Vibrational modes at finite temperature: Restoring force and mass parameters for multipole excitations

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Temperature-dependent effects upon vibrational states are calculated within a finite temperature random phase approximation. Restoring force and mass parameters, calculated for a multipole-multipole interaction, are compared with liquid drop model values at nonzero temperature. Collective contributions to the level density parameter are discussed.

I. INTRODUCTION

The study of thermal properties of finite nuclear systems has been the subject of recent publications.[1-9] Different theoretical approaches have been proposed in connection with this problem, and among them the functional integral representation of the nuclear many body grand partition function of Kerman and Troudet[5] allows for the study of nuclear properties at finite temperatures, $T\neq 0$, in terms of a field theory. In this model[5] field operators are generators of effective degrees of freedom, namely, temperature dependent fermionic and bosonic excitations. Temperature dependent fermionic degrees of freedom have been reported in connection with mean field and quasiparticle excitations,[1,3,4,8] and temperature dependent bosonic excitations have been studied, within the framework of the finite temperature random phase approximation (FTRPA), for particle-hole[7] and particle-particle[9] channels. Based on general results of Ref. 5 for vibrational contributions to the nuclear grand partition function, we can evaluate FTRPA terms associated with a separable, multipole-multipole, two body interaction.

In this paper we are going to discuss the behavior of the restoring force and mass parameters which can be extracted from FTRPA contributions, particularly for particle-hole channels. We have followed the procedure of Ref. 10, extended to finite temperatures within the FTRPA. The aim of this description is closely related to liquid drop model (LDM), parametrizations[10] at $T\neq 0$, since collective contributions to LDM terms can be calculated from the FTRPA treatment of multipole vibrations. We have calculated these contributions for the case of quadrupole and octupole vibrations in a heavy nucleus. We are also going to present some results concerning finite temperature effects produced by collective vibrations upon bosonic contributions to the excitation energy.[5]

The formalism is presented in Sec. II and results, for the case of the nucleus ^{208}Pb, are discussed in Sec. III. Some conclusions are drawn in Sec. IV.

II. FORMALISM

In this section we are going to describe the formalism, which is based on the FTRPA treatment of residual multipole-multipole interactions. The model Hamiltonian reads[14]

$$H=\sum_{j,t_z(n,p)}\epsilon_j(t_z)a_j^{+}(t_z)a_j(t_z) -\tfrac{1}{2}\sum_{\lambda\mu,t_z t_{z'}}\kappa_\lambda(t_z,t_{z'})Q^{+}_{\lambda\mu}(t_z)Q_{\lambda\mu}(t_{z'}) , \quad (1)$$

where we have used standard notation[14] both for the single particle and multipole-multipole terms. This Hamiltonian (1) can be linearized in a phonon basis with eigenstates denoted by

$$|\lambda\mu,w_i\rangle=\Gamma^{+}_{\lambda\mu}(w_i)|0\rangle , \quad (2)$$

and with $\Gamma^{+}_{\lambda\mu}(w_i)$ defined by

$$\Gamma^{+}_{\lambda\mu}(w_i)=\sum_k X^i_\lambda(k)A^{+}_{\lambda\mu}(k)-Y^i_\lambda A_{\bar{\lambda}\mu}(k) . \quad (3)$$

Forward and backward going amplitudes, X^i_λ and Y^i_λ, respectively, obey normalization conditions given by

$$\sum_k f(k,T)\{[X^i_\lambda(k)]^2-[Y^i_\lambda(k)]^2\}=1 . \quad (4)$$

In Eqs. (1)–(4) $\lambda(\mu)$ represent angular momentum (projection) associated with the multipole field; the index k stands for configurations of pairs of fermions, which at $T=0$ are particle-hole pairs and at $T\neq 0$ are particle-particle, particle-hole, and hole-hole pairs, coupled to λ, and $f(k,T)$ are temperature dependent occupation numbers defined by

$$f(k,T)=n_{k_1}(T)-n_{k_2}T) , \quad (5)$$

where $n_{k_i}(T)$ are temperature dependent Fermi-Dirac occupation numbers. The FTRPA treatment of (1) allows for the definition of $X^i_\lambda(k)$ and $Y^i_\lambda(k)$, and by w_i we are denoting the eigenvalues of the determinant[7]

$$\mathrm{Det}\begin{vmatrix} 1-\kappa_\lambda(n,n)\Theta(w_i,n) & \kappa_\lambda(n,p)\Theta(w_i,n) \\ \kappa_\lambda(n,p)\Theta(w_i,p) & 1-\kappa_\lambda(p,p)\Theta(w_i,p)\end{vmatrix}=0 , \quad (6)$$

where

$$\Theta(w_i,t_z)=\sum_k^{(t_z)}\frac{2E_k(t_z)f(k,T)}{[E_k(t_z)^2-w_i^2]}|q_\lambda(k,t_z)|^2 , \quad (7)$$

for each multipolarity λ. In Eq. (7) t_z stands for neutron and proton configurations; $q_\lambda(k,t_z)$ are reduced matrix elements of the multipole operators $Q_{\lambda\mu}$, and $E_k(t_z)$ are unperturbed energies for pairs of fermions.

With the above defined quantities we can evaluate energy weighted sum rules (EWSR) and transition probabilities $B(E\lambda,i)$ for each multipolarity λ. Therefore we can define[10] a restoring force parameter C^i_λ and a mass parameter D^i_λ, namely

$$C^i_{\lambda^\pi=2^+}=235.281\langle r\rangle^2_{(T)}w_i/B_i(E2)\ (\text{MeV})\ ,$$
$$C^i_{\lambda^\pi=3^-}=10.475\langle r^2\rangle^2_{(T)}w_i/B_i(E3)\ (\text{MeV})\ , \tag{8}$$
$$D^i_\lambda=C^i_\lambda/(w_i)^2\ (\text{MeV}^{-1})\ ,$$

respectively; $\langle r^n\rangle_{(T)}$ is the expectation value

$$\langle r^n\rangle_{(T)}=\text{Tr}(\rho r^n)/\text{Tr}(\rho)\ , \tag{9}$$

defined in terms of the single particle density matrix ρ. In addition, we can also define hydrodynamical values associated with vibrations of a charge liquid drop.[10] Surface vibrations predicted by LDM (Ref. 10) can be compared with FTRPA modes provided a temperature Coulomb term

$$C^{\text{Coul}}_\lambda=-(3/2\pi)Z^2e^2(\lambda-1)/R_c(T)(2\lambda+1)\ (\text{MeV})\ , \tag{10}$$

where $R_c(T)$ is the corresponding Coulomb radius

$$1/R_c(T)=\frac{5}{6}\frac{\int d\tau_1 d\tau_2\dfrac{\rho(r_1,T)\rho(r_2,T)}{|r_1-r_2|}}{\left[\int d\tau\rho(r,T)\right]^2}\ , \tag{11}$$

is added to the surface one,

$$C^{\text{surf}}_\lambda=(\lambda-1)(\lambda+2)R_0^2\gamma\ . \tag{12}$$

In Eq. (11) we have introduced the charge density[5]

$$\rho(r,T)=\sum_j^{(t_z=p)} n_j(T)\,|\,\phi_j(r)\,|^2\ . \tag{13}$$

In addition to fragmentation and energy splittings, which are produced by the residual multipole-multipole interactions upon unperturbed configurations, finite temperature effects would eventually lead to changes in energies and intensities of the vibrational modes. These effects will be reflected by a temperature dependence of LDM parameters, since collective contributions to them are obtained from the mass, restoring force, and radius parameters given by FTRPA calculations.

III. RESULTS AND DISCUSSIONS

In this section we are going to show the results of FTRPA calculations of quadrupole and octupole vibrations in ^{208}Pb. For the single particle basis we have adopted a Nilsson's parametrization[12] which provides a set of proton and neutron single particle energies at $T=0$. Based on the fact that self-consistent, temperature dependent Hartree Fock calculations[13] have been found to be nearly independent of T, we shall assume that the adopted set of single particle states remains unchanged in the temperature range $0\le T\le 2$ MeV. This representation of the single particle mean field, by a Nilsson's parametrization, should not be considered as a serious limitation of the present calculations, which are aimed at the analysis of bulk features of T dependent vibrational modes. With this single particle basis we have exhausted, at $T=0$, unperturbed EWSR for quadrupole and octupole particle hole configurations. They coincide, within 5% accuracy, with the corresponding mass and electric EWSR.[10] We have calculated them by performing the corresponding radial integrals, with the $T=0$ limit of the T dependent radial density.[13] The same procedure has been used for $T\neq 0$. The coupling constants $\kappa_\lambda(t_z,t_{z'})$, for $\lambda^\pi=2^+$ and 3^- channels of the residual interaction, have been adjusted in order to reproduce the observed energies of known low lying 2^+ and 3^- states in ^{208}Pb. The results for isoscalar and isovector states with the largest contributions to EWSR, at $T=0$, are shown in Table I. FTRPA results, for the same states, are shown in Table II. These values are consistent with those of Ref. 7 concerning the stability of FTRPA solutions. In fact, the energies and intensities of high lying states remain unaffected by T, although low lying portions of each spectrum are appreciably affected by thermal blocking. The corresponding restoring force and mass parameters, for states which have the largest percentages of EWSR, are shown in Table III. Shifts in the position of giant resonances are evident from these results. In the same Table III surface and

TABLE I. RPA solutions for $\lambda^\pi=2^+$ and 3^- states in ^{208}Pb, at $T=0$. Energy centroids $\bar{E}$ are shown in the second column. The third and fourth columns correspond to percentages of isoscalar f ($\tau=0$) and isovector $f(\tau=1)$ EWSR. The last two columns show electric and mass multipole moments, $B(E\lambda)_{\text{electric}}$ and $B(E\lambda)_{\text{mass}}$, respectively (Ref. 16). Single particle unit is denoted by s.p.u.

λ^π	$\bar{E}$ (MeV)	$f(\tau=0)$	$f(\tau=1)$	$B(E\lambda)_{\text{electric}}$ (s.p.u.)	$B(E\lambda)_{\text{mass}}$ (s.p.u.)
2^+	4.130	16.4	0.3	10.62	29.30
	8.015	51.4	0.3	19.91	46.58
	22.760	6.3	85.5	7.60	29.13
3^-	2.610	13.0	0.2	27.91	74.22
	16.780	15.5	0.1	5.87	13.65
	29.190	9.2	64.5	6.44	37.05

TABLE II. FTRPA solutions for $\lambda^\pi=2^+$ and 3^- modes, for $T=2$ MeV. The meaning of the values shown in columns is the same as Table I.

λ^π	$\bar{E}$ (MeV)	$f(\tau=0)$	$f(\tau=1)$	$B(E\lambda)_{\text{electric}}$ (s.p.u.)	$B(E\lambda)_{\text{mass}}$ (s.p.u.)
2^+	4.570	4.3	0.2	2.10	6.67
	8.380	46.6	0.2	16.53	38.30
	22.500	6.5	80.6	6.65	26.48
3^-	3.330	8.2	0.1	12.84	34.13
	16.820	14.7	0.2	5.26	12.09
	28.790	8.3	52.2	4.60	28.55

Coulomb contributions for LDM vibrations are shown, as a function of T. The smooth temperature dependence of these LDM values has to be attributed, mainly, to small changes in the temperature dependent surface and Coulomb radii. In the case of the surface term we have adopted a T dependent surface tension, $\gamma=\gamma\ (T=0)(1-\alpha T^2)$ with $\alpha=0.036$ MeV^{-2}. This value has been obtained from self-consistent, T dependent, Hartree-Fock plus Strutinsky shell corrections calculations of medium-heavy systems.[15] Therefore, the temperature dependence of C_λ^{surf} would reflect a competition between an increase of the radius parameter and a decrease of the surface tension. In the case of LDM vibrational modes, a smooth variation of C_λ^{surf} with T is observed, meaning that both effects are comparable, while for C_λ^i values obtained from FTRPA collective vibrations, they appear to be strongly dependent upon T. In this respect we can interpret an increase in C_λ^i as a consequence of the increase in radial expectation values and of a decrease in transition probability values at finite T. It means that for FTRPA vibrations the temperature dependence of the radius parameter dominates over the temperature dependence of surface tension coefficients.

Contributions to the excitation energy, as a function of T, are shown in Fig. 1. The results of Fig. 1 have been calculated by following standard techniques of thermal averages,[5] namely,

$$E^*_{\text{RPA}}=\frac{\text{Tr}(\rho^{\text{RPA}}H^{\text{RPA}})}{\text{Tr}(\rho^{\text{RPA}})}\ , \tag{14}$$

where the occupation numbers for bosonic degrees of freedom were calculated from FTRPA energies:

$$n_\lambda(w_i,T)=\frac{1}{e^{w_i/T}-1}\ . \tag{15}$$

In Fig. 1 we have also shown the behavior of the associated collective level density parameter. The FTRPA contributions to the level density parameter[5,11] display a peak, a feature which has also been observed for the case of finite temperature single particle excitations. We have observed that the appearance of this peak is strongly dependent upon thermal blocking of $\Delta N=0$ and $\Delta N=1$ transitions, for $\lambda^\pi=2^+$ and 3^-, respectively. This effect is a purely microscopic one, since, for an equivalent photon gas with temperature independent frequencies, one should expect to obtain a constant value for the level density parameter. Naturally the same feature is observed in FTRPA contributions to the specific heat, where dominant effects are produced by $\Delta N=0$ configurations for $\lambda^\pi=2^+$ multipole vibrations. While $\Delta N=0$ configurations for $\lambda^\pi=2^+$ produce a peak in the specific heat, at the same temperature where the peak of the collective level density parameter is obtained, contributions to it from collective $\lambda^\pi=3^-$ vibrations saturate.

TABLE III. Restoring force C_λ^i and mass D_λ^i parameters for $\lambda^\pi=2^+$ and 3^- vibrations. The states included in this table are the same as Tables I and II. LDM values, $C_\lambda^{\text{LDM}}=C_\lambda^{\text{surf}}+C_\lambda^{\text{coul}}$, are shown in last column.

λ^π	T (MeV)	$\bar{E}$ (MeV)	C_λ^i (MeV)$\times 10^{-3}$	D_λ^i (MeV^{-1})$\times 10^{-2}$	C_λ^{LDM} (MeV)$\times 10^{-2}$ surface	Coulomb
2^+	0	4.130	0.94	0.55	1.90	−1.35
		8.015	1.14	0.18		
		22.760	5.20	0.10		
	2	4.570	4.61	2.20	1.68	−1.33
		8.380	1.47	0.21		
		22.500	5.72	0.11		
3^-	0	2.610	0.36	0.53	4.77	−1.93
		15.460	11.62	0.48		
		29.190	8.08	0.09		
	2	3.330	1.03	0.93	4.20	−1.91
		16.820	14.76	0.52		
		28.790	10.70	0.13		

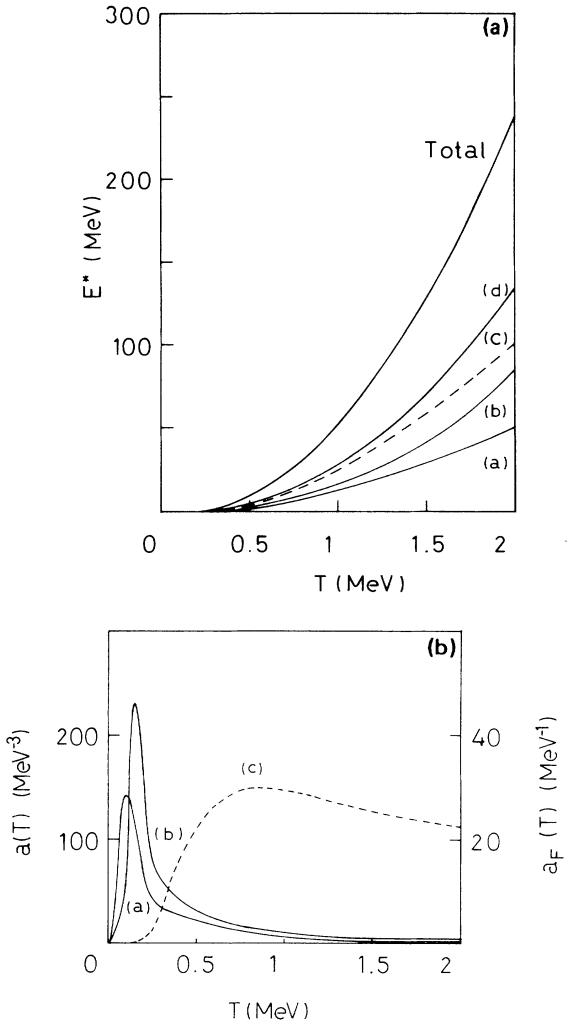

FIG. 1. (a) Fermionic and vibrational contributions to the excitation energy. The values are shown as a function of T. Curves indicated by (a) and (b) correspond to multipole fields $\lambda^{\pi}=2^{+}$ and 3^{-}, while (c) and (d) denote fermionic and bosonic $(E^{*}_{2^{+}}+E^{*}_{3^{-}})$ results, respectively. (b) Fermion and vibrational contributions to the level density parameter, as a function of T. Contributions for $\lambda^{\pi}=2^{+}$ and 3^{-} vibrational modes are denoted by (a) and (b), respectively, and their magnitudes are given by the left-hand side scale. Fermionic contributions are indicated by (c) with values given by the right-hand side scale.

It should also be noted that, as a main difference with respect to pairing vibrations at finite T (Ref. 9), FTRPA contributions to the total specific heat are larger than the fermionic ones.

The results which are shown in Fig. 1(b), where independent particle contributions to the level density parameter are compared to FTRPA ones, indicate that vibrational and single particle degrees of freedom determine, separately, the temperature dependence of the level density parameter. At low T values, $T<0.5$ MeV, vibrational contributions to $a(T)$ are dominant, while for $T>0.5$ MeV, the resulting value of $a(T)$ is given by the independent particle contribution. This result could be understood in terms of the energy scale associated to each mode. For a heavy nucleus like ^{208}Pb, vibrational modes occur at energies which are lower than the shell spacing, and they would be thermally blocked at low temperatures, as compared with single particle excitations. This conclusion should not necessarily be valid for light nuclei, like ^{40}Ca, where vibrational excitations are observed at higher energies and, consequently, higher temperatures are needed in order to produce significant changes in vibrational properties.[2] In addition, since we are not dealing with a self-consistent description of the mean field and collective excitations, our results are certainly affected by the limitations posed by the above mentioned approximations. Nevertheless, we think that the trend which can be extracted from our results would eventually persist in a more refined treatment of vibrational and single particle motions at finite temperature.

IV. CONCLUSIONS

We have reported some results concerning the temperature dependence of vibrational modes based on the FTRPA treatment of multipole-multipole interactions. While energies and intensities of giant isoscalar and isovector resonances remain almost unaffected by thermal blocking, associated values for low lying states show a more evident dependence on T. With calculated values for energies and intensities, for $\lambda^{\pi}=2^{+},3^{-}$ vibrations, we have extracted restoring force and mass parameters for isoscalar and isovector excitations and we have compared their values with LDM results, also obtained at finite T. We have found that FTRPA results are more dependent upon T than LDM ones. We have also observed a peak in the collective contributions to the level density parameter. The appearance of this peak could be interpreted in terms of the thermal blocking of $\Delta N=0$ and $\Delta N=1$ transitions. We can conclude by saying that, in order to obtain reliable descriptions of the behavior of nuclear level densities at finite T, one has to include T dependent collective degrees of freedom in the analysis of nuclear excitations at finite T.

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[16]These values have been obtained by collecting FTRPA contributions over energy intervals around dominant states. This procedure allows for the analysis of $T \neq 0$ results, particularly in view of the very large number of FTRPA roots.