PHYSICAL REVIEW C VOLUME 36, NUMBER 6 DECEMBER 1987

Quarks and the saturation properties of nuclear matter

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(Received 11 July 1986)

A two-region model of the nuclear force is developed by using quark degrees of freedom at short ranges and nucleons elsewhere. In the quark sector, the soliton bag model and generator coordinates are used so that the quark fields undergo a continuous deformation as the two nucleons overlap. This model describes the spin singlet nucleon-nucleon scattering data very well. The density and binding energy of nuclear matter are also computed in lowest-order Brueckner theory. Quark effects are found to decrease both the saturation density and binding energy.

INTRODUCTION

It is generally accepted that the theory of quarks and gluons, quantum chromodynamics, is the underlying theory of the strong interaction. Consequently, the question is not are there quarks in nuclei, but rather are quarks convenient degrees of freedom for describing nuclei? One cannot yet apply quantum chromodynamics directly to problems in nuclear physics. Instead, one uses low-energy models of quark dynamics. In recent years, many workers have used low-energy quark models to successfully describe the short-range part of the nuclear force.[1–12] In this study, we examine quark effects in both the two-nucleon and nuclear matter systems.

It is natural to apply low-energy quark models to the short-range part of the nuclear force. Since the size of the nucleon is about 1 fm, one might expect this "short-range" region to extend to separations of about 1 fm. The long-range nuclear force is known to be well described by meson exchange and is therefore more easily described with nucleon and meson degrees of freedom. Here, we develop a model to use a quark model for the short-range force and to use a nucleon-nucleon potential to describe the long-range force.

One new feature of our nucleon-nucleon scattering calculation is that the quark and nucleon degrees of freedom are matched (at ~ 1 fm) so that the quark wave functions are continuous. Furthermore this is the first soliton bag model calculation of the scattering phase shifts.

Our interest in the many-body system is spurred by the notion that the many-body system probes the off-shell nucleon-nucleon interaction. The quark model provides a microscopic description of that interaction that may have different features.

Our calculation of quark effects in nuclear matter is crude but serves as a first estimate of their importance. The matching techniques developed here for scattering are little changed in their application to the nuclear matter calculation. Indeed, the matching technique is very well suited for extension to the reference spectrum method of nuclear matter saturation calculation. We find that quark effects can be large and can possibly move the saturation point off the Coester line.

The outline of the paper is as follows. Section I describes the low-energy model, the soliton bag model[13–15] used to describe the short range part of the nuclear force. In applying this model we rely heavily upon techniques developed by Schuh and collaborators.[9] They only obtained a qualitative description of the nuclear force, since they neglected important gluon exchange effects and used a zero impact-parameter approximation.

In Sec. II a generator coordinate method for calculating the dynamics of two interacting clusters is discussed. The calculation of nucleon-nucleon scattering and the development of our matching technique are presented in Sec. III, in which the soliton bag model and the method of generator coordinates are used at short ranges. Several results are presented, including the sensitivity to the matching radius (where the quark and nucleon sectors are joined) and sensitivity to different approximations used for the method of generator coordinates. We find that all of the spin singlet phase shift data up to lab energies of 300 MeV can be adequately described with a single choice of parameters.

The nuclear force of Sec. III is applied to a reference-spectrum approximation calculation of nuclear matter properties in Sec. IV. We calculate both the nucleonic and non-nucleonic components of the short-distance two-body wave function. The results indicate that quark effects are important and lead to a decrease in the binding energy and density. Our calculation also indicates that a microscopic description should be used in the "long-range" nucleon sector.

I. SOLITON BAG MODEL

Quark dynamics are to be included in the nucleon-nucleon interaction mechanism. It is necessary to model the quark dynamics, and we use the soliton bag model (SBM).[13–15] In many ways the SBM is an evolution of the original bag model, the MIT bag model.[16] It is difficult to use the MIT bag model in its original formulation for any configurations other than spherical, static bags. The SBM overcomes this restriction, and the ability to perform calculations involving deformed and non-static bags is the reason that it is used here. In the SBM quark confinement is a dynamical feature of the model

which appears by introducing an additional scalar field, the σ field, into the model. For a review of the SBM, see the articles by Wilets and co-workers.[17]

It is possible but difficult to calculate nucleon-nucleon scattering wholly within a quark model. The problem is that the long range meson exchange force is generated by highly complicated quantum fluctuations ($q\bar{q}$ pairs). Such fluctuations have been included in nucleon and delta mass calculations.[12] Unfortunately, including such effects in a nucleon-nucleon scattering calculation would be an enormous task. Furthermore, imposing chiral symmetry, as in the cloudy bag model, leads to the necessary one and two pion exchange potentials between nucleons. Here the quark model is used only at small separations where, we expect, the most important quark effects are quark Fermi statistics and asymptotic freedom—not quantum fluctuations. The large separation force is included by using a nucleon-nucleon potential. Some features of the SBM necessary for an understanding of our calculation of the short-range force are presented next.

A. Model

The dynamics of the SBM are described by a Lagrangian density,

$$\mathcal{L}(x)=i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi-g\bar{\psi}\sigma\psi+\tfrac{1}{2}\partial_{\mu}\sigma\partial^{\mu}\sigma-U(\sigma)\ , \tag{1.1}$$

where the self-interactions of σ are given by

$$U(\sigma)=\frac{a}{2}\sigma^2+\frac{b}{3!}\sigma^3+\frac{c}{4!}\sigma^4+B\ . \tag{1.2}$$

The form of $\mathcal{L}(x)$ includes the requirements of renormalizability and translational invariance. The parameters in $U(\sigma)$ are chosen so that hadrons appear as nontopological solitons of the σ field confining the quarks. This can be achieved (for the classical solutions) by choosing $U(\sigma)$ to have a global minimum at a nonvanishing value of σ, σ_V; and to have a local minimum at or near $\sigma=0$.

The masses and sizes of the low energy baryons and mesons constrain the parameters a, b, c, and g. A variety of parameter sets which yield acceptable fits to the data have been examined by several authors.[15,17–21] In this study a single parameter set is used.

B. Effective one-gluon exchange interaction

One of the deficiencies of the SBM as defined above is that there are no color charge dependent forces in the model and color confinement must be put in as an ansatz. This can be remedied by adding explicit gluon degrees of freedom.[20,22,23] Then, color nonsinglet states develop an infinite self-energy and color confinement becomes a consequence of the model.[20]

An alternate remedy is to introduce an effective color-dependent interaction which is motivated by one-gluon exchange. This has been done routinely for nonrelativistic quark models.[3,24,25]

In this study an effective interaction will be added which is very similar to that of Harvey.[24] The following term

$$:H^{\rm eff}(x):\equiv\tfrac{2}{3}K_{\rm OGE}:\psi^{\dagger}\lambda^{(c)}\psi\psi^{\dagger}\lambda^{(c)}\psi:$$
$$+\tfrac{2}{3}K_{\rm OGE}:\psi^{\dagger}\lambda^{(c)}\Sigma\psi\psi^{\dagger}\lambda^{(c)}\Sigma\psi:-{\rm CT}\ , \tag{1.3}$$

with

$$\Sigma=\begin{bmatrix}\sigma & 0\\ 0 & \sigma\end{bmatrix}\ , \tag{1.4}$$

is added to the normal ordered Hamiltonian density. The σ are the Pauli matrices and the $\lambda^{(c)}$, the SU(3) matrices. The CT represents the counterterm which is chosen to exactly cancel the irrelevant one-body pieces of $:H^{\rm eff}:$. Sums on the quark colors and flavors are implied as well as sums on the color matrix labels and the spin matrix labels. This term would lead to a nonrelativistic potential,

$$V_{ij}^{\rm eff}=\lambda_i^{(c)}\cdot\lambda_j^{(c)}K_{\rm OGE}[\tfrac{2}{3}\delta(\mathbf{r}_{ij})+\tfrac{8}{3}\mathbf{S}_i\cdot\mathbf{S}_j\delta(\mathbf{r}_{ij})]\ , \tag{1.5}$$

where $\mathbf{S}_i$ are quark spin operators, very similar to Harvey's. The choice of $\frac{2}{3}$ for the non-spin-dependent term is made so that the self-energy contribution of $H^{\rm eff}$ vanishes for a single nucleon. The actual calculation is rather insensitive to that choice. The choice of the form of Eq. (1.3) is motivated by the nonrelativistic forms and by its simplicity.

The coupling constant, $K_{\rm OGE}$, could be fixed by fitting the nucleon-delta mass difference. However, in this study $K_{\rm OGE}$ will be chosen to obtain a best fit to the nucleon-nucleon scattering phase shifts.

C. Two-nucleon states

If one were to search for the lowest energy, spherical six-quark state of baryon number equal to two in the mean field approximation, one would find a spherical bag containing six quarks in the lowest energy orbital. However, because of one-gluon exchange contributions, the energy of this spherically symmetric state would be greater than the energy of two separated nucleons. Therefore, using a single mean field approximation state is not sufficient. Clearly, one expects this since even the deuteron is more like two nucleons than a single six-quark bag.

The fissioning of a spherical six-quark bag into two separated nucleons is described with generator coordinates. This allows a variety of baryon number equaling two states to be used in the expansion of the complete state vector. One expands the two-nucleon state vector in a set of basis states labeled by the generator coordinate. In principle one should use a complete or overcomplete set of states as basis states for a calculation using the method of generator coordinates. However, in practice one chooses a physically motivated but incomplete set to make the calculation tractable.

There are many possible choices for two-nucleon basis states. Here we use two different sets of basis states. It is found that one set (*sudden approximation*) yields an acceptable fit to the scattering phase shifts, while the other (*volume conserving*) does not. The states used here are very similar to those used by others[3,25] for the quark sector in nonrelativistic calculations. The main

difference is that the σ field must also be specified for our states. For the σ field we use a coherent state. This has the simplicity of being closely related to the mean field approximation state but is a properly defined quantum state. The definition and properties of the coherent state are described in Refs. 9, 12, and 17.

Many features of the sudden approximation set and the volume conserving set are the same. These are:

(1) The generator coordinate which labels the states corresponds closely to the relative separation distance between the two nucleons. At large separation the generator coordinate is the separation between the bag centers.

(2) The states for large values of the generator coordinate correspond to two separated mean field approximation nucleon states.

(3) To a first approximation, the σ field is assumed to adjust adiabatically to the quark density.

(4) The states are defined so that as the two bags overlap the quark states remain spatially in spherically symmetric orbitals, with each quark's state centered in its respective bag.

Next turn to the differences. In the *sudden approximation* the spherically symmetric wave functions centered at the bag centers remain fixed. Thus, for any deformation they are the same as the single-nucleon mean field approximation quark solutions. In the *volume conserving* case the quark wave functions are the same as the mean field approximation wave functions except that they are scaled so that the total enclosed volume of the two bags remains constant. To a very good approximation the zero deformation parameter state in the volume conserving basis is the same as the spherical six-quark bag in the mean field approximation.

Mathematically the states are defined as follows. For the sudden approximation,

$$|\boldsymbol{\alpha}\rangle_{\mathrm{SA}} \equiv \mathcal{A}\{[a_1^\dagger(\boldsymbol{\alpha}/2)a_2^\dagger(\boldsymbol{\alpha}/2)a_3^\dagger(\boldsymbol{\alpha}/2)]_{\mathrm{N}} \times |a_4^\dagger(-\boldsymbol{\alpha}/2)a_5^\dagger(-\boldsymbol{\alpha}/2)a_6^\dagger(-\boldsymbol{\alpha}/2)]_{\mathrm{N}}\}|\sigma_\alpha\rangle, \tag{1.6}$$

where the $a^\dagger(\mathbf{r})$ operators create a quark state with the lowest energy mean field approximation wave function centered at $\mathbf{r}$. The subscripts on the creation operators refer to the spin, isospin, and color of the quark state created. The $\mathcal{A}$ is the quark antisymmetrization operator necessary since $a_n^\dagger(\alpha/2)$ and $a_n(-\boldsymbol{\alpha}/2)$ do not obey canonical anticommutation relations. The N subscript denotes that the three quarks in the brackets are coupled to the spin, isospin, and color quantum numbers of the nucleon. The coherent σ field state $|\sigma_\alpha\rangle$ is defined so that

$$\frac{\langle\sigma_\alpha|\sigma(\mathbf{r})|\sigma_\alpha\rangle}{\langle\sigma_\alpha|\sigma_\alpha\rangle} = \sigma_\alpha(\mathbf{r}) , \tag{1.7}$$

where $\sigma_\alpha(\mathbf{r})$ is a function which approximates the adiabatic σ field configuration for the quark density of state $|\boldsymbol{\alpha}\rangle_{\mathrm{SA}}$. Following Dethier[12] $\sigma_\alpha(\mathbf{r})$ is fitted to the form

$$\sigma_\alpha(\mathbf{r}) = \sigma_V - \frac{\mu^2\sigma_1}{4\pi}\int d^3z\,\Theta_\alpha(\mathbf{z})\frac{e^{-\mu|\mathbf{r}-\mathbf{z}|}}{|\mathbf{r}-\mathbf{z}|} , \tag{1.8}$$

where $\Theta_\alpha(\mathbf{r})$ equals 1 if $|\mathbf{r}-\boldsymbol{\alpha}/2| \le R_{\mathrm{N}}$ or $|\mathbf{r}+\boldsymbol{\alpha}/2| \le R_{\mathrm{N}}$, and equals 0 otherwise.[32] The parameters μ, σ_1, and R_N are chosen so that σ_α, for large $\boldsymbol{\alpha}$, is the sigma field for two separated nucleons.

The volume conserving states, $|\boldsymbol{\alpha}\rangle_{\mathrm{VC}}$, are similarly defined except that the total volume enclosed by the two bags is kept fixed. Specifically, the σ field coherent state would be defined as in Eqs. (1.7) and (1.8) except that the theta function, $\Theta_\alpha(\mathbf{r})$, equals 1 for $|\mathbf{r}\pm\boldsymbol{\alpha}/2| \le R_\alpha$ instead of R_{N}. R_α is defined so that

$$\int d^3r\,\Theta_\alpha(\mathbf{r}) = 2\times(\tfrac{4}{3}\pi R_{\mathrm{N}}^3) , \tag{1.9}$$

for all $\boldsymbol{\alpha}$. Equation (1.9) defines the term "volume conserving." For a given $\boldsymbol{\alpha}$ the same linear scaling of R_α (namely R_α/R_{N}) is applied to the quark wave functions as well, so that the quarks are scaled so as to fill the bag.

The title "sudden approximation" used to describe the set of basis states refers to the fact that the quark wave functions are not distorted as the bags overlap. The σ fields in the basis states used here adjust adiabatically to the quark field density. We may compare the quark states used here to states used in the nonrelativistic quark model. In the nonrelativistic model the quarks have a large mass ($\approx$300 MeV) and are assumed to adjust slowly to changes in the potential. In relativistic models, such as the SBM, the quarks are massless but still contribute a similar amount of energy to the nucleon mass as do the nonrelativistic quarks. However, in relativistic models the energy arises from the quarks' kinetic energy and the quarks have large inertia. The assumption made here is that the inertia of the quarks causes them to adjust slowly to changes in the gluon field around them.

II. GENERATOR COORDINATES METHOD

The method of generator coordinates or the resonating group method[26] is a popular technique for computing the scattering of composite clusters. In hadron physics quarks cluster to form three-quark color singlet hadrons and the method of generator coordinates can be profitably applied.

The degree to which quarks cluster into three-quark color singlets as two nucleons overlap is determined by energetics. The energy involved is the energy to excite the system of six quarks into a state which does not look like two three-quark color singlet clusters. A naive estimate of this energy is the 300 MeV nucleon-delta mass splitting. This is partly a hyperfine interaction from gluons. Color confinement is caused by gluons, so this mass splitting may be indicative of other gluonic energies. The nucleon-delta mass difference is large compared to binding energies in nuclei, indicating that clustering of quarks into three-quark color singlets might be quite strong even if two nucleon bags overlap a great deal. This is a motivation for both the method of generator coordinates and the two-nucleon soliton bag model basis states discussed above.

This section covers the aspects of the method of generator coordinates used in the quark sector. The main topics are: (1) the Griffin-Hill-Wheeler equation; (2) the

effective energy which appears in the Griffin-Hill-Wheeler equation and the center of mass corrections which it contains; and (3) the direct and exchange contributions for the problem of nucleon-nucleon scattering.

A. Griffin-Hill-Wheeler equation

A set of two-nucleon basis states can be defined which are labeled by a parameter, $\boldsymbol{\alpha}$, known as the generator coordinate. These states correspond to two three-quark clusters separated by a distance $\boldsymbol{\alpha}$ along with a σ field configuration. The state vector of a two-nucleon system can be expanded as

$$|\Psi\rangle = \int d^3\alpha \, |\boldsymbol{\alpha}\rangle \phi(\boldsymbol{\alpha}) \, . \tag{2.1}$$

The Griffin-Hill-Wheeler (GHW) equation is derived by requiring that the expectation value of the normal ordered Hamiltonian is stationary with respect to variations in the generator coordinate weight function, $\phi(\boldsymbol{\alpha})$. The relevant equations are shown for the spin singlet states[27] in which case the partial waves completely decouple. Then the GHW scattering equation is

$$\int_0^R \alpha^2 d\alpha \langle \beta | :H:-E \, | \alpha \rangle_\ell \frac{u_\ell(\alpha)}{\alpha} = -\int_R^\infty \alpha^2 d\alpha \langle \beta | :H:-E \, | \alpha \rangle_\ell \frac{\bar{u}_\ell(\alpha)}{\alpha} \, . \tag{2.2}$$

The partial wave decomposition is

$$\phi(\boldsymbol{\alpha}) = \sum_\ell \frac{u_\ell(\alpha)}{\alpha} Y_{\ell 0}(\hat{\boldsymbol{\alpha}}) \, , \tag{2.3}$$

where $Y_{\ell 0}(\hat{\boldsymbol{\alpha}})$ is a spherical harmonic. For matrix elements of an operator, $\mathcal{O}$, it is

$$\langle \boldsymbol{\beta} | \mathcal{O} | \boldsymbol{\alpha} \rangle = \sum_\ell \langle \beta | \mathcal{O} | \alpha \rangle_\ell P_\ell(\hat{\boldsymbol{\beta}} \cdot \hat{\boldsymbol{\alpha}}) \, , \tag{2.4}$$

where P_ℓ is a Legendre polynomial. In Eq. (2.2) R is chosen larger than the range of interaction between clusters and the asymptotic weight-function, $\bar{u}_\ell(\alpha)$, has the form

$$\frac{\bar{u}_\ell(\alpha)}{\alpha} = j_\ell(k\alpha) + \frac{\sin(\delta_\ell)}{\cos(\delta_\ell)} n_\ell(k\alpha) \, , \tag{2.5}$$

where k is the relative momentum, δ_ℓ is the unknown phase shift, and $j_\ell(x)$ and $n_\ell(x)$ are, respectively, the regular and irregular spherical Bessel functions of order ℓ.

In this study the method of Tikhonov regularization[28] is used to eliminate the numerical divergences associated with the GHW equation. Tikhonov regularization is also used for solving all other first-kind Fredholm integral equations which arise in this study.

B. Effective energy and center of mass corrections

The energy, E, enters Eq. (2.2) as a Lagrange multiplier. It can be shown[27] that the relation between E and the relative momentum, k, depends only upon the asymptotic form of $\phi(\boldsymbol{\alpha})$ and is given by

$$E(k) = \frac{\int \alpha^2 d\alpha \langle X | :H: | \alpha \rangle_\ell j_\ell(k\alpha)}{\int \alpha^2 d\alpha \langle X | \alpha \rangle_\ell j_\ell(k\alpha)} \, , \tag{2.6}$$

for any ℓ, if $|\mathbf{X}|$ is chosen much larger than the region of interaction. $E(k)$ as defined by Eq. (2.6) is independent of both ℓ and $\mathbf{x}$. The region of integration must extend far enough above and below $\alpha = X$ to include all of the nonlocalities of the kernel.

The interpretation of $E(k)$ can now be made. In Eq. (2.6) the contributions to the integrals occur only for $\boldsymbol{\alpha} \approx \mathbf{X}$, i.e., for large $|\boldsymbol{\alpha}|$. The two-nucleon states, $|\boldsymbol{\alpha}\rangle$, for large $|\boldsymbol{\alpha}|$ are simply product states of two separated nucleons, one at $\boldsymbol{\alpha}/2$ and the other at $-\boldsymbol{\alpha}/2$. In determining $E(k)$ one is projecting onto a state of well-defined *relative momentum*. Since the mass of the total system is larger than the mass of a single nucleon, total center of mass corrections are less important than relative center of mass corrections. In fact, Eq. (2.6) is the equation for the energy of a Peierls-Yoccoz[29] projected state of relative momentum $\mathbf{k}$ for two nucleons.

These center of mass corrections are important for obtaining the correct solutions to the GHW equation. It should be mentioned that Peierls-Yoccoz projection is a low-energy approximation; a better technique is to first project the nucleon onto a state of zero momentum and then perform a Lorentz boost of the wave function.[17]

C. Direct and exchange contributions

For the two nucleon system the kernels of the GHW equation involve matrix elements of one- and two-body operators. These can be classified as direct or exchange depending on whether they are evaluated between quarks in the same nucleon cluster or quarks in different nucleon clusters. One must compute the direct and exchange matrix elements for the normalization and H between the two-nucleon states defined in Sec. I C. The technique that is used is described elsewhere,[3] so further details are not included (see also Ref. 27).

III. NUCLEON-NUCLEON SCATTERING

The problem of calculating the nucleon-nucleon interaction from meson exchange theory is a very old one. Meson exchange theories include the dynamics of the internal structure of the nucleon by expanding the model space to include excitations of the nucleon.[30,31] However, the degree to which quark substructure of the nucleon is responsible for phenomenological form factors and the nucleon resonances is not made explicit.

An alternative approach is to begin with a model for the quark dynamics. Then the influence of the quark substructure is explicit, but the long range meson exchange effects are difficult to include wholly within the framework of a quark model. One way to do this is to introduce the meson fields as fundamental fields of the model which interact with the quarks.[5] The cloudy bag model[32] and other chiral bag models[33] include explicit pions. Yang and Zhong[10] calculated the nucleon-nucleon interaction in the cloudy bag model and obtained the one-pion exchange tail at large separation.

The approach here is to use quark dynamics only at small separation and to use explicit nucleon degrees of freedom interacting via a potential at large separations. The new aspects of our nucleon-nucleon scattering work are contained in this section. Namely, we develop and apply a technique for matching a small separation description in terms of quarks with a large separation description in terms of nucleons.

A. Matching quarks and nucleons

There are several techniques of incorporating quark dynamics into the nucleon-nucleon interaction. F-matrix or P-matrix calculations are popular.[34–37] A drawback of most F-matrix calculations is that the quarks are only explicitly included in a single spherical six-quark bag. The generator coordinate method avoids this by allowing a series of bag states which undergo a smooth deformation from a spherical six-quark bag into two separated three-quark bags to exist.

For the nonrelativistic many-body problem one can separate the relative, center of mass, and internal coordinates for two clusters. If one writes the state vector of the two-cluster system as an unknown wave function of the relative coordinate times two fixed internal wave functions of the internal coordinates, then one is led to the equations of the resonating group method. Moreover, the resonating group wave function can be related to the generator coordinate weight function. In general the wave function and the weight function are related to each other by an integral transform.[38]

For the relativistic many-body problem it is not known how to separate relative, center of mass, and internal coordinates. However, at a large separation the distance between the geometric centers of the two nucleon clusters should be approximately the relative nucleon-nucleon coordinate.

Here we assume that for large separations the distance between the geometric centers of two nucleon clusters is the relative nucleon-nucleon coordinate. Moreover, we assume that the nucleon-nucleon wave function can be related to the generator coordinate weight function via an integral transform like the one which relates the resonating group wave function to the generator coordinate weight function for nonrelativistic calculations.

The state vector is related to the generator coordinate weight function by

$$|\Psi\rangle = \int d^3\alpha\, |\boldsymbol{\alpha}\rangle \phi(\boldsymbol{\alpha}) \ . \tag{3.1}$$

To obtain the nucleon-nucleon wave function, one projects the state vector onto a state $|\mathbf{r}\rangle$ of two nucleons separated by $\mathbf{r}$. The wave function, $\psi(\mathbf{r})$, is given by

$$\psi(\mathbf{r}) \equiv \langle \mathbf{r}|\Psi\rangle = \int d^3\alpha \langle \mathbf{r}|\boldsymbol{\alpha}\rangle \phi(\boldsymbol{\alpha}) \ . \tag{3.2}$$

The kernel of the integral transform relating the wave function and the weight function is $\langle \mathbf{r}|\boldsymbol{\alpha}\rangle$. In order to obtain this kernel the state $|\boldsymbol{\alpha}\rangle$ should be projected onto the state of two nucleons separated by $\mathbf{r}$. For large separation, the basis states $|\boldsymbol{\alpha}\rangle$ are just such states [see Eq. (1.6)]. For intermediate separations the basis state $|\boldsymbol{\alpha}\rangle$ differs from the state $|\mathbf{r}\rangle$ mainly by deformations in the σ field. In this calculation the basis states $|\boldsymbol{\alpha}\rangle$ will be used as the two nucleon projection states $|\mathbf{r}\rangle$ in obtaining the integral transform of Eq. (3.1).[39] The kernel of this transformation is nonlocal, just as in the nonrelativistic problem, because the states $|\boldsymbol{\alpha}\rangle$ for different $\boldsymbol{\alpha}$ are not orthogonal. Thus

$$\psi(\mathbf{r}) = \int d^3\alpha \langle \boldsymbol{\beta}=\mathbf{r}|\boldsymbol{\alpha}\rangle \phi(\boldsymbol{\alpha}) \ . \tag{3.3}$$

The notation $|\boldsymbol{\beta}=\mathbf{r}\rangle$ indicates that the ket which is used is a basis state which has previously been denoted with Greek vectors, $|\boldsymbol{\alpha}\rangle$ or $|\boldsymbol{\beta}\rangle$; but here the ket is used in an integral transform which relates the weight function, $\phi(\boldsymbol{\alpha})$, to the wave function, $\psi(\mathbf{r})$.

With this transformation, both the wave function and the weight function are defined over all of space. However, the physics at small separation should be determined only by the quarks and the physics at large separation should be determined only by the nucleon-nucleon potential. These two requirements are implemented by requiring that the wave function, $\psi(\mathbf{r})$, satisfy the Schrödinger equation with the Reid nucleon-nucleon potential[40] for $|\mathbf{r}| > R_M$, R_M being the matching radius between the quark sector and the nucleon sector. On the other hand, the weight function, $\phi(\boldsymbol{\alpha})$, is required to solve the Griffin-Hill-Wheeler equation for $|\boldsymbol{\alpha}| < R_M$. These two requirements and the transformation between $\phi(\boldsymbol{\alpha})$ and $\psi(\mathbf{r})$ completely determine both $\phi(\boldsymbol{\alpha})$ and $\psi(\mathbf{r})$ for all values of $\boldsymbol{\alpha}$ and $\mathbf{r}$.

The equations determining the solution are: (1) the Schrödinger equation,

$$-\frac{1}{2m}\nabla^2\psi(\mathbf{r}) + V_R(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r}) \ , \tag{3.4}$$

for $|\mathbf{r}| > R_M$; (2) the Griffin-Hill-Wheeler equation,

$$\int_{|\boldsymbol{\alpha}|<R_M} d^3\alpha \langle \boldsymbol{\beta}| :H:-E\, |\boldsymbol{\alpha}\rangle \phi(\boldsymbol{\alpha}) = -\int_{|\boldsymbol{\alpha}|>R_M} d^3\alpha \langle \boldsymbol{\beta}| :H:-E\, |\boldsymbol{\alpha}\rangle \bar{\phi}(\boldsymbol{\alpha}) \ , \tag{3.5}$$

for $|\boldsymbol{\beta}| < R_M$; and (3) the transformation between $\phi(\boldsymbol{\alpha})$ and $\psi(\mathbf{r})$,

$$\psi(\mathbf{r}) = \int_{|\boldsymbol{\alpha}|<R_M} d^3\alpha \langle \boldsymbol{\beta}=\mathbf{r}|\boldsymbol{\alpha}\rangle \phi(\boldsymbol{\alpha}) + \int_{|\boldsymbol{\alpha}|>R_M} d^3\alpha \langle \boldsymbol{\beta}=\mathbf{r}|\boldsymbol{\alpha}\rangle \bar{\phi}(\boldsymbol{\alpha}) \ , \tag{3.6}$$

for $|\mathbf{r}| > R_M$. The function $\bar{\phi}(\boldsymbol{\alpha})$ is the generator coordinate weight function for $|\boldsymbol{\alpha}| > R_M$ and it is determined by Eq. (3.6). The weight function is required to be continuous at $|\boldsymbol{\alpha}| = R_M$, i.e.,

$$\phi(\boldsymbol{\alpha}=R_M\cdot\hat{\boldsymbol{\alpha}}) = \bar{\phi}(\boldsymbol{\alpha}=R_M\cdot\hat{\boldsymbol{\alpha}}) \ , \tag{3.7}$$

$\hat{\boldsymbol{\alpha}}$ is the unit vector in the direction of $\boldsymbol{\alpha}$. Equations (3.4)–(3.7) must be solved self-consistently.

In practice a partial wave decomposition of the equations is made and the resulting equations are solved by iteration. For each partial wave the phase shift, δ_ℓ, is determined as a function of energy. For details of the iteration procedure, see Ref. 27.

This technique of including both the nucleon-nucleon physics at large separation and the quark physics at

short separation requires one to obtain both a wave function and a weight function. There are four points which should be emphasized:

(1) The calculation does not depend on the nucleon-nucleon potential, V_R, for $|\mathbf{r}| < R_M$.

(2) The calculation also does not depend on the kernel of the Griffin-Hill-Wheeler equation for $|\boldsymbol{\alpha}|$ and $|\boldsymbol{\beta}| > R_M$. The kernel of the GHW equation is calculated using the model of quark dynamics, the soliton bag model. This shows that the quark dynamics are used only at small separations.

(3) The integral transform which relates the weight function to the wave function is used to find the wave function only for $|\mathbf{r}| > R_M$. The use of the generator coordinate $\boldsymbol{\alpha}$ to represent the nucleon-nucleon coordinate should be more reliable at larger separations.

(4) This technique for matching the nucleon-nucleon sector with the quark sector will be applied to the nuclear matter calculation.

B. Numerical techniques

The solution of the equations presented in the previous section requires the numerical evaluation of the GCM basis states, the GCM kernels, the solution of the Schrödinger equation, and the solution of the coupled integral equations, Eqs. (3.5) and (3.6). Since these evaluations may introduce errors into the solutions, one must check the accuracy of the individual calculations and of the procedure.

The SBM parameters defined in Sec. I used here are

$$\begin{aligned} a &= 30.23 \text{ fm}^{-2} , \\ b &= -612.3 \text{ fm}^{-1} , \\ c &= 4000 , \\ B &= 9.69 \text{ MeV fm}^{-3} , \\ g &= 10.0 . \end{aligned} \tag{3.8}$$

These are slightly rescaled parameters taken from previous work[17] which yield the correct nucleon mass after center-of-mass projection. These parameters are not adjusted at all. The remaining parameter in the SBM, the effective one-gluon exchange interaction coupling constant, K_{OGE}, is discussed in the next section.

All of the equations are solved in coordinate space with a mesh size of approximately $\frac{1}{6}$ fm. The only approximation made in the evaluation of the kernel is to omit a small spin-dependent term. Since tensor forces from one-gluon exchange have also been neglected we will not examine the spin triplet phase shifts closely. Nor will we include quark effects in the spin-triplet states in our nuclear matter calculation in the next section. There are many checks for individual steps in our calculation which are described in Ref. 27.

A final check on the entire calculation is to calculate the phase shifts for nucleon-nucleon scattering in which all the interactions between the two nucleons are "turned off." To do this one must calculate basis states in the quark sector which are product states of two nucleons separated by the generator coordinate, $\boldsymbol{\alpha}$. These free two-nucleon states are defined so that parity is a good quantum number. The quarks within each nucleon are in an antisymmetric state; however, there is no antisymmetrization between quarks in different nucleons.

The kernels of the GHW equation are evaluated as in the full calculation except that there are no interactions between the quarks and σ field of the different nucleons. The effective energy in the GHW kernel is unchanged since it is an asymptotic property.

In the nucleon sector of the problem, the potential in the Schrödinger equation is set to zero. The problem is solved using Eqs. (3.4)–(3.7) exactly as it is solved in the full problem: the same mesh is used, the same regularization scheme and parameters are used, the same iteration procedure is used, and the same technique for obtaining the phase shift is used. Thus, this constitutes a problem solved with exactly the same technique, but the exact solution for this problem is known. For the exact solution the phase shifts vanish at all energies for all partial waves. The calculation of the phase shifts yields all phase shifts to be smaller than 0.001 rad for all partial waves and all lab energies up to 352 MeV. This is the strongest single check of the accuracy of this calculational program.

C. Results

The nucleon-nucleon scattering calculation is carried out with two different choices of basis states in the quark sector: (1) the sudden approximation basis states for which the quark orbitals are kept equal to the single nucleon quark orbitals even as the two nucleons overlap, and (2) the volume conserving basis states for which the overlapping bags expand as they overlap (so as to keep the total enclosed volume of the two bags constant). Otherwise the volume conserving basis states are the same as the sudden approximation states.

There are two parameters in this calculation which are adjusted to fit the scattering phase shifts: (1) the coupling constant, K_{OGE}, for the effective one-gluon exchange interaction, and (2) the matching radius, R_M, between the quark sector and the nucleon sector. The value of K_{OGE} can be chosen close to the value $K_{\text{N}\Delta}$, which produces the proper nucleon-delta mass splitting, but is not. There are several reasons for this: First, this calculation is not a determination of the prediction of the soliton bag model for nucleon-nucleon scattering. Rather, it is a model of the nucleon-nucleon interaction motivated by quark physics at short separation and nucleon physics at large separation. It is important that this model fit the nucleon-nucleon scattering data. Only then can it be applied to a calculation of nuclear matter and compared with other calculations which do not include quarks. Second, the effective one-gluon exchange interaction is an effective operator which may not affect six-quark systems and three-quark systems in the same way as true one- or many-gluon exchange. Third, there are other contributions to the nucleon-delta mass splitting which are not included; pion exchange is an example. Fourth, in this calculation the center of mass energy of the nucleons at large separation is subtracted; how-

ever, the center of mass of the entire two-nucleon system is fixed in the basis states used here. Thus, there are corrections to the kernels of the Griffin-Hill-Wheeler equation for the motion of the entire two-nucleon center of mass. These corrections should be smaller than the corrections for the single nucleon center of mass motion because the two nucleon system is heavier. The value of $K_{\rm OGE}$ which fits the nucleon-delta mass splitting in this model is $K_{N\Delta} \approx -0.9$ fm^2.

The matching radius between the quark sector and the nucleon sector is also adjusted to fit the scattering data. Earlier workers who have separated the nucleon-nucleon interaction into an interior quark sector and an exterior nucleon sector have found matching radii of approximately $R_M \approx 0.8$–0.9 fm; see, for example, the work by Kisslinger and Miller,[41] Lomon,[42] or Simonov.[43]

The parameters $K_{\rm OGE}$ and R_M are fitted by calculating the $^1S_0$ phase shift and attempting to fit the experimental phase shifts in the 0–300 MeV lab energy region important in nuclear matter. Once the parameters are determined, all other partial waves are calculated with the same parameters.

First, consider the volume conserving basis set in which the quark wave functions expand as the bags overlap. This leads to an increase in the confinement size and a decrease in quark kinetic energies. Thus, one expects less repulsion in the volume conserving basis than in the sudden approximation basis. We find that the $^1S_0$ phase shift cannot be accurately described using the volume conserving basis with any reasonable $K_{\rm OGE}$ and R_M even though the qualitative features can be obtained. Figure 1 is a plot of the calculated $^1S_0$ phase shifts versus lab energy for three different sets of parameters. In Fig. 1 the solid line shows the Reid potential phase shifts which accurately describe the experimental results; the dotted lines are drawn through the calculated phase shifts for the three parameter sets. The three parameter sets shown are typical of results obtained with the volume conserving basis set. In all of the results shown, the decrease with energy is too small, indicating insufficient repulsion. We need to describe accurately the $^1S_0$ phase shift. The volume conserving basis set leads only to a qualitative fit, so we will not consider it further.

For the sudden approximation basis states the $^1S_0$ phase shift can be very well described, as shown in Fig. 2. In Fig. 2 the effective gluon coupling constant is fixed at $K_{\rm OGE} = -1.55$ fm^2 and three values for the matching radius are shown: (1) for $R_M = 0.670$ fm the phase shifts are slightly too large but the slope is quite close to the experimental value, (2) for $R_M = 0.837$ fm the calculated points lie nearly on top of the data (the best fit), and (3) for $R_M = 1.004$ fm the calculated points are close to the data for $E_{\rm lab} < 100$ MeV. However, the magnitude of the slope is too great and for large energies the points lie below the experimental values. This figure also shows the sensitivity to the matching radius. If the same dynamics were included in both the quark sector and the nucleon sector, then there should be no sensitivity to the matching radius. In this calculation different dynamics are included in the two sectors, so sensitivity to R_M is expected. However, the sensitivity is seen to be rather small.

Other parameter sets do not give a better description of the $^1S_0$ phase shifts. If the matching radius is made smaller than 0.837 fm, then the phase shifts become rather insensitive to $K_{\rm OGE}$ and a very large value would be required to produce a good fit.

The choice of matching radius $R_M = 0.837$ fm gives the best description of the "data," which occurs for an effective one-gluon exchange coupling constant $K_{\rm OGE} = -1.55$ fm^2. A reduction of the magnitude of $K_{\rm OGE}$ from -1.55 fm^2 to -0.9 fm^2 (from the Δ-n splitting) would result in insufficient repulsion. It has been stated by many other workers; see, for example, Oka,[3] that magnetic one-gluon exchange is largely responsible for the repulsion in the nucleon-nucleon interaction.

With the matching radius and the effective one-gluon exchange coupling constant fixed by the $^1S_0$ phase shifts,

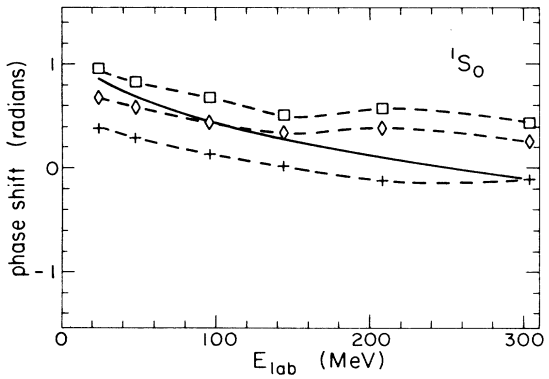

FIG. 1. $^1S_0$ phase shifts for the volume conserving basis. The matching radius is $R_M = 0.837$ fm. Phase shifts for three values of $K_{\rm OGE}$ are shown: $K_{\rm OGE} = -1.2$ fm^2, □; $K_{\rm OGE} = -1.3$ fm^2, ◇; $K_{\rm OGE} = -1.5$ fm^2, +. The dotted lines are drawn to guide the eye (in this and Fig. 2). The solid line represents the Reid potential phase shifts in this and Figs. 2–8.

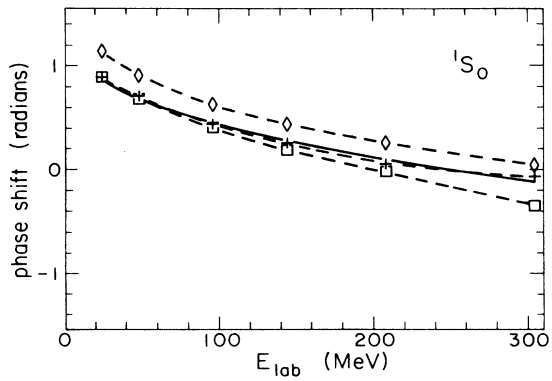

FIG. 2. $^1S_0$ phase shifts for the sudden approximation basis. The effective coupling constant is $K_{\rm OGE} = -1.55$ fm^2. Phase shifts for three values of R_M are shown: $R_M = 0.670$ fm, ◇; $R_M = 0.837$ fm, +; $R_M = 1.004$ fm, □. The sudden approximation basis is used in this and Figs. 3–10.

one can compute the phase shifts for the other partial waves. Recall that some spin dependent contributions to the quark sector have been omitted so there is a larger theoretical error in the spin triplet phase shifts.

Figures 3 through 8 show the phase shifts for all partial waves with $L \leq 2$ as well as for the $^3F_2$. For the coupled channel phase shifts and mixing parameters the bar phase shift conventions are used. Note that all spin singlet phase shifts ($^1S_0$, $^1P_1$, and $^1D_2$) agree remarkably well with the data. The $L=J$ spin triplet phase shifts ($^3P_1$ and $^3D_2$) are also in quite good agreement. The rest of the spin triplet phase shifts are only good at low energy, if at all. The low energy agreement is probably due to the dominance of the long-range interaction.

IV. QUARK EFFECTS IN NUCLEAR MATTER

The formalism for a quark-nucleonic description of the nucleon-nucleon interaction is developed in Secs. II and III. Here, we apply this model to nuclear matter to see if quarks matter. We find that quark effects can be large and can move the saturation point off the Coester line.

Our model for nucleon-nucleon scattering is based upon four equations: (1) the Schrödinger equation for the nucleon sector ($r > R_M$); (2) the Griffin-Hill-Wheeler equation for the quark sector ($r < R_M$); (3) the integral transform which relates the nucleon sector wave function to the generator coordinate weight function; and (4) a regularity condition for the weight function.

We proceed by obtaining appropriate generalizations of these four steps. Then the reference-spectrum method[44] is employed to simplify this first calculation. Pauli corrections are implemented by assuming that the energy shift due to Pauli corrections is not modified by quark effects. This is a crude approximation, but work by Green and Niskanen[45] indicates that such an approximation is reasonable in the similar case when deltas are the non-nucleonic degrees of freedom.

A. Nucleon sector

We expect that the influence of quarks will be to provide short-distance corrections to conventional nucleonic treatments. Thus we state the standard procedures for computing the binding energy and density of nuclear matter.

One starts with the independent pair approximation. The plane wave functions are labeled by $|\phi_k\phi_l\rangle$. Interactions between the independent pair are included in the wave function $|\psi_{kl}\rangle$, which satisfies the Bethe-Goldstone equation

$$|\psi_{kl}\rangle = |\phi_k\phi_l\rangle + \frac{Q}{e_{kl}-H_0} v\,|\psi_{kl}\rangle \ . \tag{4.1}$$

We simplify the calculation by using the reference

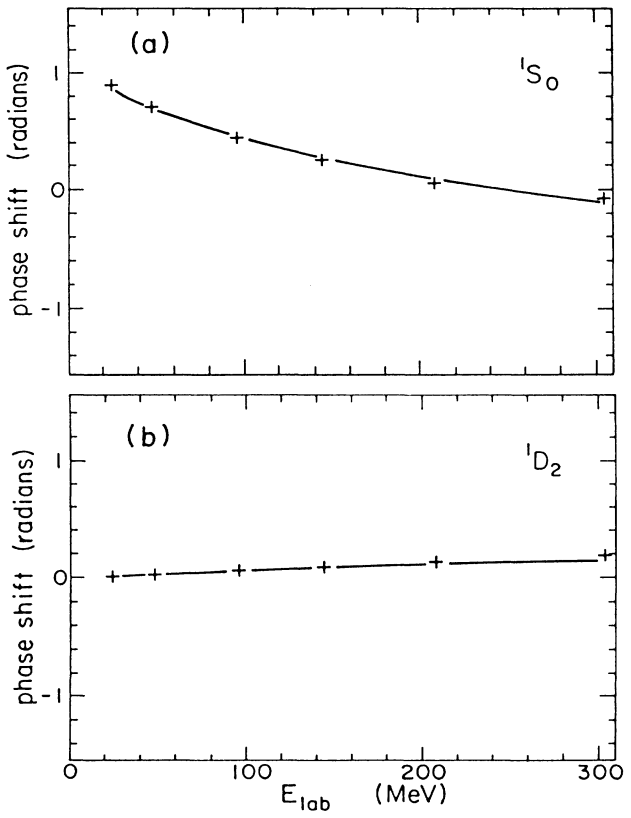

FIG. 3. (a) $^1S_0$ phase shifts. (b) $^1D_2$ phase shifts. The effective coupling constant is $K_{OGE} = -1.55$ fm^2. The matching radius is $R_M = 0.837$ fm. These values are used in this and in Figs. 4–8. The + represents the calculated phase shifts in this and all subsequent figures.

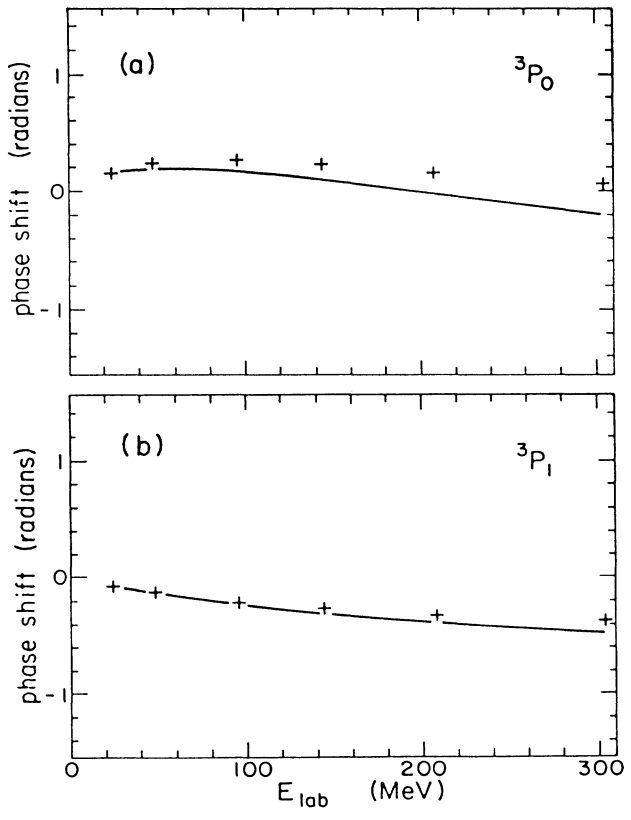

FIG. 4. (a) $^3P_0$ phase shifts. (b) $^3P_1$ phase shifts.

spectrum method[44] of Bethe, Brandow, and Perschek (BBP). It has the simplicity of leading to a differential equation. The resulting equation is expressed in terms of the defect wave function, ξ_{kl}, where $|\xi_{kl}\rangle = |\phi_{kl}\rangle - |\psi_{kl}\rangle$. One writes

$$\left[-\frac{1}{M}\nabla^2+\gamma^2\right]\xi_{kl}(\mathbf{r})=V(\mathbf{r})\psi_{kl}(\mathbf{r}) , \tag{4.2}$$

in the coordinate space representation. The number γ^2 in Eq. (4.2) is related to the energy denominator in the Bethe-Goldstone equation; γ^2 is given by

$$\gamma^2=\frac{P^2}{M}-\left[2V_0+\frac{P^2+K^2}{Mm^*}\right] , \tag{4.3}$$

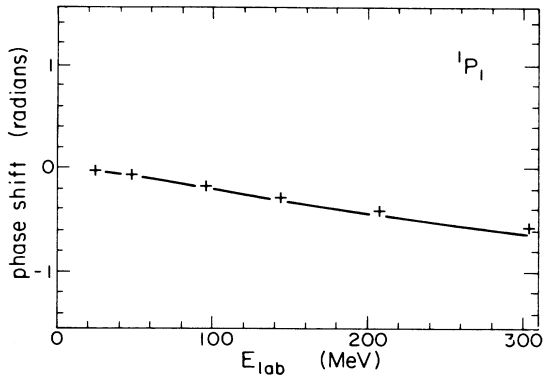

FIG. 6. $^1P_1$ phase shifts.

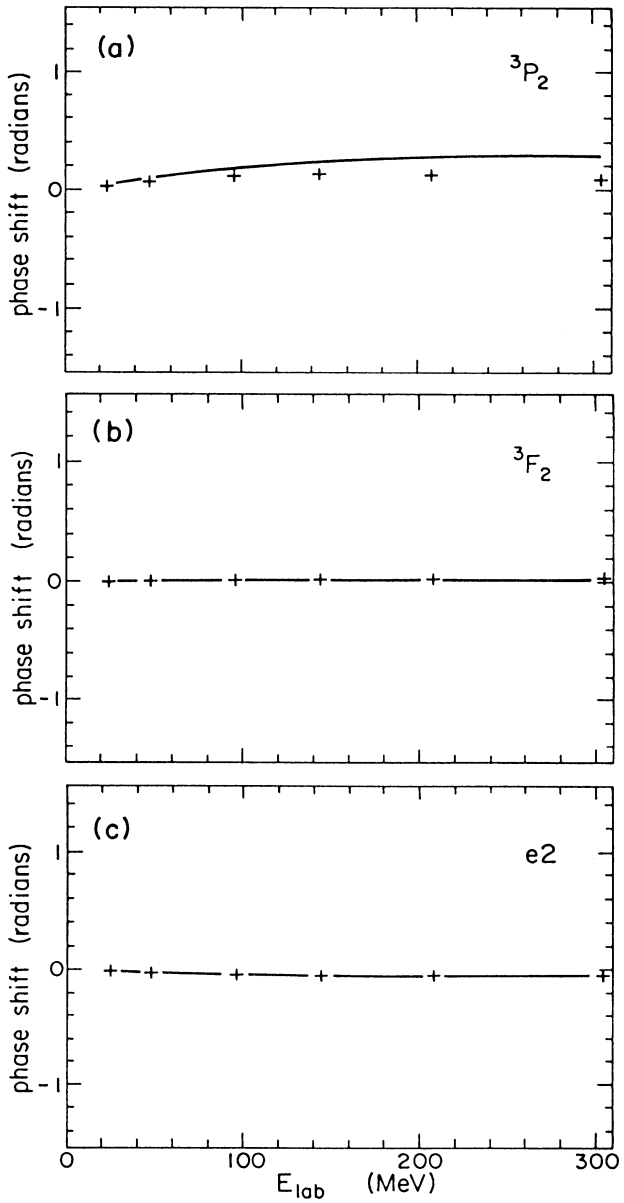

FIG. 5. (a) $^3P_2$ phase shifts. (b) $^3F_2$ phase shifts. (c) $e2$ mixing parameter.

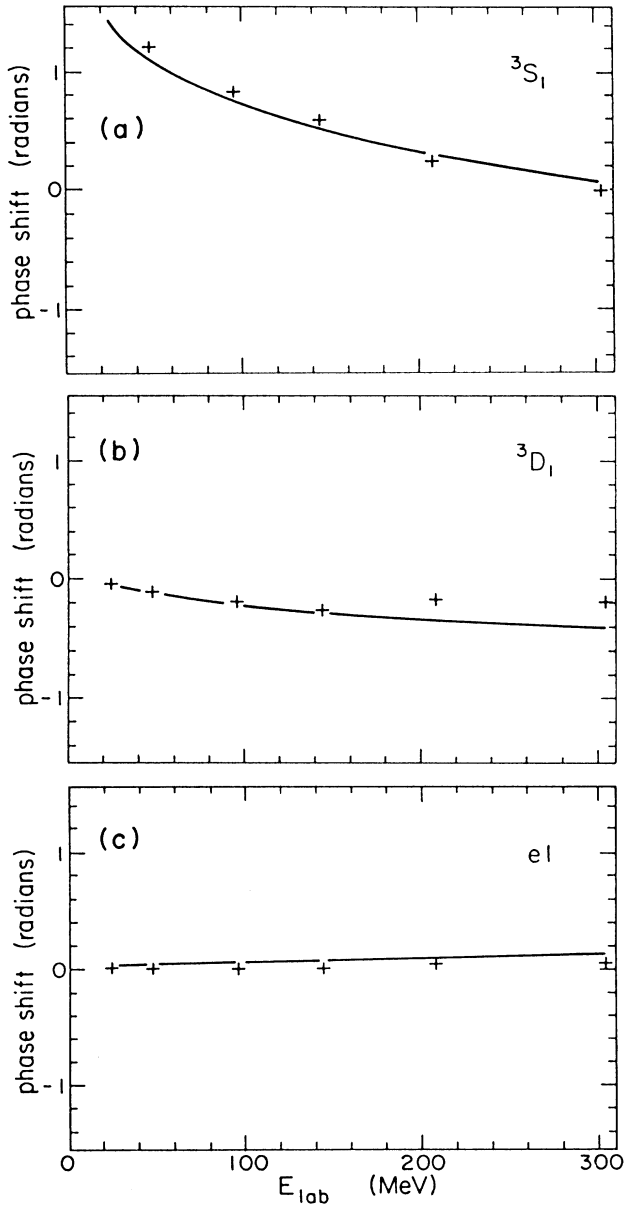

FIG. 7. (a) $^3S_1$ phase shifts. (b) $^3D_1$ phase shifts. (c) $e1$ mixing parameter.

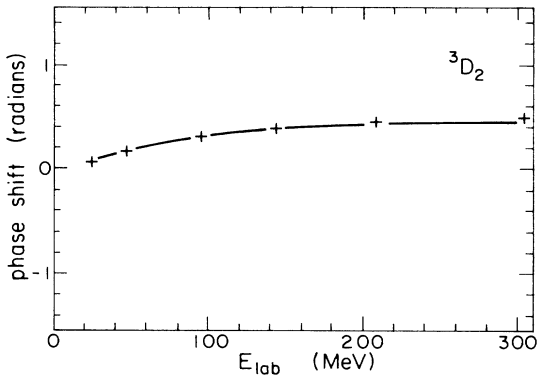

FIG. 8. $^3D_2$ phase shifts.

where the average momentum, $\mathbf{P}$, is given by $\mathbf{P}=(\mathbf{k}_k+\mathbf{k}_l)/2$, and the relative momentum, $\mathbf{K}$, is given by $\mathbf{K}=(\mathbf{k}_i-\mathbf{k}_j)/\angle$. V_0 and m^* are parameters describing the hole energy spectrum which are chosen to obtain self-consistency,

$$e_h(k_i)\equiv k_i^2/2m^*+V_0 \ .$$

Partial wave decompositions of the wave functions ψ_{kl}, ϕ_{kl}, and ξ_{kl} can be made, and the reference-spectrum equation for uncoupled partial waves is found to be

$$\left[\frac{1}{M}\left[\frac{\partial^2}{\partial r^2}-\frac{L(L+1)}{r^2}\right]-\gamma^2\right]\chi_L(r)=-V_L(r)u_L(r) \ , \tag{4.4}$$

or

$$\left[-\frac{1}{M}\left[\frac{\partial^2}{\partial r^2}-\frac{L(L+1)}{r^2}\right]+V_L+\gamma^2\right]u_L(r) = \left[\frac{K^2}{M}+\gamma^2\right]F_L(Kr) \ . \tag{4.5}$$

The functions u_L, F_L, and χ_L are, respectively, the radial wave functions of the decompositions for ψ_{kl}, ϕ_{kl}, and ξ_{kl}. These are chosen so the $F_L(Kr)=Krj_L(K_r)$ (where j_L is the regular spherical bessel function), $u_L(r)\to F_L(Kr)$ as $r\to\infty$, and $\chi_L=F_L-u_L$. The coupled partial wave equations are similar.

The contribution of an uncoupled partial wave to the G matrix can be written as

$$\langle K'JLT \mid G(P) \mid KJLT\rangle = \frac{4\pi}{KK'}\int_0^\infty dr\, F_L(K'r)V_L(r)u_L(K,P,r) \ , \tag{4.6}$$

where JLT are quantum numbers of the nucleon pair. The dependence of u_L and the G-matrix element upon the relative and average momenta K and P has been made explicit. The single-particle hole energies, $V(k_i)$, and the average binding energy, $\bar{V}$, can be calculated from the G matrix.

So far we have repeated standard lore. The present calculation proceeds by employing (4.2) only for $r>R_M$. As a result the functions $u_L(Kr)$ depend upon the boundary condition imposed at $r=R_M$. This is discussed in the next section. Even when quarks are included, we compute G matrix elements such as in Eq. (4.6) for the calculation of $V(k_i)$. However, quark degrees of freedom are used to determine the necessary integrands for $r<R_M$.

B. Quark sector

We again require that Eq. (4.2) be satisfied, only now quarks are to be used. This equation can be written as

$$[H_{\rm rel}+(T_{\rm c.m.}-e_{k\ell})]\mid\psi_{k\ell}\rangle=(T_{\rm c.m.}+T_{\rm rel}-e_{k\ell})\mid\phi_k\phi_\ell\rangle \ , \tag{4.7}$$

where $T_{\rm c.m.}$ and $T_{\rm rel}$ are, respectively, the center of mass and relative kinetic energy operators. In this case, the correlated state vector ψ_{kl} is expanded in the generator coordinate sudden approximation basis set used in the scattering problem. The free two-nucleon state $\mid\phi_k\phi_l\rangle$ is expanded in free two-nucleon basis states $\mid\boldsymbol{\alpha};\mathrm{NN}\rangle$ (states formed by taking the product of two nucleons separated by $\boldsymbol{\alpha}$). Then, after the center of mass degrees of freedom are removed, the states are given by

$$\begin{aligned}\mid\psi_{kl}\rangle&=\int d^3\alpha\mid\boldsymbol{\alpha}\rangle\phi(\boldsymbol{\alpha}) \ ,\\ \mid\phi_k\phi_l\rangle&=\int d^3\alpha\mid\boldsymbol{\alpha};\mathrm{NN}\rangle\phi^{\rm free}(\boldsymbol{\alpha}) \ .\end{aligned} \tag{4.8}$$

We require Eq. (4.7) to be satisfied to within small variations of $\mid\psi_{kl}\rangle$; thus

$$\frac{\delta}{\delta\phi^*}\langle\psi_{kl}\mid\{:H:-E_0+\gamma^2\mid\psi_{kl}\rangle -(K^2/M+\gamma^2)\mid\phi_k\phi_l\rangle\}=0 \ . \tag{4.9}$$

(E_0 is the $K=0$ effective energy discussed in Sec. II.) Thus the Griffin-Hill-Wheeler equation is replaced by

$$\int d^3\alpha\langle\boldsymbol{\beta}\mid:H:-E_0+\gamma^2\mid\boldsymbol{\alpha}\rangle\phi(\boldsymbol{\alpha}) -(K^2/M+\gamma^2)\langle\boldsymbol{\beta}\mid\boldsymbol{\alpha};\mathrm{NN}\rangle\phi^{\rm free}(\boldsymbol{\alpha})=0 \ . \tag{4.10}$$

The function $\phi^{\rm free}$ is an input to this equation and is determined by solving the Griffin-Hill-Wheeler equation in which all the interactions between the two nucleons are "turned off." As in the scattering problem, we only require Eq. (4.10) to be satisfied for $\mid\boldsymbol{\beta}\mid<R_M$. The weight-function for $\mid\boldsymbol{\beta}\mid>R_M$ is determined from the integral transform which relates it to the wave function, see Eq. (3.7). We also require the regularity condition on the weight function, Eq. (3.7). Thus, Eqs. (4.2), (4.10), (3.6), and (3.7) completely determine the correlated wave function and weight function in the reference-spectrum approximation.

To evaluate the G-matrix element, one must know the nucleon-nucleon wave function for $r<R_M$. This is obtained by projecting out the nucleon-nucleon components of the state vector. Thus, we use the free nucleon-nucleon states, $\mid\mathbf{r};\mathrm{NN}\rangle$, described above, and obtain

$$\psi_{\rm NN}(r<R_M)=\int d^3\alpha\langle \mathbf{r};{\rm NN}\,|\,\boldsymbol{\alpha}\rangle\phi(\boldsymbol{\alpha})\ . \tag{4.11}$$

The G-matrix element is determined from an overlap between $\psi_{\rm NN}$ $(r<R_M)$, the effective potential $\tilde{V}$, and the undistorted free wave function. Since we do not know $\tilde{V}$ in the quark sector, we try to eliminate it. We assume that the wave function in the quark sector satisfies a coupled-channel Schrödinger equation with completely unspecified potentials, except that any nonlocalities connecting the interior and exterior sectors are prohibited. The term with the effective potential acting on the two baryon wave function is replaced by $(\gamma^2+\nabla^2/M)\psi_{\rm NN}$. After integration by parts we find that the G-matrix element for an uncoupled partial wave is given by

$$\begin{aligned}\langle KJL\,|\,G(P)\,|\,KJL\rangle &= \frac{4\pi}{K^2}\left[\left[\frac{K^2}{M}+\gamma^2\right]\int_0^{R_M} dr\,F_L(F_L-u_{\rm NN}^{(L)})\right.\\ &\quad+\frac{1}{M}\left[F_L\frac{\partial u_{\rm NN}^{(L)}}{\partial r}-\frac{\partial F_L}{\partial r}u_{\rm NN}^{(L)}\right]\Bigg|_{r=R_M}\\ &\quad\left.+\int_{R_M}^{\infty} dr\,F_L(V_L^R u_{\rm Reid}^{(L)})\right]\ ,\end{aligned} \tag{4.12}$$

where $F_L=Krj_L(Kr)$ and the interior wave function, $u_{\rm NN}^{(L)}$, is the projection of the nucleon-nucleon component of the state vector of (4.8) determined in the quark sector for the Lth partial wave. The exterior wave function, $u_{\rm Reid}^{(L)}$, is determined using the Reid potential, V_L^R. This completely specifies the quark-nucleon reference-spectrum calculation.

Before discussing the actual calculation and results, let us examine the difference between Eqs. (4.11) and (3.6). Equation (4.11) results from projecting out the nucleon-nucleon components of the state vector. It is these components which must be used to evaluate the G matrix. Equation (3.6) is used to match the weight function with the wave function. However, in deriving Eq. (3.6) we assume that the solution of the Schrödinger equation with the Reid potential should be matched with a wave function which contains all of the components (nucleon-nucleon, delta-delta, etc.) as are in the state vector for the quark sector. Since the Reid potential is a phenomenological potential, we do not know which components we should use in the integral transform. Thus, we have projected onto our basis states to obtain Eq. (3.6). If we find that the non-nucleonic components of the state vector are large at the matching radius, then our treatment of the nucleon-nucleon sector using the Reid potential is an oversimplification. In that case, an accurate description can only be obtained by using a microscopic theory in the nucleon sector which contains explicit nucleon excitations. This point is discussed below.

C. Results

The binding energy for spin singlet states is obtained in the reference-spectrum approximation, first using only the Reid potential (i.e., $R_M=0$), and second, using our full model. The difference between these energies is taken to be the difference between our model and the full Reid calculation.[46]

There are several technical points which should be mentioned. When solving the scattering problem one first obtains two linearly independent solutions of the Schrödinger equation for $r>R_M$, $\psi^{(1)}$, and $\psi^{(2)}$. These two wave functions are used to obtain two weight functions, $\phi^{(1)}$ and $\phi^{(2)}$, by solving Eqs. (3.5) and (3.6) self-consistently. The regular solution, $\psi=\psi^{(1)}+A\psi^{(2)}$ (or $\phi=\phi^{(1)}+A\phi^{(2)}$) was obtained using Eq. (3.7), the regularity condition for the weight function.

In the nuclear matter calculation, we first obtain two linearly independent solutions, $\psi_{ij}^{(1)}$ and $\psi_{ij}^{(2)}$, of the reference-spectrum equation, Eq. (4.2), for $r>R_M$. This is done by solving Eq. (4.2) for the defect solution, with the correct boundary condition at $r\to\infty$ [namely $\zeta_{ij}(r\to\infty)=0$], both with and without the inhomogeneous term. This yields ζ_{ij}^H and ζ_{ij}^I, the homogeneous and inhomogeneous defect solutions. Two solutions are then chosen to be

$$\psi_{ij}^{(1)}=\phi_{ij}-(\zeta_{ij}^I+\zeta_{ij}^H)$$

and

$$\psi_{ij}^{(2)}=\phi_{ij}-(\zeta_{ij}^I-\zeta_{ij}^H)\ .$$

These are two independent solutions of Eq. (4.2) which obey the correct boundary condition at $r=\infty$. The general solution is given by

$$\psi_{ij}=(1-A)\psi_{ij}^{(1)}+A\psi_{ij}^{(2)}\ .$$

(For the Reid-only problem, one would determine A by requiring ψ_{ij} to be regular at the origin.) The solutions $\psi_{ij}^{(1)}$ and $\psi_{ij}^{(2)}$ are then used to find two independent weight functions by using Eqs. (4.10) and (3.6). The regularity condition, Eq. (3.7), is then used to determine A. Thus, the numerical techniques are almost identical in the nuclear matter and scattering problems.

There are several results from this calculation. The final result is the binding energy versus density. Let us discuss some intermediate results first.

In this calculation, Pauli corrections are not modified by the quarks. The motivation for this is that the reference-spectrum wave function is known to yield accurate results at short distances. For the Reid potential the wave function is excluded from the origin by the large repulsion. Since the wave function is highly distorted at short distances, it contains mostly high-momentum components. Thus, the Pauli operator has little effect on the short-distance wave function. In our calculations we have observed that the reference-spectrum wave function accurately describes the correct wave function except in the region 1 fm $<r<$ 3 fm. Thus, if one found no significant change in the shape of the wave function when quarks are included, then one could be confident that Pauli effects are negligible in the quark sector. The Pauli corrections discussed here should not be confused with quark antisymmetrization in the two-nucleon problem. Our two-nucleon states have complete antisymmetry among the six quarks. The Pauli corrections being neglected involve Pauli blocking

by other nucleons in the system—not the two interacting nucleons.

However, the wave functions found from the quark model exhibit significant qualitative changes. The $^1S_0$ wave functions from the full calculation are plotted in Fig. 9. The dashed line is the Reid potential wave function; the solid line is the wave function in the quark model which contains all of the components from the quark sector; i.e., it is found from Eq. (3.6) even for $r < R_M$. The dash-dot line is the nucleon-nucleon component of $|\psi_{kl}\rangle$ for $r < R_M$, and the dotted line represents the non-nucleonic components (the difference between the solid and dash-dot lines). The values of k_F, K, and P used here are $k_F = 1.4$ fm^{-1}, $K = \frac{3}{5} k_F$, and P is its angle averaged value, $\bar{P}$. The qualitative features seen here are typical of those for other k_F, K, and P as well.

Several features of these functions should be noticed. First, the non-nucleonic components are large even at the matching radius. This implies that non-nucleonic degrees of freedom should be explicitly treated in the nucleon sector. Second, the nucleon-nucleon wave function for $r < R_M$ is not highly distorted (as it is for the Reid potential). Thus there is a qualitative change in the wave function due to the presence of quarks. This change increases as k_F is increased, and in all cases causes a decrease in the binding contribution from the nucleon sector. Furthermore, Pauli corrections at short distance may be more important than would otherwise be expected. Third, the wave function in the nucleon sector ($r > R_M$) is changed only slightly.

It is important to understand why the nucleon-nucleon and non-nucleonic components of the wave function appear as they do here. It is known that the nucleon-nucleon component of a spherical six-quark bag is small; i.e., $\langle \alpha = 0; \mathrm{NN} \mid \alpha = 0 \rangle \ll \langle \alpha = 0 \mid \alpha = 0 \rangle$. Also, one can show that the nucleon-nucleon components of our deformed basis states at 1 fm separation are predominantly nucleon-nucleon states; i.e.,

$$\langle \alpha; \mathrm{NN} \mid \alpha \rangle \mid_{\alpha = 1 \text{ fm}} \sim \langle \alpha \mid \alpha \rangle \mid_{\alpha = 1 \text{ fm}} .$$

One might then expect that the nucleon-nucleon components of the wave function at $r = 0$ fm would be small, and also that the non-nucleonic components at $r \sim 1$ fm would be small. In Fig. 9 this is shown to be false. The reason for this discrepancy is that the transformation of the weight function to the wave function is *nonlocal.*

The approximate relation

$$\langle \alpha; \mathrm{NN} \mid \alpha \rangle \mid_{\alpha = 1 \text{ fm}} \sim \langle \alpha \mid \alpha \rangle \mid_{\alpha = 1 \text{fm}}$$

was a motivation for our choice of the integral transform, Eq. (3.6). However, we now see that the nonlocality of the integral transform produces large non-nucleonic components in the wave function even for $r \sim 1$ fm (and, similarly, produces large nucleon-nucleon components for $r \sim 0$ fm).

The qualitative change in the nucleon-nucleon wave function at short distances results in a change in the G matrix. Three terms contribute to the G matrix, see Eq. (4.12): the interior term, the surface term, and the exterior term. The interior contribution is repulsive and depends on the size of the defect wave function. One can see that for $r < 0.5$ fm the defect wave function is decreased by quark effects (because the wave function is larger and, hence, closer to the free wave function), so that there is less repulsion. However, for 0.5 fm $< r < R_M$ the defect wave function is increased and one obtains more repulsion. Since the defect wave function is multiplied by $\sin(Kr)$ (for $L = 0$ states), the region 0.5 fm $< r < R_M$ is weighted more than the region $r < 0.5$ fm. Thus, the overall repulsion from the interior is increased.

For a matching radius of $R_M \approx 1$ fm the surface contribution is repulsive as well. This is because the slope of the correlated wave function is larger (and the magnitude of the wave function is smaller) than that for the free wave function. There are two partially canceling effects from the quarks in the surface contribution. The nucleon-nucleon wave function at the matching radius is reduced; this increases the G matrix. However, the slope of the wave function is decreased; this decreases the G matrix. It turns out that the entire surface contribution is less repulsive.

The exterior contribution to the G matrix is always attractive for the $^1S_0$ channel. The quark dynamics always tend to decrease the wave function in the nucleon sector; thus, the attraction is decreased. This decrease in attraction is larger at high densities. This effect is due to a change in the off-shell T matrix and should not be very sensitive to the interpretation of the wave function in the nucleon sector.

Typical numbers for a $^1S_0$ G-matrix element are the following: Without quarks "interior" $= 0.27$ fm^2, "surface" $= 1.06$ fm^2, "exterior" $= -3.30$ fm^2, "total" $= -1.97$ fm^2. With quarks one has "interior" $= 0.70$ fm^2, "surface" $= 0.59$ fm^2, "exterior" $= -3.13$ fm^2, "total" $= -1.84$ fm^2. These are the values for the same k_F, K, and P used for Fig. 9. One sees here all the effects mentioned above.

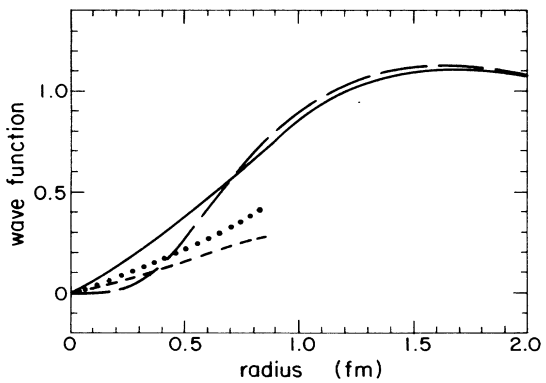

FIG. 9. Nuclear matter wave functions with quarks. The solid line represents the full quark calculation wave function, including all components. The long dashed line represents the Reid potential wave function. The dotted line represents the nucleon-nucleon components of the quark calculation wave function, and the short dashed line represents the non-nucleonic components. Here, $k_F = 1.4$ fm^{-1}, $K = \frac{3}{5} k_F$, and $P = \bar{p}$.

The density dependence of the interior, surface, and exterior contributions are not the same. Specifically, the decrease in repulsion due to changes in the surface contribution dominates at small densities and the increases in repulsion from the interior and exterior dominate at large densities. The plot of binding energy versus Fermi momentum is shown in Fig. 10. The solid line represents the Reid-only calculation of Siemens;[46] the plotted points (□) and error bars represent our calculation; the dotted line is drawn to guide the eye; the arrow and large circle show the shift and uncertainty due to three-body and four-body cluster contributions as estimated by Day[47] for the Reid potential. The single point with error bars in k_F and E/A represents the empirical region.[47] The error bars on our points represent numerical errors as estimated by comparing our Reid-only reference-spectrum calculation with the exact Reid calculation in the independent-pair model.[46] These error bars *do not* represent uncertainties due to model assumptions.

Figure 10 shows that the saturation point is shifted to a smaller binding energy and smaller density. This is qualitatively similar to the results obtained by including the delta resonance.

There are several corrections or improvements which could be made to our results. The most important one would be to use a microscopic theory in the nucleon sector. The problems with our assumption concerning the components of the Reid wave function have been pointed out above, see Fig. 10. It is not known whether this improvement would change the qualitative features of our results. One would expect that including deltas in the nucleon sector would shift our curve upward toward smaller values of the binding energy. This would not be desired since it might move our saturation point back into the saturation band. A second important improvement would be to calculate the three-body cluster contribution. Wiringa *et al.*[31] have estimated that three-body cluster contributions may lead to collapse in some models of nucleons and deltas in nuclear matter which do not have a large repulsive core. Three-body cluster calculations in a model with composite particles would be a formidable task.

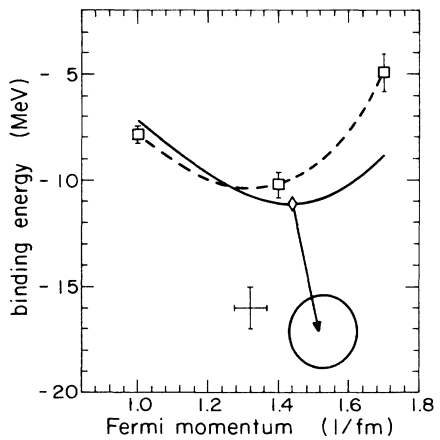

FIG. 10. Quark effects on nuclear matter saturation. The solid line represents the Reid potential saturation curve. The plotted points (□) represent the binding energies calculated with the quark model. The arrow represents the shift in the Reid potential saturation point due to three- and four-body clusters. The empirical values are indicated by the large error barred region; other features are described in the text.

Our quark model gives a small decrease in the binding energy and a large decrease in the density in a reference-spectrum approximation calculation of the saturation properties of nuclear matter. This is a desirable change; however, our results indicate that non-nucleonic components for $r > 1$ fm should be included as well. The decrease in binding energy is, in part, due to projecting out the nucleon-nucleon components of the wave function for $r < R_M$, thereby increasing the defect wave function. This is similar to repulsion found from delta models. The nucleon-nucleon wave functions obtained for $r < R_M$ show a qualitative difference from the Reid-only calculation. Such differences are not seen in calculations using delta models[48] and are due to the nonlocal transformation which relates the quark sector weight function to the wave function. This new effect may be responsible for the unusual density dependence of the binding energy which moves the saturation point slightly off the saturation band in the right direction. Inclusion of deltas in the region with $r > R_M$ could move the saturation point back into the saturation band.

V. SUMMARY

This study examines quark effects in the two-nucleon and many-body systems. Quark dynamics produces the short-range part of the nuclear force via the soliton bag model and the generator coordinates method. For simplicity, the Reid potential has been used for the long-range force. In Sec. III it is shown that our model describes the nucleon-nucleon scattering data in the spin singlet channels. The matching radius (the separation at which the quark and nucleon sectors are matched), the truncation of the generator coordinate basis set, and the value of the one-gluon exchange coupling constant are the adjusted parameters. The best value for the matching radius is about $\frac{5}{6}$ fm, consistent with earlier estimates.[41] Since an adequate description of spin singlet scattering data was obtained with a single basis set and choice of parameters, the model was applied to the many-body problem.

The application of our model to the many-body system leads to new results and new questions. The saturation density and binding energy of nuclear matter are determined by using our model in a reference-spectrum calculation. Quark effects cause a large decrease in the saturation density and a small decrease in the binding energy. More importantly, the non-nucleonic components of the two-body correlated wave function are calculated from the quark configurations and are found to be large. This indicates a deficiency in our treatment of the nucleon sector. One should include non-nucleonic components of the wave function and use a microscopic theory for the nuclear force in the long-range ($r > 1$ fm)

region. One should also improve the calculations of Pauli blocking effects and self-consistency corrections.

It should be emphasized that there is a difference between using quark and nucleonic (nucleons, deltas, etc.) degrees of freedom to describe the short-distance physics. This difference arises because in any calculation one truncates the basis set. For example, in our quark calculation we have truncated our basis set for the quarks to a small number of states. These quark configurations could be expanded in terms of baryonic degrees of freedom, but would require a very large basis (more than just nucleons and deltas, in fact, even color octet-octet states). Conversely, if one expanded simple nucleonic two-body states with a set of properly antisymmetrized quark states, then one would require a large basis of quark states. Thus, there are real differences between the two descriptions and many-body calculations should help distinguish the two.

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