PHYSICAL REVIEW C VOLUME 36, NUMBER 6 DECEMBER 1987

Effective hadron theories from a quark model

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(Received 19 June 1987)

In the context of the quark exchange model of Lenz *et al.*, effective hadron theories are constructed and the resulting physical observables compared to their "experimental" values as defined by the calculations using the exact quark model operators. This model study illustrates convergence issues for an effective hadron basis in a quark model in which all physical observables can be computed exactly.

I. INTRODUCTION

The quark exchange model of Lenz *et al.*[1] has confining, saturating forces and provides an unambiguous model context in which the convergence of effective hadron theories can be examined. Traditionally, such theories have been successful in describing long range, low energy nuclear observables with a small number of hadronic degrees of freedom. However, their success in describing short range phenomena has not been firmly established, particularly since there are a large number of variable parameters, such as the meson-baryon coupling constants and form factors. The quark model to be used here as the basis of a model study exhibits the qualitative features of low energy nuclear phenomenology and can be solved exactly. It is desirable to confront convergence issues in the context of such an exact model, as the presumably correct underlying theory, quantum chromodynamics (QCD), is yet intractable at nucleonic distance scales. The large N_c limit of QCD strengthens the notion that, at low energies, QCD must be equivalent to some meson theory.[2,3] However, a picture of nucleons and mesons as extended composite objects makes it difficult to understand the success of the traditional nuclear phenomenology and makes one question the efficacy of a meson theory at extremely small distance scales. We discuss here the methodology for "hadronizing" the quark model and focus upon application of the resulting theory to the bound state form factor at large momentum transfer, a direct measure of short-distance structure.

The model used is a many body potential model with quark exchange dynamics, the minimal dynamics consistent with confinement. The quark exchange model is a dynamical extension to the multihadron regime of nonrelativistic potential quark models, which generally have had substantial success in quantitatively reproducing static hadronic properties such as masses and decay widths.[4] It is based upon a partitioning of color and configuration space at the hadrons's quark rearrangement surface, such that only color singlet hadron configurations are permitted outside the interaction region. In addition, unlike the case of two body potential models, confinement in this model can be enforced without the usual long range van der Waals forces, which empirical evidence has shown to be nonphysical.[5,6] As noted already, the model yields a surprisingly rich set of phenomena.[1] In the specific example of a $q^2\bar{q}^2$ system, which is technically simplest and yet contains the qualitative features of more complicated systems, one finds that the scattering from the system exhibits resonances at the inelastic hadronic thresholds and that a weak deuteronlike bound state exists with a binding energy of only a few percent of the hadron excitation energy scale.

The calculations which follow have been performed for the $q^2\bar{q}^2$ system in the limit of U(1) color. The quark level scattering and bound state properties of this system have been examined in great detail in Ref. 1. In this many-body model, the SU(N_c) interactions are not specified by confinement, and a parameter specifying the relative strength of color nonsinglet and color singlet confining forces is thus needed. The U(1) limit contains only the confining dynamics determined from isolated hadron spectroscopy and, most important, is shown in Ref. 1 to display all the qualitative features seen for $N_c > 1$ when the color nonsinglet forces are chosen "reasonably." Consequently, we have every expectation that this simple model is rich enough to answer the most important questions and defer to a later and more extensive report results for $N_c = 3$.

II. QUARK MODEL CALCULATIONS

We consider a $q^2\bar{q}^2$ system in the quark exchange model of Lenz *et al.* with harmonic confinement. The $\bar{q}$'s are not required to be antiquarks but merely objects distinguishable from quarks. Labeling the position of quarks 1,2 and antiquarks 3,4 with $\mathbf{r}_i$, the quark Hamiltonian is

$$h(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) = -\frac{1}{2\mu}\sum_{i=1}^{4}\Delta_i + \tfrac{1}{2}\mu\omega^2 \min\{r_{13}^2+r_{24}^2, r_{14}^2+r_{23}^2\} , \qquad (1)$$

where $\mu = m_q/2$, $\mathbf{r}_{ij} \equiv \mathbf{r}_i - \mathbf{r}_j$. The Hamiltonian is confining and saturating by ansatz as the quarks and antiquarks are always paired in the lowest potential energy hadronic state. A two-dimensional reduction can be effected by introducing center of mass and relative coordinates:

$$\mathbf{r}_{\rm c.m.}=\tfrac{1}{4}(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4)\ , \tag{2a}$$

$$\mathbf{x}=\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)-\tfrac{1}{2}(\mathbf{r}_3+\mathbf{r}_4)\ , \tag{2b}$$

$$\mathbf{y}=\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_3)-\tfrac{1}{2}(\mathbf{r}_2+\mathbf{r}_4)\ , \tag{2c}$$

$$\mathbf{z}=\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_4)-\tfrac{1}{2}(\mathbf{r}_2+\mathbf{r}_3)\ . \tag{2d}$$

As a consequence of the harmonic confinement, the dependence on the confined variable $\mathbf{x}$ factorizes, and the y,z orbital angular momenta decouple. One expects the effect of quark exchange dynamics to be maximized for s waves; and since that channel contains a bound state, we consider only the y,z channel Hamiltonian for relative angular momentum $l=0$ and obtain

$$h(y,z)=-\partial_y^2-\partial_z^2+4[z^2\theta(y-z)+y^2\theta(z-y)] \tag{3}$$

in units of $2m_q=1$, $b_0^2=(\mu\omega)^{-1}=1$. Solutions to Eq. (3) can be classified with respect to their symmetry under the spatial exchange of y,z, as the internal degrees of freedom have been neglected. Consideration can thus be restricted to the $y\geq z$ triangle in the $y-z$ scattering plane. The symmetric state satisfies $(\partial_y-\partial_z)\psi(y,z)=0$ at the rearrangement surface $y=z$ and has a bound state; this is the state whose properties shall be considered. Using functions which satisfy the $y>z$ Hamiltonian, one can use Green's identity to find an integral equation for $\psi(y,z)$. The scattering phase shifts, as well as the bound state energy and form factor, can be calculated; see Ref. 1 for details. The bound state form factor is

$$F(q)=2e^{-q^2/32}\int_0^\infty dy\int_0^y dz\, j_0(qy/2)j_0(qz/2)\psi_B^2(y,z) \tag{4}$$

where $\psi_B(y,z)$ is the bound state wave function generated by the Hamiltonian of Eq. (3) and the decoupled x oscillator is in its ground state. This form factor generated by the quark model will serve as the reference point for the "equivalent" hadron theories derived below.

III. HADRONIZATION

One now wants to construct an equivalent model in a hadronic basis. This procedure necessarily involves identifying and projecting out the hadron internal degrees of freedom and thus is necessarily nonunique. This follows as no unique definition of a hadron exists at distances less than the interhadron separation and as one cannot identify which hadron contains which quark. Equivalently, no unique prescription exists for the channel variable, which specifies the hadron relative separation. The procedure to be illustrated will work for any quark model operator, though the charge operator will be used as an example. The charge form factor can be written formally as

$$F(q)=\langle\psi_B\,|\,\hat\rho_q\,|\,\psi_B\rangle \tag{5}$$

where $\hat\rho_q$ is the quark model charge operator. Any projector which satisfies $\sum_n P_n^2=1$, $P_nP_m=\delta_{nm}P_n$ yields

$$F(q)=\sum_{n,m}\langle\psi_B\,|\,P_n(P_n\hat\rho_qP_n)P_n\,|\,\psi_B\rangle\ . \tag{6}$$

Thus, $P_m\,|\,\psi_B\rangle$ defines an effective hadronic wave function $|\,\chi_m\rangle$ and $P_n\hat\rho_qP_m$ an effective hadronic charge operator $[\hat\rho_q]_{nm}$, where the m subscript corresponds to a hadron in its mth internal state. Self-consistency of the hadronic operator and wave function expansion is implicit. (One could consider different expansions for the operators and wave functions, using projectors P and $\tilde P$, say, but this is unnatural as $\langle P_n\,|\,\tilde P_m\rangle\neq\delta_{nm}$ in general.) The calculation is also self-consistent in that inclusion of higher level terms in hadronic excitation requires solution of coupled channel equations to obtain the appropriate effective wave functions. Clearly, a sum over all terms in the hadronic expansion will yield the quark model result; however, interest resides in the possible rapid convergence of this expansion, particularly at large q. Certainly, nuclear phenomenology appears to be successful with a rather small number of hadronic degrees of freedom.

One can straightforwardly "hadronize" the model by merely picking either y or z as the confined coordinate. This corresponds to using the hadronic states at asymptotic separation as the hadronic basis for all separation. With z as the confined coordinate, the projector becomes

$$P_n(y,z,y',z')=\delta(y-y')\Phi_n(z)\Phi_n(z') \tag{7}$$

where $\Phi_n(z)$ satisfies

$$\left[-\frac{d^2}{dz^2}+4z^2\right]\Phi_n(z)=\varepsilon_n\Phi_n(z)\ ,$$
$$\varepsilon_n=\left[2n+\tfrac{3}{2}\right]\omega=8n+6\ . \tag{8}$$

The x coordinate is ignored in the above projector and in all subsequent projectors since it decouples from $h(y,z)$.

Using the $P_n\hat\rho_qP_m$ prescription, one finds the effective charge operator,

$$[\rho_q(y)]_{nm}=f_{00}(q)f_{nm}(q)j_0(qy/2) \tag{9}$$

where $f_{nm}(q)=\int_0^\infty dz\,\Phi_n(z)j_0(qz/2)\Phi_m(z)$ and $f_{00}(q)$ comes from the x integration. As the isolated hadron form factor with internal state n is $|\,f_{nn}(q)\,|^2$, the point charge operator is modified by the elementary hadron form factor to lowest order of truncation. This is rather appealing since this is the standard charge operator used in nuclear physics. The effective wave functions are obtained by inserting the projector of Eq. (7) in the energy matrix element:

$$\varepsilon=\sum_{n,m}\int_0^\infty dy\,\chi_n(y)\times\left[\delta_{nm}(-\partial_y^2+\varepsilon_n)+\int_y^\infty dz\,4(y^2-z^2)\Phi_n(z)\Phi_m(z)\right]\chi_m(y) \tag{10a}$$

$$\equiv\sum_{n,m}\int_0^\infty dy\,\chi_n(y)h_{nm}(y)\chi_m(y) \tag{10b}$$

where $\chi_m(y) \equiv \int_0^\infty dz\, \Phi_m(z)\psi_B(y,z)$. $h_{nm}(y)$ yields a local effective coupled-channel potential; for example, the ground state channel Hamiltonian is just

$$h_{00}(y) = -\partial_y^2 + V_{\text{eff}}(y) ,$$
$$V_{\text{eff}}(y) = -4\int_y^\infty dz(z^2-y^2)\Phi_0^2(z) . \quad (11)$$

The potential is everywhere attractive. As $y \to \infty$, $V_{\text{eff}} \sim -4(2/\pi)^{1/2} y \exp(-2y^2)$, and as $y \to 0$, $V_{\text{eff}} \sim -4\langle z^2\rangle = -3$. Direct integration of this equation yields a scattering length of -2.07; there is no bound state to this level of truncation. This is wholly consistent with the slow convergence of the hadronic state occupation probability as noted in Ref. 1 and, given that the bound state in the original model came from quark exchange dynamics, is not at all surprising. Clearly, this approach based upon labeling which quarks are assigned to which hadron yields a simple but entirely unacceptable equivalent theory.

Incorporation of the quark exchange dynamics in the projection operator is apparently crucial. This can be done by defining confined wave functions which recognize the quark rearrangement embedded in the imposed rearrangement surface boundary conditions. Thus, the definition of a hadron is modified from its free value for finite separation; this modification is surely arbitrary as the hadron definition is itself arbitrary when hadrons overlap. We start with one specific choice. For $y \geq z$, we define $\varphi_n(z;y)$ such that

$$(-\partial_z^2 + 4z^2)\varphi_n(z;y) = \varepsilon_n(y)\varphi_n(z;y) , \quad (12)$$

where

$$\partial_z \varphi_n(z;y)\,|_{z=y} = 0$$

and

$$\int_0^y dz\, \varphi_n(z;y)\varphi_m(z;y) = \delta_{nm} .$$

One would like to embed the exact condition at $z=y$ in $\varphi_n(z;y)$ and we return to such a choice below; however, for these coordinates, the normal derivative at $z=y$ couples the confined and channel wave functions. Thus, for technical convenience, only a constraint on the z derivative is made at $z=y$. One can now define the projector

$$\begin{aligned} P_n(y,z,y',z') = [&\varphi_n(z;y)\varphi_n(z';y')\delta(y-y') \\ &\times\theta(y-z)\theta(y'-z') \\ &+\varphi_n(y;z)\varphi_n(y';z')\delta(z-z') \\ &\times\theta(z-y)\theta(z'-y')] , \end{aligned} \quad (13)$$

which can be used to "hadronize" $h(y,z)$. Defining

$$\chi_n(y) \equiv \sqrt{2}\int_0^y dz\, \varphi_n(z;y)\psi_B(z,y)$$

so that

$$\psi_B(z,y) = \frac{1}{\sqrt{2}}\sum_n [\chi_n(y)\varphi_n(z;y)\theta(y-z) + \chi_n(z)\varphi_n(y;z)\theta(z-y)] , \quad (14)$$

one has

$$\varepsilon = \tfrac{1}{4}\sum_{n,m} [\langle \chi_m\varphi_m \,|\, h\varphi_n\chi_n\rangle + \langle h\chi_m\varphi_m \,|\, \varphi_n\chi_n\rangle] . \quad (15)$$

A symmetric prescription in the hadronic expansion is necessary and sufficient to yield a Hermitian effective Hamiltonian. Integration by parts with respect to y, in conjunction with Eq. (12) and the $\varphi_n(z;y)$ orthogonality relations, yields

$$\begin{aligned} h_{mn}(y) = [&-\partial_y^2 + \varepsilon_n(y)]\delta_{nm} + \varphi_n(y;y)\partial_y\varphi_m(y;y) \\ &+ \int_0^y dz\{\partial_y\varphi_n(z;y)\partial_y\varphi_m(z;y) + \tfrac{1}{2}[\varphi_n(z;y)\partial_y^2\varphi_m(z;y) - \varphi_m(z;y)\partial_y^2\varphi_n(z;y)]\} \\ &+ \int_0^y dz[\varphi_n(z;y)\partial_y\varphi_m(z;y) - \varphi_m(z;y)\partial_y\varphi_n(z;y)]\partial_y . \end{aligned} \quad (16)$$

To lowest order of truncation, one finds a local effective Hamiltonian:

$$h_{00}(y) = -\partial_y^2 + V_{00}(y) \quad (17a)$$

and

$$\begin{aligned} V_{00}(y) = &\varepsilon_0(y) - \varepsilon_0(\infty) + \varphi_0(y;y)\partial_y\varphi_0(y;y) \\ &+ \int_0^y dz[\partial_y\varphi_0(z;y)]^2 . \end{aligned} \quad (17b)$$

The energy has been defined relative to $\varepsilon_0(\infty)=6$. The potentials through $n=m=1$ are shown in Fig. 1(a), and V_{00} can be seen to have a shape familiar from the usual meson-nucleon phenomenologies: there exists a short range "hard core" repulsion and intermediate range attraction. Analytic limits can be extracted as $y \to 0$ or as $y \to \infty$. That is, as $y \to 0$,

$$\varphi_0(z;y) \sim \left[\frac{2}{y}\right]^{1/2} \sin\left[\frac{\pi z}{2y}\right]$$

and thus,

$$V_{00} \sim \left[\frac{\pi^2}{3} - \frac{3}{4}\right]\frac{1}{y^2} - 6 .$$

The $y \to \infty$ asymptotic form follows from the formal solution for $\varphi_0(z;y)$ in terms of the confluent hypergeometric function and its asymptotics.[7] One finds that

$$V_{00} \sim -64(2/\pi)^{1/2} y^3 \exp(-2y^2)$$

as $y \to \infty$. The Gaussian damping as $y \to \infty$ is a conse-

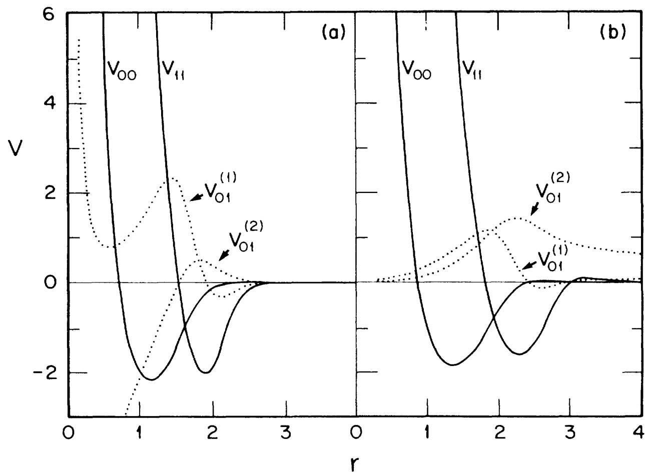

FIG. 1. Effective hadronic potentials through $n=m=1$. The scale in r is set by $\bar{r}=\sqrt{\frac{3}{2}}$, the isolated hadron ground-state rms radius. $V_{01}(y)\equiv V_{01}^{(1)}(y)+V_{01}^{(2)}\partial_y$. (a) (y,z) scheme [Eq. (16)]; (b) (r,θ) scheme [Eq. (24)].

quence of harmonic confinement; the effective $n=m=0$ potential in this model is intrinsically of much shorter range than the phenomenological Yukawa potential. However, it is important to remember that the "long range" tail in the model arises from the hadron overlap, or quark exchange dynamics, while that of the Yukawa potential actually comes from meson exchange. As one examines the potentials to higher order of truncation, it appears that the diagonal potentials are qualitatively similar to $V_{00}(y)$; the minima occur at larger y with increasing n, which is due, in part, to the larger rms radius of a hadron with excitation state n. The novel feature of the higher truncation calculations is the ∂_y nonlocal term in the channel couplings; it is clear, though, from the overlap of the potential wells in Fig. 1(a), that the effect of successive higher channels is rather small. The result of solving Eq. (16) as an eigenvalue problem for successive truncations yields the following bound state energies: $\varepsilon^{(0)}=-0.0877$, $\varepsilon^{(1)}=-0.118$, and $\varepsilon^{(2)}=-0.129$, all relative to the isolated hadron ground state energy of $\varepsilon_0(\infty)=6$. The quark model value, in the same units, is $-0.160=-0.04\omega$. It is noteworthy that even the lowest order truncation has a bound state, since the quark model binding is weak relative to the confinement scale.

The scattering of the $n=m=0$ system has been calculated as well. As the hadronic calculation contains merely one channel, it is incapable of reproducing the resonances in the quark model phase shift above the first inelastic threshold at $k=2\sqrt{2}$;[1] and even below that threshold the agreement is still rather modest. In particular, the scattering length and effective range are $a=4.42$, $r_0=1.63$, whereas the quark model values are $a^{(q)}=3.52$, $r_0^{(q)}=1.51$.

To calculate the form factor, one obtains the effective charge operator using the $P_n\hat{\rho}_qP_m$ prescription for $y\geq z$. Rewriting this in terms of one-body and exchange pieces, one has

$$[\rho_q(y)]_{nm}=[\rho_q^{\text{1-body}}(y)]_{nm}+[\rho_q^{\text{exch}}(y)]_{nm} , \tag{18a}$$

with

$$[\rho_q^{\text{1-body}}(y)]_{nm}\equiv f_{00}(q)f_{nm}(q)j_0(qy/2), \tag{18b}$$

$$[\rho_q^{\text{exch}}(y)]_{nm}=f_{00}(q)j_0(qy/2)\times\left[\int_0^y dz[\varphi_n(z;y)j_0(qz/2)\times\varphi_m(z;y)]-f_{nm}(q)\right] . \tag{18c}$$

The exchange charge operator gives zero contribution as $q\to 0$ from charge conservation. Nor is there a contribution as $y\to\infty$ since the hadrons approach infinite separation. The one-body charge operator is the effective charge operator of the projector of Eq. (7). The resulting form factor contributions are

$$F_{nm}^{\text{1-body}}(q)=\langle\chi_n^{(\nu)}\,|\,[\rho_q^{\text{1-body}}]_{nm}\,|\,\chi_m^{(\nu)}\rangle , \tag{19a}$$

$$F_{nm}^{\text{exch}}(q)=\langle\chi_n^{(\nu)}\,|\,[\rho_q^{\text{exch}}]_{nm}\,|\,\chi_m^{(\nu)}\rangle , \tag{19b}$$

and

$$F^{(\nu)}(q)=\sum_{n,m}^{\nu}[F_{nm}^{\text{1-body}}(q)+F_{nm}^{\text{exch}}(q)] \tag{19c}$$

where the channel wave functions $\chi_n^{(\nu)}(y)$ depend on the original level of truncation, ν. Figure 2 shows the convergence by truncation of the form factor of the "hadronized" system to the quark model calculation. The agreement of the $\nu=2$ form factor is impressive; this is particularly surprising as the channel occupation probability is $O(10^{-3})$ for the first excited state and $O(10^{-4})$ for the second excited state. Figure 3 shows the sensi-

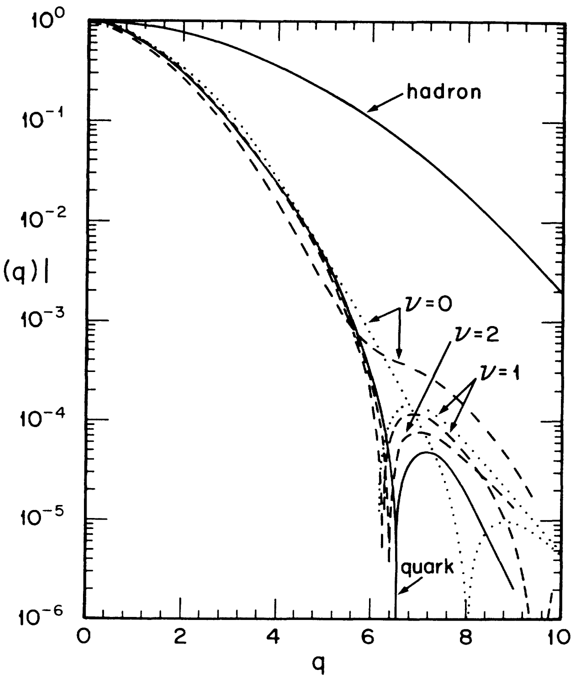

FIG. 2. Effective form factor by truncation. "Quark" is the exact $F(q)$ as per Eq. (4). "Hadron" is $|f_{00}(q)|^2$. Dashes: (y,z). Dots: (r,θ).

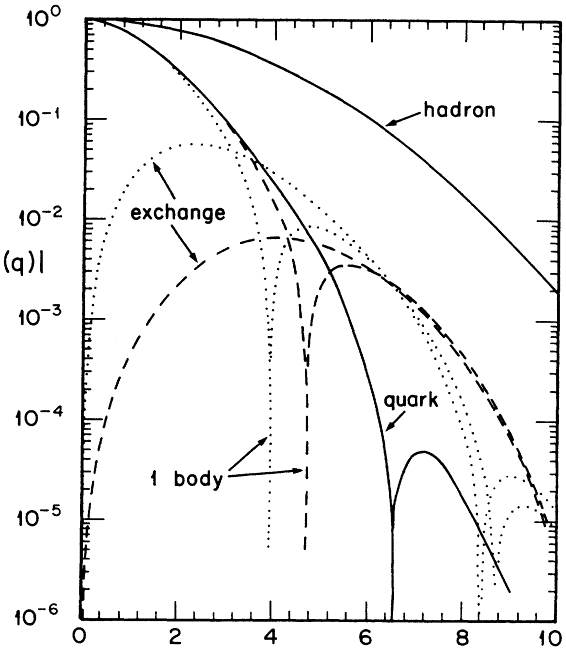

FIG. 3. One-body and exchange contributions to the $\nu=1$ effective form factor. "Quark" and "hadron" are as in Fig. 2. Dashes: (y,z). Dots: (r,θ).

tive cancellation of the one-body and exchange contributions in the example of the $\nu=1$ truncation.

The previous projector [Eq. (13)] incorporated the internal rearrangement, but not the full exchange dynamics, as the confined wave functions did not incorporate the correct boundary condition at $z=y$. The natural step is now to construct a projector which does incorporate this boundary condition. This can generally be done by transforming to other coordinate systems in which the normal derivative constraint at the rearrangement surface can be imposed in terms of the "confined" coordinate. The most naive transformation is a mapping to polar coordinates, so that for $y \geq z$,

$$y = r\cos\theta\ ,\qquad z = r\sin\theta\ . \tag{20}$$

In analogy to Eq. (12) in the limit of $z \ll y$, confined wave functions $\varphi_n(\theta;r)$ are introduced for $\theta \leq \pi/4$ such that

$$\left[-\frac{1}{r^2}\partial_\theta^2 + 4r^2\theta^2\right]\varphi_n(\theta;r) = \varepsilon_n(r)\varphi_n(\theta;r)\ , \tag{21}$$

where

$$\partial_\theta \varphi_n(\theta;r)\,|_{\,\theta=\pi/4} = 0$$

and

$$\int_0^{\pi/4} r\, d\theta\, \varphi_m(\theta;r)\varphi_n(\theta;r) = \delta_{nm}\ .$$

The projector

$$P_n(\theta,r,\theta',r') = \sqrt{r}\,\sqrt{r'}\varphi_n(\theta;r)\varphi_n(\theta';r')\delta(r-r') \tag{22}$$

can then be used to "hadronize" $h(\theta,r)$. This projector holds for all θ; as for $\theta \geq \pi/4$, one makes the identification $z=r\cos\theta$ and $y=r\sin\theta$, so that the internal rearrangement is incorporated in the coordinates. The $1/r$ kinetic energy pieces that occur in the polar quark Hamiltonian suggest that the resulting effective hadronic potentials will be long range; nevertheless, we go on to investigate the convergence of this scheme. Now

$$\chi_n(r) \equiv \sqrt{2}\int_0^{\pi/4} d\theta\, \varphi_n(\theta;r)\psi_B(\theta,r)$$

implies that for $\theta \leq \pi/4$

$$\psi_B(\theta,r) = \frac{1}{\sqrt{2}}\sum_n \left[\chi_n(r)\varphi_n(\theta;r)\Theta\left[\frac{\pi}{4}-\theta\right]\right]\ , \tag{23}$$

and a symmetric prescription for the energy as described above yields the effective Hamiltonian

$$\begin{aligned} h_{nm}(r) = {} & \left[-\partial_r^2 + \left[\varepsilon_n(r) - \varepsilon_0(\infty) - \frac{1}{2r^2}\right]\right]\delta_{nm} \\ & + \int_0^{\pi/4} r\, d\theta \left[4r^2(\sin^2\theta - \theta^2)\varphi_n\varphi_m + \partial_r\varphi_m\partial_r\varphi_n + \frac{1}{2r}(\varphi_m\partial_r\varphi_n - \varphi_n\partial_r\varphi_m) + \tfrac{1}{2}(\varphi_m\partial_r^2\varphi_n - \varphi_n\partial_r^2\varphi_m)\right] \\ & + \int_0^{\pi/4} r\, d\theta(\varphi_m\partial_r\varphi_n - \varphi_n\partial_r\varphi_m)\partial_r\ , \end{aligned} \tag{24}$$

where the $\varphi_n(\theta;r)$ arguments have been suppressed and, as before, the energy is defined relative to $\varepsilon_0(\infty)=6$. The Hamiltonian is again local to lowest truncation with

$$V_{00}(r) = \varepsilon_0(r) - \varepsilon_0(\infty) - \frac{1}{2r^2} + \int_0^{\pi/4} r\, d\theta[4r^2(\sin^2\theta - \theta^2)\varphi_0^2 + (\partial_r\varphi_0)^2]\ . \tag{25}$$

The potentials through $\nu=1$ are shown in Fig. 1(b). The 00 potential is qualitatively similar to the 00 potential of the (y,z) confinement scheme with internal rearrangement [Eq. (13)]; however, it is of much longer range, though the numerical falloff for $r\to\infty$ is not $O(r^{-2})$ as Eq. (25) naively indicates. In addition, as $r\to 0$,

$$\varphi_0(\theta;r) \sim \left[\frac{8}{\pi}\right]^{1/2}\sin(2\theta)$$

and, thus, $V_{00}(r) \sim 15/4r^2-6$; hence, $V_{00}(r)$ is much stiffer as the channel variable goes to zero. For $\nu=1$, of particular interest is the long range nonlocal channel coupling: $V_{01}^{(2)} \sim O(1/r)$ as $r \to \infty$. One sees a substantial lowering of the bound state energy with truncation: $\varepsilon^{(0)}=-0.108$ and $\varepsilon^{(1)}=-0.150$. Direct integration of the $\nu=0$ potential yields a slightly more attractive scattering length than in the (y,z) scheme, $a=4.27$. However, the stiffer repulsion of the potential as $r \to 0$ yields poorer agreement to the quark model phase shifts for large k as seen in a comparison of the effective ranges: $r_0^{(r,\theta)}=1.81$, $r_0^{(y,z)}=1.63$, and $r_0^{(q)}=1.51$.

With the $P_n \hat{\rho}_q P_m$ prescription, one obtains the modified charge operator. Pulling out the one body piece, one has for the exchange part

$$[\rho_q^{\text{exch}}(r)]_{nm}=f_{00}(q)\left\{\int_0^{\pi/4} r\,d\theta \left[\varphi_n j_0\left[\frac{qr}{2}\cos\theta\right] j_0\left[\frac{qr}{2}\sin\theta\right]\varphi_m\right]-f_{nm}(q)j_0(qr/2)\right\}. \tag{26}$$

The resulting form factor contributions are calculated as in Eq. (19), and the convergence by truncation is shown in Fig. 2. The one-body charge operator is identical to Eq. (18b); however, the resulting contribution to the form factor is very different in this confinement scheme as the channel wave functions are different. The one-body and exchange form factor contributions are shown for $\nu=1$ in Fig. 3. As in the (y,z) case, the cancellation of the one-body and exchange contributions at high q yields the total form factor's convergence with truncation. Here, though, the exchange contribution is extremely large, due to the exchange charge's $O(1/r)$ behavior as $r \to \infty$. Overall, the agreement of the $\nu=1$ form factor with the quark level result is rather good, which is surprising not only because the first excited state occupation probability is $O(10^{-3})$, but also because the channel couplings and effective charge operator display an "unphysical" long range behavior. The long range nature of the channel couplings and exchanged charge is particularly pathological when one considers that the original quark Hamiltonian had no such van der Waals forces, so that the convergence of this scheme to the various quark observables studied is all the more remarkable.

IV. CONCLUSIONS

The above results show that a parameter-free hadronic description with a small number of hadronic degrees of freedom can reproduce the quark model observables with remarkable accuracy. This success persists even in extreme kinematic regimes; in particular, the form factor is well described by an hadronic basis even when the isolated hadron form factor has fallen by two orders of magnitude. The most remarkable feature is the rather close agreement with observables sensitive to different physics, such as the low energy scattering parameters and the high momentum transfer bound state form factor, of phenomenologies which themselves appear to be very different. Of course, they have the underlying common feature of building a signature of the quark exchange into the hadronization. Consequently, this study does not provide encouragement that any distinctive quark signatures are to be found in low energy nuclear observables. On the other hand, it does give credibility to programs which would attempt to calculate "observed" hadronic phenomenological parameters from the underlying theory. In addition, it is to be emphasized that, regardless of the sophistication of such attempts, a consistent, understood hadronization scheme is a necessary underpinning of any calculation. This applies equally well to a lattice calculation as to a potential quark model.

This work was supported in part by funds provided by the U.S. Department of Energy under Contract No. DE-AC02-76ER03069.

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