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# Electromagnetic interaction of an off-shell nucleon

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The electromagnetic vertex for an off-shell nucleon is investigated. To get an idea of the magnitude of the off-shell variation, which is usually neglected, a one-pion loop model is used. For simplicity, only the half-off-shell case is considered. Variations up to 10% relative to the on-shell case are found in the form factors for kinematics occurring in intermediate energy reactions. The magnitude of the off-shell effects is also investigated within a commonly used recipe for the electron-bound nucleon cross section and found to be of comparable size.

## I. INTRODUCTION

Electron scattering has been one of the most important and accurate tools for nuclear structure research. The main reason is that the electromagnetic interaction of the electron is well known and can be used for precise studies of the nuclear structure. The standard theoretical approach to analyze the data has been to assume that the current operator for the nucleus is the sum of the *free* nucleon operators. In contrast, many recent investigations have focused on the following question: How well do we actually know the interaction of an electron with a *bound* nucleon? A good example are the (e,e′p) coincidence experiments[1–3] in the quasifree knockout region. The purpose of these measurements is to search for deviations of the electron-bound nucleon interaction from that of a free nucleon. A variety of mechanisms has been proposed which yield such medium modifications of the electromagnetic interaction, for example, relativistic effects on the nucleon spinors due to scalar and vector meson exchanges[4–6] or a "swelling" of the nucleons,[7–9] typically in the order of 10%. Interpretations of the latter type are thought to be connected with explanations of the European Muon Collaboration (EMC) data through changes in the quark distribution of a bound nucleon.[10]

A problem in these studies is that a reference point is needed that allows one to uniquely identify such genuine medium effects. This cannot be done on the basis of the free nucleon current: The relation between energy and momentum for a bound "off-shell" nucleon simply makes it impossible to relate its electromagnetic current to that of a free nucleon in a model-independent way. Nevertheless, several recipes exist,[11,12] and differences among them can be large. However, one class of recipes,[11] where current conservation is imposed at the vertex, yields only small differences in the off-shell behavior. All the prescriptions are based on the assumption that the current is characterized by two form factors which depend only on the four-momentum of the virtual photon. But in general, the electromagnetic vertex for an off-shell nucleon involves more form factors, which depend on, in addition to the square of photon momentum, other scalar variables. To obtain this general vertex, one needs a dynamical model that describes the electromagnetic structure of the nucleon. Constraints for realistic models are, for example, the observed electric and magnetic form factors of the free nucleon. A meaningful investigation of possible medium modifications of individual nucleons such as the recently discussed "swollen nucleons" cannot be done without knowing the full electromagnetic vertex of the off-shell nucleon.

In this paper, we want to study the general structure of the off-shell vertex in a simple field theoretical model which is certainly sufficient for a qualitative discussion. We believe that the model is also realistic enough to give the right order of magnitude of effects involved in the analysis of (e,e′) and (e,e′p) experiments.

The general form of the electromagnetic vertex of an off-shell Dirac particle was already given over twenty years ago by Bincer,[13] who discussed the use of dispersion relations to obtain the electromagnetic structure of a single nucleon. This formalism was subsequently applied by Drell and Pagels[14] to compute the anomalous magnetic moment of electron, muon, and nucleon. The only application of this approach to a reaction was by Nyman,[15] who calculated the cross section for proton-proton bremsstrahlung. The poor description of the data was ascribed by Nyman to assumptions made in the dispersion approach. Since then, the problem of the electromagnetic current of an off-shell nucleon in its general form has to our knowledge not been addressed again. We think that, in light of the recent investigations of the electron-bound nucleon cross section, it is important to look into this problem in more detail.

In Sec. II, we discuss the electromagnetic vertex of an off-shell nucleon and describe the one-pion loop model which we use for our calculations. Results for the form factors and examples of their dependence on the kinematic variables are given in Sec. III; in the framework of the model we also examine the modifications of the off-shell electron-nucleon cross section obtained from a commonly used recipe. A summary and conclusions are presented in Sec. IV.

 

## II. THE ELECTROMAGNETIC VERTEX

### A. General formalism

We start with the most general form for the photon-nucleon vertex. Using the form of Bincer[13] (but the notation of Bjorken and Drell[16]), we have

$$\Gamma_\mu = \sum_{j,k=0,1} (\gamma\cdot p')^j (\gamma_\mu A_1^{jk} + i\sigma_{\mu\nu} q^\nu A_2^{jk} + q_\mu A_3^{jk})(\gamma\cdot p)^k \ . \tag{1}$$

The kinematics are defined in Fig. 1. The 12 functions $A_i^{jk}$ depend on the three scalar variables at the vertex, e.g., $q^2$, and $W'^2 \equiv p'^2$, and $W^2 \equiv p^2$. (We omit isospin labels.) To keep the discussion simple, we will focus here only on particular linear combinations of these $A_1^{jk}$ that occur in the case when the final nucleon is on shell,

$$\bar{u}(p',s')(p'\cdot\gamma - M) = 0 \ ,$$
$$p'^2 = M^2 \ . \tag{2}$$

In that case, the half-off-shell vertex operator can be written as

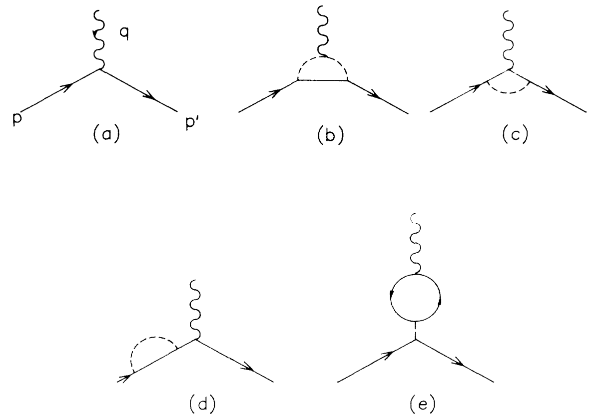


FIG. 1. Types of Feynman diagrams for the photon-nucleon vertex to order $g^2$.

$$\bar{u}(p',s')\Gamma_\mu = e\bar{u}(p',s')\left[\left(\gamma_\mu f_1^{(+)} + \frac{i\sigma_{\mu\nu}}{2M} q^\nu f_2^{(+)} + q_\mu f_3^{(+)}\right)\Lambda_+ + \left(\gamma_\mu f_1^{(-)} + \frac{i\sigma_{\mu\nu}}{2M} q^\nu f_2^{(-)} + q_\mu f_3^{(-)}\right)\Lambda_-\right] , \tag{3}$$

where $f_i^{(\pm)} = f_i^{(\pm)}(q^2, W, M)$. We define $W>0$. The projection operators $\Lambda_\pm$ are defined as

$$\Lambda_\pm = (\pm p\cdot\gamma + W)/2W \ , \tag{4a}$$

and satisfy

$$\Lambda_+ + \Lambda_- = 1 \ , \tag{4b}$$

$$\Lambda_\pm^2 = \Lambda_\pm \ . \tag{4c}$$

In the on-shell case, $W=M$, these operators are the usual projections on the free positive and negative energy states. This type of half-off-shell vertex, Eq. (3), occurs, for example, in the (e,e'N) reaction when the initial nucleon is taken as bound (off shell) and the knocked-out nucleon is assumed to be in a plane wave (on-shell) state. Such an assumption is made in several of the theoretical treatments of nucleon knockout and inclusive electron scattering.[11] We use it here only to keep the discussion brief. For the main points we want to illustrate, this simpler version, Eq. (3), is sufficient.

We can further reduce the number of invariant functions by using the Ward-Takahashi identity,[17] i.e., by requiring gauge invariance,

$$(p'-p)_\mu \Gamma_{\rm irr}^\mu(p',p) = S_F'(p')^{-1} - S_F'(p)^{-1} \ . \tag{5}$$

Here $\Gamma_{\rm irr}^\mu$ is the irreducible photon-nucleon vertex and $S_F'$ the full nucleon propagator. For the half-off-shell case above, we can relate the irreducible vertex to the (reducible) vertex in Eq. (3) by

$$\bar{u}(p',s')\Gamma^\mu(p',p) = \bar{u}(p',s')\Gamma_{\rm irr}^\mu(p',p) S_F'(p) S_F^{-1}(p) \ , \tag{6}$$

with $S_F(p)$ the bare propagator. From Eq. (3) we obtain a separate relation for the functions multiplying the projection operators $\Lambda_\pm$,

$$f_1^{(\pm)} = e_{\rm N} + \frac{q^2}{\pm W - M} f_3^{(\pm)} \ , \tag{7}$$

where $e_{\rm N}$ is the nucleon charge in units of $|e|$.

Using Eqs. (4) and (7), we can write Eq. (3) as

$$\bar{u}(p',s')\Gamma_\mu = e\bar{u}(p',s')\left\{\left[\gamma_\mu\left(e_{\rm N} + \frac{q^2}{W-M}\right) f_3^{(+)} + i\frac{\sigma_{\mu\nu}}{2M} q^\nu f_2^{(+)} + q_\mu f_3^{(+)}\right]\Lambda_+ + \left[\gamma_\mu\left(e_{\rm N} - \frac{q^2}{W+M}\right) f_3^{(-)} + i\frac{\sigma_{\mu\nu}}{2M} q^\nu f_2^{(-)} + q_\mu f_3^{(-)}\right]\Lambda_-\right\} . \tag{8}$$

The usual form factors, $F_1$ and $F_2$, that occur in the on-shell vertex for a free nucleon,

$$\bar{u}(p',s')\Gamma_\mu u(p,s)=e\bar{u}(p',s')\left[\gamma_\mu F_1(q^2)+i\frac{\sigma_{\mu\nu}}{2M}q^\nu F_2(q^2)\right]u(p,s)\ , \tag{9}$$

are related to the quantities in Eq. (8) through

$$F_1(q^2)=e_{\rm N}+q^2\left[\frac{\partial}{\partial W}f_3^{(+)}(q^2,W,M)\right]_{W=M}\ , \tag{10}$$

$$F_2(q^2)=f_2^{(+)}(q^2,M,M)\ . \tag{11}$$

In the situation where the final nucleon is on shell, we are dealing with four invariant functions, which are depending on $q^2$ and $W$. This is in contrast to the free case, Eq. (9), where two functions occur which depend only on $q^2$.

We would like to stress that questions concerning the off-shell behavior of the electromagnetic vertex of the nucleon already play a role in Compton scattering on a *free* nucleon. Since the intermediate nucleon is necessarily off shell, a vertex of the type in Eq. (3) is involved. For example, the vertex for emitting a real photon $k'$ is

$$\bar{u}(p',s')\Gamma_\mu=e\bar{u}(p',s')\left\{\left[\gamma_\mu e_{\rm N}-\frac{i\sigma_{\mu\nu}}{2M}k'^\nu f_2^{(+)}(0,W,M)\right]\Lambda_+ +\left[\gamma_\mu e_{\rm N}-\frac{i\sigma_{\mu\nu}}{2M}k'^\nu f_2^{(-)}(0,W,M)\right]\Lambda_-\right\}\ . \tag{12}$$

For this vertex, one needs a model to extend the form factor $F_2(q^2)$, Eq. (11), off the mass shell to $f_2^{(+)}(0,W,M)$. Furthermore, the form factor $f_2^{(-)}$, which only occurs in the propagation of the $\Lambda_-$ components of the intermediate nucleon, must be known. Therefore, Compton scattering provides a test for dynamical models of the nucleon's electromagnetic structure that goes beyond testing on-shell nucleon properties.

### B. A simple model

To evaluate the form factors $f_i^{(\pm)}$, we use a well-known field theoretical model. It consists of a pion and nucleon field coupled through the pseudoscalar interaction

$$L_{\pi\rm NN}=-ig\bar{\psi}\gamma_5\tau\cdot\phi\psi\ , \tag{13}$$

with

$$\frac{g^2}{4\pi}=14.3\ .$$

We couple the electromagnetic field in the (isospin-dependent) minimal way, which ensures gauge invariance. This model is known to be renormalizable. As in earlier applications,[18] we calculate the electromagnetic vertex up to order $eg^2$. Feynman diagrams of this order are shown in Fig. 1. Diagram (e) is known not to contribute.[19] The terms represented by diagrams (a) and (d) only contribute for the proton vertex. The diagrams we have to evaluate are two irreducible vertex corrections, (b) and (c), and a nucleon self-energy needed for (d). The one-loop integrals in these diagrams are divergent and we have to renormalize. We use the dimensional regularization method[20] and have to evaluate the following expressions for the vertex corrections:

$$\bar{u}(p')\Gamma_\mu^b({\rm p}',{\rm p},{\rm n})=\frac{-2eg^2}{(2\pi)^4}(2\pi M_0)^{4-n}\bar{u}(p')\int d_nk\frac{2k_\mu k_\nu-q_\mu k_\nu}{(k^2-2k\cdot p'+i\varepsilon)(k^2-\mu^2+i\varepsilon)[(k-q)^2-\mu^2+i\varepsilon]}\gamma^\nu\ , \tag{14a}$$

$$\begin{aligned}\bar{u}(p')\Gamma_\mu^c({\rm p}',{\rm p},{\rm n})=\frac{-2eg^2}{(2\pi)^4}(2\pi M_0)^{4-n}\bar{u}(p')\Bigg\{&\int d_nk\frac{k_\alpha k_\beta}{(k^2-2k\cdot p'+i\varepsilon)(k^2-\mu^2+i\varepsilon)[(p-k)^2-M^2+i\varepsilon]}\gamma^\alpha\gamma_\mu\gamma^\beta\\&+\int d_nk\frac{k_\alpha}{(k^2-2k\cdot p'+i\varepsilon)(k^2-\mu^2+i\varepsilon)[(p-k)^2-M^2+i\varepsilon]}\\&\times\gamma^\alpha\gamma_\mu(M-p_\beta\gamma^\beta)\Bigg\}\ .\end{aligned} \tag{14b}$$

The self-energy is given by

$$\begin{aligned}-i\Sigma({\rm p},{\rm n})=\frac{3g^2}{(2\pi)^4}(2\pi M_0)^{4-n}\Bigg\{&\int d_nk\frac{k_\alpha}{(k^2-M^2+i\varepsilon)[(k-p)^2-\mu^2+i\varepsilon]}\gamma^\alpha\\&-\int d_nk\frac{M}{(k^2-M^2+i\varepsilon)[(k-p)^2-\mu^2+i\varepsilon]}\Bigg\}\ .\end{aligned} \tag{15}$$

$M_0$ is an arbitrary reference mass needed to ensure that the expressions are of the right dimensions when $n \neq 4$. Of course, the final result does not depend on this mass. We use the standard Feynman identities to combine the propagators appearing in Eqs. (14) and (15). The resulting denominator for each diagram is then free of poles for spacelike photons and $p^2 = W^2 \leq M^2$, which ensures that the pion remains virtual (off shell).

The isospin structure of the vertex is such that the contributions from the diagrams for the irreducible vertex are for a neutron

$$\Gamma_\mu^{\rm irr} = \Gamma_\mu^{(b)} + \Gamma_\mu^{(c)} , \tag{16a}$$

and for a proton

$$\Gamma_\mu^{\rm irr} = \Gamma_\mu^{(a)} - \Gamma_\mu^{(b)} + \tfrac{1}{2}\Gamma_\mu^{(c)} . \tag{16b}$$

The divergences for the neutron case, Eq. (16a), cancel as is necessary. For the proton this cancellation does not occur and the charge is renormalized. Finally, the divergent part of the self-energy is canceled by a mass counterterm. As in all applications, we adopt the choice that the renormalized charge and mass parameters have the physical values. The two-dimensional integrals over the Feynman parameters are evaluated numerically. As mentioned above, the model is gauge invariant. Therefore, it is a check of the whole calculation that one indeed obtains only four linearly independent functions for the vertex as in Eq. (8). Another test is that the form factors, $f_1^{\pm}(q^2, W, M)$, vanish for $q^2 = 0$ for the neutron.

To assess how "realistic" this model is, we compare some properties of the predicted on-shell form factors, defined in Eqs. (10) and (11), with the experimental values. For any gauge invariant model the "Dirac" form factor, $F_1(q^2)$, at $q^2 = 0$ is determined by the nucleon charge, i.e., $F_1^{\rm p}(0) = 1$ and $F_1^{\rm n}(0) = 0$. For the rms radius, obtained from the slope of $F_1(q^2)$ near the photon point $q^2 = 0$, we obtain $\langle r_{\rm p}^2 \rangle^{1/2} = 0.55$ fm, while the observed value is 0.8 fm. The Pauli form factor, $F_2(q^2)$, yields at $q^2 = 0$ the anomalous magnetic moment of the nucleon. For the proton, we obtain $F_2^{\rm p}(0) = \kappa_{\rm p} = 0.51$ compared to an experimental value of $\kappa_{\rm p} = 1.79$. The predicted rms radius for $F_2^{\rm p}$ is $\langle r_{\rm p}^2 \rangle^{1/2} = 0.58$ fm, while the data yield 0.85 fm. The anomalous magnetic moment of the neutron obtained from the model is $\kappa_{\rm n} = -3.7$, while the experimental value is $\kappa_{\rm n} = -1.91$.

Clearly, this simple model only provides a qualitative description[21] of the observed nucleon properties. This is sufficient for the purpose of our paper where we only focus on relative off-shell effects *within* the model. We expect that the order of magnitude of the effects we obtain below will be the same in a more realistic model.

## III. RESULTS

### A. The off-shell vertex

We now discuss results for the electromagnetic vertex of a nucleon where the initial nucleon is bound and the final nucleon is on shell. This situation is encountered in (e,e′p) calculations, where the knocked out nucleon is described by a plane wave. This expression is given in Eq. (8). For simplicity, we only present examples for a proton.

We start with the form factors $f_1^{(\pm)}$, which are constrained to go to 1 as $q^2 \to 0$. Figure 2 shows $f_1^{(+)}$, as a function of $Q^2 = -q^2$ and for several off-shell situations of the initial nucleon, characterized by $W < M$. As a reference curve, we also show the free form factor $F_1$ predicted by the model. The $Q^2$ dependence is not changed significantly as one goes off shell. For a given $Q^2$ the magnitude of the form factor increases as one goes further off shell. At $Q^2 = 10$ fm$^{-2}$, the increase from $W = M$ to $W = 700$ MeV is about 8%. Since for a bound nucleon

$$W^2 = (M - E_S)^2 - \mathbf{p}^2 ,$$

one can go off shell by increasing the separation energy, $E_S$, or the initial momentum $\mathbf{p}$. The curve $W = 700$ MeV corresponds to already a rather extreme situation: For a binding of 60 MeV, the initial nucleon momentum is 532 MeV/$c$, about twice the Fermi momentum. Figure 3 shows $f_1^{(-)}$, which is of about the same order of magnitude over the range of four-momenta considered here. The off-shell variation of $f_1^{(-)}$ is much stronger than for $f_1^{(+)}$. In both cases, we see that by going off shell the slope of the form factor decreases, corresponding to a smaller rms radius.

Figures 4 and 5 show the form factors $f_2^{(\pm)}$. When going off shell to $W = 700$ MeV, the form factor $f_2^{(+)}$ increases by about 10%; in contrast, $f_2^{(-)}$ decreases by 30%. While $f_1^{(\pm)}$ was subject to the gauge constraint $f_1^{(\pm)}(0, W, M) = 1$, there is no such condition for $f_2^{(\pm)}(0, W, M)$. We therefore already see an off-shell effect of $f_2^{(\pm)}$ at the photon point. The off-shell variation for $f_2^{(-)}$ at $q^2 = 0$ is much larger than that of $f_2^{(+)}$.

The form factors $f_i^{(-)}$, which are absent in the on-shell case, are of a similar magnitude as the $f_i^{(+)}$ form

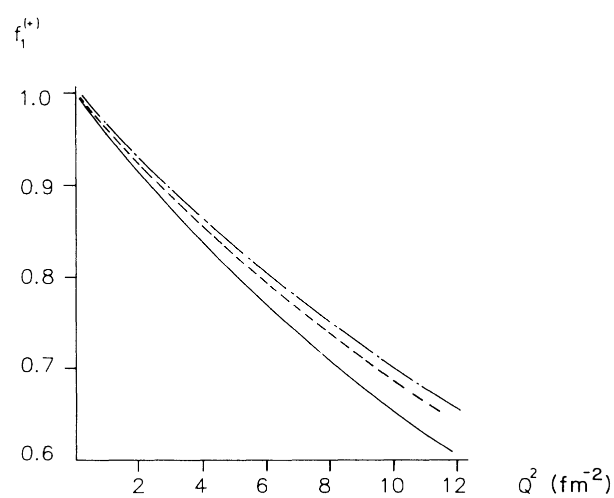


FIG. 2. The form factor $f_1^{(+)}(q^2, W, M)$ defined in Eq. (7) as a function of $Q^2 = -q^2$. Solid curve, $W = M$; dashed, $W = 800$ MeV; dot-dashed, $W = 700$ MeV.

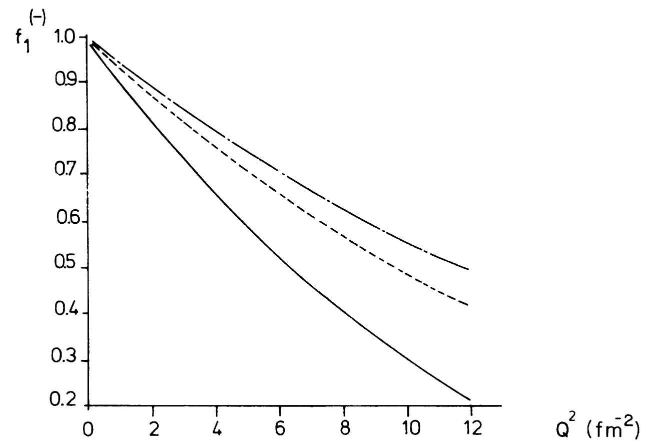


FIG. 3. The form factor $f_1^{(-)}(q^2,W,M)$, defined in Eq. (7). Labeling as in Fig. 2.

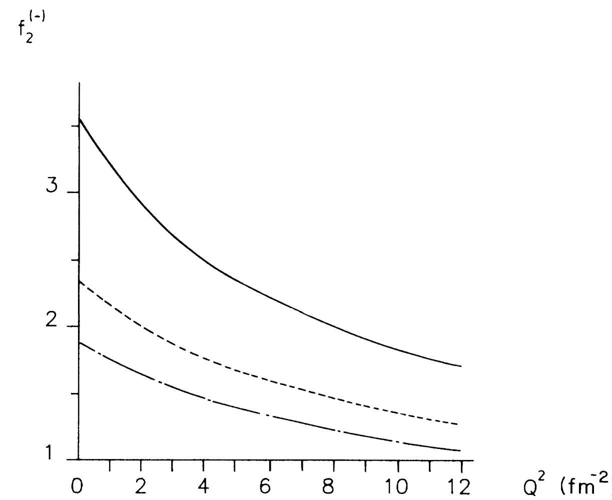


FIG. 5. The form factor $f_2^{(-)}(q^2,W,M)$, defined in Eq. (3). Labeling as in Fig. 2.

factors. How much the $f_i^{(+)}$ and $f_i^{(-)}$ will contribute to a physical amplitude if the initial nucleon is off shell depends, of course, on the dynamics of the system, i.e., the $\Lambda_+$ and $\Lambda_-$ components of the initial nucleon wave function.

A linear combination of the Dirac and Pauli form factors, $F_1$ and $F_2$, that is used most commonly are the electric and magnetic Sachs form factors $G_E$ and $G_M$, respectively. We extend their definition also to the half-off-shell case,

$$G_E^{(\pm)}(q^2,W,M)=f_1^{(\pm)}(q^2,W,M)+\frac{q^2}{4M^2}f_2^{(\pm)}(q^2,W,M)\ , \tag{17a}$$

$$G_M^{(\pm)}(q^2,W,M)=f_1^{(\pm)}(q^2,W,M)+f_2^{(\pm)}(q^2,W,M)\ . \tag{17b}$$

In several recent experiments, there were attempts to extract the deviation of these form factors for a bound nucleon from the free form factors due to medium

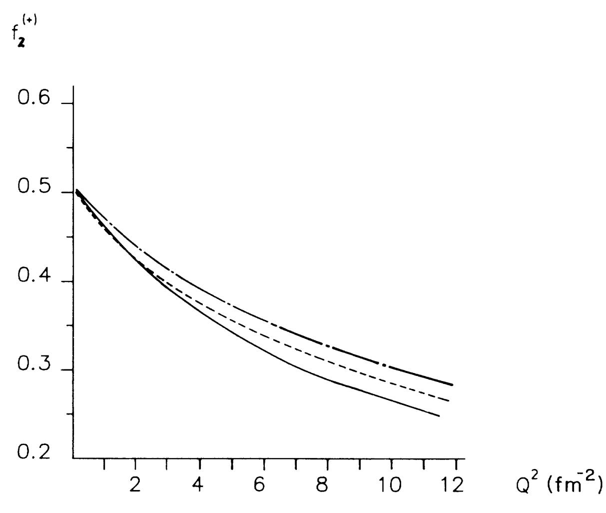


FIG. 4. The form factor $f_2^{(+)}(q^2,W,M)$, defined in Eq. (3). Labeling as in Fig. 2.

modifications. Figure 6 shows our results for the off-shell to on-shell ratio for $G_E^{(+)}$ and $G_M^{(+)}$. Except in the vicinity of $Q^2=0$, the ratios increase for increasing $Q^2$ over the range considered here. However, the curve with $W=900$ MeV, which is the least off shell, can be seen to start to decrease again. Figure 6 shows that the increase of the ratio is larger for $G_M$ than for $G_E$. In an (e,e'p) experiment[1] investigating the bound proton electromagnetic interaction, the ratio $G_M/G_E$ was investigated for $Q^2$ up to 6.5 fm$^{-2}$. This ratio is shown in Fig. 7 for our model. The off-shell variation is rather small and amounts to not more than 3% over the $Q^2$ range considered; this is smaller than the variation in the form factors separately. It is much less than the size of the medium effects (22%) reported in Ref. 1.

### B. Cross sections

To calculate a cross section (or an amplitude) for an electromagnetic nuclear reaction, we must besides the vertex also know both initial and final nuclear wave functions. These wave functions determine how far a nucleon is off shell or, for example, how important the $\Lambda_-$ components of the vertex are. The many-body dynamics which yields the wave functions also will give rise to medium modifications of the electromagnetic interaction beyond the single nucleon off-shell effects discussed here. Such a complete treatment, which depends on details of the nuclear dynamics, is beyond the scope of this paper. We therefore conclude this section by looking only at off-shell effects in the electron-bound nucleon cross section obtained from a commonly used recipe. This is the prescription of De Forest,[11] who evaluates electromagnetic vertex operators between free nucleon spinors (determined by the three-momentum). For an initially bound nucleon, this necessarily leads to a violation of current conservation. The recipe to restore current conservation is to use a longitudinal current matrix element obtained via the continuity equation rather

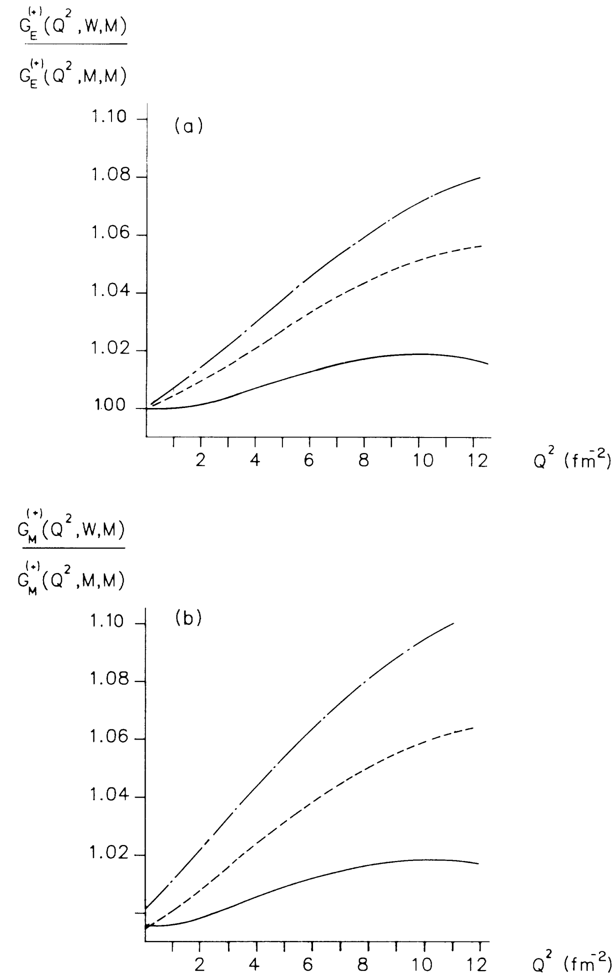


FIG. 6. (a) The ratio $G_E^{(+)}(q^2,W,M)/G_E^{(+)}(q^2,M,M)$ as a function of $Q^2=-q^2$. Solid curve, $W=900$ MeV; dashed, 800 MeV; dot-dashed, $W=700$ MeV. (b) The ratio $G_M^{(+)}(q^2,W,M)/G_M^{(+)}(q^2,M,M)$. Labeling as in (a).

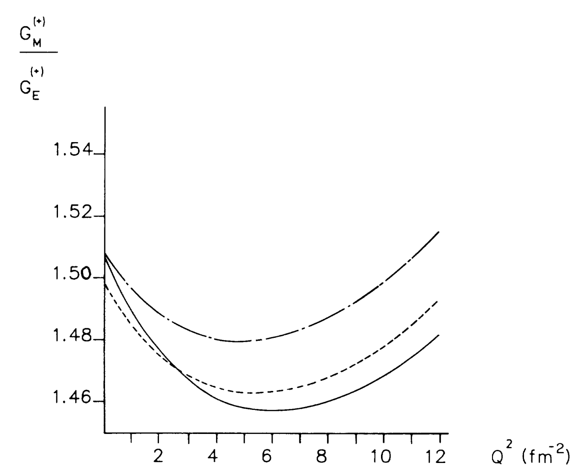


FIG. 7. The ratio $G_M^{(+)}(q^2,W,M)/G_E(q^2,W,M)$ as a function of $Q^2=-q^2$. Solid curve, $W=M$; dashed, $W=800$ MeV; dot-dashed, $W=700$ MeV.

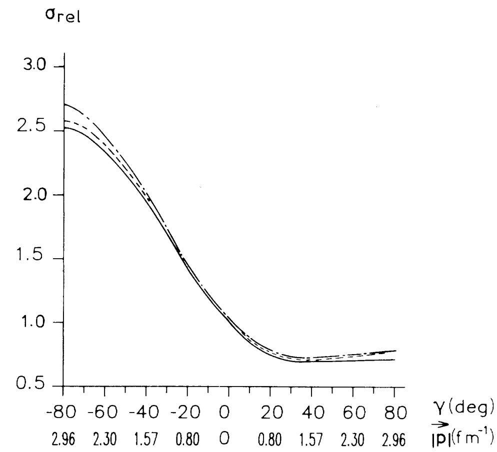


FIG. 8. The relative off-shell cross section defined in the text for scattering of 320 MeV electrons at $|\mathbf{q}|=2.3$ fm$^{-1}$. The angle between **q** and **p** is $\gamma$. Solid curve, $\sigma_{cc2}$; dashed, without form factors $f_i^{(-)}$; dot-dashed, full calculation.

than calculating it from the operator with the free spinors. With these matrix elements the cross section for a nucleon, $\sigma_{cc}$ (cc denotes current conservation), is then calculated in the standard way. While this popular recipe contains no further input from nuclear dynamics, we use it here with our vertex operator to test its sensitivity to off-shell variations.

We have calculated the same kinematical situations considered in Ref. 11, but discuss only one typical example. Figure 8 shows the cross sections for a nucleon with initial momentum **p** and for a given separation energy, $E_S=60$ MeV. The final nucleon is treated as free, has a momentum of $|\mathbf{p}'|=2.3$ fm$^{-1}$, and is assumed to be in the electron scattering plane. Varying $|\mathbf{p}|$ away from $|\mathbf{p}|=0$ corresponds to going further off shell. Our reference curve is the cross section based on the free current in Eq. (9). It corresponds to the cross section $\sigma_{cc2}$ of de Forest, but with the free form factors $F_1$ and $F_2$ obtained from our model.[22] All curves in the figure are normalized such that $\sigma_{cc2}=1$ at $|\mathbf{p}|=0$. The cross section resulting from the full vertex $\Gamma_\mu$, Eq. (8), is larger. This is not surprising in view of the behavior of the form factors discussed above. The relative increase is smallest for $|\mathbf{p}|=0$ (about 2%) and increases to 8% at $|\mathbf{p}|=3$ fm$^{-1}$. Figure 8 also shows that the contribution from the form factors $f_i^-$ is not negligible for $\gamma < -40°$, even though free spinors are used. (Note that for $W=M$ one has $\Lambda_- u=0$.) However, if one uses the free operator structure of Eq. (9), but with the form factors $f_1^{(+)}$ and $f_2^{(+)}$ instead of $F_1$ and $F_2$, one reproduces the result of the full calculation within 3%. This suggests that—for the kinematics considered here—one might neglect the $\Lambda_-$ part and use $\Lambda_+ \to 1$ in Eq. (8) when working with this recipe.

## IV. SUMMARY AND CONCLUSION

For interpretation of the electromagnetic reactions on nuclei and in some cases also on free nucleons (e.g., Compton scattering and pion photoproduction) one must know the photon off-shell nucleon vertex. To obtain this off-shell operator, including the form factors, it is necessary to have a microscopic dynamical model for the structure of the nucleon. We have for our qualitative study of the vertex chosen a renormalizable one pion loop model with pseudoscalar coupling. While this only yields rough agreement with the on-shell nucleon observables, we believe that this model is sufficient to study sizes of the off-shell variations. To keep the discussion simple, we only consider the case where one of the nucleons is on shell. This is an assumption often made in, e.g., calculations of quasifree electron scattering. The requirement of gauge invariance then further reduces the number of independent vertex functions to four, in contrast to the on-shell case when there are only two. Furthermore, these half-off-shell form factors depend on, in addition to $Q^2$, another scalar variable, $W$, the invariant mass of the off-shell nucleon. (This is in contrast to, for example, the "swollen nucleon" radius, which is a fixed property of a nucleon in a given nucleus.)

The order of magnitude of the form factors $f_i^{(+)}$, $i=1,2$, which in the on-shell limit go to the Dirac and Pauli form factors, and the form factors $f_i^{(-)}$, $i=1,2$, is comparable. For all four form factors, the $Q^2$ dependence is not changed significantly as one goes off shell. However, the variation as one goes from on-shell ($W=M$) to off-shell situations that occur in intermediate energy nuclear reactions can be considerable. For example, for $W=700$ MeV and $Q^2=10$ fm$^{-2}$, one form factor varies up to 30%. In almost all cases considered, going off shell increased the magnitude of the form factors. We have also examined the off-shell behavior of the ratio of form factors $G_M/G_E$, which has been studied recently with the (e,e′p) reaction. We found this ratio to be rather insensitive to off-shell effects. This indicates that in this case such effects are not likely to be responsible for the reported medium modification of the virtual photon-proton coupling in nuclei.

As an application of our half-off-shell form factors, we calculated the e-N cross section for a bound nucleon in the framework of a commonly used recipe, which makes simplifying assumptions about the nucleon wave functions and current structure. For our kinematics, we found that when our off-shell effects are included in this prescription, the cross section increases by up to 8%. We would like to stress again that the above recipe does not involve any nuclear dynamics. To get a better estimate of how large the off-shell effects in a nuclear reaction really are, one has to use correct (relativistic) nuclear wave functions. The usually neglected contribution of the $f_i^{(-)}$, $i=1,2$, form factors in the vertex depends crucially on the wave function. If one goes beyond the one-nucleon treatment, the nuclear many-body dynamics will not only yield the nuclear wave functions and many-body electromagnetic currents, but also consistently determine the photon-bound nucleon vertex.

We believe that our discussion in the simple one-pion loop model has indicated the order of magnitude of the off-shell effects. For a nucleon sufficiently far off shell, the size of these effects can be considerable, for example, comparable to the more exotic medium modifications of the nucleon form factors used to explain recent (e,e′) and (e,e′p) data. Therefore they should in general be incorporated in precise studies of electromagnetic reactions where a nucleon off its mass shell is involved.

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