Meson-nucleus interactions using polarized nuclei

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Under certain conditions the asymmetry resulting from single charge exchange reactions arises only from the (quantum mechanical) distortion effects of the incoming and outgoing waves. If the distortion of the incident wave is well known, some basic properties of the hadronic interaction in the final state can be inferred. Examples for (π^+,π^0) and (π^+,η) reactions are given.

Recent innovations in experimental techniques have opened up the possibility of using polarized targets in nuclear reactions. At a recent workshop on polarized targets, we discussed aspects of pion single charge exchange from polarized nuclei.[1] In this paper we present an extension of that work and examine in more detail meson reactions which cause a transition from a spin-J initial state to a spin-0 final state. These kinds of reactions have a unique feature. Under certain conditions the left-right asymmetry depends primarily on the difference in the interaction strengths of the incoming and outgoing projectiles with the nuclear medium. This allows a direct assessment of this property. The purpose of this work is twofold: First, to point out and to discuss the special situation that exists for these asymmetry measurements; second, to present specific calculations for possible future experiments.

While any practical polarization technique will provide a distribution of azimuthal quantum numbers, for the reaction considered here where the final state has $J=0$ and there is no parity change, only transitions from initial states of even m to the obligatory $m=0$ final state are possible. We assume for simplicity a pure $m=+2$ initial state.

We will confine our discussion to the reactions $^{10}\vec{\mathrm{B}}(\pi^+,\pi^0)^{10}\mathrm{C}$ and $^{10}\vec{\mathrm{B}}(\pi^+,\eta)^{10}\mathrm{C}$, but similar mathematical treatment will apply to other spin-0 projectiles. In particular, we calculate the left-right asymmetry measured in the plane perpendicular to the initial boron polarization axis. The nuclear medium effects are included via the distorted wave impulse approximation (DWIA), which is briefly discussed in the next section. In the remaining sections we give qualitative properties and results of these calculations.

DWIA FORMALISM

Detailed aspects of the DWIA formalism are presented in many books[2] and articles,[3] and here we present the final expressions as applied to pion reactions. The amplitude for the π^+ charge-exchange reaction, F_{if}, is given by

$$F_{if}=\langle \chi_f^{(-)} \mid \langle \Psi_f \mid t_{\rm cx} \mid \Psi_i \rangle \mid \chi_i^{(+)} \rangle \ , \quad (1)$$

where $\mid \Psi_i \rangle$ and $\mid \Psi_f \rangle$ are the initial and final nuclear states, and $\chi_i^{(+)}$ and $\chi_f^{(-)}$ correspond to the distorted waves of the incoming π^+ and outgoing π^0 (or η), respectively. These distorted waves are obtained by solving an appropriate wave equation in which the π-nucleus interaction is described by an optical potential which represents an approximate solution to the $A+1$ body problem. We treat the charge-exchange operator $t_{\rm cx}$ as the quasilocal one-body operator (zero range)

$$t_{\rm cx}(E,\mathbf{k},\mathbf{k}')=\tau^+[\lambda_0(E)+\lambda_1(E)\mathbf{k}\cdot\mathbf{k}'+\lambda_{\rm SF}(E)\boldsymbol{\sigma}\cdot(\mathbf{k}\times\mathbf{k}')] \quad (2)$$

in momentum space. The gradient operators $\mathbf{k}$ and $\mathbf{k}'$ act on the pion-nucleus relative-motion wave functions. The complex amplitudes $\lambda(E)$, which we obtain from pion-nucleon data, correspond to the s-wave (λ_0), p-wave (λ_1), and the spin-dependent $(\lambda_{\rm SF})$ components of the interaction. For pion kinetic energies less than 200 MeV, the $l=0$ and $l=1$ partial waves given above are adequate to describe pion-nucleon interactions.

The axis of quantization is arbitrary in these calculations. However, if it is chosen perpendicular to the scattering plane, in the direction $\hat{\mathbf{n}}=\hat{\mathbf{k}}\times\hat{\mathbf{k}}'$, certain restrictions are imposed on the allowed values of $\Delta m=M_i$ (the initial nuclear projections) minus M_f (the final nuclear projection). They are due to the assumed invariance of the interaction under parity, which is incorporated in Eq. (2), and are a consequence of the Bohr reflection theorem.[4] For a spin-0 projectile, Bohr's theorem states

$$\pi_{if}=(-1)^{\Delta m} \ , \quad (3)$$

where π_{if} is the product of the initial and final parity of the nuclear states. This result is general and not limited to the DWIA approach. We will take the axis of polarization along $\hat{\mathbf{n}}$ throughout our calculations.

The distorted waves of the incoming and outgoing pion can be combined and expressed in a form which is composed of terms, which operate on the nuclear states and transform as tensors under rotation. Equation (1) becomes

$$F_{if}=\langle \Psi_f(J_fM_f)\,|\,t_{00}(\theta;r)Y_{00}(\hat{\mathbf{r}})+t_{1-\mu}(\theta;r)Y_{1\mu}(\hat{\mathbf{r}})+t_{2-\mu}(\theta;r)Y_{2\mu}(\hat{\mathbf{r}})+\cdots+\sum_{JK} t_{JK-\mu}(\theta;r)(Y_K\times\sigma)_{J\mu}\,|\,\Psi_i(J_iM_i)\rangle \ . \qquad (4)$$

where the integration over d^3r is implied. Here the nucleon creation and annihilation operators are implicit and θ, which does not act on the nuclear wave functions, is the scattering angle, i.e., the angle between the π^+ and π^0 asymptotic momenta. We use the notation of Ref. 3 in labeling the various components as $J(KS)$, where J is the orbital angular momentum change of the pion-nucleus system, K is the orbital angular momentum transfer to the constituents of the nucleus, and $S=0,1$ correspond to spin-independent and spin-dependent components of the transition operator, respectively. The form of Eq. (4) has several advantages. The nuclear and reaction mechanism aspects are separated. The dependence on nuclear structure enters the calculation in the form of the reduced matrix elements $\langle \Psi_f\|Y_K\|\Psi_i\rangle$ and $\langle \Psi_f\|(Y_K\times\sigma)_J\|\Psi_i\rangle$. The functions $t_{K\mu}(\theta;r)$ and $t_{JK\mu}(\theta;r)$ depend on the reaction mechanism, the transition density, and the pion's interaction with the other nucleons. One would like to understand one of these ingredients and learn about the other.

For reactions in which $J_F=0$, the calculation is greatly simplified since only terms in which $J=J_i$ contribute to the scattering amplitude in Eq. (4). In addition, parity invariance requires $(-1)^K=\pi_{\rm if}$. Thus the reaction $^{10}\vec{\rm B}(\pi^+,\pi^0)^{10}$C proceeds only via the $(Y_2\times\sigma)_3$ and $(Y_4\times\sigma)_3$ transitions, and from Eq. (3), ΔM equals $+2$, 0, or -2. We will assume ^{10}B to be completely in the $M_i=+2$ state and calculate the left-right symmetry symmetry $A_Y(\theta)$ in the scattering plane normal to the quantization axis.

We define the asymmetry as

$$A_Y(\theta)\equiv\frac{\sigma_R(\theta)-\sigma_L(\theta)}{\sigma_R(\theta)+\sigma_L(\theta)}\ , \qquad (5)$$

where σ_L and σ_R denote the left and right cross sections (i.e., σ_L is for $\mathbf{k}\times\mathbf{k}'$ in the $+m$ direction). In this ratio the reaction amplitude $\lambda_{\rm SF}(E)$ and the nuclear structure dependence largely cancel between the numerator and denominator. In fact, if the nuclear model space is restricted to the p shell, the reaction proceeds only via the $(Y_2\times\sigma)_3$ operator and the cancellation is exact. On the experimental side as well, normalization errors are greatly diminished in this ratio. Therefore, the uncertainties in the data taking and theoretical analysis are greatly reduced, and the asymmetry is mainly dependent on the projectiles interaction with the nucleus. It is this dependence which we investigate in the next section.

QUALITATIVE FEATURES OF ASYMMETRY IN SPIN-J TO SPIN-0 TRANSITIONS

Usually an asymmetry in the left and right scattering arises from an interference between the spin-dependent and spin-independent components of the scattering amplitude from a polarized target. For an initial state in which $-(-1)^J=\pi_{if}$, however, only the spin-dependent part of the interaction contributes. The asymmetry is of a different nature. It arises not from such an amplitude interference, but from an asymmetry in the interaction of the incoming and outgoing projectiles. This can be seen by examining the amplitudes for left and right scattering from the DWIA formalism.

The amplitudes for scattering to the left and right can be written as

$$F_{\rm left}(\theta)=\lambda_{\rm SF}\langle{}^{10}{\rm C}\|[Y_2\times\sigma]_3\|{}^{10}{\rm B}\rangle/\sqrt{7}\sum_{\substack{ll'\\mm'}} I_{ll'}P_l^mP_{l'}^{m'}e^{im'\theta}\left[\begin{matrix} l' & l\\ m' & m\end{matrix}\middle|\begin{matrix}3\\2\end{matrix}\right]\ , \qquad (6)$$

$$F_{\rm right}(\theta)=\lambda_{\rm SF}\langle{}^{10}{\rm C}\|[Y_2\times\sigma]_3\|{}^{10}{\rm B}\rangle/\sqrt{7}\sum_{\substack{ll'\\mm'}} I_{ll'}P_l^mP_{l'}^{m'}e^{-im'\theta}\left[\begin{matrix} l' & l & 3\\ m' & m & 2\end{matrix}\right]\ , \qquad (7)$$

or, alternatively,

$$F_{\rm right}(\theta)=\lambda_{\rm SF}\langle{}^{10}{\rm C}\|[Y_2\times\sigma]_3\|{}^{10}{\rm B}\rangle/\sqrt{7}e^{-i2\theta}\sum_{\substack{ll'\\mm'}} I_{l'l}P_l^mP_{l'}^{m'}e^{im'\theta}\left[\begin{matrix} l' & l\\ m' & m\end{matrix}\middle|\begin{matrix}3\\2\end{matrix}\right] \qquad (8)$$

with the terms defined in the Appendix. If the distorted waves of the incoming and outgoing projectiles are identical [i.e., $\chi_i(\mathbf{r})=\chi_f(\mathbf{r})$], then $I_{ll'}$ equals $I_{l'l}$. The amplitude for scattering to the left [Eq. (6)] has the same magnitude as that for scattering to the right [Eq. (8)] and the asymmetry is zero. This property is not limited to the DWIA model, but stems from symmetry requirements. The asymmetry will also be zero when the distorted waves are unequal if $I_{ll'}$ is real, since from Eq. (6) and Eq. (7), $|F_{\rm left}|^2$ equals $|F_{\rm right}|^2$. This situation would occur if the initial and final scattering states were plane waves with perhaps different momentum. An asymmetry results therefore when the distorted waves are different, but not both plane waves. This difference is

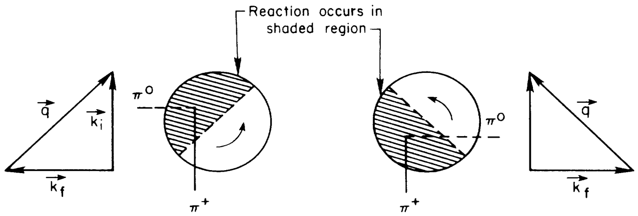

FIG. 1. Diagram describing "Newns polarization."

caused by different absorptive properties, different interaction strengths, and different momenta in the nuclear medium. Next we discuss how these aspects, under certain conditions, affect the asymmetry.

Consider the case of an asymmetry caused by different absorptive properties for the incoming and outgoing projectiles. The classical explanation of this phenomenon was given by Newns,[5] Tobocman,[6] and Newns and Refai[7] for the case of deuteron stripping, and we present the corresponding explanation for pions in Fig. 1. The initial boron nucleus in the figure is taken to be polarized out of the page, up being the positive direction. Since the final state has $J=0$, negative angular momentum must be transferred during the process. Thus the momentum-transfer vector **q** must pass to the left of the center of the nucleus so that $\mathbf{r}\times\mathbf{q}$ is negative. The region of the reaction, therefore, will be determined by the direction of the momentum transfer **q**, shown in the figure. For scattering to the right, **q** points to the upper left, and the reaction occurs in the lower left hemisphere of the nucleus in order to produce the necessary angular momentum transfer. As illustrated in Fig. 1, the distance traveled in the nucleus by the π^+ (solid line) is longer than for the π^0 (dashed line). The situation is reversed for scattering to the left. An asymmetry will result, therefore, if the incoming and outgoing particles have different absorptive properties, and the magnitude will scale with the difference.

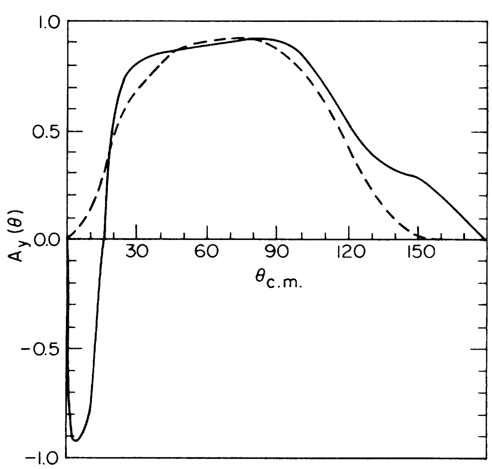

FIG. 2. Asymmetry for He $^{10}\vec{\mathrm{B}}(\pi^+,\pi^0)^{10}\mathrm{C}$ reaction at 165 MeV. The solid curve is the DWIA calculation with π^+ distorted waves and π^0 plane waves. The dashed curve is the result of Eq. (10).

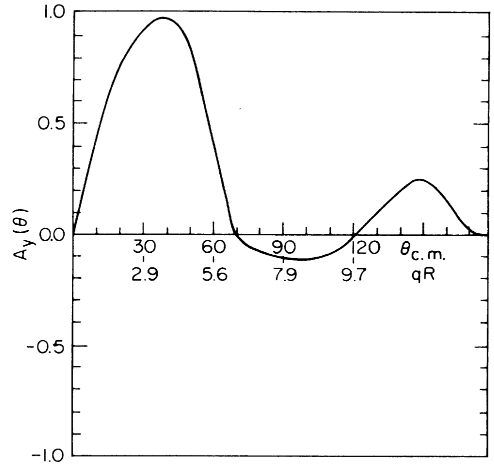

FIG. 3. The result of Eq. (10) for $KR=5.6$.

The quantum mechanical treatment (within certain regions of momentum transfer) gives results in qualitative agreement with the classical description. For comparison consider the case in which the incoming particle has a short mean-free path and the outgoing one very long. We will represent this situation by using distorted waves for the π^+ and plane waves for the π^0. The laboratory energy $T_\pi=165$ MeV is chosen since the π^+ is highly reactive near the $P33$ resonance. The resulting asymmetry for this extreme example is plotted in Fig. 2 (solid line). Except for small angles, where quantum mechanical effects are enhanced, the result is as expected from the classical arguments.

We digress, in this paragraph, to discuss the overall features of the DWIA calculation in a semiclassical approach. To do this we evaluate the reaction integral assuming that the interaction takes place only on the surface of the forward hemisphere of a sphere. For single multipole transition (l), the left and right scattering amplitudes are approximately

$$F_{\substack{L\\R}}(qR)=\lambda(E)\int_{\substack{\text{forward}\\\text{hemisphere}}} e^{i\mathbf{q}\cdot\mathbf{R}}\,Y_{l\,\substack{+m\\-m}}(\theta,\phi)\,d\Omega\ , \tag{9}$$

$$F_{\substack{L\\R}}(qR)=\int_0^{\pi}\int_{-1}^{1}\exp\{i\,[qR\,\sin\theta\cos(\alpha/2-\phi)\pm m\phi]\}\times P_l^m(\theta)\,d\cos\theta\,d\phi\ , \tag{10}$$

where α is the scattering angle. As discussed in Ref. 5, in the classical limit (i.e., large m) the major contribution to this integral will come when the phase in the exponent is stationary over the ϕ integral. This occurs when

$$\frac{d}{d\phi}[-qR\,\sin\theta\cos(\alpha/2-\phi)\pm m\phi]=0$$

or

$$qR\,\sin\theta\sin(\alpha/2-\phi)=\pm m\ ,$$

which results in scattering primarily to the right as expected from the classical analog of Fig. 1. The classical limit is not reached, however, for reactions in which qR is small [i.e., $^{10}\vec{B}(\pi^+,\pi^0)^{10}C$ for $T_\pi < 200$ MeV]. Nevertheless, we show in Figs. 2 and 3 that $A_Y(\theta)$ is substantial and in qualitative agreement with these simple ideas. The asymmetry,

$$A_Y(\theta)=(|F_R|^2-|F_L|^2)/(|F_R|^2+|F_R|^2) ,$$

calculated from Eq. (10) for $l=2$, is plotted in Fig. 2 for comparison with the full DWIA calculation. In Fig. 3 $A_Y(\theta)$ [as calculated from Eq. (10)] is also plotted for larger momenta. As seen in this figure, the sign of the polarization depends on the value of qR. For $qR < 6.0$ the sign of the polarization is the same as the classical interpretation of Fig. 1, but there are regions in which the sign is reversed. A good approximation to this integral is obtained by setting $\alpha=0$ and $\theta=\pi/2$ and integrating only over ϕ. This gives

$$A(qR)=\frac{8\pi J_2(qR)j_1(qR)}{\pi^2 J_2^2(qR)+16j_1^2(qR)} , \tag{11}$$

where j_1 and J_2 are the spherical and regular Bessel functions. For this highly asymmetric absorptive case, there is a direct relationship between $A_Y(\theta)$ and the mean-free path.

Next consider the case in which there is no absorption. This is represented by a real optical potential, and results in focusing and bending of the projectile wave functions. In Fig. 4 we plot two situations, one for an attractive and the other for a repulsive potential, at $T_\pi=165$ MeV for the full DWIA calculation. The outgoing particle is described by a plane wave. It is seen that substantial polarization can occur even in the absence of attenuation. In this case, the polarization reflects the sign of the interaction. In the next section we present calculations for pion single charge exchange and coherent eta production.

RESULTS

As a first example consider the reaction $^{10}\vec{B}(\pi^+,\pi^0)^{10}C$ at $T_\pi=70$ MeV. At this low energy blocking effects tend to reduce the pions interaction within the nucleus. The extent of this blocking and other second order correlation effects is not completely understood, and an asymmetry measurement could be useful. Results of pion single-charge exchange experiments near 50 MeV suggest that the pion interacts very weakly, almost to the extent of being a plane wave in the nucleus.[8] If this were the case, then $A_Y(\theta)$ would be zero for all angles. Note that since $|\mathbf{k}|\neq|\mathbf{k}'|$, $A_Y(\theta)$ will be zero only if $I_{ll'}$ is real in Eq. (6). Deviations from this plane wave approximation would cause an asymmetry, and such a measurement could give information about the nature of the blocking effects. In Fig. 5 we plot calculations of $A_Y(\theta)$ with (solid line) and without (dashed line) Pauli blocking effects, which are included using the model of Ref. 9. The net effect is to reduce the imaginary part of the optical potential. The dashed curve corresponds to using a finite range (t-rho) optical potential which does not include medium modifications. The asymmetries are small since the π^+ and π^0 have roughly the same interaction with the nucleus. However, if exotic effects were to occur, such as might be associated with a nonzero hadronization distance or a shorter lifetime for the π^0 in the nuclear medium, then the asymmetries could be as dramatic as those shown in Fig. 2.

As a second example, we consider the eta-production reaction $^{10}\vec{B}(\pi^+,\eta)^{10}C$. For $T_{\pi+}=460$ MeV, the eta has relatively low energy and the s-wave eta-nucleon amplitude is the dominant one. Since there is a large difference between the incoming and outgoing waves, one expects the possibility of large asymmetries and a sensitivity to the eta-nucleon interaction. We have cal-

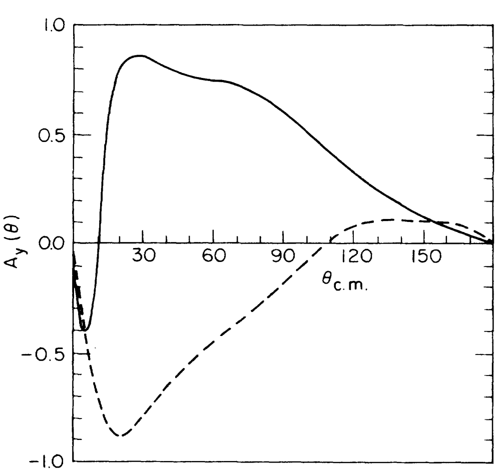

FIG. 4. Asymmetry for the reaction $^{10}\vec{B}(\pi^+,\pi^0)^{10}C$ at 165 MeV for the full DWIA calculation. The solid (dashed) curve is for $b_0=-3$ $(+3)$ and $b_1=-7$ $(+7)$. The π^0 wave is undistorted in both cases.

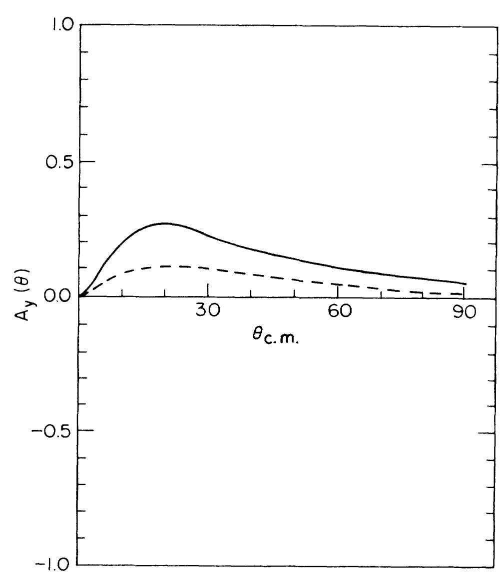

FIG. 5. Asymmetry for the reaction $^{10}\vec{B}(\pi^+,\pi^0)^{10}C$ at 70 MeV for the full DWIA calculation. The solid (dashed) line corresponds to a blocked (unblocked) optical potential.

calculated $A_Y(\theta)$ for the case in which the eta-nucleon scattering length is $+0.1$ fm, -0.1 fm, and zero (i.e., plane wave final state), and the results are plotted in Fig. 6. The sensitivity to the scattering length is encouraging. For this calculation, the pion distorted waves were calculated using a t-rho finite range optical potential. These distorted waves have been tested successfully with pion single charge exchange calculations at this energy. The eta distorted waves were also obtained from a t-rho finite range optical potential with strength determined by the scattering length. This optical potential is perhaps unrealistic at low energies since it is real and ignores inelastic η-nucleon exothermic channels which would give rise to imaginary pieces. Nonetheless, these exploratory results indicate that this might be a sensitive method to extract this eta-nucleon parameter. We also note that the spin averaged cross section does not uniquely determine the eta-nucleon scattering length, and also has a strong dependence on the transition amplitude $\lambda_{SF}(E)$, the form factor of the transition density, and the nuclear structure reduced matrix element.

In summary, we have shown that meson reactions from an odd-integer spin initial state to a spin-0 final state of the same parity provide potentially very useful experiments. The left-right asymmetry perpendicular to the polarization axis depends on the difference in the interaction of the incoming and outgoing projectiles. The asymmetry for the reaction $^{10}\vec{B}(\pi^+,\eta)^{10}C$ is sensitive to the η-nucleon scattering lengths.

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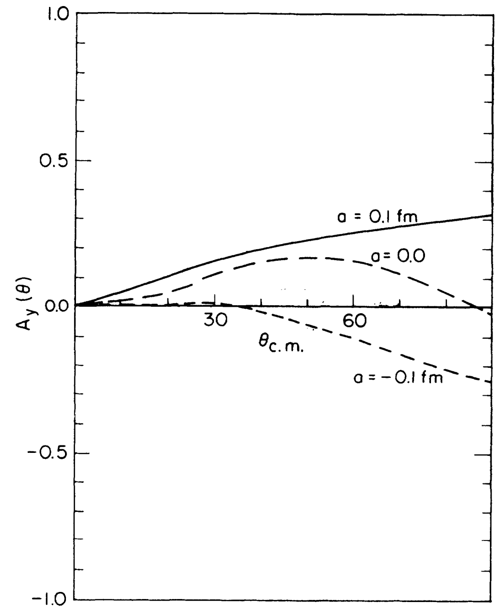

FIG 6. Asymmetry for the $^{10}\vec{B}(\pi^+,\eta)^{10}C$ reaction at 461 MeV for the full DWIA calculation. The η-nucleus optical potential is derived corresponding to the three scattering lengths (a) indicated in the figure.

APPENDIX

Only the spin-dependent part of the operator in Eq. (4) will cause transitions between the ground states of ^{10}B and ^{10}C. This operator can be decomposed into the following form in coordinate space:

$$\begin{aligned}
\sigma\cdot(\nabla\Psi^f\times\nabla\Psi^i) &= i\sqrt{6}\sum_{\substack{JK\\ ll'}} i^{l-l'}\sqrt{2K+1}[Y_K\times\sigma]_J\cdot[Y_{l'}(\hat{\mathbf{k}}')\times Y_l(\hat{\mathbf{k}})]_J \\
&\times\left[[l'l(2l-1)(2l'-1)]^{1/2}\begin{Bmatrix} l' & l & J\\ 1 & 1 & 1\\ l'-1 & l-1 & K\end{Bmatrix}\begin{bmatrix} l-1 & l'-1 & K\\ 0 & 0 & 0\end{bmatrix} D^-[\Psi^f_{l'}]D^-[\Psi^i_l]\right.\\
&\quad +[(l'+1)(l+1)(2l'+3)(2l+3)]^{1/2}\begin{Bmatrix} l' & l & J\\ 1 & 1 & 1\\ l'+1 & l+1 & K\end{Bmatrix}\begin{bmatrix} l+1 & l'+1 & K\\ 0 & 0 & 0\end{bmatrix} D^+[\Psi^f_{l'}]D^+[\Psi^i_l]\\
&\quad +[l'(l+1)(2l'-1)(2l+3)]^{1/2}\begin{Bmatrix} l' & l & J\\ 1 & 1 & 1\\ l'-1 & l+1 & K\end{Bmatrix}\begin{bmatrix} l+1 & l'-1 & K\\ 0 & 0 & 0\end{bmatrix} D^-[\Psi^f_{l'}]D^+[\Psi^i_l]\\
&\quad \left.+[l(l'+1)(2l'+3)(2l-1)]^{1/2}\begin{Bmatrix} l' & l & J\\ 1 & 1 & 1\\ l'+1 & l-1 & K\end{Bmatrix}\begin{bmatrix} l-1 & l'+1 & K\\ 0 & 0 & 0\end{bmatrix} D^+[\Psi^f_{l'}]D^-[\Psi^i_l]\right]\\
&\equiv\sum_{\substack{JK\\ ll'}} G_{JKll'}(r)[Y_K\times\sigma]_J\cdot[Y_{l'}(\hat{\mathbf{k}}')\times Y_l(\hat{\mathbf{k}})]_J\\
&\equiv\sum_{JK} t_{JK\,-\mu}(\theta;r)(Y_K\times\sigma)_{J\mu}\ ,
\end{aligned}\tag{A1}$$

where $D^+(\Psi_l)$ and $D^-(\Psi_l)$ are defined as

$$D^+(\Psi_l)\equiv\frac{d\Psi_l}{dr}-\frac{l}{r}\Psi_l$$

and

$$D^-(\Psi_l)\equiv\frac{d\Psi_l}{dr}+\frac{(l+1)}{r}\Psi_l\ . \qquad \text{(A2)}$$

Here Ψ_l^f and Ψ_l^i represent the radial wave function for the l'th partial wave, $\hat{\mathbf{k}}'$ and $\hat{\mathbf{k}}$ the direction of the outgoing and incoming projectile, and θ is the angle between $\mathbf{k}'$ and $\mathbf{k}$. For the $^{10}\vec{\mathrm{B}}$ reaction J equals 3. If we limit our discussion to the p-shell basis, then $K=2$ and the operator becomes

$$\sigma\cdot(\nabla\Psi^f\times\nabla\Psi^i)=\sum_{ll'}G_{32ll'}(r)[Y_2\times\sigma]_3\cdot[Y_{l'}(\hat{\mathbf{k}}')\times Y_l(\hat{\mathbf{k}})]_3\ . \qquad \text{(A3)}$$

The matrix elements $F_M(\theta)$ between the initial ^{10}B and final ^{10}C nuclear states is then

$$\begin{aligned}F_M(\theta)&=\lambda_{\mathrm{SF}}(E)\langle\,^{10}\mathrm{C}(J=0)\,|\,\sigma\cdot(\nabla\Psi^f\times\nabla\Psi^i)\,|\,^{10}\vec{\mathrm{B}}(J=3,M)\rangle\\&=\lambda_{\mathrm{SF}}(E)\langle\,^{10}\mathrm{C(g.s.)}\|[Y_2\times\sigma]_3\|^{10}\mathrm{B(g.s.)}\rangle/\sqrt{7}(-1)^M\sum_{ll'}I_{ll'}[Y_{l'}(\hat{\mathbf{k}}')\times Y_l(\hat{\mathbf{k}})]_{3M}\ ,\end{aligned} \qquad \text{(A4)}$$

where $I_{ll'}\equiv\int G_{32ll'}(r)\rho_t(r)r^2dr$. In this expression $\rho_t(r)$ is the nuclear transition density. Choosing the axis of quantization as $\hat{\mathbf{n}}=\hat{\mathbf{k}}\times\hat{\mathbf{k}}'$, the matrix element for the reaction becomes

$$F_M(\theta)=\lambda_{\mathrm{SF}}\langle\,^{10}\mathrm{C}\|[Y_2\times\sigma]_3\|^{10}\mathrm{B}\rangle/\sqrt{7}\times\sum_{\substack{ll'\\mm'}}I_{ll'}P_l^mP_{l'}^{m'}e^{im'\theta}\begin{bmatrix}l' & l & 3\\ m' & m & M\end{bmatrix} \qquad \text{(A5)}$$

to within an overall phase. In this equation $P_l^m\equiv P_l^m(90^\circ)$ and $\sin\theta=\hat{\mathbf{k}}\times\hat{\mathbf{k}}'$. For $M=2$ the amplitude, for scattering to the left, is given by

$$F_{\mathrm{left}}(\theta)=F_2(\theta)=\lambda_{\mathrm{SF}}\langle\,^{10}\mathrm{C}\|[Y_2\times\sigma]_3\|^{10}\mathrm{B}\rangle/\sqrt{7}\times\sum_{\substack{ll'\\mm'}}I_{ll'}P_l^mP_{l'}^{m'}e^{im'\theta}\left[\begin{array}{cc|c}l' & l & 3\\ m' & m & 2\end{array}\right]\ , \qquad \text{(A6)}$$

and scattering to the right by

$$\begin{aligned}F_{\mathrm{right}}(\theta)&=F_2(-\theta)\\&=\lambda_{\mathrm{SF}}\langle\,^{10}\mathrm{C}\|[Y_2\times\sigma]_3\|^{10}\mathrm{B}\rangle/\sqrt{7}\times\sum_{\substack{ll'\\mm'}}I_{ll'}P_l^mP_{l'}^{m'}e^{-im'\theta}\left[\begin{array}{cc|c}l' & l & 3\\ m' & m & 2\end{array}\right]\ .\end{aligned} \qquad \text{(A7)}$$

Or alternatively for scattering to the right, $M=-2$ in Eq. (A5). In this case, $F_{\mathrm{right}}(\theta)$ is given by

$$\begin{aligned}F_{\mathrm{right}}(\theta)&=F_{-2}(\theta)\\&=\lambda_{\mathrm{SF}}\langle\,^{10}\mathrm{C}\|[Y_2\times\sigma]_3\|^{10}\mathrm{B}\rangle/\sqrt{7}\times\sum_{\substack{ll'\\mm'}}I_{ll'}P_l^mP_{l'}^{m'}e^{im'\theta}\left[\begin{array}{cc|c}l' & l & 3\\ m' & m & -2\end{array}\right]\ .\end{aligned} \qquad \text{(A8)}$$

But $\left[\begin{smallmatrix}l' & l\\ m' & m\end{smallmatrix}\middle|\begin{smallmatrix}3\\ -2\end{smallmatrix}\right]$ equals $(-1)\left[\begin{smallmatrix}l' & l\\ -m' & -m\end{smallmatrix}\middle|\begin{smallmatrix}3\\ 2\end{smallmatrix}\right]$ which is equal to $\left[\begin{smallmatrix}l & l'\\ -m & -m'\end{smallmatrix}\middle|\begin{smallmatrix}3\\ 2\end{smallmatrix}\right]$, since $l\pm l'$ is even in our case when $K=2$, from Eq. (A1). Upon the interchange of $l\leftrightarrow l'$ and $-m\leftrightarrow m'$, Eq. (A8) becomes

$$F_{\mathrm{right}}(\theta)=\lambda_{\mathrm{SF}}\langle\,^{10}\mathrm{C}\|[Y_2\times\sigma]_3\|^{10}\mathrm{B}\rangle/\sqrt{7}e^{-i2\theta}\times\sum_{\substack{ll'\\mm'}}I_{l'l}P_l^mP_{l'}^{m'}e^{im'\theta}\left[\begin{array}{cc|c}l' & l & 3\\ m' & m & 2\end{array}\right]\ , \qquad \text{(A9)}$$

since now $m+m'=2$.

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