Effect of neutron-proton mass difference on charge symmetry breaking in neutron-proton elastic scattering

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The nucleon mass difference contribution to charge symmetry breaking in neutron-proton elastic scattering is calculated in a relativistic formalism based upon a covariant representation of the NN amplitude. The charge symmetry breaking amplitude is separated into two terms: a piece which involves the on-shell charge symmetric T matrix with charge symmetry breaking arising due to effects in external wave functions, and a term involving off-shell T matrices with charge symmetry breaking associated with mass difference effects in internal nucleon propagators. We find that most of the charge symmetry breaking arising from the neutron-proton mass difference comes from the latter term.

I. INTRODUCTION

Charge symmetry breaking of the nucleon-nucleon interaction, i.e., invariance under isospin transformation of neutrons to protons (and vice versa), is satisfied to a large degree. It is broken only by effects due to isospin violating mass differences and by electromagnetic interactions. In this paper we present a new calculation of charge symmetry breaking in elastic neutron-proton scattering due to the neutron-proton mass difference. This calculation was motivated by a recent experiment at TRIUMF which has, for the first time, observed the existence of charge symmetry breaking (CSB) in the neutron-proton system.[1] The experiment measured the difference in analyzing power

$$\Delta A = A_{\rm n}(\theta_{\rm n}) - A_{\rm p}(\theta_{\rm p}) , \tag{1}$$

in np scattering. Here $A_{\rm n}(p)$ is the analyzing power for scattering of a polarized (unpolarized) neutron beam from an unpolarized (polarized) proton target. For the kinematics used in the experiment, neutron laboratory kinetic energy of 477 MeV and $\theta_{\rm c.m.}=70°$, the analyzing power is going through zero, so that a null experiment is possible.[1] The experimental result is

$$\Delta A_{\rm exp} = (37\pm17\pm8)\times10^{-4} . \tag{2}$$

This quantity would be exactly zero, if charge symmetry were exact.

In the past charge symmetry breaking in the neutron-proton system has been discussed in terms of non-charge-symmetric potentials. However, it seems more natural for the study of nucleon mass difference effects to focus explicitly on the nucleon propagator and wave functions. We can do this in a covariant formalism. Then with a reasonably mild assumption about the two-nucleon interaction kernel, we can calculate the effect of the neutron-proton mass difference entirely in terms of scattering T matrices. The details are given in Sec. II.

Before going into details, let us first consider a convenient parametrization of NN data in terms of Pauli spin operators. The NN elastic scattering matrix can be represented by[2]

$$\begin{aligned} M(\mathbf{p}',\mathbf{p}) = \tfrac{1}{2}[&(a+b)+(a-b)\boldsymbol{\sigma}_1\cdot\hat{\mathbf{n}}\,\boldsymbol{\sigma}_2\cdot\hat{\mathbf{n}} \\ &+(c+d)\boldsymbol{\sigma}_1\cdot\hat{\mathbf{q}}\,\boldsymbol{\sigma}_2\cdot\hat{\mathbf{q}}+(c-d)\boldsymbol{\sigma}_1\cdot\hat{\mathbf{P}}\,\boldsymbol{\sigma}_2\cdot\hat{\mathbf{P}} \\ &+e(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\hat{\mathbf{n}}+f(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\hat{\mathbf{n}}] , \end{aligned} \tag{3}$$

with

$$\hat{\mathbf{P}}=\frac{\mathbf{p}+\mathbf{p}'}{|\mathbf{p}+\mathbf{p}'|},\quad \hat{\mathbf{q}}=\frac{\mathbf{p}-\mathbf{p}'}{|\mathbf{p}-\mathbf{p}'|},\quad \hat{\mathbf{n}}=\frac{\mathbf{p}\times\mathbf{p}'}{|\mathbf{p}\times\mathbf{p}'|} , \tag{4}$$

where $\mathbf{p}$ and $\mathbf{p}'$ are the initial and final momenta in the center-of-mass system (CMS).

In this parametrization, ΔA is given by

$$\Delta A = \frac{4\,{\rm Re}(b^*f)}{|a|^2+|b|^2+|c|^2+|d|^2+|e|^2+|f|^2} . \tag{5}$$

As a consequence, ΔA is directly related to the charge-symmetry breaking f amplitude and is a measure for CSB in np scattering.

II. FORMALISM

Let T be the neutron-proton scattering operator in the covariant representation. The scattering amplitude T is evaluated between Dirac spinors. We separate from T a piece T_0 which describes neutron-proton scattering in the equal mass limit.

$$T = T_0 + \Delta T , \tag{6}$$

where T_0 is constructed using the familiar invariants S,T,V,A,P. Additional invariants arising because of the neutron-proton mass difference, or off-mass extrapolation would appear in ΔT. The rationale behind this separation is that when T_0 is evaluated between neutron and proton spinors (including the mass difference in the spinors) it gives the only model independent contribution to charge symmetry breaking. It is model independent in the sense that its calculation requires no information

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about the interaction potential or kernel but only the on-shell scattering phase shifts.

The operator ΔT is model dependent and, in general, its calculation requires that we make some statement about the interaction. To motivate our model we start with the Bethe-Salpeter description of the two-nucleon system. In the CMS the scattering equation takes the form

$$T(p',p)=V(p',p)+\int d^4k\ V(p',k)G(k)T(k,p)\ , \qquad (7)$$

or schematically

$$T=V+VGT\ ,$$

where G is the two-nucleon propagator, that is, the product of one-nucleon Feynman propagators which obviously depend on nucleon masses. The operator V is the two-particle irreducible kernel. We will assume that the difference between neutron and proton masses can be neglected in the kernel. This is exactly true for the lowest order kernel corresponding to single boson exchanges with momentum independent couplings and we assume that it is a good approximation for the full kernel. Then we define another scattering operator T_0' by the equation

$$T_0'=V+VG_0T_0'\ , \qquad (8)$$

where G_0 is the two-nucleon propagator for equal nucleon masses ($\Delta m=m_n-m_p=0$). The two scattering operators T and T_0' are related by

$$T=T_0'+T_0'(G-G_0)T\ . \qquad (9)$$

Now to obtain the leading order (in Δm) contribution it is sufficient to expand Eq. (9) to first order in $(G-G_0)$ and we can also identify the terms in T_0' constructed from the invariants S,T,V,A,P with T_0. Writing $T_0'=T_0+\delta T_0$, where δT_0 contains invariants whose coefficients vanish when $\Delta m=0$, we get

$$T=T_0+\delta T_0+T_0'(G-G_0)T_0'\ , \qquad (10a)$$

$$\Delta T=\delta T_0+T_0'(G-G_0)T_0'\ . \qquad (10b)$$

To calculate ΔT we have to make a model and in this paper we consider an approximation which represents one extreme of simplification, that is, to drop the terms δT_0. This gives

$$T=T_0+T_0(G-G_0)T_0\ , \qquad (11a)$$

$$\Delta T=T_0(G-G_0)T_0\ . \qquad (11b)$$

This approximation represents the best that we can do without having to make specific use of the interaction kernel. The operator ΔT is model dependent in that one still needs an off-shell extrapolation of T_0 to evaluate Eqs. (11).

III. CALCULATION

Using the notation of Tjon and Wallace[3] the invariant (Dirac) representation of the NN scattering operator has the form

$$F=F_S 1^{(1)}1^{(2)}+F_V\gamma_\mu^{(1)}\gamma^{\mu(2)}+F_T\sigma_{\mu\nu}^{(1)}\sigma^{\mu\nu(2)} +F_P\gamma_5^{(1)}\gamma_5^{(2)}+F_A\gamma_5^{(1)}\gamma_\mu^{(1)}\gamma_5^{(2)}\gamma^{\mu(2)}\ , \qquad (12)$$

in the absence of charge symmetry breaking effects. We can then make the identification

$$\hat{F}=\frac{1}{2p}\frac{m^2}{[2\pi(s)^{1/2}]}T_0\ , \qquad (13)$$

where $s=4(p^2+m^2)$. Taking matrix elements of Eq. (12) with nucleon Dirac spinors,

$$u(\mathbf{p},\sigma_i)=\left[\frac{E+m}{2m}\right]^{1/2}\begin{bmatrix}1\\ \dfrac{\sigma\cdot\mathbf{p}}{E+m}\end{bmatrix}\chi_i\ , \qquad (14)$$

it is straightforward to obtain the Pauli amplitudes of Eq. (3). The relationship between invariant amplitudes (F_S,F_V,F_T,F_A,F_P) and charge-symmetric Pauli amplitudes (a,b,c,d,e) is given in Appendix A.

The charge symmetry breaking amplitude [f in Eq. (3)] is obtained by evaluating the right-hand side of Eq. (11) between nucleon Dirac spinors and keeping the terms (in Pauli form) proportional to Δm. The calculation of the second term in Eq. (11a) requires an off-mass-shell amplitude T_0. As a simple model of the off-shell behavior we use Horowitz's[4] relativistic analog of the Love-Franey model. In this model the invariant amplitudes are parametrized with the functional form of one-meson-exchange amplitudes. The (complex) meson couplings and form factors are adjusted to fit empirical NN phase shifts on mass shell. The principal contributions to the total amplitude are in the pseudoscalar, scalar, and vector terms corresponding to π, σ, and ω exchange in Horowitz's parametrization. The advantage of using the amplitude of Ref. 4 is that the second term of Eq. (11a) then takes the form of a Feynman box diagram which greatly simplifies its computation.

The first term in Eq. (11a) is a charge symmetry T_0 operator, so that the resulting CSB contribution will originate from the nucleon mass difference in the Dirac spinors. Both charge symmetric and CSB Pauli amplitudes are calculated in Appendix A. These amplitudes and the corresponding contribution to ΔA are given by the on-shell relativistic amplitudes. The result for the CSB amplitude $f^{(1)}$ [from the first term of Eq. (11a)] is

$$f^{(1)}=f_S(F_S-F_V+2F_T+F_A)\ . \qquad (15)$$

The pseudoscalar amplitude F_P does not contribute to $f^{(1)}$. $f^{(1)}$ is therefore dominantly determined by the scalar and vector amplitude. Note that these two amplitudes appear with a negative relative sign in Eq. (15). The cancellation between the scalar and vector amplitude, which is observed for the charge symmetric Pauli amplitudes, does not hold for $f^{(1)}$. Nonetheless it turns out that the CSB contribution from $f^{(1)}$ is small. At the energy and scattering angle of the TRIUMF experiment, Horowitz's parametrization[5] for $T_{\rm lab}=400$ MeV gives

$$f^{(1)}=(-9.1,3.0)\times10^{-5}\ \text{fm}^2\ ,$$

which gives $\Delta A^{(1)}=0.76\times10^{-4}$.

As noted above, the CSB contribution from the first term of Eq. (11a) does not require the off-shell behavior of the T operator. Therefore, instead of using Horowitz's representation, we can calculate the required relativistic amplitudes directly from a phase-shift analysis. When we calculate the invariant amplitudes from Arndt's program SAID,[6] we get

$$f^{(1)}=(-7.9,3.3)\times10^{-5}\ \text{fm}^2$$

and $\Delta A^{(1)}=2.0\times10^{-4}$.

Note that the above results for $\Delta A^{(1)}$ are different, although the corresponding values for $f^{(1)}$ are more or less comparable. This reflects a strong cancellation in the evaluation of Eq. (5). [At the given energy and scattering angle, $b=(0.024,0.083i)$ fm^2.] In the calculation of $\mathrm{Re}(b^*f)$ the product of the real parts almost cancels the product of the imaginary parts. Note that $\mathrm{Im}f$ and thus higher-order Born contributions to T_0 are crucial for this cancellation. This makes the resulting value of $\Delta A^{(1)}$ rather sensitive to the choice of on-shell amplitude. However, by comparison with the experimental value of ΔA [Eq. (2)], we can see that $\Delta A^{(1)}$, which describes CSB due to external propagation, is negligible.

Now let us consider the second term of Eq. (11a). In this case the CSB contributions come from the nucleon mass difference in the propagator. Due to the form of Horowitz's representation (which describes T_0 as a sum of effective complex one-boson-exchange diagrams), this calculation requires the evaluation of two-boson-exchange box amplitudes. As stated above, these contributions involve the off-shell behavior of T_0 and are expected to be model dependent.

We calculate the corresponding CSB amplitude, denoted $f^{(2)}$, by evaluating the various box diagrams involving pseudoscalar, scalar and vector particle exchange and keeping the terms of order $\Delta m/m$.

The calculation of the four-dimensional loop integration was facilitated by the use of the Blankenbecler-Sugar reduction[7]

$$\frac{1}{(K_1^2-m_1^2+i\varepsilon)}\frac{1}{(K_2^2-m_2^2+i\varepsilon)} \Rightarrow i\pi\frac{\omega_1+\omega_2}{\omega_1\omega_2}\frac{\delta\left[K_0-\dfrac{\omega_1-\omega_2}{2}\right]}{(\omega_1+\omega_2)^2-s-i\varepsilon}\,, \tag{16}$$

where

$$K_0=(K_{1,0}-K_{2,0}),\quad \mathbf{K}=\frac{(\mathbf{K}_1-\mathbf{K}_2)}{2}\,, \tag{17}$$

$$\omega_{1,2}=(\mathbf{K}^2+m_{1,2}^2)^{1/2},\quad \sqrt{s}=(\mathbf{p}^2+m_1^2)^{1/2}+(\mathbf{p}^2+m_2^2)^{1/2}\,.$$

Detailed expressions giving the f amplitude for various box diagrams are presented in Appendix B. Using parameters of Horowitz, the results for (the CSB quantity) ΔA are given in Table I for the kinematical configurations of present and planned experiments.

Compared with $\Delta A^{(1)}$ as given above, we see that large contributions to ΔA can arise from nucleon mass difference effects in the relativistic two-nucleon propagator. The largest contribution to ΔA comes from the "box diagrams" involving pseudoscalar-scalar and pseudoscalar-vector parts of $T_0(G-G_0)T_0$. However, the contribution to ΔA from these appear with opposite signs.

TABLE I. $10^4\Delta A$ for various box diagrams.

	$T_{\text{lab}}=477$ MeV $\theta_{\text{c.m.}}=70°$	$T_{\text{lab}}=350$ MeV $\theta_{\text{c.m.}}=72°$	$T_{\text{lab}}=188$ MeV $\theta_{\text{c.m.}}=96°$
P,P	3.05	1.2	-2.5
P,S	-10.8	-11.7	-0.3
P,V	12.9	15.8	4.3
S,S	3.6	2.3	-0.4
S,V	-0.006	0.06	0.4
V,V	0.29	1.6	-0.5
Total	8.5	9.3	1.3

In Table II we have added to ΔA the contributions from electromagnetic interactions and ρ-ω mixing. These numerical values are from Ref. 8. Our total charge symmetry breaking is somewhat smaller than the presently available experimental result at 477 MeV.

In Fig. 1 we have plotted the angular distribution of ΔA at 477 MeV for the charge symmetry breaking amplitude f calculated in the plane wave one pion exchange approximation. The charge symmetric amplitudes were obtained from the SAID program (this is also true for all other calculations in this paper). We see that at small center of mass angles ΔA is negative. It has a maximum near 70°, where it has a value -14×10^{-14} and is zero at 90°. For larger angles it is large and positive and has a peak value of $+43\times10^{-4}$ at 130°. This backward peak is clearly due to the exchange nature of the process as the OPE contribution to ΔA comes from u-channel diagram.

In Fig. 2 is shown the angular distribution of ΔA from the full calculations. Notice two important points in comparison to OPE contribution, i.e., Fig. 1. Firstly, the large and positive contribution at large angles has been greatly reduced in full calculations. Second, ΔA at smaller angles has changed sign and has become positive. The comparison of Fig. 2 with Fig. 1 reveals the quantitative effects we expect in a calculation with distortions. The effect of distortions (which is our full cal-

TABLE II. $10^4\Delta A$. The separate contributions of electromagnetic, proton-neutron mass difference (calculated using Horowitz's parametrization), and ρ-ω interactions are given.

T_{lab} $\theta_{\text{c.m.}}$	477 MeV 70°	350 MeV 72°	188 MeV 96°
Electromagnetic	6	3	10
Proton-neutron mass difference	10.5	8.1	-2.6
$\rho\omega$	5 ± 1.2	1.6 ± 0.6	-2.3 ± 0.8
Total	21.5 ± 1.2	12.7 ± 0.6	5.1 ± 0.8
Experiment	$37\pm17\pm8$		

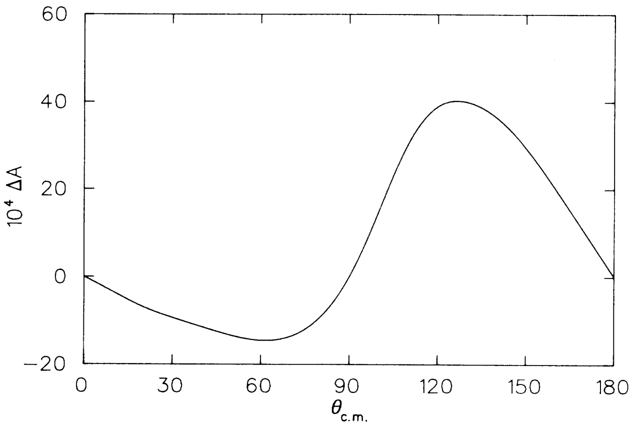

FIG. 1. Contribution to $[10^{-4}\Delta A(\theta)]$ from plane wave one-pion exchange.

culation) is to reduce the amplitude at high momentum transfer and enhance it in regions of lower momentum transfer. The structure in the angular distribution of Fig. 1 gets washed out when the charge symmetry breaking amplitudes acquires a complex phase.

For the kinematics of the TRIUMF experiment the contribution to ΔA from proton-neutron mass difference is of the order of 10×10^{-4}. This is smaller than the previous calculations[8,9] based on a distorted-wave formation. In these calculations distortions play an important role in making ΔA large and positive. Although our numerical results show the qualitative features of distortion we have not been able to reproduce the same degree of enhancement of ΔA at small angles. An obvious place to look for this discrepancy is in the simple off-shell extrapolation of the T matrix that has been used here.

IV. CONCLUSIONS

We have presented a new calculation of charge symmetry breaking effects in neutron-proton scattering due to the neutron-proton mass difference. The calculation uses a covariant formulation which allows one to focus explicitly on the mass difference effect in nucleon propagation. The calculation is carried out in terms of T matrices, without recourse to a potential, although a model is required for the off-shell behavior.

We find a small charge symmetry breaking effect in the analyzing power due to nucleon mass differences with most of the effect coming from differences in external wave functions and in propagation in intermediate states. The smallness of our calculated effect is due to cancellations between different parts of the charge symmetric T matrices. The final numbers are quite sensitive to the exact value of the T matrix but the fact that strong cancellations occur seems to be model independent. We have not investigated the sensitivity of this calculation to the particular choice of the Blankenbecler-Sugar reduction to evaluate the box diagrams nor to the choice of the off-shell T matrix. This would be an interesting question for further study.

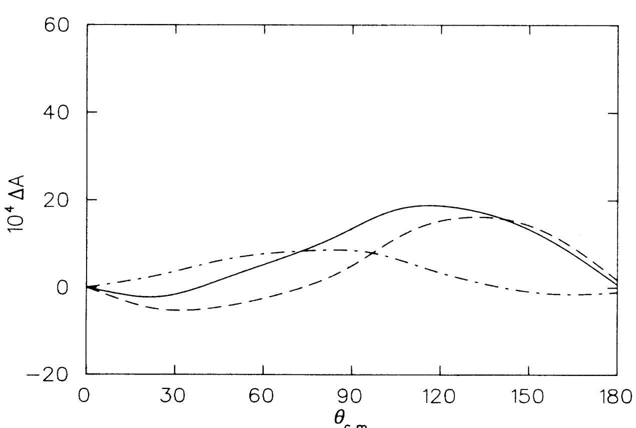

FIG. 2. Total contribution to $10^4\Delta A(\theta)$ from charge dependent NN T matrix. Dash line is the contribution from T_0. Dash-dot line is the contribution from $T_0(G-G)T_0$. Solid line is the full contribution.

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APPENDIX A

In the notation of Gersten,[2] the NN scattering operator is written as [see Eq. (3)]

$$\hat{M}=\tfrac{1}{2}[(a+b)+(a-b)\boldsymbol{\sigma}_1\cdot\hat{\mathbf{n}}\,\boldsymbol{\sigma}_2\cdot\hat{\mathbf{n}}+(c+d)\boldsymbol{\sigma}_1\cdot\hat{\mathbf{q}}\,\boldsymbol{\sigma}_2\cdot\hat{\mathbf{q}}$$
$$+(c-d)\boldsymbol{\sigma}_1\cdot\hat{\mathbf{P}}\,\boldsymbol{\sigma}_2\cdot\hat{\mathbf{P}}+e(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\hat{\mathbf{n}}$$
$$+f(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\hat{\mathbf{n}}]\ , \qquad \text{(A1)}$$

where different units vectors are defined in the main text. Equating the matrix elements of $\hat{M}$ taken between Pauli spinors to the Dirac spinor matrix elements of the covariant operator $\hat{F}$ [defined in Eq. (10)], we have

$$\bar{u}_{\rm p}(\mathbf{p}',s_1')\bar{u}_{\rm n}(-\mathbf{p}',s_2')\hat{F}u_{\rm p}(\mathbf{p},s_1)u_{\rm n}(-\mathbf{p},s_2)$$
$$=\chi_{s_1'}^{\dagger}\chi_{s_2'}^{\dagger}\hat{M}\chi_{s_1}\chi_{s_2}\ . \qquad \text{(A2)}$$

Here subscript p (n) on the Dirac spinors stands for proton (neutron). From Eq. (A2) we can write the a, b, c, d, e, and f amplitudes in terms of F_S, F_V, F_T, F_P, and F_A amplitudes. We get

$$\frac{a+b}{2}=\alpha_S F_S+\alpha_V F_V+\alpha_T F_T+\alpha_P F_P+\alpha_A F_A\ ,$$
$$\frac{a-b}{2}=\beta_S F_S+\beta_V F_V+\beta_T F_T+\beta_P F_P+\beta_A F_A\ ,$$
$$\frac{c+d}{2}=\gamma_S F_S+\gamma_V F_V+\gamma_T F_T+\gamma_P F_P+\gamma_A F_A\ , \qquad \text{(A3)}$$
$$\frac{c-d}{2}=\delta_S F_S+\delta_V F_S+\delta_V F_V+\delta_T F_T+\delta_P F_P+\delta_A F_A,$$
$$e=e_S F_S+e_V F_V+e_T F_T+e_P F_P+e_A F_A\ ,$$
$$f=f_S F_S+f_V F_V+f_T F_T+f_P F_P+f_A F_A\ .$$

The coefficients of expansion are

$$\alpha_S=\frac{1}{4m_pm_n}\left[(E_p+m_p)-\frac{p^2\cos\theta_{c.m.}}{(E_p+m_p)}\right]\left[(E_n+m_n)-\frac{p^2\cos\theta_{c.m.}}{(E_n+m_n)}\right],$$

$$\alpha_V=\frac{1}{4m_pm_n}\left\{\left[(E_p+m_p)+\frac{p^2\cos\theta_{c.m.}}{(E_p+m_p)}\right]\left[(E_n+m_n)+\frac{p^2\cos\theta_{c.m.}}{(E_n+m_n)}\right]+2p^2(1+\cos\theta_{c.m.})\right\},$$

$$\alpha_T=-\frac{1}{4m_pm_n}2\left[2p^2(1-\cos\theta_{c.m.})+\frac{p^4\sin^2\theta_{c.m.}}{(E_p+m_p)(E_n+m_n)}\right],$$

$$\alpha_P=0\ ,$$

$$\alpha_A=\frac{1}{4m_pm_n}\frac{1}{(E_p+m_p)(E_n+m_n)}p^4\sin^2\theta_{c.m.}\ ,$$

$$\beta_S=-\frac{1}{4m_pm_n}\frac{1}{(E_p+m_p)(E_n+m_n)}p^4\sin^2\theta_{c.m.}\ ,$$

$$\beta_V=-\frac{1}{4m_pm_n}\left[\frac{p^4\sin^2\theta_{c.m.}}{(E_p+m_p)(E_n+m_n)}+2p^2(1-\cos\theta_{c.m.})\right],$$

$$\beta_T=\frac{1}{4m_pm_n}2\left[2p^2(1+\cos\theta_{c.m.})+(E_p+m_p)(E_n+m_n)+\left[\frac{E_n+m_n}{E_p+m_p}+\frac{E_p+m_p}{E_n+m_n}\right]p^2\cos\theta_{c.m.}\right.$$
$$\left.+\frac{p^4\cos^4\theta_{c.m.}}{(E_p+m_p)(E_n+m_n)}\right],$$

$$\beta_P=0\ ,$$

$$\beta_A=-\frac{1}{4m_pm_n}\left[(E_p+m_p)(E_n+m_n)-\left[\frac{E_n+m_n}{E_p+m_p}+\frac{E_p+m_p}{E_n+m_n}\right]p^2\cos\theta_{c.m.}+\frac{p^4\cos^2\theta_{c.m.}}{(E_p+m_p)(E_n+m_n)}\right].$$

They continue as

$$\gamma_S=0\ ,$$

$$\gamma_V=0\ ,$$

$$\gamma_T=\frac{1}{4m_pm_n}2\left[2p^2(1+\cos\theta_{c.m.})+(E_p+m_p)(E_n+m_n)+\left[\frac{E_n+m_n}{E_p+m_p}+\frac{E_p+m_p}{E_n+m_n}\right]p^2+\frac{1}{(E_p+m_p)(E_n+m_n)}p^4\right],$$

$$\gamma_P=-\frac{1}{4m_pm_n}2p^2(1-\cos\theta_{c.m.})\ ,$$

$$\gamma_A=-\frac{1}{4m_pm_n}\left[(E_p+m_p)(E_n+m_n)-\left[\frac{E_n+m_n}{E_p+m_p}+\frac{E_p+m_p}{E_n+m_n}\right]p^2+\frac{p^4}{(E_p+m_p)(E_n+m_n)}\right],$$

$$\delta_S=0\ ,$$

$$\delta_V=-\frac{1}{4m_pm_n}2p^2(1-\cos\theta_{c.m.})\ ,$$

$$\delta_T=\frac{1}{4m_pm_n}2\left[(E_p+m_p)(E_n+m_n)-\left[\frac{E_n+m_n}{E_p+m_p}+\frac{E_p+m_p}{E_n+m_n}\right]p^2+\frac{p^4}{(E_p+m_p)(E_n+m_n)}\right],$$

$$\delta_P=0\ ,$$

$$\delta_A=-\frac{1}{4m_pm_n}\left[2p^2(1+\cos\theta_{c.m.})+(E_p+m_p)(E_n+m_n)+\left[\frac{E_n+m_n}{E_p+m_p}+\frac{E_p+m_p}{E_n+m_n}\right]p^2+\frac{p^4}{(E_p+m_p)(E_n+m_n)}\right].$$

And, continuing,

$$e_S=\frac{1}{4m_pm_n}\frac{i}{2}\left\{\left[(E_p+m_p)-\frac{p^2\cos\theta_{c..m}}{(E_p+m_p)}\right]\frac{p^2\sin\theta_{c.m.}}{(E_n+m_n)}+\left[(E_n+m_n)-\frac{p^2\cos\theta_{c.m.}}{(E_n+m_n)}\right]\frac{p^2\sin\theta_{c.m.}}{(E_p+m_p)}\right\},$$

$$e_V=-\frac{1}{4m_pm_n}\frac{i}{2}\left\{\left[(E_p+m_p)+\frac{p^2\cos\theta_{c.m.}}{(E_p+m_p)}\right]\frac{p^2\sin\theta_{c.m.}}{(E_p+m_p)}\right.$$
$$\left.+\left[(E_n+m_n)+\frac{p^2\cos\theta_{c.m.}}{(E_n+m_n)}\right]\frac{p^2\sin\theta_{c.m.}}{(E_p+m_p)}+4p^2\sin\theta_{c.m.}\right\},$$

$$e_T=\frac{1}{4m_pm_n}2i\left[-2p^2\sin\theta_{c.m.}-\frac{1}{2}\left[\frac{E_n+m_n}{E_p+m_p}+\frac{E_p+m_p}{E_n+m_n}\right]p^2\sin\theta_{c.m.}\right.$$
$$\left.-\frac{1}{(E_p+m_p)(E_n+m_n)}p^4\sin\theta_{c.m.}\cos\theta_{c.m.}\right],$$

$$e_P=0\ ,$$

$$e_A=-\frac{1}{4m_pm_n}\left[\frac{i}{2}\right]\left[\left[\frac{E_n+m_n}{E_p+m_p}+\frac{E_p+m_p}{E_n+m_n}\right]p^2\sin\theta_{c.m.}-\frac{1}{(E_p+m_p)(E_n+m_n)}2p^4\sin\theta_{c.m.}\cos\theta_{c.m.}\right],$$

$$f_S=\frac{1}{4m_pm_n}\left[\frac{i}{2}\right]\left[\left[\frac{E_n+m_n}{E_p+m_p}-\frac{E_p+m_p}{E_n+m_n}\right]\right]p^2\sin\theta_{c.m.}\ ,$$

$$f_V=-f_S\ ,$$
$$f_T=2f_S\ ,$$
$$f_P=0\ ,$$
$$f_A-f_S\ .$$

APPENDIX B

Using the Blankenbecler-Sugar reduction[10] for the two-nucleon propagator, the f-amplitude for two-boson-exchange box diagrams can be written as

$$f(p,\theta_{c.m.})=\frac{i}{4p\sqrt{s}}\frac{m^2}{(2\pi)^4}\int d^3K\frac{(\mathbf{K}^2+m^2)^{-1/2}}{\mathbf{p}^2-\mathbf{K}^2-i\varepsilon}I_{M_1M_2}(\mathbf{p},\mathbf{K},\mathbf{p}') \tag{B1}$$

with $m=(m_p+m_n)/2$ the average nucleon mass. Below are given the integrand factors for different contributions,

$$I_{M_1M_2}(\mathbf{p},\mathbf{K},\mathbf{p}')=\frac{g_{M_1}^2g_{M_2}^2F_{M_1}^2(\mathbf{p}-\mathbf{K})F_{M_2}^2(\mathbf{p}'-\mathbf{K})}{[(\mathbf{p}-\mathbf{K})^2+m_1^2][\mathbf{p}'-\mathbf{K})^2+m_2^2]}(\tau_1\cdot\tau_2)^{I_{M_1}+I_{M_2}}R_{M_1M_2}(\mathbf{p},\mathbf{K},\mathbf{p}')\ . \tag{B2}$$

Here $M_{1,2}=\pi,\eta,\sigma,\omega$; $m_{1,2}$ is the corresponding meson mass, $I_{M_{1,2}}$ the isospin of the indicated meson, and $\tau_{1,2}$ are nucleon isospin matrices. $F_{M_{1,2}}$ is a form factor and $g_{M_{1,2}}^2$ the coupling constants. Furthermore, we have ($E=(\mathbf{p}^2+m^2)^{1/2}$, $\Delta\omega=[(K^2+m_n^2)^{1/2}-(K^2+m_p^2)^{1/2}]$),

$$R_{\pi^+\pi^-}=i\left[\frac{E+m}{4m^2}\right]\left[\frac{\Delta m}{2}\left\{\left[1-\frac{\mathbf{p}\cdot\mathbf{p}'}{(E+m)^2}\right](\mathbf{p}-\mathbf{K})\times\mathbf{q}\cdot\hat{\mathbf{n}}-\frac{p^2\sin\theta_{c.m.}}{(E+m)^2}(\mathbf{p}-\mathbf{K})\cdot\mathbf{p}\right\}\right.$$
$$\left.-\frac{\Delta\omega}{2}\left\{\left[1+\frac{\mathbf{p}\cdot\mathbf{p}'}{(E+m)^2}\right](\mathbf{p}-\mathbf{K})\times\mathbf{q}\cdot\hat{\mathbf{n}}+\frac{p^2\sin\theta_{c.m.}}{(E+m)^2}(\mathbf{p}-\mathbf{K})\cdot\mathbf{p}\right\}\right],$$

$$R_{\pi^0\pi^0}=-R_{\pi^+\pi^-}\ ,$$
$$R_{\pi^0\pi^+}=R_{\pi^+\pi^0}=0\ .$$

Apart from isospin the expressions for π^0 can be applied for η,

$$R_{\sigma\sigma}=i\frac{\Delta\omega}{2m}p^2\sin\theta_{\text{c.m.}}+i\left[\frac{E+M}{4m^2}\right]\left[\frac{\Delta m}{2}\left\{\left[1-\frac{\mathbf{p}\cdot\mathbf{p}'}{(E+m)^2}\right](\mathbf{p}-\mathbf{K})\times\mathbf{q}\cdot\hat{\mathbf{n}}-\frac{p^2\sin\theta_{\text{c.m.}}}{(E+m)^2}(\mathbf{p}-\mathbf{K})\cdot\mathbf{P}\right\}\right.$$
$$\left.+\frac{\Delta\omega}{2}\left\{\left[1+\frac{\mathbf{p}\cdot\mathbf{p}'}{(E+m)^2}\right](\mathbf{p}-\mathbf{K})\times\mathbf{q}\cdot\hat{\mathbf{n}}+\frac{p^2\sin\theta_{\text{c.m.}}}{(E+m)^2}(\mathbf{p}-\mathbf{K})\cdot\mathbf{P}\right\}\right],$$

$$R_{\pi^+\sigma}=i\left[\frac{E+m}{4m^2}\right]\left[\frac{\Delta m}{2}\left\{\left[1+\frac{\mathbf{p}\cdot\mathbf{p}'}{(E+m)^2}\right](\mathbf{p}-\mathbf{K})\times\mathbf{P}\cdot\hat{\mathbf{n}}+\frac{p^2\sin\theta_{\text{c.m.}}}{(E+m)^2}(\mathbf{p}-\mathbf{K})\cdot\mathbf{q}\right\}\right.$$
$$\left.+\frac{\Delta\omega}{2}\left\{\left[1+\frac{\mathbf{p}\cdot\mathbf{p}'}{(E+m)^2}\right](\mathbf{p}-\mathbf{K})\times\mathbf{q}\cdot\hat{\mathbf{n}}+\frac{p^2\sin\theta_{\text{c.m.}}}{(E+m)^2}(\mathbf{p}-\mathbf{K})\cdot\mathbf{P}\right\}\right],$$

$$R_{\sigma\pi^+}=i\left[\frac{E+m}{4m^2}\right]\left[\frac{\Delta m}{2}\left\{\left[1+\frac{\mathbf{p}\cdot\mathbf{p}'}{(E+m)^2}\right](\mathbf{p}'+\mathbf{K})\times\mathbf{P}\cdot\hat{\mathbf{n}}+\frac{p^2\sin\theta_{\text{c.m.}}}{(E+m)^2}(\mathbf{p}'+\mathbf{K})\cdot\mathbf{q}\right\}\right.$$
$$\left.-\frac{\Delta\omega}{2}\left\{\left[1+\frac{\mathbf{p}\cdot\mathbf{p}'}{(E+m)^2}\right](\mathbf{p}'+\mathbf{K})\times\mathbf{q}\cdot\hat{\mathbf{n}}+\frac{p^2\sin\theta_{\text{c.m.}}}{(E+m)^2}(\mathbf{p}'+\mathbf{K})\cdot\mathbf{P}\right\}\right],$$

$$R_{\pi^0\sigma}=R_{\sigma\pi^0}=0\ ,$$

$$R_{\pi^0\omega}=R_{\omega\pi^0}=R_{\pi^0\pi^0}\ ,$$

$$R_{\pi^+\omega}=i\left[\frac{E+m}{4m^2}\right]\left[\frac{\Delta m}{2}\left\{\left[1-\frac{\mathbf{p}\cdot\mathbf{p}'}{(E+m)^2}\right](\mathbf{p}-\mathbf{K})\times\mathbf{q}\cdot\hat{\mathbf{n}}-\frac{p^2\sin\theta_{\text{c.m.}}}{(E+m)^2}(\mathbf{p}-\mathbf{K})\cdot\mathbf{P}-2\mathbf{K}\times\mathbf{p}\cdot\hat{\mathbf{n}}\right.\right.$$
$$\left.-\frac{1}{(E+m)^2}[\mathbf{p}\cdot\mathbf{p}'(\mathbf{p}-\mathbf{K})\times\mathbf{p}\cdot\hat{\mathbf{n}}+p^2(\mathbf{p}-\mathbf{K})\times\mathbf{q}\cdot\hat{\mathbf{n}}-p^2\sin\theta_{\text{c.m.}}(\mathbf{p}-\mathbf{K})\cdot\mathbf{q}]\right\}$$
$$+\frac{\Delta\omega}{2}\left[1-\frac{\mathbf{p}\cdot\mathbf{p}'}{(E+m)^2}\right](\mathbf{p}-\mathbf{K})\times\mathbf{p}\cdot\hat{\mathbf{n}}-\frac{p^2\sin\theta_{\text{c.m.}}}{(E+m)^2}(\mathbf{p}-\mathbf{K})\cdot\mathbf{q}+2\mathbf{K}\times\mathbf{p}\cdot\hat{\mathbf{n}}$$
$$\left.+\frac{1}{(E+m)^2}[\mathbf{p}\cdot\mathbf{p}'(\mathbf{p}-\mathbf{K})\times\mathbf{p}\cdot\hat{\mathbf{n}}+p^2(\mathbf{p}-\mathbf{K})\times\mathbf{q}\cdot\hat{\mathbf{n}}-p^2\sin\theta_{\text{c.m.}}(\mathbf{p}-\mathbf{K})\cdot\mathbf{q}]\right],$$

$$R_{\omega\pi^+}=i\left[\frac{E+m}{4m^2}\right]\left[\frac{\Delta m}{2}\left\{\left[1-\frac{\mathbf{p}\cdot\mathbf{p}'}{(E+m)^2}\right](\mathbf{p}'+\mathbf{K})\times\mathbf{q}\cdot\hat{\mathbf{n}}-\frac{p^2\sin\theta_{\text{c.m.}}}{(E+m)^2}(\mathbf{p}'+\mathbf{K})\cdot\mathbf{p}-2\mathbf{K}\times\mathbf{p}'\cdot\hat{\mathbf{n}}\right.\right.$$
$$\left.-\frac{1}{(E+m)^2}[\mathbf{p}\cdot\mathbf{p}'(\mathbf{p}'+\mathbf{K})\times\mathbf{p}\cdot\hat{\mathbf{n}}+p^2(\mathbf{p}'+\mathbf{K})\times\mathbf{q}\cdot\hat{\mathbf{n}}+p^2\sin\theta_{\text{c.m.}}(\mathbf{p}'+\mathbf{K})\cdot\mathbf{q}]\right\}$$
$$+\frac{\Delta\omega}{2}\left\{\left[1-\frac{\mathbf{p}\cdot\mathbf{p}'}{(E+m)^2}\right](\mathbf{p}'+\mathbf{K})\times\mathbf{p}\cdot\hat{\mathbf{n}}-\frac{p^2\sin\theta_{\text{c.m.}}}{(E+m)^2}(\mathbf{p}'+\mathbf{K})\cdot\mathbf{q}-2\mathbf{K}\times\mathbf{p}'\cdot\hat{\mathbf{n}}\right.$$
$$\left.\left.-\frac{1}{(E+m)^2}[\mathbf{p}\cdot\mathbf{p}'(\mathbf{p}'+\mathbf{K})\times\mathbf{p}\cdot\hat{\mathbf{n}}+p^2(\mathbf{p}'+\mathbf{K})\times\mathbf{q}\cdot\hat{\mathbf{n}}+p^2\sin\theta_{\text{c.m.}}(\mathbf{p}'+\mathbf{K})\cdot\mathbf{p}]\right\}\right],$$

$$R_{\sigma\omega}=-i\frac{\Delta\omega}{2m}p^2\sin\theta_{\text{c.m.}}+i\left[\frac{E+m}{4m^2}\right]\left[\frac{\Delta m}{2}\left\{\left[1+\frac{\mathbf{p}\cdot\mathbf{p}'}{(E+m)^2}\right](\mathbf{p}-\mathbf{K})\times\mathbf{P}\cdot\hat{\mathbf{n}}+\frac{p^2\sin\theta_{\text{c.m.}}}{(E+m)^2}(\mathbf{p}-\mathbf{K})\cdot\mathbf{q}\right.\right.$$
$$\left.-2\left[1+\frac{p^2}{(E+m)^2}\right]\mathbf{K}\times\mathbf{p}\cdot\hat{\mathbf{n}}\right\}$$
$$+\frac{\Delta\omega}{2}\left\{\left[1-\frac{\mathbf{p}\cdot\mathbf{p}'}{(E+m)^2}\right](\mathbf{p}-\mathbf{K})\times\mathbf{P}\cdot\hat{\mathbf{n}}-\frac{p^2\sin\theta_{\text{c.m.}}}{(E+m)^2}(\mathbf{p}-\mathbf{K})\cdot\mathbf{q}\right.$$
$$\left.\left.+2\left[1-\frac{p^2}{(E+m)^2}\right]\mathbf{K}\times\mathbf{p}\cdot\hat{\mathbf{n}}\right\}\right],$$

$$R_{\omega\sigma}=-i\frac{\Delta\omega}{2m}p^2\sin\theta_{\text{c.m.}}+i\left[\frac{E+m}{4m^2}\right]\left[\frac{\Delta m}{2}\left\{\left[1+\frac{\mathbf{p}\cdot\mathbf{p}'}{(E+m)^2}\right](\mathbf{p}'-\mathbf{K})\times\mathbf{P}\cdot\hat{\mathbf{n}}+\frac{p^2\sin\theta_{\text{c.m.}}}{(E+m)^2}(\mathbf{p}-\mathbf{K})\cdot\mathbf{q}\right.\right.$$
$$\left.-2\left[1+\frac{p^2}{(E+m)^2}\right]\mathbf{K}\times\mathbf{p}'\cdot\hat{\mathbf{n}}\right\}$$
$$+\frac{\Delta\omega}{2}\left\{\left[1-\frac{\mathbf{p}\cdot\mathbf{p}'}{(E+m)^2}\right](\mathbf{p}'-\mathbf{K})\times\mathbf{P}\cdot\hat{\mathbf{n}}-\frac{p^2\sin\theta_{\text{c.m.}}}{(E+m)^2}(\mathbf{p}'-\mathbf{K})\cdot\mathbf{q}\right.$$
$$\left.\left.+2\left[1-\frac{p^2}{(E+m)^2}\right]\mathbf{K}\times\mathbf{p}'\cdot\hat{\mathbf{n}}\right\}\right],$$

$$R_{\omega\omega}=i\left[\frac{E+m}{4m^2}\right]\left[2(\Delta m-\Delta\omega)\mathbf{K}\times\mathbf{q}\cdot\hat{\mathbf{n}}+\frac{\Delta m}{(E+m)^2}(p^2\mathbf{K}\times\mathbf{q}\cdot\hat{\mathbf{n}}+p^2\sin\theta_{\text{c.m.}}\mathbf{K}\cdot\mathbf{P})+\frac{\Delta\omega}{(E+m)^2}(\mathbf{p}^2\mathbf{K}\times\mathbf{q}\cdot\hat{\mathbf{n}})\right].$$

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