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Unitary theory of pion photoproduction in the chiral bag model

M. Araki* and I. R. Afnan
School of Physical Sciences, The Flinders University of South Australia, Bedford Park, South Australia 5042, Australia
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We present a multichannel unitary theory of single pion photoproduction from a baryon B. Here, B is the nucleon or $\Delta(1232)$, with possible extension to include the Roper resonance and strange baryons. We treat the baryon as a three-quark state within the framework of the gauge and chiral Lagrangian, derived from the Lagrangian for the chiral bag model. By first exposing two-body, and then three-body unitarity, taking into consideration the $\pi\pi$B and $\gamma\pi$B intermediate states, we derive a set of equations for the amplitudes both on and off the energy shell. The Born term in the expansion of the amplitude has the new feature that the vertices in the pole diagram are undressed, while those in the crossed, contact, and pion pole diagrams are dressed.

I. INTRODUCTION

The recent interest in the interaction of electromagnetic probes with nuclei,[1] has been primarily motivated by the theoretical progress[2-5] in our understanding of the meson-nuclear interaction, based on models which are consistent with quantum chromodynamics (QCD).[6] Although gluons and quarks are the only ingredients of QCD, there is a consensus that both the mesonic[2-5] and the gluon-quark description of the nuclear interaction[7] are possible. This idea is supported not only by the success of the old meson theory of Yukawa, but also by the theories[2-5] of meson-nuclear interactions based on QCD. Due to the absence of the complexity of the nucleon-nucleon overlap, the single pion photoproduction from a single nucleon is the simplest reaction amongst the electromagnetic interactions with nuclei. Therefore, this is an ideal place to study the interplay of quarks, gluons, and mesons in the nucleon.

The single pion photoproduction from a nucleon has been studied to date in the following four approaches. (i) The dispersion relation approach of Chew, Goldberger, Low, and Nambu (CGLN),[8] where unitarity and analyticity are strictly imposed. (ii) The Lagrangian approach of Blomqvist and Laget (BL),[9] in which the amplitude is the nonrelativistic limit of the Olsson and Osypowski[10] amplitude but includes the Δ resonance, with a width, in the S-channel propagators. In one of their approaches which is popular in nuclear physics literature, unitarity is violated. In their second approach, they showed a way to recover unitarity, and then examined it in the $M1$ multipole amplitude. (iii) The on-shell multichannel approach of Olsson,[11] where special attention is paid to the separation of the amplitude into a resonant and nonresonant part, within a unitary two-channel formalism. The relative phase of the two amplitudes is then determined by the Watson theorem. (iv) The off-shell multichannel approach of Tanabe and Ohta (TO),[12] in which the pion photoproduction amplitude is written in terms of the two-channel (γN,πN) formalism, with the πN amplitude determined by coupling the πN and $\pi\Delta$ channels using separable potentials. This approach gives an off-shell amplitude that could be used in heavier systems such as $\gamma d \rightarrow \pi^0 d$, $\gamma t \rightarrow {}^3\mathrm{He}\pi^- \ldots$.

Recently, Wittman, Davidson, and Mukhopadhyay[13] examined the M_{1+} and E_{1+} transitions in single pion photoproduction using the first three of the above four methods. Their conclusions were (a) the nonunitary approach of BL violates the Watson Theorem, (b) the E_{1+} and M_{1+} amplitudes resulting from BL method gives poor agreement with experiment, and (c) unitarizing the BL amplitudes improves the agreement with the data and brings the results in line with the theoretical analysis of Olsson.[11]

The unitary method of CGLN (Ref. 8) and Olsson (Ref. 11) can be formulated within the framework of a Lagrangian whose coupling constants may be compared with the observed values or with the prediction of the quark models. However, the connection between the multiple scattering formalism and the quark model is not clear. In particular, if the Lagrangian is an effective Lagrangian to be used in the tree approximation, then the coupling constants, which are already renormalized, can be determined from experiment. This approach does not satisfy unitarity which plays an important role in pion photoproduction. On the other hand, if the Lagrangian is not an effective Lagrangian, i.e., the coupling constants have to be renormalized, then the renormalization has to be carried out in such a manner as to incorporate, or at least be consistent with, unitarity.

With the recent success of the quark meson models (QMM),[2-5] that are consistent with QCD in describing the baryon spectrum [in particular, the nucleon (N) and Δ] in terms of three quarks, it is desirable to establish a unitary theory of pion photoproduction based on QMM. In this way the Δ and possibly the Roper (R) will be treated in terms of their quark content rather than their traditional representation as a $\pi-$N resonance. We therefore need a theory, based on the Lagrangian from the QMM with minimal electromagnetic coupling, that includes the following. (i) The coupling between the γB and πB channels with B$=$N,Δ,R,... given in terms of their quark substructure. (ii) The coupling constants and vertex functions calculated from the QMM Lagrangian which is gauge and chirally invariant. (iii) The renormalization of the propagators and vertices which is carried out in a manner that is consistent with unitarity. In other words, the same equations are used to calculate both the scattering amplitude and the renormalization constants. In this way we have a formulation of single pion photoproduction that is consistent with QCD and includes the quark content of the N,Δ,R, . . . , yet satisfies unitarity which is demanded by previous analyses.[13]

In Sec. II we first consider the Lagrangian for the chiral bag model. The photon-quark and photon-pion coupling is induced by demanding $U(1)$ gauge invariance. This Lagrangian and the corresponding Hamiltonian is projected onto the baryon space which is truncated to include the asymptotic states $|B\rangle$, $|\pi B\rangle$, $|\gamma B\rangle$, $|\gamma\pi B\rangle$, and $|\pi\pi B\rangle$. To render a problem that is manageable, we neglect the direct coupling between the $|B\rangle$ and $|\gamma\pi B\rangle$ and also the coupling between $|B\rangle$ and $|\pi\pi B\rangle$. We then proceed in Sec. III to exposing the two body unitarity cuts. This allows us to write equations that satisfy two-body unitarity for the reactions $B(\pi,\pi)B'$ and $B(\gamma,\pi)B'$. These equations are similar to those derived by Tanabe and Ohta[12] except for the following facts. (i) Tanabe and Ohta do not treat the N and Δ on equal footing in terms of their quark substructure. (ii) Their renormalization is not carried out consistently. (iii) The Lagrangian they use is not self-contained in that the parameters in the $(\gamma\pi B)$ vertex functions are adjusted to fit the calculated E_{1+} and M_{1+} amplitudes to experiment in the region of the Δ resonance. We hope that our more general formulation will be valid even away from the Δ resonance. At this stage the Born term in our equation includes all the diagrams in Fig. 1. The diagrams in Figs. 1(a)–1(d) are included in the theories of CGLN,[8] BL,[9] and Olsson;[11] and, in addition, BL include the diagram in Fig. 1(e). At this stage we make no distinction between the πBB vertices in the different diagrams. However, when we expose three-body unitarity in Sec. IV, we find the surprising result that the vertices in the pole diagram [shown in Fig. 1(a)] are undressed while the vertices in the other Born diagrams are dressed, due to the $\pi-q$ interaction. In this way the renormalization and scattering amplitude are calculated in a consistent manner; in that the final amplitude gives both the renormalization constant and scattering amplitude. For completeness, we present in Sec. V explicit expressions for the Born diagrams in Fig. 1, while Sec. VI is devoted to a partial wave expansion of our equation and a discussion of their content.

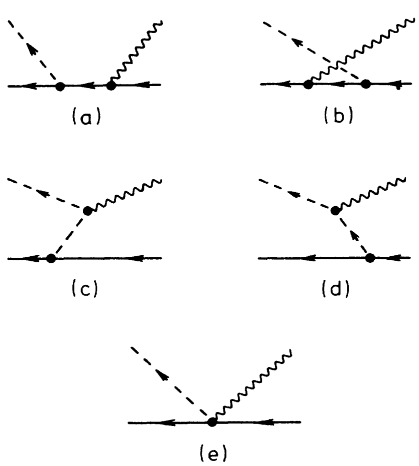

FIG. 1. The time ordered diagrams that contribute to the Born amplitude for pion photoproduction. Note time goes from right to left.

II. THE EFFECTIVE HAMILTONIAN

Since we are to deal with both the strong and electromagnetic interactions, it is essential that we start from a Lagrangian that is chirally symmetric and gauge invariant. In particular, we take the chiral bag model[14–16] Lagrangian that has the pseudovector πqq coupling. At the tree level calculation, it is known that this Lagrangian reproduces the current algebra results[17] in π-N scattering[15] and pion photoproduction.[18] Therefore, this Lagrangian is considered to be more adequate than the Lagrangian with the pseudoscalar coupling.[19,20] Although our final equations do not depend on the details of the Lagrangian, it is advantageous to specify the form of the Lagrangian in use. This is necessary, first in order to carry out the actual calculation and second, in order to make the formulation tractable.[21,22]

We commence with the chiral bag model Lagrangian[14–16] and expand the interaction term to order $g^2=(2f_\pi)^{-2}$, where f_π is the pion decay constant at the tree level. To introduce the electromagnetic coupling, we impose $U(1)$ gauge invariance on the truncated Lagrangian and keep terms to order eg, where e is the charge of the baryon or meson. In other words, we keep the terms to order g, g^2, e, and eg, and discard the eg^2 term with the hope that it is negligible. This truncation does lead to the breaking of both the gauge and chiral symmetries at higher orders. On the other hand we gain the ability to impose two- and three-body unitarity. It is always possible to treat the neglected terms in perturbation at a later stage, particularly since we expect them to be small. The resultant Lagrangian may be written as

$$\mathcal{L}=\mathcal{L}_{\rm MIT}+\mathcal{L}_\pi+\mathcal{L}_\gamma+\mathcal{L}_I \;, \tag{2.1}$$

where

$$\mathcal{L}_{\rm MIT}=(\tfrac{1}{2}\bar{q}\gamma^\mu\overleftrightarrow{\partial}_\mu q-B)\theta_v-\tfrac{1}{2}\bar{q}q\Delta_s \;, \tag{2.2}$$

$$\mathcal{L}_\pi=\tfrac{1}{2}[(\partial_\mu\boldsymbol{\phi})^2-m_\pi^2\boldsymbol{\phi}^2] \;, \tag{2.3}$$

$$\mathcal{L}_\gamma=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \;, \tag{2.4}$$

and the interaction Lagrangian is given by

$$\mathcal{L}_I=\mathcal{L}_{qq\pi}+\mathcal{L}_{qq\pi\pi}+\mathcal{L}_{\pi\pi\pi\pi}+\mathcal{L}_{qq\gamma}+\mathcal{L}_{qq\pi\gamma}+\mathcal{L}_{\pi\pi\gamma} \tag{2.5}$$

with

$$\mathcal{L}_{qq\pi}=g\bar{q}\gamma^\mu\gamma_5\boldsymbol{\tau}\cdot(\partial_\mu\boldsymbol{\phi})q\theta_v \;, \tag{2.6}$$

$$\mathcal{L}_{qq\pi\pi}=-g^2\bar{q}\gamma^\mu\boldsymbol{\tau}\cdot(\boldsymbol{\phi}\times\partial_\mu\boldsymbol{\phi})q\theta_v \;, \tag{2.7}$$

$$\mathcal{L}_{qq\gamma}=Qe\bar{q}\gamma^\mu qA_\mu\theta_v \;, \tag{2.8}$$

$$\mathcal{L}_{qq\pi\gamma}=eg\bar{q}\gamma^\mu\gamma_5(\boldsymbol{\tau}\times\boldsymbol{\phi})_3 qA_\mu\theta_v \;, \tag{2.9}$$

$$\mathcal{L}_{\pi\pi\gamma}=eF_\pi(\partial^\mu\boldsymbol{\phi}\times\boldsymbol{\phi})_3A_\mu \;, \tag{2.10}$$

where Q is the quark charge. For the π-π Lagrangian $\mathcal{L}_{\pi\pi\pi\pi}$, we have a choice of two models. The cloudy bag model[15,19] gives

$$\mathcal{L}_{\pi\pi\pi\pi}=-\tfrac{2}{3}g^2[(\partial_\mu\boldsymbol{\phi})^2\boldsymbol{\phi}^2-(\boldsymbol{\phi}\cdot\partial_\mu\boldsymbol{\phi})^2] , \tag{2.11a}$$

while the model of Eisenberg and Kalberman[14] gives

$$\mathcal{L}_{\pi\pi\pi\pi}=-g^2[(\partial_\mu\boldsymbol{\phi})^2\boldsymbol{\phi}^2-m_\pi^2\boldsymbol{\phi}^4] . \tag{2.11b}$$

Recently, Roberts *et al.*[23] have shown that the bosonization of QCD does in fact lead to the sum of these two terms, and this is required if we are to get the current algebra results for the $\pi-\pi$ scattering length. In Eq. (2.10) we have introduced a pion form factor F_π. If the pion is regarded as a Goldstone Boson or elementary, then $F_\pi=1$. However, if we regard the pion field[20] as an approximation for the center of mass of a $\bar{q}q$ composite pion, then $F_\pi=F_\pi(k^2)\le 1$. The pion form factor may be obtained in the bag model, either by boosting the bag wave function[24] or by considering the sequential decay $\gamma\to\rho\to 2\pi$. In the pion photoproduction reaction,[25] it is expected that one can get more information about F_π.

We now quantize our pion and photon fields following the convention of Bjorken and Drell;[26] the pion field is given by

$$\phi_\alpha(x)=\frac{1}{(2\pi)^{3/2}}\int\frac{d^3q}{\sqrt{2\omega_q}}(e^{-iq\cdot x}a_{\alpha,q}+e^{iq\cdot x}a^\dagger_{\alpha,q}) \tag{2.12}$$

and the photon field

$$A^\lambda_\mu(x)=\frac{1}{(2\pi)^{3/2}}\int\frac{d^3k}{\sqrt{2k_0}}(e^{-ik\cdot x}\epsilon^\lambda_\mu c^\lambda_k+e^{ik\cdot x}\epsilon^{\lambda *}_\mu c^{\lambda\dagger}_k) . \tag{2.13}$$

On the other hand, the quark field can be written by excluding antiquarks, as

$$q(x)=\sum_{n\nu}\psi_{n\nu}b_{n\nu} , \tag{2.14}$$

where $b_{n\nu}$ are the annihilation operators for the quarks.

We now have to project our Lagrangian $\mathcal{L}$, given in Eq. (2.5) in terms of the quark field, onto the space of hadrons to get the corresponding effective Hamiltonian. Here, we restrict our basis to one-, two-, and three-particle states $|\mathrm{B}\rangle$, $|\pi\mathrm{B}\rangle$, $|\gamma\mathrm{B}\rangle$, $|\pi\pi\mathrm{B}\rangle$, and $|\gamma\pi\mathrm{B}\rangle$, where $|\mathrm{B}\rangle$ is a bare baryon composed of three quarks. In the Fock space representation, the effective Hamiltonian (written in terms of creation and annihilation operators for the baryons, meson, and photons) is

$$\hat{H}=\sum_n\epsilon_n B^\dagger_n B_n+\sum_\alpha\int d^3q\,\omega_q a^\dagger_{\alpha q}a_{\alpha q}+\sum_\lambda\int d^3k\,k_0 C^{\lambda\dagger}_k C^\lambda_k+\hat{H}_I , \tag{2.15}$$

where the first term in the mass term of the baryon, with ϵ_n possibly including the kinetic energy, by expecting a proper prescription for the c.m. correction. The interaction Hamiltonian $\hat{H}_I$ is the sum of the following twelve terms,

$$\begin{aligned}\hat{H}_I=&\langle \mathrm{B}|\hat{H}|\mathrm{B}\pi\rangle+\langle \mathrm{B}\pi|\hat{H}|\mathrm{B}\pi\rangle+\langle\pi\pi|\hat{H}|\pi\pi\rangle\\&+\langle \mathrm{B}|\hat{H}|\mathrm{B}\gamma\rangle+\langle \mathrm{B}\pi|\hat{H}|\mathrm{B}\gamma\rangle+\langle\pi\pi|\hat{H}|\gamma\rangle\\&+\langle\pi|\hat{H}|\pi\gamma\rangle+\text{five Hermitian conjugate terms} .\end{aligned} \tag{2.16}$$

For example, $\langle \mathrm{B}\pi|\hat{H}|\mathrm{B}\gamma\rangle$ may be written, in terms of the Hamiltonian at the quark level, as

$$\langle \mathrm{B}\pi|\hat{H}|\mathrm{B}\gamma\rangle=\sum_{\substack{mn\\ \alpha\lambda}}\int d^3q\,d^3k\,\langle m,\alpha q\,|H_{\mathrm{qq}\pi\gamma}|\,n,\lambda k\rangle\times B^\dagger_m B_n a^\dagger_{\alpha q}C^\lambda_k . \tag{2.17}$$

We have chosen to truncate our interaction Hamiltonian in such a way as to avoid any direct coupling between the single particle state $|\mathrm{B}\rangle$ and the three-body state $|\mathrm{B}\pi\pi\rangle$ or $|\mathrm{B}\pi\gamma\rangle$. This truncation was utilized in writing Eq. (2.16) and was introduced to render our final result simple and manageable from a computational point of view. Also, we can see that $\hat{H}_I$ is not covariant because we have neglected terms such as $\langle \mathrm{B}|\hat{H}|\mathrm{B}\pi\gamma\rangle$. These neglected terms can always be estimated in perturbation.

III. TWO-BODY UNITARITY

Let us now derive the equations which satisfy two- and three-body unitarity. To accomplish this, we use the last-cut lemma, which was first introduced by Taylor[27] and later used by other groups for the three-body problem, in the πNN system[28,29] and in the $\pi\pi$N system.[21,22] We refer the reader particularly to Refs. 22 and 28 for the details. However, for the sake of completeness, we outline this procedure below.

To derive equations for the amplitude for a given process, we need to classify the diagrams that contribute, in perturbation theory, to this amplitude according to their irreducibility using the last-cut-lemma. To achieve this we need to first define a k cut as an arc that separates the initial state from the final state, in a given diagram, and cuts k-particle lines with at least one line being internal. Second, an amplitude is r-particle irreducible if all diagrams that contribute to this amplitude will not admit any k cuts with $k\le r$. With these two definitions, we can introduce the last-cut-lemma which states that for a given amplitude that is $(r-1)$-particle irreducible, there is a unique way of obtaining an internal r-particle cut, closest to the final (initial) state for all diagrams that contribute to the amplitude. By virtue of this lemma, we can expose one-, two-, and three-particle intermediate states and the corresponding unitarity cuts and, in this way, derive equations for the amplitude that satisfy unitarity. From the statement of the lemma it is clear that one needs to expose n-particle unitarity before exposing the $(n+1)$-particle unitarity.

Here, we will treat the πB scattering and the pion photoproduction as coupled channels. However, if we restrict our analysis to include amplitudes to lowest order in the electromagnetic coupling (i.e., to order e), then we need to carry out the analysis for the π-B amplitude and the pion photoproduction amplitude separately. Also in the approximation, the S matrix for Compton scattering is unity, and all radiative corrections to the $\pi-\mathrm{B}$ amplitude are neglected. Since the analysis for the $\pi-\mathrm{B}$ amplitude has been discussed in detail in previous works[21,22] at least at the level of two-body unitarity,[28,29] we will present here a summary of the results. For $\pi-\mathrm{B}$ scattering with no coupling to the photoproduction channel, the amplitude is given by

$$t^{(0)}=t^{(1)}+f^{(1)\dagger}d_{\mathrm{B}}f^{(1)}\ , \tag{3.1}$$

where t is the $\pi-\mathrm{B}$ amplitude, f is the $\mathrm{B}\leftarrow\pi\mathrm{B}$ amplitude, and d_{B} is the dressed baryon propagator. The superscript gives the irreducibility of the amplitude, e.g., $t^{(n)}$ is the n-particle irreducible $\pi\mathrm{B}\leftarrow\pi\mathrm{B}$ amplitude. The one-particle irreducible $\pi\mathrm{B}\leftarrow\pi\mathrm{B}$ amplitude satisfies the Lippmann-Schwinger equation

$$t^{(1)}=t^{(2)}+t^{(2)}gt^{(1)}=t^{(2)}+t^{(1)}gt^{(2)} \tag{3.2}$$

with $g=d_{\mathrm{B}}d_{\pi}$, the π-B propagator. To get an explicit form for the two-particle irreducible amplitude $t^{(2)}$, we need to expose the three-particle intermediate states. The $\mathrm{B}\leftarrow\pi\mathrm{B}$ amplitude $f^{(1)}$ is given by

$$f^{(1)}=f^{(2)}+f^{(2)}gt^{(1)}=f^{(2)}+f^{(1)}gt^{(2)}\ , \tag{3.3}$$

while the dressed baryon propagator is given by

$$d_{\mathrm{B}}^{-1}=d_0^{-1}-\Sigma^{(1)}\ , \tag{3.4}$$

where $\Sigma^{(1)}$, the self-energy, is given by

$$\Sigma^{(1)}=\Sigma^{(2)}+f^{(1)}gf^{(2)\dagger}=\Sigma^{(2)}+f^{(2)}gf^{(1)\dagger}\ . \tag{3.5}$$

In Eq. (3.4), the bare propagator d_0 given by

$$d_0^{-1}=E-m_{\mathrm{B}}^{(0)}\ , \tag{3.6}$$

where $m_{\mathrm{B}}^{(0)}$, the bare mass of the baryon, is predicted by the MIT bag model. Considering the fact that we have neglected the direct coupling between the $|\mathrm{B}\rangle$ and $|\pi\pi\mathrm{B}\rangle$ states, then $\Sigma^{(2)}=0$ in Eq. (3.5) (see Sec. IV). Equations (3.1)–(3.6) are the basic equations for the π-B scattering below the threshold for pion production. The basic input into these equations are the two-particle irreducible amplitudes $t^{(2)}$ and $f^{(2)}$. To determine these amplitudes in terms of the interaction Lagrangian, we need to examine the three-body unitarity structure of these two amplitudes. This we leave for Sec. IV.

Turning to the amplitude for single pion photoproduction, i.e., $\pi+\mathrm{B}\leftarrow\mathrm{B}+\gamma$, we can classify the diagrams that contribute to this amplitude into two classes: those that are one-particle irreducible, which we denote by $\tilde{t}^{(1)}$, and those that are one-particle reducible. The one-particle reducible diagrams can be further divided into two classes: those of order e, i.e., first order in the electromagnetic coupling, and the rest. Those of order e can be written using the last-cut-lemma as $f^{(0)\dagger}d_0\tilde{f}^{(1)}=f^{(1)\dagger}d_{\mathrm{B}}\tilde{f}^{(1)}$. Here, $\tilde{f}^{(1)}$ is the one-particle irreducible amplitude for $\mathrm{B}\leftarrow\gamma\mathrm{B}$. If we neglect the higher order terms in the electromagnetic coupling, then

$$\tilde{t}^{(0)}=\tilde{t}^{(1)}+f^{(0)\dagger}d_0\tilde{f}^{(1)}=\tilde{t}^{(1)}+f^{(1)\dagger}d_B\tilde{f}^{(1)}\ . \tag{3.7}$$

In writing the second equation we have made use of the fact that $f^{(0)\dagger}d_0=f^{(1)\dagger}d_{\mathrm{B}}$ (Ref. 28). In Eq. (3.7) we have divided the photoproduction ampltiude into a pole or resonance part and a nonpole part. Since d_{B} is the dressed baryon propagator, it includes not only the nucleon but the Δ and possibly the Roper resonance (provided the bare Roper is included in terms of its bare three-quark structure). The one-particle irreducible $\pi\mathrm{B}\leftarrow\mathrm{B}$ amplitude $f^{(1)\dagger}$, in the absence of radiative correction, is given by Eq. (3.3).

Turning to the one-particle irreducible photoproduction amplitude $\tilde{t}^{(1)}$, we can classify the diagrams that contribute to this amplitude into two groups: (i) those that are two-particle irreducible which we denote by $\tilde{t}^{(2)}$, and (ii) the two-particle reducible diagrams that contribute to $\tilde{t}^{(1)}$. Here again, if we maintain terms that are of first order in the electromagnetic coupling, the two-particle reducible diagrams are of the form $t^{(1)}g\tilde{t}^{(2)}$, where g is the $\pi\mathrm{B}$ propagator and $t^{(1)}$ is the $\pi-\mathrm{B}$ amplitude that is a solution of Eq. (3.2). Thus, to first order in the electromagnetic coupling, we have

$$\begin{aligned}\tilde{t}^{(1)}&=\tilde{t}^{(2)}+t^{(1)}g\tilde{t}^{(2)}\\&=\tilde{t}^{(2)}+t^{(2)}g\tilde{t}^{(1)}\ .\end{aligned} \tag{3.8}$$

The second of these equations demonstrates that $\tilde{t}^{(1)}$ includes the contribution from the $\pi-\mathrm{B}$ unitarity cut.

To complete our definition of the photoproduction amplitude, we need to examine the one-particle irreducible amplitude for $\mathrm{B}\leftarrow\gamma\mathrm{B}$, $\tilde{f}^{(1)}$. The diagrams that contribute to this amplitude are divided into two classes: those that are two-particle irreducible we denote by $\tilde{f}^{(2)}$, while the two particle reducible diagrams can again be divided into two groups. Those of order e can be written using the last-cut-lemma as $f^{(1)}g\tilde{t}^{(2)}=f^{(2)}g\tilde{t}^{(1)}$. The rest of the diagrams, which are of higher order in the electromagnetic coupling, and include all radiative corrections to this amplitude, we neglect. Thus, to lowest order in the electromagnetic coupling, we have

$$\tilde{f}^{(1)}=\tilde{f}^{(2)}+f^{(1)}g\tilde{t}^{(2)}=\tilde{f}^{(2)}+f^{(2)}g\tilde{t}^{(1)}\ . \tag{3.9}$$

In Eqs. (3.7)–(3.9), which are illustrated diagrammatically in Fig. 2, we have the amplitude for single pion photoproduction $\tilde{t}^{(0)}$ expressed in terms of the two particle irreducible amplitudes $f^{(2)}$ for $\mathrm{B}\leftarrow\pi\mathrm{B}$, $\tilde{f}^{(2)}$ for $\mathrm{B}\leftarrow\gamma\mathrm{B}$, $t^{(2)}$ for $\pi\mathrm{B}\leftarrow\pi\mathrm{B}$, and $\tilde{t}^{(2)}$ for $\pi\mathrm{B}\leftarrow\gamma\mathrm{B}$. Although these amplitudes can be defined, at this stage, in terms of the corresponding elements of the interaction Hamiltonian in Eq. (2.16), we will show in Sec. IV the advantages of exposing the three-body unitarity cuts. In particular, we find that $t^{(2)}$ and $\tilde{t}^{(2)}$ includes more than one expects, from just taking the matrix elements of $\tilde{H}_I$.

The above result can be recast in terms of a distorted wave Born approximation. This is achieved by using Eqs. (3.8) and (3.9) in Eq. (3.7) to get

$$\tilde{t}^{(0)}=(t^{(0)}g+1)\tilde{t}^{(2)}+f^{(1)\dagger}d_{\mathrm{B}}\tilde{f}^{(2)}\ . \tag{3.10}$$

The second term on the right-hand side (rhs) can be recast using Eqs. (3.1), (3.3), and (3.4) to the form

$$f^{(1)\dagger}d_{\mathrm{B}}\tilde{f}^{(2)}=(t^{(0)}g+1)f^{(2)\dagger}d_0\tilde{f}^{(2)}\ . \tag{3.11}$$

Combining the results of Eqs. (3.10) and (3.11), we get

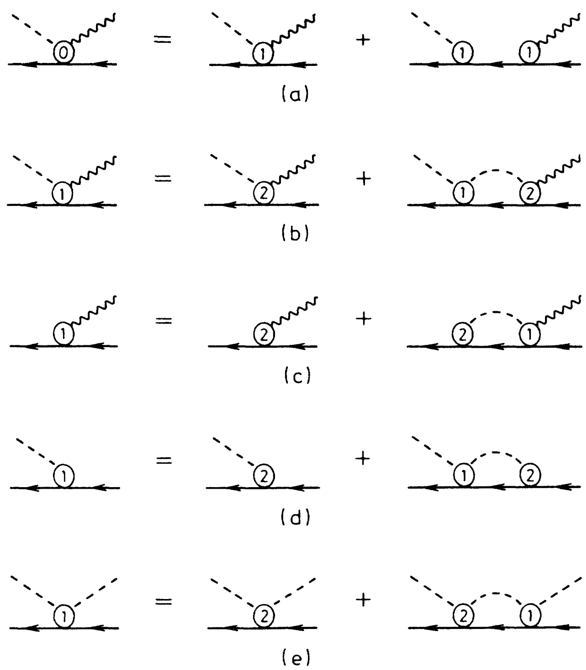

FIG. 2. A diagrammatic representation of the equations to be solved for pion photoproduction. The numbers in the circle represent the irreducibility of each amplitude. Here (a) is Eq. (3.7), (b) is Eq. (3.8), (c) is Eq. (3.9), (d) is Eq. (3.3), and (e) is Eq. (3.2).

$$\tilde{t}^{(0)}=(t^{(0)}g+1)\tilde{v} \ , \tag{3.12}$$

where

$$\tilde{v}=\tilde{t}^{(2)}+f^{(2)\dagger}d_0\tilde{f}^{(2)} \ . \tag{3.13}$$

The basic results in this section are similar to those of Tanabe and Ohta.[12] However, some of the details are different, particularly since we start from a Lagrangian and use that Lagrangian to determine all the free parameters of the theory. Numerical results for the $\pi-$B sector[21] give a good description of p-wave scattering and we expect this to carry over to the photoproduction. At this level, $t^{(2)}$ and $\tilde{t}^{(2)}$ are not uniquely determined by the Lagrangian. Their determination will require the examination of three-body unitarity, which is the subject of the next section.

IV. THREE-BODY UNITARITY

In Sec. III we showed that the amplitude for $\pi-$B elastic scattering and pion photoproduction can be written in terms of the two particle irreducible amplitudes $\Sigma^{(2)}$ for B$\leftarrow$B, $f^{(2)}$, for B$\leftarrow$Bπ, $\tilde{f}^{(2)}$ for B$\leftarrow$Bγ, $t^{(2)}$ for πB$\leftarrow\pi$B, and $\tilde{t}^{(2)}$ for πB$\leftarrow\gamma$B. The determination of these amplitudes in terms of the interaction Hamiltonian in Eq. (2.16) will require the exposure of three-body intermediate states. To this end, we use the last-cut-lemma to expose the three-body unitarity cut for $\Sigma^{(2)}$. This gives

$$\Sigma^{(2)}=\Sigma^{(3)}+\Gamma^{(3)}G^{(2)}\Gamma^{(2)\dagger}+\tilde{\Gamma}^{(3)}\tilde{G}^{(2)}\tilde{\Gamma}^{(2)\dagger} \ , \tag{4.1}$$

where $G^{(2)}=d_\pi d_\pi d_{\rm B}$ and $\tilde{G}^{(2)}=d_\gamma d_\pi d_{\rm B}$. Here, $\Gamma^{(3)}$, $(\tilde{\Gamma}^{(3)})$ and $\Sigma^{(3)}$ are the three-particle irreducible amplitudes for B$\leftarrow\pi\pi$B (B$\leftarrow\gamma\pi$B) and B$\leftarrow$B. Note that in writng Eq. (4.1) we have neglected two photon intermediate states. In fact, for the Hamiltonian under consideration, and taking only the diagrams that are to lowest order in the electromagnetic coupling, the last term on the rhs of Eq. (4.1) should not be included. Both $\Gamma^{(3)}$ and $\tilde{\Gamma}^{(3)}$ involve the coupling of the B to the $\pi\pi$B and $\gamma\pi$B Hilbert spaces, respectively, with at least four-particle intermediate states. Since there are no terms that directly couple the B with $\pi\pi\pi$B, $\gamma\pi\pi$B, $\gamma\gamma\pi$B, or $\gamma\gamma\gamma$B Hilbert spaces in the Hamiltonian, then $\Gamma^{(3)}=\langle {\rm B}\,|\,\hat{H}\,|\,\pi\pi{\rm B}\rangle$ and $\tilde{\Gamma}^{(3)}=\langle {\rm B}\,|\,\hat{H}\,|\,\gamma\pi{\rm B}\rangle$. But such terms are absent from the interaction Hamiltonian in Eq. (2.6) and thus $\Gamma^{(3)}=\tilde{\Gamma}^{(3)}=0$. We therefore have $\Sigma^{(2)}=\Sigma^{(3)}$. In a similar manner we can show that $\Sigma^{(2)}=\Sigma^{(3)}=\Sigma^{(4)}=\cdots=\langle {\rm B}\,|\,\hat{H}\,|\,{\rm B}\rangle$. The last equality results from the contribution of diagrams with *no* intermediate states. Thus, in the absence of any baryon counter terms in $\hat{H}_I$ we have $\Sigma^{(2)}=0$.

In a similar manner, if we apply the last-cut-lemma to the two-particle irreducible amplitude for B$\leftarrow\pi$B, we obtain

$$\begin{aligned} f^{(2)}&=f^{(3)}+\Gamma^{(3)}G^{(2)}F^{(2)\dagger}+\tilde{\Gamma}^{(3)}\tilde{G}^{(2)}\tilde{F}_3^{(2)\dagger} \\ &=f^{(3)}+\Gamma^{(2)}G^{(2)}F^{(3)\dagger}+\tilde{\Gamma}^{(2)}\tilde{G}^{(2)}\tilde{F}_3^{(3)\dagger} \ , \end{aligned} \tag{4.2}$$

where $F^{(n)\dagger}(\tilde{F}_3^{(n)\dagger})$ is the n-particle irreducible amplitude for $\pi\pi$B$\leftarrow\pi$B ($\gamma\pi$B$\leftarrow\pi$B). Here again, the last term on the rhs does not contribute if we only maintain terms to first order in the electromagnetic coupling. Since both $\Gamma^{(3)}$ and $\tilde{\Gamma}^{(3)}$ are zero, as shown above, then $f^{(2)}=f^{(3)}$. In this way we can show that $f^{(2)}=f^{(3)}=f^{(4)}=\cdots=\langle {\rm B}\,|\,\hat{H}\,|\,\pi{\rm B}\rangle$. Here again, the last equality is the result of the contribution from diagrams with *no* intermediate states. In other words, the two-body irreducible amplitude for B$\leftarrow\pi$B is in fact the πBB vertex in the interaction Hamiltonian.

We now turn to the two-particle irreducible B$\leftarrow\gamma$B amplitude, $\tilde{f}^{(2)}$. With the application of the last-cut-lemma, this can be written as

$$\tilde{f}^{(2)}=\tilde{f}^{(3)}+\Gamma^{(2)}G^{(2)}\tilde{F}_1^{(3)\dagger}+\tilde{\Gamma}^{(2)}\tilde{G}^{(2)}\tilde{F}_2^{(3)\dagger} \ , \tag{4.3}$$

where $\tilde{F}_1^{(n)}$ $(\tilde{F}_2^{(n)})$ is the n-particle irreducible amplitude for γB$\leftarrow\pi\pi$B (γB$\leftarrow\gamma\pi$B). By classifying the diagrams that contribute to $\Gamma^{(2)}$ and $\tilde{\Gamma}^{(2)}$ according to their irreducibility, and using the last-cut-lemma, we can show that $\Gamma^{(2)}\propto\Gamma^{(3)}$ and $\tilde{\Gamma}^{(2)}\propto\tilde{\Gamma}^{(3)}$. Since both $\Gamma^{(3)}$ and $\tilde{\Gamma}^{(3)}$ are zero, we have that $\tilde{f}^{(2)}=\tilde{f}^{(3)}$. Repeating this procedure will give us $\tilde{f}^{(2)}=\tilde{f}^{(3)}=\tilde{f}^{(4)}=\cdots=\langle {\rm B}\,|\,\hat{H}\,|\,\gamma{\rm B}\rangle$. In this way we have related the two-particle irreducible amplitudes $\Sigma^{(2)}$, $f^{(2)}$, and $\tilde{f}^{(2)}$ to the terms in the interaction Hamiltonian given in Eq. (2.16). In particular, we have shown that three-body unitarity has no contribution to these amplitudes for the $\hat{H}_I$ under consideration.

In the absence of coupling to the photon channels, the contribution of three-body unitarity to $t^{(2)}$ has been examined in detail by Afnan and Pearce.[22] Here, we summarize their results for completeness. Making use of the last-cut-lemma, we can expose the three-particle intermediate states for $t^{(2)}$ to get

$$t^{(2)}=t^{(3)}+\{F^{(3)}G^{(2)}F^{(2)\dagger}\}_c$$
$$=t^{(3)}+\{F^{(2)}G^{(2)}F^{(3)\dagger}\}_c \ , \tag{4.4}$$

where the subscript c implies that only connected diagrams are to be included when the last-cut-lemma is applied. Making use of the fact that $F^{(n)}$ has both connected and disconnected parts, i.e.,

$$F^{(n)}=F_c^{(n)}+F_d^{(n)} \ , \tag{4.5}$$

both of which contribute to the second term on the rhs of Eq. (4.4), we can write

$$F^{(2)}=F^{(3)}+F^{(3)}G^{(2)}M^{(2)}=F^{(3)}+F^{(2)}G^{(2)}M^{(3)} \ , \tag{4.6}$$

where $M^{(n)}$ is the n-particle irreducible $3\rightarrow3$ amplitude for the reaction $\pi\pi\mathrm{B}\rightarrow\pi\pi\mathrm{B}$. Here again the $3\rightarrow3$ amplitude can be divided into connected and disconnected parts, i.e.,

$$M^{(n)}=M_c^{(n)}+M_d^{(n)} \ . \tag{4.7}$$

If we now take $M_c^{(3)}=0$, i.e., no three-body forces, then the $3\rightarrow3$ amplitude $M^{(2)}$ can be written in terms of the Alt-Grassberger-Sandhas[30] (AGS) amplitude $U_{\alpha\beta}$, as

$$M^{(2)}=\sum_{\alpha\beta}[M_d^{(2)}(\alpha)\delta_{\alpha\beta}+M_d^{(2)}(\alpha)G^{(2)}U_{\alpha\beta}G^{(2)}M_d^{(2)}(\beta)] \ , \tag{4.8}$$

where we have made use of the fact that

$$M_d^{(2)}=\sum_{\alpha}M_d^{(2)}(\alpha) \tag{4.9}$$

with

$$M_d^{(2)}(\alpha)=\sum_i\bar{\delta}_{ij}d_\pi^{-1}(i)t^{(1)}(j) \quad \text{for } \alpha=j=1,2 \ ,$$
$$=d_{\mathrm{B}}^{-1}t^{(1)}(3) \quad \text{for } \alpha=3 \ . \tag{4.10}$$

Here $t^{(1)}(j)$, $j=1,2$, is the one-particle irreducible π-B elastic amplitude, while $t^{(1)}(3)$ is the $\pi-\pi$ one-particle irreducible amplitude. In writing Eqs. (4.8)–(4.10), we have labeled particles 1 and 2 to be pions while particle 3 is the baryon. We have also used the convention that $t(j)$ is the interaction of the jth pion with the baryon. If we now substitute Eq. (4.6) into Eq. (4.4), make use of the fact that

$$F_d^{(n)}=\sum_{j=1,2}F_d^{(n)}(j)=\sum_{ij}\bar{\delta}_{ij}d_\pi^{-1}(i)f^{(n-1)}(j) \ , \tag{4.11}$$

and use the AGS equations[30]

$$U_{\alpha\beta}=G^{(2)-1}\bar{\delta}_{\alpha\beta}+\sum_\gamma\bar{\delta}_{\alpha\gamma}M_d^{(2)}(\gamma)G^{(2)}U_{\gamma\beta}$$
$$=G^{(2)-1}\bar{\delta}_{\alpha\beta}+\sum_\gamma U_{\alpha\gamma}G^{(2)}M_d^{(2)}(\gamma)\bar{\delta}_{\gamma\beta} \ , \tag{4.12}$$

to regroup the resultant multiple scattering series, we obtain

$$t^{(2)}=t^{(3)}+\sum_{ij}F_d^{(2)}(i)G^{(2)}U_{ij}G^{(2)}F_d^{(2)\dagger}(j) \ , \tag{4.13}$$

where $\bar{\delta}_{\alpha\beta}=1-\delta_{\alpha\beta}$. Here, and through the rest of this paper, the $i,j,\ldots$ sum runs over 1 and 2, while $\alpha,\beta,\gamma,\ldots$ runs over 1, 2, and 3. A diagrammatic representation of Eq. (4.13), in its multiple scattering series form, is given in Fig. 3. The first term $t^{(3)}$ is determined by the contact term resulting from $\mathcal{L}_{qq\pi\pi}$ in Eq. (2.5). The second term in the series in Fig. 3 has the u-channel baryon pole (i.e., the crossed pion diagram) with the special feature that the πBB vertices are one-particle irreducible, as compared to the s-channel pole term which has two-particle irreducible vertices at the potential level. It is only at the amplitude level (i.e., after dressing the πBB vertices and baryon propagator) that the s-channel pole term has one-particle irreducible vertices. Here, we observe that if we take $t^{(1)}=t^{(2)}$, then the final amplitude $t^{(0)}$ is crossing symmetric. The third term in the series in Fig. 3 gives the dressing to the contact term $t^{(3)}=\langle \mathrm{B}\pi|\hat{H}|\mathrm{B}\pi\rangle$. The other terms in the series give contributions from multiple scattering including the effect of the π-π interaction. To include the full contribution of the multiple scattering series, we need to solve a three-body problem that couples the πB to the $\pi\pi$B channels.

If we now turn on the coupling to the photon channels, then the only contributions are of second order or higher in the electromagnetic coupling. This is clearly the case as both our initial and final states in $t^{(2)}$ are π-B channels. To maintain consistency with Sec. III we will neglect the contribution of the coupling to the photon channels in $t^{(2)}$.

Finally, we turn to the two-particle irreducible amplitude for $\pi\mathrm{B}\leftarrow\gamma\mathrm{B}$, $\tilde{t}^{(2)}$. The diagrams that contribute to this amplitude are either three-particle irreducible, which we denote by $\tilde{t}^{(3)}$, or three-particle reducible. Using the last-cut-lemma we can write the three-particle reducible diagrams as

$$\{F^{(2)}G^{(2)}\tilde{F}_1^{(3)\dagger}\}_c+\{\tilde{F}_3^{(2)}\tilde{G}^{(2)}\tilde{F}_2^{(3)\dagger}\}_c \ . \tag{4.14}$$

In writing this expression we have neglected the intermediate states with two or more photons, on the grounds that they are of higher order in the electromagnetic cou-

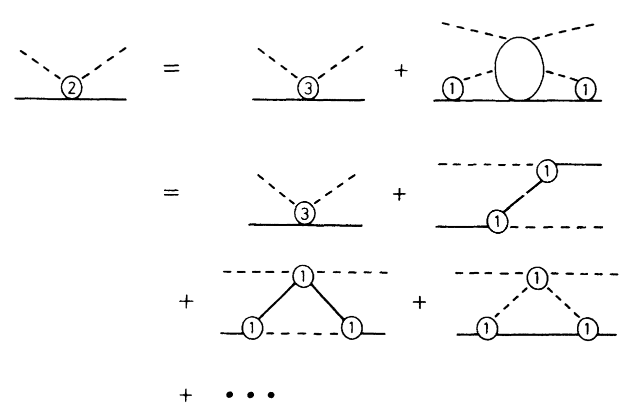

FIG. 3. Diagrammatic representation of Eq. (4.13), and the lowest order multiple scattering contribution to $t^{(2)}$.

pling. The three-particle irreducible amplitudes $\tilde F_1^{(3)}$ and $\tilde F_2^{(3)}$ consist of connected and disconnected parts, both of which can contribute to the above expression. The connected parts of $\tilde F_i^{(3)}$ $(i=1,2)$ are zero because there is no direct coupling between the γB and either the $\pi\pi$B or $\gamma\pi$B channels. Also, there is no direct coupling to the four particle intermediate states via $\hat H_I$. Thus the only contribution to $\tilde F_i^{(3)}$ $(i=1,2)$ comes from the disconnected part. If we only maintain the terms up to first order in the electromagnetic coupling then

$$\tilde F_1^{(3)\dagger}=\tilde F_{1,d}^{(3)\dagger}=d_{\rm B}^{-1}\langle \pi\pi\,|\,\hat H_I\,|\,\gamma\rangle \equiv d_{\rm B}^{-1}\tilde f_a^{(2)\dagger} \tag{4.15}$$

and

$$\tilde F_2^{(3)\dagger}=\tilde F_{2,d}^{(3)\dagger}=d_{\gamma}^{-1}\langle \pi{\rm B}\,|\,\hat H_I\,|\,{\rm B}\rangle \equiv d_{\gamma}^{-1} f^{(2)\dagger} \ . \tag{4.16}$$

We now can write the two-particle irreducible amplitude $\tilde t^{(2)}$, as

$$\tilde t^{(2)}=\tilde t^{(3)}+\{F^{(2)}G^{(2)}\tilde F_{1,d}^{(3)\dagger}\}_c+\{\tilde F_3^{(2)}\tilde G^{(2)}\tilde F_{2,d}^{(3)\dagger}\}_c \ . \tag{4.17}$$

To fully expose the contribution of three-body unitarity, we need to write the two-particle irreducible amplitudes $F^{(2)}$ and $\tilde F_3^{(2)}$ in terms of the basic interaction Hamiltonian $\hat H_I$. For $F^{(2)}$ this was done in the absence of coupling to the photon channels by Afnan and Pearce,[22] and the results are summarized in Eqs. (4.6) and (4.8). The inclusion of coupling to the electromagnetic interaction to first order will not change this result since both the initial and final channels involve only pions and baryons. Thus, the second term on the rhs of Eq. (4.17) is of the form

$$\{F^{(2)}G^{(2)}\tilde F_{1,d}^{(3)\dagger}\}_c=\{F^{(3)}G^{(2)}\tilde F_{1,d}^{(3)\dagger}\}_c+\{F^{(3)}G^{(2)}M^{(2)}G^{(2)}\tilde F_{1,d}^{(3)\dagger}\}_c \ . \tag{4.18}$$

Because $F^{(3)}$ is the amplitude for $\pi{\rm B}\leftarrow\pi\pi{\rm B}$, with at least four-body intermediate states, and there are no terms in the Hamiltonian that couples the πB and $\pi\pi\pi$B spaces, we have that $F^{(3)}=F_d^{(3)}$. Furthermore, because of the fact that the initial photon creates two pions through $\tilde F_{1,d}^{(3)\dagger}$ and one of these pions gets absorbed by the baryon via $F^{(3)}$, we can drop the subscript c and write our final result as

$$\{F^{(2)}G^{(2)}\tilde F_{1,2}^{(3)\dagger}\}_c=\sum_i F_d^{(3)}(i)G^{(2)}\tilde F_{1,d}^{(3)\dagger}+\sum_{i\alpha}F_d^{(3)}(i)G^{(2)}M_d^{(2)}(\alpha)G^{(2)}\tilde F_{1,d}^{(3)\dagger}+\sum_{i\alpha\beta}F_d^{(3)}(i)G^{(2)}M_d^{(2)}(\alpha)G^{(2)}U_{\alpha\beta}G^{(2)}M_d^{(2)}(\beta)G^{(2)}\tilde F_{1,d}^{(3)\dagger} \ . \tag{4.19}$$

Here we note that the first term on the rhs can be combined with the second term for $\alpha=i$, to replace $F_d^{(3)}(i)$ by $F_d^{(2)}(i)$. This can also be achieved for the $F_d^{(3)}(i)$ in the third term, using the AGS equations for $U_{\alpha\beta}$. In this way we can replace $F_d^{(3)}(i)$ by $F_d^{(2)}(i)$ in Eq. (4.19) to get

$$\{F^{(2)}G^{(2)}\tilde F_{1,d}^{(3)\dagger}\}_c=\sum_i F_d^{(2)}(i)G^{(2)}\tilde F_{1,d}^{(3)\dagger}+\sum_{i\alpha}F_d^{(2)}(i)G^{(2)}U_{i\alpha}G^{(2)}M_d^{(2)}(\alpha)G^{(2)}\tilde F_{1,d}^{(3)\dagger} \ . \tag{4.20}$$

This result can be further simplified by dressing the $\pi\pi\leftarrow\gamma$ vertex, $\tilde F_{1,d}^{(3)\dagger}$. Making use of the AGS equation and the fact that

$$\tilde F_{1,d}^{(2)\dagger}=d_{\rm B}^{-1}\tilde f_a^{(1)\dagger} \tag{4.21}$$

with

$$\tilde f_a^{(1)\dagger}=\tilde f_a^{(2)\dagger}+t^{(1)}(3)d_\pi(1)d_\pi(2)\tilde f_a^{(2)\dagger} \ , \tag{4.22}$$

we can rewrite Eq. (4.20) as

$$\{F^{(2)}G^{(2)}\tilde F_{1,d}^{(3)\dagger}\}_c=\sum_i F_d^{(2)}(i)G^{(2)}U_{i3}G^{(2)}\tilde F_{1,d}^{(2)\dagger} \ . \tag{4.23}$$

Now, by iterating the AGS equation, we can generate a multiple scattering series for the contribution of this class of diagrams to $\tilde t^{(2)}$. In Fig. 4 we present diagrammatically the first few terms in the series for $\{F^{(2)}G^{(2)}\tilde F_{1,d}^{(3)\dagger}\}_c$. Note that in Eq. (4.23) only the one-particle irreducible amplitudes and vertices are included.

We now turn to the last term on the rhs of Eq. (4.17). Here, we have to examine the two-particle irreducible amplitude for $\pi{\rm B}\leftarrow\gamma\pi{\rm B}$, $\tilde F_3^{(2)}$. This consists of two parts—a connected and a disconnected part, i.e.,

$$\tilde F_3^{(2)}=\tilde F_{3,c}^{(2)}+\tilde F_{3,d}^{(2)} \ , \tag{4.24}$$

where the disconnected part is given by

$$\tilde F_{3,d}^{(2)}=d_{\rm B}^{-1}\tilde f_b^{(1)}+d_\pi^{-1}\tilde f^{(1)} \ , \tag{4.25}$$

with $\tilde f^{(1)}$ given by Eq. (3.9) and $\tilde f_b^{(1)}$, the $\pi\leftarrow\gamma\pi$ amplitude, given in terms of the interaction Hamiltonian by

$$\tilde f_b^{(1)}=\tilde f_b^{(2)}=\langle \pi\,|\,\hat H_I\,|\,\gamma\pi\rangle \ . \tag{4.26}$$

The first equality in Eq. (4.26) is the result of including the electromagnetic interaction to first order only.

We now turn to the class of connected diagrams that contribute to the $\pi{\rm B}\leftarrow\gamma\pi{\rm B}$ amplitude, $\tilde F_{3,c}^{(2)}$. These diagrams can be divided into two classes: those that are three-particle irreducible which we denote by $\tilde F_{3,c}^{(3)}$. To the rest of the diagrams that contribute to $\tilde F_{3,c}^{(2)}$, and are

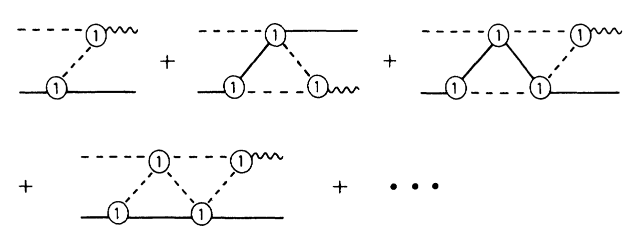

FIG. 4. The lowest order multiple scattering contribution to $\{F^{(2)}G^{(2)}\tilde F_{1,d}^{(3)\dagger}\}_c$. See Eq. (4.23).

three-particle reducible, we apply the last-cut-lemma to get

$$\tilde{F}^{(2)}_{3,c}=\tilde{F}^{(3)}_{3,c}+\{F^{(2)}G^{(2)}\tilde{M}^{(3)}_{A}\}_c+\{\tilde{F}^{(2)}_{3}\tilde{G}^{(2)}\tilde{M}^{(3)}_{B}\}_c \quad (4.27a)$$

$$=\tilde{F}^{(3)}_{3,c}+\{F^{(3)}G^{(2)}\tilde{M}^{(2)}_{A}\}_c+\{\tilde{F}^{(3)}_{3}\tilde{G}^{(2)}\tilde{M}^{(2)}_{B}\}_c \ . \quad (4.27b)$$

In writing Eq. (4.27) we have dropped the intermediate states with two photons as they would only result from including the electromagnetic interaction in second or higher order. Here, $\tilde{M}^{(n)}_{A}$ and $\tilde{M}^{(n)}_{B}$ are the n-particle irreducible amplitudes for the reactions $\pi\pi B\leftarrow\gamma\pi B$ and $\gamma\pi B\leftarrow\gamma\pi B$, respectively. To lowest order in the electromagnetic coupling, the amplitude $\tilde{M}^{(n)}_{B}$ is disconnected, i.e.,

$$\tilde{M}^{(n)}_{B}=\tilde{M}^{(n)}_{B,d}=d_{\gamma}^{-1}t^{(n-1)} \ . \quad (4.28)$$

On the other hand, the $\pi\pi B\leftarrow\gamma\pi B$ amplitude $\tilde{M}^{(2)}_{A}$ has both a connected and a disconnected part, i.e.,

$$\tilde{M}^{(2)}_{A}=\tilde{M}^{(2)}_{A,c}+\tilde{M}^{(2)}_{A,d}=\tilde{M}^{(2)}_{A,c}+d_{\pi}^{-1}\,\tilde{t}^{(1)} \ . \quad (4.29)$$

The connected amplitude $\tilde{M}^{(2)}_{A,c}$ for $\pi\pi B\leftarrow\gamma\pi B$ should be written, to first order in the electromagnetic coupling, in terms of the AGS amplitudes $U_{\alpha\beta}$, the basic two-body amplitude $t^{(1)}(\alpha)$, $\alpha=1,2,3$, and $\tilde{t}^{(1)}(i)$, $i=1,2$. This can be achieved by classifying the diagrams that contribute to $\tilde{M}^{(2)}_{A,c}$ according to their irreducibility using the last-cut-lemma. This gives us

$$\tilde{M}^{(2)}_{A,c}=\tilde{M}^{(3)}_{A,c}+\{M^{(3)}G^{(2)}\tilde{M}^{(2)}_{A}\}_c+\{\tilde{M}^{(3)}_{A}\tilde{G}^{(2)}\tilde{M}^{(2)}_{B}\}_c$$

$$=\tilde{M}^{(3)}_{A,c}+\{M^{(2)}G^{(2)}\tilde{M}^{(3)}_{A}\}_c+\{\tilde{M}^{(2)}_{A}\tilde{G}^{(2)}\tilde{M}^{(3)}_{B}\}_c \ . \quad (4.30b)$$

Here again we have limited our analysis to exclude two or more photon intermediate states on the grounds that they are the result of including the electromagnetic interaction to second or higher order. Because our interaction Hamiltonian does not admit the coupling between n and $n+2$ pion states, $\tilde{M}^{(3)}_{A,c}=0$.

In writing Eq. (4.30), we have chosen two different implementations of the last-cut-lemma. In Eq. (4.30a) we have used the lemma to expose the last (furthest to the left) three-particle intermediate state. In this case we get a set of coupled equations for the two-particle irreducible $\pi\pi B\leftarrow\gamma\pi B$ amplitude, $\tilde{M}^{(2)}_{A,c}$. The second decomposition, corresponding to Eq. (4.30b), will give us the amplitude $\tilde{M}^{(2)}_{A,c}$ in terms of the AGS amplitude $U_{\alpha\beta}$ for the $\pi\pi N$ three-body system. Making use of Eqs. (4.8), (4.28), and (4.29), and after some algebra that involves replacing $\tilde{t}^{(2)}(i)$ by $\tilde{t}^{(1)}(i)$ with the help of the AGS equations (4.12), we can write Eq. (4.30b) as

$$\tilde{M}^{(2)}_{A,c}=\sum_{ij}\tilde{t}^{(1)}(i)d_{B}\bar{\delta}_{ij}t^{(2)}(j)+\tilde{M}^{(2)}_{A,c}\,\tilde{G}^{(2)}d_{\gamma}^{-1}t^{(2)}+\sum_{\alpha j}M_d^{(2)}(\alpha)G^{(2)}U_{\alpha j}d_{\pi}(j)d_{B}\tilde{t}^{(1)}(j) \ . \quad (4.31)$$

Here $i,j=1,2$, while $\alpha=1,2,3$. This equation can be solved formally for $\tilde{M}^{(2)}_{A,c}$ to give

$$\tilde{M}^{(2)}_{A,c}=\sum_{ij}\tilde{t}^{(1)}(i)d_{B}\bar{\delta}_{ij}t^{(1)}(j)+\sum_{\alpha ij}M_d^{(2)}(\alpha)G^{(2)}U_{\alpha i}d_{\pi}(i)d_{B}\tilde{t}^{(1)}(i)\bar{\delta}_{ij}(1+d_{\pi}(j)d_{B}t^{(1)}(j)) \ . \quad (4.32)$$

In this way we have written the amplitude $\tilde{M}^{(2)}_{A,c}$ for ($\pi\pi B\leftarrow\gamma\pi B$), in terms of the AGS amplitude $U_{\alpha i}$ and the two-body one-particle irreducible amplitudes $t^{(1)}$ and $\tilde{t}^{(1)}$.

The results of Eqs. (4.28), (4.29), and (4.32) will enable us to write the two-particle irreducible amplitude for $\pi B\leftarrow\gamma\pi B$, $\tilde{F}^{(2)}_{3,c}$, in Eq. (4.27) as

$$\tilde{F}^{(2)}_{3,c}=\tilde{f}^{(2)}_{b}d_{\pi}t^{(1)}+\tilde{f}^{(1)}d_{B}t^{(1)}+\sum_{ijk}f^{(1)}(i)d_{\pi}(i)d_{B}U_{ij}d_{\pi}(j)d_{B}\tilde{t}^{(1)}(j)\bar{\delta}_{jk}(1+d_{\pi}(k)d_{B}t^{(1)}(k)) \ . \quad (4.33)$$

This in turn will allow us to write the last term on the rhs of Eq. (4.17) as

$$\{\tilde{F}^{(2)}_{3}\tilde{G}^{(2)}\tilde{F}^{(3)\dagger}_{2,d}\}_c=\tilde{f}^{(1)}_{b}d_{\pi}f^{(1)\dagger}+\tilde{f}^{(1)}d_{B}f^{(1)\dagger}+\sum_{ijk}f^{(1)}(i)d_{\pi}(i)d_{B}U_{ij}d_{\pi}(j)d_{B}\tilde{t}^{(1)}(j)\bar{\delta}_{jk}d_{\pi}(k)d_{B}f^{(1)\dagger}(k) \ . \quad (4.34)$$

In Fig. 5, we illustrate the lowest order contributions to $\{\tilde{F}^{(2)}_{3}\tilde{G}^{(2)}\tilde{F}^{(3)\dagger}_{2,d}\}$. We now can combine the results of Eqs. (4.23) and (4.34) to get

$$\tilde{t}^{(2)}=\tilde{t}^{(3)}+\tilde{f}^{(1)}_{b}d_{\pi}f^{(1)\dagger}+\tilde{f}^{(1)}d_{B}f^{(1)\dagger}+\sum_{i}F_d^{(2)}(i)G^{(2)}U_{i3}G^{(2)}\tilde{F}^{(2)\dagger}_{1,d}+\sum_{ij}F_d^{(2)}(i)G^{(2)}U_{ij}G^{(2)}\tilde{M}^{(2)}_{A,d}(j)\tilde{G}^{(2)}\tilde{F}^{(2)\dagger}_{2,d} \ . \quad (4.35)$$

From the above result we see that the amplitude $\tilde{t}^{(2)}$, which plays the role of a potential in Eq. (3.8) and has a t-channel pole, is written in terms of the dressed (i.e., one-particle irreducible) form factors $f^{(1)}$, $\tilde{f}^{(1)}$, $\tilde{f}^{(1)}_{b}$, and $\tilde{f}^{(1)}_{a}$. This is similar to the situation for pion elastic scattering where $t^{(2)}$ (which includes the crossed diagram) is written in terms of the dressed πNN vertex. Finally, if we examine the lowest order contributions to $\tilde{t}^{(2)}$, as illustrated in Figs. 4 and 5, we observe that the contact term ($\tilde{t}^{(3)}=\langle B\pi\,|\,\hat{H}\,|\,B\gamma\rangle$) gets also dressed, due to the contribution from the third term in Fig. 5.

The analysis in this section has shown us how we can write the basic input to the equations in Sec. III, in terms of the interaction Hamiltonian $\hat{H}_I$. In addition, we find that a proper treatment of the multiple scattering series within the framework of Faddeev-AGS theory requires

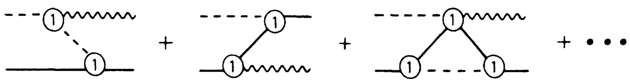

FIG. 5. The lowest order multiple scattering contribution to $\{\tilde{F}_3^{(2)}\tilde{G}^{(2)}\tilde{F}_{2,d}^{(3)\dagger}\}$. See Eq. (4.34).

that $\tilde{t}^{(2)}$ be written in terms of the one-particle irreducible two-body amplitude. This, to a certain extent, renders the problem to be nonlinear, in that we need to know $t^{(1)}$, $\tilde{t}^{(1)}$, $f^{(1)}$, and $\tilde{f}^{(1)}$ in order to calculate $t^{(2)}$ and $\tilde{t}^{(2)}$ which in turn are needed to calculate $t^{(1)}$ and $\tilde{t}^{(1)}$. This requirement of self-consistency could be overcome in the initial stage by parametrizing the amplitudes required to calculate $\tilde{t}^{(2)}$. In the event that we have a spectator pion in the amplitudes in $F_d^{(2)}$, $F_{1,d}^{(2)}$, $F_{2,d}^{(2)}$, $M_d^{(2)}$, and $\tilde{M}_d^{(2)}$, then the input amplitudes $\tilde{t}^{(1)}$, $t^{(1)}$ are at an energy $\omega=(p^2+m_\pi^2)^{1/2}$ lower than that at which $\tilde{t}^{(2)}$ is calculated. This, in a sense, removes part of the nonlinearity in the problem. However, we need to maintain self-consistency.

Finally, a comparison of the lowest order diagrams that contribute to $\tilde{t}^{(2)}$ (see Figs. 4 and 5), with the diagrams in Fig. 1, (the Born amplitudes) shows that the diagrams corresponding to Figs. 1(b)–1(e) have their vertices or amplitudes one-particle irreducible. On the other hand, the contribution corresponding to Fig. 1(a) has vertices that are two-particle reducible. This difference between Fig. 1(a) and the other diagrams in Fig. 1 has not been taken into consideration in the past.

V. THE VERTEX FUNCTIONS AND BORN TERMS

In the last section we showed that three-body unitarity in conjunction with a partial summation of the multiple scattering series determines the form of the nonpole Born terms $t^{(2)}$ and $\tilde{t}^{(2)}$, in terms of one-particle irreducible form factors and subamplitudes. These, in turn, can be written in terms of the interaction Hamiltonian H_I as given in Eq. (2.16). At this stage we as yet have no explicit form for the different matrix elements of $\hat{H}$ in terms of which $\hat{H}_I$ is written in Eq. (2.16). In this section we present the matrix elements of $\hat{H}$ by considering the Lagrangian in Eqs. (2.1)–(2.11). In particular, we need to get explicit expressions for $f^{(2)}$, $\tilde{f}^{(2)}$, $t^{(3)}$, and $\tilde{t}^{(3)}$.

To establish the normalization of our basis state, we write the S matrix for $\mathrm{B}'\leftarrow\pi\mathrm{B}$ in terms of $f^{(2)}$ as

$$\langle \mathrm{B}' \mid S \mid \mathrm{B}\pi(\mathbf{q})\rangle = i(2\pi)\delta^4(p'-p-q) f^{(2)}_{\mathrm{B'B}\pi}(\mathbf{q}) , \tag{5.1}$$

where

$$\begin{aligned}\langle m \mid f^{(2)}_{\mathrm{B'B}\pi}(\mathbf{q}) \mid n,\alpha\rangle &= -\langle m \mid \hat{H}^{PS}_{\mathrm{qq}\pi} \mid n,\alpha\mathbf{q}\rangle && (5.2\mathrm{a})\\ &= \int d^3x \langle \mathrm{B}'(m) \mid \hat{\mathcal{L}}^{PV}_{\mathrm{qq}\pi}(0) \mid \mathrm{B}(n);\pi_\alpha(\mathbf{q})\rangle . && (5.2\mathrm{b})\end{aligned}$$

The equivalence of Eqs. (5.2a) and (5.2b) can be shown by applying Gauss' theorem to the axial vector current, by using the Dirac equation for a massless quark and by employing the boundary condition that

$$i\gamma^\mu n_\mu q = q .$$

By inserting the MIT bag wave function and Eq. (2.6) into Eq. (5.2), we can derive

$$f^{(2)}_{\mathrm{B'B}\pi}(q) = \frac{i}{[(2\omega)(2\pi)^3]^{1/2}}\sqrt{4\pi} \times \frac{f_{\pi\mathrm{NN}}}{m_\pi}\frac{3j_1(qR)}{qR} C_{\mathrm{B'B}}\langle \mathrm{B}' \mid \mathbf{\Sigma}\cdot\mathbf{q}\mathcal{T} \mid \mathrm{B}\rangle \tag{5.3}$$

where $\mathbf{\Sigma}$ and $\mathcal{T}$, the spin and isospin operators, and $C_{\mathrm{B'B}}$, are summarized in Table I. The spin operators in Table I have matrix elements given in terms of the Clebsch-Gordan coefficients as

$$\begin{aligned}\langle \tfrac{1}{2}M' \mid \sigma_\mu \mid \tfrac{1}{2}M\rangle &= \sqrt{3}(\tfrac{1}{2}M,1\mu \mid \tfrac{1}{2}M') ,\\ \langle \tfrac{3}{2}M' \mid S_\mu \mid \tfrac{1}{2}M\rangle &= (\tfrac{1}{2}M,1\mu \mid \tfrac{3}{2}M') ,\\ \langle \tfrac{3}{2}M' \mid \sigma_{\Delta,\mu} \mid \tfrac{3}{2}M\rangle &= \sqrt{15}(\tfrac{3}{2}M,1\mu \mid \tfrac{3}{2}M') .\end{aligned} \tag{5.4}$$

The isospin operators τ, $\mathbf{T}$, and τ_Δ are also defined in exactly the same way.

Employing the same procedure, we can determine the photon-baryon vertex $\tilde{f}^{(2)}$, which is defined in terms of the baryon states by

$$\begin{aligned}\langle m \mid \tilde{f}^{(2)}_{\mathrm{B'B}\gamma}(\mathbf{k}) \mid n,\lambda\rangle &= \int d^3x \langle \mathrm{B}'(m) \mid \mathcal{L}_{\mathrm{qq}\gamma}(0) \mid \mathrm{B}(n);\gamma_\lambda(\mathbf{k})\rangle\\ &= \frac{e}{[(2k_0)(2\pi)^3]^{1/2}}\{\langle \mathrm{B}' \mid J_0 \mid \mathrm{B}\rangle\epsilon^0_\lambda - \langle \mathrm{B}' \mid \mathbf{J} \mid \mathrm{B}\rangle\cdot\epsilon_\lambda\} .\end{aligned} \tag{5.5}$$

The explicit forms of J_0 and $\mathbf{J}$ are given in Table II. In this table, the electric and magnetic form factors are defined as

$$G_E^N = \left[\frac{1+\tau_3}{2}\right] G_E^0 , \tag{5.6a}$$

$$G_M^N = \left[\frac{1+5\tau_3}{6}\right] G_M^0 , \tag{5.6b}$$

TABLE I. The explicit form of the operators $\mathbf{\Sigma}$ and $\mathcal{T}$ and the value of the coefficient $C_{\mathrm{B'B}}$ needed in Eq. (5.3), in the case when B,B′ are the N or Δ.

B′	B	$\mathbf{\Sigma}$	$\mathcal{T}$	$C_{\mathrm{B'B}}$
N	N	σ	τ	1
Δ	N	$\mathbf{S}$	$\mathbf{T}$	$\frac{6\sqrt{2}}{5}$
N	Δ	$\mathbf{S}^\dagger$	$\mathbf{T}^\dagger$	$\frac{6\sqrt{2}}{5}$
Δ	Δ	σ_Δ	τ_Δ	$\frac{1}{5}$

TABLE II. The explicit expression for the electromagnetic currents in Eq. (5.5) for the case when the baryon is an N or Δ.

B′	B	$\langle B' \mid J_0 \mid B\rangle$	$\langle B' \mid \mathbf{J} \mid B\rangle$
N	N	G_E^N	$\frac{1}{2m_N}[G_E^N(\mathbf{p}'+\mathbf{p})+iG_M^N(\boldsymbol{\sigma}\times\mathbf{k})]$
Δ	N	0	$i\frac{m_\Delta+3m_N}{4m_N m_\Delta}G_M^{\Delta N}(\mathbf{S}\times\mathbf{k})$
N	Δ	0	$i\frac{m_\Delta+3m_N}{4m_N m_\Delta}G_M^{\Delta N}(\mathbf{S}^\dagger\times\mathbf{k})$
Δ	Δ	G_E^Δ	$\frac{1}{2m_\Delta}[G_E^\Delta(\mathbf{p}'+\mathbf{p})+iG_M^\Delta(\boldsymbol{\sigma}_\Delta\times\mathbf{k})]$

$$G_M^{N\Delta}=\sqrt{2}T_3G_M^0 \ , \tag{5.6c}$$

$$G_M^{\Delta N}=G_M^{N\Delta^\dagger} \ , \tag{5.6d}$$

$$G_E^\Delta=\left[\frac{1+t_3}{2}\right]G_E^0 \ , \tag{5.6e}$$

$$G_M^\Delta=\left[\frac{1+t_3}{6}\right]G_M^0 \ , \tag{5.6f}$$

where $\tau_3=\tau_0$, $T_3=T_0$, and $t_3=\tau_{\Delta,0}$.

In Eq. (5.6), G_E^0 and G_M^0 are expressed in terms of the MIT bag wave function as follows:

$$G_E^0(k^2)=R_0(j_0^2+j_1^2;k) = 1-\tfrac{1}{6}k^2\langle r^2\rangle+\tfrac{1}{120}k^4\langle r^4\rangle-\cdots \ , \tag{5.7}$$

$$G_M^0(k^2)=\frac{4m_N}{k}R_1(j_0j_1;k) = 2m_N(\mu_p^0-\tfrac{1}{15}k^2I_2(R)+\cdots) \ , \tag{5.8}$$

where the function R_l is given by

$$R_l(f;k)\equiv N^2\int_0^R dr\, r^2 j_l(kr) f\left[\frac{\omega r}{R}\right] \tag{5.9}$$

with

$$N^2=\frac{\omega}{2(\omega-1)}\frac{1}{R^3j_0^2(\omega)} \ . \tag{5.10}$$

In Eq. (5.8), μ_p^0 is the proton magnetic moment and is given by

$$\mu_p^0=\tfrac{2}{3}R_0(rj_0j_1;0) \ , \tag{5.11}$$

and

$$I_n(R)=R_0(r^{2n-1}j_0j_1;0) \ . \tag{5.12}$$

The derivation of the N−N current, i.e., $\langle N \mid J_\mu \mid N\rangle$, is given in Ref. 18. The factor of $(1+k^2/4m_N^2)^{-1}$ is approximated to 1 in the present paper because of the nonrelativistic treatment of our theory. Derivation of the rest of the currents shown in Table II, is briefly given in Appendix A.

At this stage we should point out that, as a result of our analysis of three-body unitarity, the diagrams in Figs. 1(b)−1(e) have dressed vertices, while the diagram in Fig. 1(a) has bare vertices. If we are to calculate pion photoproduction below the threshold for the two pion final state, then $f^{(1)}$ and $\bar{f}^{(1)}$ are real, and we may neglect their energy dependence.[21] Furthermore, the momentum dependence of $f^{(2)}$ and $f^{(1)}$ are very similar.[21] This suggests that we may use Eq. (5.3) for both $f^{(2)}$ and $f^{(1)}$. However, the coupling constant $f_{\pi NN}$ is not the same for the dressed $f^{(1)}$ and bare $f^{(2)}$ vertices. Since the physical coupling constant is related to the residue of the $\pi-N$ amplitude at the nucleon pole, the $f_{\pi NN}$ for $f^{(1)}$ should be the physical coupling constant, while that in $f^{(2)}$ ($f_{\pi NN}^0$) is a parameter of the Lagrangian to be adjusted so that the dressed form factor $f^{(1)}$ gives the physical coupling constant. The bare coupling constant $f_{\pi NN}^0$ is related to the parameter of the Lagrangian $g=(2f_\pi)^{-1}$ by

$$\frac{f_{\pi NN}^0}{m_\pi}=\frac{5}{9}\frac{\omega}{\omega-1}\frac{1}{2f_\pi} \tag{5.13}$$

with $\omega=2.043$. This parameter in turn determines the strength of both the contact term [Eq. (2.7)] and the seagull term [Eq. (2.9)]. Thus by adjusting $f_{\pi NN}^0$ to give the physical coupling constant $f_{\pi NN}$, we have determined the strength of both the contact and the seagull term. However, here again the partial summation of the multiple scattering series, as carried out in the last section, give dressing to both the contact and seagull terms. This allows us to take the strength of these terms from experiment other than the πNN coupling constant. Thus for $\pi-N$ scattering it was found that the contact term should have a strength corresponding to $f_\pi=93$ MeV.

We now turn to the determination of the contact and the seagull terms. Here again to define our normalization, we write the S matrix as

$$\langle B',\pi \mid S \mid B,\pi \text{ or } \gamma\rangle = 2\pi i\delta^4(P_f-P_i)\langle B',\pi \mid T \mid B,\pi \text{ or } \gamma\rangle \ . \tag{5.14}$$

For the contact term, we take the matrix element of Eq. (2.7) between baryon states. This gives

$$\langle B',\pi_\beta(q') \mid T \mid B,\pi_\alpha(q)\rangle = \frac{(-2i)}{[4\omega\omega'(2\pi)^6]^{1/2}}g^2\frac{N^2}{4\pi}\epsilon_{\alpha\beta\gamma} \times\left\langle B' \left| \int d^3x\,\theta_V[(\omega'+\omega)(j_0^2+j_1^2)\delta_{B'B}-\tfrac{10}{3}C_{B'B}j_0j_1\boldsymbol{\Sigma}\cdot\mathbf{p}\times\mathbf{r}]\mathcal{T}_\gamma \right| B\right\rangle e^{i(\mathbf{q}-\mathbf{q}')\cdot\mathbf{x}} \ , \tag{5.15}$$

where $\mathbf{p}=\mathbf{q}'+\mathbf{q}$, and where $\boldsymbol{\Sigma}$, $\tilde{\mathcal{T}}$, and $C_{B'B}$ are given in Table II. If this amplitude is to include all the dressing that arises from exposing three-body unitarity, then the coupling strength is $g=\frac{1}{2}f_\pi$ with $f_\pi=93$ MeV. On the other hand, if we explicitly include the contribution to the renormalization of the contact term from the multiple scattering series, then f_π is a parameter.

Turning to the crossed diagram, this can be written in terms of the dressed vertices $f^{(1)}$ as

$$\langle \mathrm{B}',\pi_\beta(q') \mid T_C \mid \mathrm{B},\pi_\alpha(q)\rangle = \frac{f^{(1)}_{\mathrm{B}'C\pi,\beta}(\mathbf{q}) f^{(1)\dagger}_{\pi C\mathrm{B},\alpha}(\mathbf{q}')}{W - E(\mathbf{q}+\mathbf{q}') - \omega(\mathbf{q}) - \omega(\mathbf{q}')} , \tag{5.16}$$

where C is an intermediate baryon state. Here, $f^{(1)}_{\mathrm{B}'C\pi}$ and $f^{(1)\dagger}_{\pi C\mathrm{B}}$ are given in Eq. (5.3) with $f_{\pi\mathrm{NN}}$ being the physical coupling constant.

The seagull diagram [Fig. 1(e)] which corresponds to the projection of $\mathcal{L}_{\mathrm{qq}\pi\gamma}$, as given in Eq. (2.9), onto the baryon states, is given by

$$\langle \mathrm{B}',\pi_\alpha(q) \mid T \mid \mathrm{B},\gamma(k)\rangle = \frac{5eg}{3[4\omega k_0(2\pi)^6]^{1/2}} \epsilon_{\alpha\beta 3} C_{\mathrm{B}'\mathrm{B}} \frac{N^2}{4\pi} \left\langle B' \left| \int d^3x [(j_1^2 - j_0^2)(\mathbf{\Sigma}\cdot\boldsymbol{\epsilon}) - 2j_1^2(\boldsymbol{\epsilon}\cdot\hat{\mathbf{r}})(\mathbf{\Sigma}\cdot\hat{\mathbf{r}})]\mathcal{T}_\beta \right| \mathrm{B} \right\rangle e^{i(\mathbf{k}-\mathbf{q})\cdot\mathbf{x}} . \tag{5.17}$$

On the other hand, the crossed diagram for pion photoproduction [Fig. 1(b)] is given by

$$\langle \mathrm{B}',\pi_\alpha(q) \mid T \mid \mathrm{B},\gamma(k)\rangle = \frac{\tilde{f}^{(1)}_{\mathrm{B}'C\gamma}(\mathbf{k}) f^{(1)\dagger}_{\pi C\mathrm{B},\alpha}(\mathbf{q})}{W - E(\mathbf{q}+\mathbf{k}) - \omega(\mathbf{q}) - k_0} . \tag{5.18}$$

Here again, C is the intermediate baryon state, while k_0 is the energy of the photon.

To complete our results for the Born term as given in Fig. 1, we need to obtain the expressions for the diagrams in Figs. 1(c) and 1(d). These are given by

$$\langle \mathrm{B}',\pi_\alpha(q) \mid T \mid \mathrm{B},\gamma(k)\rangle$$
$$= i\frac{5eF_\pi(k^2)}{3[4\omega k_0(2\pi)^6]^{1/2}} \frac{C_{\mathrm{B}'\mathrm{B}}\epsilon_{\alpha\beta 3}(2q\cdot\epsilon)}{\omega(\mathbf{k}-\mathbf{q}) \pm [\omega(\mathbf{q}) - k_0]} \times \frac{1}{2\omega(\mathbf{k}-\mathbf{q})} f^{(1)}_{\mathrm{B}'\mathrm{B}\pi,\beta}(\mathbf{k}-\mathbf{q}) , \tag{5.19}$$

where the + sign corresponds to Fig. 1(c) and the − sign corresponds to Fig. 1(d). Here again $f^{(1)}$ has the dressed πNN coupling constant as shown in the last section. For Fig. 1(c), F_π must be replaced by $\tilde{f}_a^{(1)\dagger}$ as given in Eq. (4.22) since the $\pi\pi\gamma$ vertex is dressed due to π-π interaction. On the other hand, for Fig. 1(d), F_π is taken to be the bare coupling $\tilde{f}_a^{(2)}$ which is in $\mathcal{L}_{\pi\pi\gamma}$. Here, the dressing is of higher order in the electromagnetic coupling, and is neglected. We should note here that $\tilde{f}_a^{(1)\dagger}$ in Eq. (5.19) [for Fig. 1(c)] is real if the center-of-mass energy is below the threshold for two-pion production.

VI. ANGULAR MOMENTUM DECOMPOSITION

To reduce the dimensionality of our integrals, and integral equations developed in Secs. III and IV, we need to decompose our amplitudes in one of three ways. (i) In terms of partial waves with orbital angular momentum L, spin S, and total angular momentum J. (ii) Using helicity states as a basis for the expansion of the amplitude. (iii) An expansion in terms of the multipoles in the electromagnetic coupling. For π-N scattering at low to medium energies, the partial wave expansion is the most common approach. On the other hand, for pion photoproduction, the analyses are performed in terms of either multipole[8] or helicity states.[31] Since in the present investigation we need to consider both π-N scattering and pion photoproduction at low to medium energies, we will use partial wave expansion in terms of the states with definite L, S, and J. These in turn can be transformed to multipole states.[32]

To calculate the amplitude for pion photoproduction, we need to use either Eqs. (3.12) and (3.13) or Eqs. (3.7), (3.8), and (3.9). In both cases we need to expand the amplitudes $t^{(n)}$, $\tilde{t}^{(n)}$, $f^{(n)}$, and $\tilde{f}^{(n)}$ in terms of partial waves. Since the two-body channels involved are the $|\pi\mathrm{B}\rangle$ and $|\gamma\mathrm{B}\rangle$ channels, we need to expand these in terms of the states with definite total angular momentum and isospin. For the $|\pi\mathrm{B}\rangle$ channel we have

$$|\mathbf{q};tm_t;\tau m_\tau s m_s\rangle = \sum_{\substack{Lm_L\\ Jm_J\\ Tm_T}} Y^*_{Lm_L}(\hat{\mathbf{q}})(Lm_L s m_s \mid Jm_J)(tm_t \tau m_\tau \mid Tm_T) \mid q;(t\tau)T,(Ls)Jm_J\rangle , \tag{6.1}$$

where t, τ, and s are the isospins of the pion and baryon and the spin of the baryon, respectively. Here m_t, m_τ, and m_s are the corresponding projection operators. For the $|\gamma\mathrm{B}\rangle$ channel, the corresponding expansion is

$$|\mathbf{k};\lambda;s m_s\rangle = \sum_{\substack{lm_l\\ Lm_L\\ Jm_J}} Y^*_{lm_l}(\hat{\mathbf{k}})(lm_l 1\lambda \mid Lm_L)(Lm_L s m_s \mid Jm_J) \mid k;[(l1)Ls]Jm_J\rangle , \tag{6.2}$$

where λ is the polarization of the photon, and L is the total angular momentum of the photon. Since isospin is not conserved in electromagnetic transition we have not included explicitly the isospin quantum numbers in the γ-B channel. However, charge conservation requires that the z component of isospin be conserved, and in this channel this corresponds to the z component of the isospin of the baryon.

If we now label the π-B states with quantum numbers (LJT) by $\alpha,\beta,\gamma,\ldots$, (and the corresponding γ-B channels with quantum numbers (lLJ) by $\tilde{\alpha},\tilde{\beta},\tilde{\gamma},\ldots$,), then we can carry out a partial wave decomposition of the equations in Sec. III to get one dimensional equations. For the π-B scattering amplitude, we have[21]

$$t_{\alpha\beta}^{(0)}(q',q)=t_{\alpha\beta}^{(1)}(q',q)+f_{\alpha}^{(1)\dagger}(q')d_{\rm B}f_{\beta}^{(1)}(q) \ . \tag{6.3}$$

Since $d_{\rm B}$ is a diagonal matrix of dimension 2×2 for B=N and Δ, then $f_{\beta}^{(1)}(q)$ is a column matrix of dimension 2. The one-particle irreducible amplitude $t_{\alpha\beta}^{(1)}$ satisfies the set of coupled equations

$$t_{\alpha\beta}^{(1)}(q,q')=t_{\alpha\beta}^{(2)}(q,q') +\sum_{\gamma}\int_0^\infty dq''q''^2 t_{\alpha\gamma}^{(2)}(q',q'')g_{\gamma}(q'')t_{\gamma\beta}^{(1)}(q'',q') \ , \tag{6.4}$$

where we have assumed that $g=d_{\rm B}d_{\pi}$ is diagonal. This in fact is the case if B=N,Δ. However, if one includes the Roper as a three quark state, then g is not diagonal, in general.[21] The one-particle irreducible πBB form factor is, after partial wave expansion, given by

$$f_{\alpha}^{(1)}(q)=f_{\alpha}^{(2)}(q)+\sum_{\gamma}\int_0^\infty dq''q''^2 f_{\gamma}^{(2)}(q'') \times g_{\gamma}(q'')t_{\gamma\alpha}^{(1)}(q'',q) \ . \tag{6.5}$$

Thus, given the two-particle irreducible amplitudes we can calculate the π-B amplitude.

Turning to pion photoproduction, we can carry out the partial wave expansion in analogy with the π-B case to obtain

$$\tilde{t}_{\alpha\beta}^{(0)}(q,k)=\tilde{t}_{\alpha\beta}^{(1)}(q,k)+f_{\alpha}^{(1)\dagger}(q)d_{\rm B}\tilde{f}_{\beta}^{(1)}(k) \ , \tag{6.6}$$

where

$$\tilde{t}_{\alpha\beta}^{(1)}(q,k)=\tilde{t}_{\alpha\beta}^{(2)}(q,k) +\sum_{\gamma}\int_0^\infty dq''q''^2 t_{\alpha\gamma}^{(1)}(q,q'')g_{\gamma}(q'')\tilde{t}_{\gamma\beta}^{(2)}(q'',k) \tag{6.7}$$

and

$$\tilde{f}_{\beta}^{(1)}(k)=\tilde{f}_{\beta}^{(2)}(k) +\sum_{\gamma}\int_0^\infty dq''q''^2 f_{\gamma}^{(1)}(q'')g_{\gamma}(q'')\tilde{t}_{\gamma\beta}^{(2)}(q'',k) \ . \tag{6.8}$$

To get the relation between the full and partial wave amplitudes, we make use of Eqs. (6.1) and (6.2) to write

$$\langle \mathbf{q},tm_t;\tau m_\tau,sm_s\,|\,\tilde{t}^{(n)}\,|\,\mathbf{k},\lambda;\tau'm_\tau',s'm_s'\rangle = \sum_{\substack{Lm_L\\ lm_l\\ L'm_L'\\ Jm_J}} Y_{Lm_L}(\hat{\mathbf{q}})Y_{lm_l}^{*}(\hat{\mathbf{k}})(Lm_Lsm_s\,|\,Jm_J)(L'm_L's'm_s'\,|\,Jm_J)(lm_l1\lambda\,|\,L'm_L')\sum_T(tm_t\tau m_\tau\,|\,Tm_\tau')\tilde{t}_{\alpha\beta}^{(n)}(q,k) \ . \tag{6.9}$$

Here, the requirement of parity conservation leads to the fact that $L=l$ or $l\pm2$ (see Ref. 32).

Finally, the determination of both the baryon propagator $d_{\rm B}$ and the πB propagator g_γ required in Eqs. (6.6)–(6.8) has been given in detail by Pearce and Afnan[21] in their analysis of π-N scattering.

VII. CONCLUSION

In this paper, we have presented the first and complete off-shell multichannel unitary theory of pion photoproduction from a single baryon B, where B is N, Δ, or N*. The inclusion of strange baryons is possible by extending the SU(2) flavor to SU(3), as was done for π-N scattering.[33] In the present formulation we have coupled the $|B\rangle$, $|\pi B\rangle$, $|\gamma B\rangle$, $|\gamma\pi B\rangle$, and $|\pi\pi B\rangle$ channels, but no direct couplings between $|B\rangle$ and $|\pi\pi B\rangle$ and $|\gamma\pi B\rangle$. This restriction on the coupling is required to render a set of equations that are computationally viable. In the above formulation we have achieved a unification of the renormalization of the nucleon with pion elastic scattering and photoproduction while maintaining two- and three-body unitarity. The new feature of our result is that the Born amplitude for pion photoproduction can be divided into the following two parts. (i) A baryon pole part [Fig. 1(a)] which has an s-channel pole. The πBB and γBB vertices in this contribution are those in the Lagrangian and have bare coupling constants. (ii) The nonpole contribution [Figs. 1(b)–1(e)] has the u-channel baryon pole and a t-channel meson pole. More important, is the fact that all diagrams that contribute to the nonpole Born term have their vertices and amplitudes one-particle irreducible [except for the $\gamma\pi\pi$ vertex in Fig. 1(d) which is two particle irreducible], i.e., the vertices are renormalized to the extent that the coupling constants are not those in the Lagrangian, but are the physically observed coupling constants. In effect, we have shown that the vertices in Fig. 1(a) are the bare vertices, while those in Figs. 1(b)–1(e) have the dressed vertices. This result was achieved by going beyond two-body unitarity and exposing three-body unitarity as we did in Sec. IV.

Although the analysis was done for a special Lagrangian given in Eqs. (2.1)–(2.10), the results of Secs. III and IV are general to the extent that we need not specify the form of the Lagrangian as in Eqs. (2.6)–(2.10). Thus, if at any future date, we were able to derive a Lagrangian of the form given in Eqs. (2.1) and (2.5) from QCD, then the analysis of Secs. III and IV would still hold. However, the results of Sec. V would need to be appropriately modified.

We hope to use the present formulation to investigate a number of unresolved problems. (i) The threshold amplitude for pion photoproduction has been studied almost exclusively in the covariant tree (Born) approximation. Here gauge invariance determines the charged pion photoproduction, while the partial conserved axial vector current (PCAC) governs the neutral pion photoproduction amplitude. In fact, the nonzero threshold amplitude for neutral pion photoproduction comes only from the antiparticle contribution, since the Kroll-Ruderman[34] term is absent. However, there remains the question whether higher multiple scattering terms can produce the correct sign and magnitude for this amplitude, even when the antiparticle contribution is ignored, as in the present formulation. Recently, Hoodbhoy[35] showed that the photon-Skyrmion interaction reproduces the right magnitude for the neutral pion photoproduction amplitude at threshold. In the Skyrme-Witton model,[2] however, the quark degrees of freedom are integrated. Thus, the physical content of Hoodbhoy's calculation is not yet clear. It is a very interesting and challenging problem to determine the mechanism of threshold pion photoproduction. (ii) If the πNN vertex $f^{(2)}$ is parametrized by the monopole type of form factor, then the range of this form factor is about 0.7–0.8 GeV both in the Cloudy Bag Model,[19] and in the chiral potential model.[36] On the other hand, one-boson-exchange models[37,38] of N-N scattering demand a range of 1.1–2.0 GeV for the πNN vertex.

We expect that the analysis of pion photoproduction, based on the present formulation, will give a measure of the range of the πNN vertex. (iii) There is a large discrepancy between theory and experiment,[11,14,39] in the value of the $E2$ amplitude. We expect that our present theory will put some constraint on the prediction of the $E2$ amplitudes. (iv) Since our formulation incorporates both two- and three-body unitarity, we can use the theory above the threshold for two pion production. This will allow us to investigate the recently observed structure in the region of the Roper resonance in pion photoproduction.[39]

Finally, by taking advantage of the off-shell amplitude, which the present theory provides, we should be able to study the reactions $\gamma d \to \pi d$, $\gamma d \to pn$, and $pp \to pp\gamma \cdots$ in a consistent manner. A unified theory of these reactions will be reported elsewhere.

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APPENDIX A

Here we present the isobar electromagnetic currents. The main contribution to the Δ-N currents comes from[40,41]

$$\langle \Delta(p') \mid J_\mu \mid N(p)\rangle = \bar{U}^{\nu}(p')[g_{\nu\mu}(m_\Delta + m_N) + p_\nu \gamma_\mu]\gamma_5 U(p) \ . \tag{A1}$$

The time and space components of the Rarita-Schwinger wave function can be written as[41]

$$U^0(p,s) = \left[\frac{D_\Delta}{2m_\Delta}\right]^{1/2} \begin{bmatrix} 1 \\ \dfrac{\sigma\cdot \mathbf{p}}{D_\Delta} \end{bmatrix} \frac{\mathbf{S}^\dagger \cdot \mathbf{p}}{m_\Delta}\psi_s \ , \tag{A2}$$

$$U^k(p,s) = \left[\frac{D_\Delta}{2m_\Delta}\right]^{1/2} \begin{bmatrix} 1 \\ \dfrac{\sigma\cdot \mathbf{p}}{D_\Delta} \end{bmatrix} \left[(S^\dagger)^k + \frac{(\mathbf{S}^\dagger\cdot\mathbf{p})p^k}{m_\Delta D_\Delta}\right]\psi_s \ , \tag{A3}$$

where $D_\Delta = m_\Delta + (m_\Delta^2 + \mathbf{p}^2)^{1/2}$.

If we keep terms up to order p/m_Δ in Eqs. (A2) and (A3), in the same manner used to approximate $(1 + \mathbf{p}^2/4m_N^2)$ by 1 in $\langle N \mid J_\mu \mid N\rangle$ (see Sec. V), then the above expression becomes

$$U^0(p,s) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{\mathbf{S}^\dagger\cdot\mathbf{p}}{m_\Delta}\psi_s \tag{A4}$$

and

$$U^k(p,s) = \begin{bmatrix} 1 \\ \dfrac{\sigma\cdot\mathbf{p}}{2m_\Delta} \end{bmatrix} (\mathbf{S}^\dagger)^k \psi_s \ . \tag{A5}$$

By inserting (A4) and (A5) into (A1) and by neglecting the second rank tensor terms, we find

$$\langle \Delta \mid J_0 \mid N\rangle = \left[\frac{-i}{2}\right]\left[\frac{m_\Delta + 3m_N}{2m_\Delta m_N}\right]\psi_{s'}^\dagger \mathbf{S}\cdot(\mathbf{p}'\times\mathbf{p})\chi_s \tag{A6}$$

and

$$\langle \Delta \mid \mathbf{J} \mid N\rangle = \left[\frac{i}{2}\right]\left[\frac{m_\Delta + 3m_N}{2m_N}\right]\psi_{s'}^\dagger \mathbf{S}\times\left[\mathbf{p} - \frac{m_N}{m_\Delta}\mathbf{p}'\right]\chi_s \ . \tag{A7}$$

The evaluation of the vertex function $\tilde{f}^{(2)}$ from Eq. (5.3) gives us, however, $\langle \Delta \mid J_0 \mid N\rangle = 0$. This is due to the fact that J_0, derived from Eq. (2.8), is spin independent, and thus $\langle \Delta \mid \sum_\alpha Q_\alpha J_0 \mid N\rangle = 0$. We therefore take $\langle \Delta \mid J_0 \mid N\rangle = 0$ in Table II. It should also be noted here that $\langle \Delta \mid J_0 \mid N\rangle = 0$ if we take the static limit for U^μ. In general, the Δ-Δ current can be expressed as

$$\langle \Delta(p') \mid J_\mu \mid \Delta(p)\rangle = \bar{U}^{\rho}(p',s')O_{\rho\mu\sigma}(p',p)U^\sigma(p,s) \ . \tag{A8}$$

If we take the simplest form $g_{\rho\sigma}\gamma_\mu$ for $O_{\rho\mu\sigma}$ then (A8) becomes, with the help of (A4) and (A5),

$$\langle \Delta \mid J_0 \mid \Delta\rangle = \psi_{s'}^\dagger \psi_s = \delta_{s's} \ , \tag{A9}$$

$$\langle \Delta \mid \mathbf{J} \mid \Delta\rangle = \psi_{s'}^\dagger \frac{1}{2m_\Delta}\left[(\mathbf{p}'+\mathbf{p}) + \frac{i}{3}\sigma_\Delta\times(\mathbf{p}'-\mathbf{p})\right]\psi_s \ . \tag{A10}$$

*Present address: Institut für Kernphysik, Universität Mainz, Postfach 3980, 6500 Mainz 1, Federal Republic of Germany.

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