_1\bar B_2}\delta_{j_{B_1}j_{B_2}}\delta_{l_{B_1}l_{B_2}}\delta_{j_Bj_{B'}}\delta_{m_Bm_{B'}}R_{\bar B\bar B_1}R_{\bar B'\bar B_2}\\ &\times\sum_{JT}\frac{2J+1}{2j_B+1}\Big[\rho^{\rm nn}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac12,\tfrac12)\rangle_{JT}+\tfrac12\rho^{\rm pp}_{\bar B_1\bar B_2}(\delta_{T0}+\delta_{T1})\langle H_2(\tfrac12,\tfrac12)\rangle_{JT}\\ &+\tfrac14\rho^{\Delta^0\Delta^0}_{\bar B_1\bar B_2}(\delta_{T1}+3\delta_{T2})\langle H_2(\tfrac32,\tfrac32)\rangle_{JT}-\tfrac12\rho^{\rm n\Delta^0}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac12,\tfrac32)\rangle_{JT}\\ &-\tfrac12\rho^{\Delta^0{\rm n}}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac32,\tfrac12)\rangle_{JT}+\tfrac12\rho^{\Delta^+\Delta^+}_{\bar B_1\bar B_2}(\delta_{T1}+\delta_{T2})\langle H_2(\tfrac32,\tfrac32)\rangle_{JT}\\ &-\tfrac12\rho^{\rm p\Delta^+}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac12,\tfrac32)\rangle_{JT}-\tfrac12\rho^{\Delta^+{\rm p}}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac32,\tfrac12)\rangle_{JT}\Big]\cdots\,,\end{aligned}\tag{A2}$$

where we used the abbreviation

$$\langle\bar B(\tau_B)\bar B_1(\tau_{B_1})\,|\,H_2\,|\,\bar B'(\tau_{B'})\bar B_2(\tau_{B_2})\rangle_{JT}=\langle H_2(\tau_{B_1},\tau_{B_2})\rangle_{JT}\,.$$

Using the same argument we get the following expressions for the rest of the above mentioned cases:

$$\begin{aligned}\langle B(\Delta^0)\,|\,h\,|\,B'(\Delta^0)\rangle=&\langle B(\Delta^0)\,|\,H_1\,|\,B'(\Delta^0)\rangle+\sum_{\bar B_1\bar B_2}\delta_{j_{B_1}j_{B_2}}\delta_{l_{B_1}l_{B_2}}\delta_{j_Bj_{B'}}\delta_{m_Bm_{B'}}R_{\bar B\bar B_1}R_{\bar B'\bar B_2}\\ &\times\sum_{JT}\frac{2J+1}{2j_B+1}\Big[\tfrac14\rho^{\rm nn}_{\bar B_1\bar B_2}(\delta_{T1}+3\delta_{T2})\langle H_2(\tfrac12,\tfrac12)\rangle_{JT}+\tfrac15\rho^{\Delta^0\Delta^0}_{\bar B_1\bar B_2}(2\delta_{T1}+3\delta_{T3})\langle H_2(\tfrac32,\tfrac32)\rangle_{JT}\\ &-\frac{1}{\sqrt{10}}\rho^{\rm n\Delta^0}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac12,\tfrac32)\rangle_{JT}-\frac{1}{\sqrt{10}}\rho^{\Delta^0{\rm n}}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac32,\tfrac12)\rangle_{JT}\\ &+\tfrac12\rho^{\rm pp}_{\bar B_1\bar B_2}(\delta_{T1}+\delta_{T2})\langle H_2(\tfrac12,\tfrac12)\rangle_{JT}+\tfrac14\rho^{\Delta^+\Delta^+}_{\bar B_1\bar B_2}\\ &\times(\delta_{T0}+\tfrac15\delta_{T1}+\delta_{T2}+\tfrac95\delta_{T3})\langle H_2(\tfrac32,\tfrac32)\rangle_{JT}+\rho^{\rm p\Delta^+}_{\bar B_1\bar B_2}\left[\frac{1}{\sqrt{40}}\delta_{T1}-\frac{1}{\sqrt8}\delta_{T2}\right]\\ &\times\langle H_2(\tfrac12,\tfrac32)\rangle_{JT}+\rho^{\Delta^+{\rm p}}_{\bar B_1\bar B_2}\left[\frac{1}{\sqrt{40}}\delta_{T1}-\frac{1}{\sqrt8}\delta_{T2}\right]\langle H_2(\tfrac32,\tfrac12)\rangle_{JT}\Big]\,,\end{aligned}\tag{A3}$$

$$\langle B(\mathrm{n})|h|B'(\Delta^0)\rangle=\sum_{\bar B_1\bar B_2}\delta_{j_{B_1}j_{B_2}}\delta_{l_{B_1}l_{B_2}}\delta_{j_Bj_{B'}}\delta_{m_Bm_{B'}}R_{\bar B\bar B_1}R_{\bar B'\bar B_2}$$
$$\times\sum_{JT}\frac{2J+1}{2j_B+1}\Bigg[\tfrac{1}{2}\rho^{\mathrm{nn}}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac12,\tfrac12)\rangle_{JT}+\frac{1}{\sqrt{10}}\rho^{\Delta^0\Delta^0}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac32,\tfrac32)\rangle_{JT}$$
$$-\sqrt{2/5}\,\delta_{T1}\rho^{\mathrm{n}\Delta^0}_{\bar B_1\bar B_2}\langle H_2(\tfrac12,\tfrac32)\rangle_{JT}-\tfrac14\rho^{\Delta^0\mathrm{n}}_{\bar B_1\bar B_2}(\delta_{T1}-3\delta_{T2})\langle H_2(\tfrac32,\tfrac12)\rangle_{JT}$$
$$-\tfrac12\rho^{\mathrm{pp}}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac12,\tfrac12)\rangle_{JT}+\rho^{\Delta^+\Delta^+}_{\bar B_1\bar B_2}\left[\frac{1}{\sqrt{40}}\delta_{T1}-\frac{1}{\sqrt{8}}\delta_{T2}\right]\langle H_2(\tfrac32,\tfrac32)\rangle_{JT}$$
$$-\rho^{\mathrm{p}\Delta^+}_{\bar B_1\bar B_2}\left[\frac{1}{\sqrt{8}}\delta_{T0}+\frac{1}{\sqrt{40}}\delta_{T1}\right]\langle H_2(\tfrac12,\tfrac32)\rangle_{JT}$$
$$+\tfrac12\rho^{\Delta^+\mathrm{p}}_{\bar B_1\bar B_2}(\delta_{T1}+\delta_{T2})\langle H_2(\tfrac32,\tfrac12)\rangle_{JT}\Bigg], \tag{A4}$$

$$\langle B(\Delta^0)|h|B'(\mathrm{n})\rangle=\sum_{\bar B_1\bar B_2}\delta_{j_{B_1}j_{B_2}}\delta_{l_{B_1}l_{B_2}}\delta_{j_Bj_{B'}}\delta_{m_Bm_{B'}}R_{\bar B\bar B_1}R_{\bar B'\bar B_2}$$
$$\times\sum_{JT}\frac{2J+1}{2j_B+1}\Bigg[\tfrac{1}{2}\rho^{\mathrm{nn}}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac12,\tfrac12)\rangle_{JT}+\frac{1}{\sqrt{10}}\rho^{\Delta^0\Delta^0}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac32,\tfrac32)\rangle_{JT}$$
$$-\tfrac14(\delta_{T1}-3\delta_{T2})\rho^{\mathrm{n}\Delta^0}_{\bar B_1\bar B_2}\langle H_2(\tfrac12,\tfrac32)\rangle_{JT}-\sqrt{2/5}\,\rho^{\Delta^0\mathrm{n}}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac32,\tfrac12)\rangle_{JT}$$
$$-\tfrac12\rho^{\mathrm{pp}}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac12,\tfrac12)\rangle_{JT}+\left[\frac{1}{\sqrt{40}}\delta_{T1}-\frac{1}{\sqrt{8}}\delta_{T2}\right]\rho^{\Delta^+\Delta^+}_{\bar B_1\bar B_2}\langle H_2(\tfrac32,\tfrac32)\rangle_{JT}$$
$$+\tfrac12\rho^{\mathrm{p}\Delta^+}_{\bar B_1\bar B_2}(\delta_{T1}+\delta_{T2})\langle H_2(\tfrac12,\tfrac32)\rangle_{JT}$$
$$-\left[\frac{1}{\sqrt{8}}\delta_{T0}+\frac{1}{\sqrt{40}}\delta_{T1}\right]\rho^{\Delta^+\mathrm{p}}_{\bar B_1\bar B_2}\langle H_2(\tfrac32,\tfrac12)\rangle_{JT}\Bigg], \tag{A5}$$

$$\langle B(\mathrm{p})|h|B'(\mathrm{p})\rangle=\sum_{\bar B_1\bar B_2}\delta_{j_{B_1}j_{B_2}}\delta_{l_{B_1}l_{B_2}}\delta_{j_Bj_{B'}}\delta_{m_Bm_{B'}}R_{\bar B\bar B_1}R_{\bar B'\bar B_2}$$
$$\times\sum_{JT}\frac{2J+1}{2j_B+1}\Bigg[\tfrac12\rho^{\mathrm{nn}}_{\bar B_1\bar B_2}(\delta_{T0}+\delta_{T1})\langle H_2(\tfrac12,\tfrac12)\rangle_{JT}+\tfrac12\rho^{\Delta^0\Delta^0}_{\bar B_1\bar B_2}(\delta_{T1}+\delta_{T2})\langle H_2(\tfrac32,\tfrac32)\rangle_{JT}$$
$$-\tfrac12\rho^{\mathrm{n}\Delta^0}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac12,\tfrac32)\rangle_{JT}-\tfrac12\rho^{\Delta^0\mathrm{n}}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac32,\tfrac12)\rangle_{JT}$$
$$+\rho^{\mathrm{pp}}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac12,\tfrac12)\rangle_{JT}+\tfrac14\rho^{\Delta^+\Delta^+}_{\bar B_1\bar B_2}(\delta_{T1}+3\delta_{T2})\langle H_2(\tfrac32,\tfrac32)\rangle_{JT}$$
$$-\tfrac12\rho^{\mathrm{p}\Delta^+}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac12,\tfrac32)\rangle_{JT}-\tfrac12\rho^{\Delta^+\mathrm{p}}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac32,\tfrac12)\rangle_{JT}\Bigg], \tag{A6}$$

$$\langle B(\Delta^+)|h|B'(\Delta^+)\rangle=\langle B(\Delta^+)|H_1|B'(\Delta^+)\rangle+\sum_{\bar B_1\bar B_2}\delta_{j_{B_1}j_{B_2}}\delta_{l_{B_1}l_{B_2}}\delta_{j_Bj_{B'}}\delta_{m_Bm_{B'}}R_{\bar B\bar B_1}R_{\bar B'\bar B_2}$$
$$\times\sum_{JT}\frac{2J+1}{2j_B+1}\Bigg[\tfrac{1}{2}\rho^{\rm nn}_{\bar B_1\bar B_2}(\delta_{T1}+\delta_{T2})\langle H_2(\tfrac12,\tfrac12)\rangle_{JT}+\tfrac14\rho^{\Delta^0\Delta^0}_{\bar B_1\bar B_2}(\delta_{T0}+\tfrac15\delta_{T1}+\delta_{T2}$$
$$+\tfrac95\delta_{T3})\langle H_2(\tfrac32,\tfrac32)\rangle_{JT}+\rho^{\rm n\Delta^0}_{\bar B_1\bar B_2}\left[\frac{1}{\sqrt{40}}\delta_{T1}-\frac{1}{\sqrt8}\delta_{T2}\right]\langle H_2(\tfrac12,\tfrac32)\rangle_{JT}$$
$$+\rho^{\Delta^0\rm n}_{\bar B_1\bar B_2}\left[\frac{1}{\sqrt{40}}\delta_{T1}-\frac{1}{\sqrt8}\delta_{T2}\right]\langle H_2(\tfrac32,\tfrac12)\rangle_{JT}$$
$$+\tfrac14\rho^{\rm pp}_{\bar B_1\bar B_2}(\delta_{T1}+3\delta_{T2})\langle H_2(\tfrac12,\tfrac12)\rangle_{JT}+\tfrac15\rho^{\Delta^+\Delta^+}_{\bar B_1\bar B_2}(2\delta_{T1}+3\delta_{T3})\langle H_2(\tfrac32,\tfrac32)\rangle_{JT}$$
$$-\frac{1}{\sqrt{10}}\rho^{\rm p\Delta^+}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac12,\tfrac32)\rangle_{JT}-\frac{1}{\sqrt{10}}\rho^{\Delta^+\rm p}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac32,\tfrac12)\rangle_{JT}\Bigg]\,, \tag{A7}$$

$$\langle B({\rm p})|h|B'(\Delta^+)\rangle=\sum_{\bar B_1\bar B_2}\delta_{j_{B_1}j_{B_2}}\delta_{l_{B_1}l_{B_2}}\delta_{j_Bj_{B'}}\delta_{m_Bm_{B'}}R_{\bar B\bar B_1}R_{\bar B'\bar B_2}$$
$$\times\sum_{JT}\frac{2J+1}{2j_B+1}\Bigg[\tfrac12\rho^{\rm nn}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac12,\tfrac12)\rangle_{JT}+\rho^{\Delta^0\Delta^0}_{\bar B_1\bar B_2}\left[\frac{1}{\sqrt{40}}\delta_{T1}-\frac{1}{\sqrt8}\delta_{T2}\right]\langle H_2(\tfrac32,\tfrac32)\rangle_{JT}$$
$$-\rho^{\rm n\Delta^0}_{\bar B_1\bar B_2}\left[\frac{1}{\sqrt8}\delta_{T0}+\frac{1}{\sqrt{40}}\delta_{T1}\right]\langle H_2(\tfrac12,\tfrac32)\rangle_{JT}$$
$$+\tfrac12\rho^{\Delta^0\rm n}_{\bar B_1\bar B_2}(\delta_{T1}+\delta_{T2})\langle H_2(\tfrac32,\tfrac12)\rangle_{JT}$$
$$-\tfrac12\rho^{\rm pp}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac12,\tfrac12)\rangle_{JT}+\frac{1}{\sqrt{10}}\rho^{\Delta^+\Delta^+}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac32,\tfrac32)\rangle_{JT}$$
$$-\sqrt{2/5}\rho^{\rm p\Delta^+}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac12,\tfrac32)\rangle_{JT}-\tfrac14\rho^{\Delta^+\rm p}_{\bar B_1\bar B_2}(\delta_{T1}-3\delta_{T2})\langle H_2(\tfrac32,\tfrac12)\rangle_{JT}\Bigg]\,, \tag{A8}$$

$$\langle B(\Delta^+)|h|B'({\rm p})\rangle=\sum_{\bar B_1\bar B_2}\delta_{j_{B_1}j_{B_2}}\delta_{l_{B_1}l_{B_2}}\delta_{j_Bj_{B'}}\delta_{m_Bm_{B'}}R_{\bar B\bar B_1}R_{\bar B'\bar B_2}$$
$$\times\sum_{JT}\frac{2J+1}{2j_B+1}\Bigg[\tfrac12\rho^{\rm nn}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac12,\tfrac12)\rangle_{JT}+\rho^{\Delta^0\Delta^0}_{\bar B_1\bar B_2}\left[\frac{1}{\sqrt{40}}\delta_{T1}-\frac{1}{\sqrt8}\delta_{T2}\right]\langle H_2(\tfrac32,\tfrac32)\rangle_{JT}$$
$$+\tfrac12\rho^{\rm n\Delta^0}_{\bar B_1\bar B_2}(\delta_{T1}+\delta_{T2})\langle H_2(\tfrac12,\tfrac32)\rangle_{JT}$$
$$-\rho^{\Delta^0\rm n}_{\bar B_1\bar B_2}\left[\frac{1}{\sqrt8}\delta_{T0}+\frac{1}{\sqrt{40}}\delta_{T1}\right]\langle H_2(\tfrac32,\tfrac12)\rangle_{JT}-\tfrac12\rho^{\rm pp}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac12,\tfrac12)\rangle_{JT}$$
$$+\frac{1}{\sqrt{10}}\rho^{\Delta^+\Delta^+}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac32,\tfrac32)\rangle_{JT}-\tfrac14\rho^{\rm p\Delta^+}_{\bar B_1\bar B_2}(\delta_{T1}-3\delta_{T2})\langle H_2(\tfrac12,\tfrac32)\rangle_{JT}$$
$$-\sqrt{2/5}\rho^{\Delta^+\rm p}_{\bar B_1\bar B_2}\delta_{T1}\langle H_2(\tfrac32,\tfrac12)\rangle_{JT}\Bigg]\,. \tag{A9}$$

*Permanent address: Al-Najah National University, Nablus, West Bank, Via, Israel.
†Permanent address: Iowa State University, Ames, IA 50011.

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