Possible existence of new $K^\pi = 0^+$ low-lying excited bands in ^{168}Er

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Using the angular momentum projection approach with the l^2-depressing parameter μ_p in the Nilsson model reduced by 15%, the band heads of the 0_2^+, 0_3^+, and 0_4^+ bands in ^{168}Er are calculated to lie at 1221, 1426, and 1823 keV, respectively, in good agreement with recent experimental data. An additional low-lying 0^+ band with a band head at 1966 keV is also predicted. A general criterion for the existence of a low-lying 0^+ excited band is described and invoked to substantiate the calculated results.

There have been many investigations and discussions recently concerning the nuclear structure of ^{168}Er, stimulated largely by the detailed experimental data of Davidson *et al.*[1] on the low energy spectrum of this nucleus obtained from a high resolution γ-ray study following neutron capture. In order to reproduce the detailed structure of the 0_2^+ and 0_3^+ bands with both the band heads and moments of inertia being given correctly, Lin *et al.*[2] used the method of angular momentum projection based on wave functions obtained from a Nilsson-BCS calculation. In the calculation, the Nilsson model was adopted with all single-particle parameters taken from Nilsson *et al.*,[3] except for a slight modification of omitting the np-dependent factor α_τ for the value of κ. In this way Lin *et al.*[2] interpreted the 0^+ levels at 1217 and 1422 keV as mainly two-quasiparticle excited bands and concluded that there could be no other low-lying 0^+ bands. The calculated results agreed very well with the experimental observations of Davidson *et al.*[1] which were at that time reported to be complete below 2.2 MeV.

However, a new 0_4^+ band has since been found by Burke *et al.*[4] with a band head at 1833 keV having a large $\pm\frac{1}{2}(411)$ proton quasiparticle configuration, contrary to the conclusion of Lin *et al.*[2] This certainly puzzles many authors to the extent that some of them regard the disagreement as a drawback of the theory.

In this Brief Report, we wish to report some preliminary results from further investigation of the problem. It is realized that the deficiency of the theory may come from a Nilsson parameter which needs to be readjusted slightly. We shall also discuss a general criterion for the existence of a low-lying 0^+ excited band from which we suggest that if the 0_4^+ band exists, there should also be at least one more near-lying 0_5^+ band for the present case. As a matter of fact, our preliminary calculation indicates that there are two bands, 0_5^+ and 0_6^+, which call for further searching experimentally.

Since the Hamiltonian of a nuclear system is rotationally invariant, its eigenfunctions must have good angular momentum. However, wave functions obtained in many well-known approximation methods are not eigenfunctions of the angular momentum operator. For instance, the Hartree-Bogoliubov calculation of Baranger,[5] the phonon calculation of Dumitrescu and Hamamoto,[6] and the phonon calculation of Soloviev and Shirikova[7] do not conserve angular momentum. To recover the rotational symmetry, the method of angular momentum projection was given by Peierls and Yoccoz,[8] and has been discussed and applied to realistic calculations by many authors to treat the ground-state rotational structure of a heavy deformed nucleus.[9–13] Lately, this method has been extended by Lin and Faessler[14,15] to treat excited states with an even number of quasiparticles.

The disadvantage of using the angular momentum projection method is that it is very time consuming, even when the numerical computations are performed on a computer. For low-lying excited states in a heavy deformed nucleus, a fast approximation method has been developed recently by Lin[16] such that an accurate calculation can be easily carried out. This can certainly provide a useful tool for studying the structure of these states with an angular momentum conserving theory. However, as was already stated in Lin *et al.*,[2] there is another difficulty in performing this kind of calculation at present when we are dealing with detailed quantitative structure concerning near-lying levels, and the difficulty is accentuated when the levels are not already known experimentally. This is due to the fact that a quantitatively reliable single-particle model with accurate parameters is not available to meet our purpose because the parameters in the existing single-particle models are all fitted for calculations with wave functions which do not conserve angular momentum. Fortunately, if we take a well-established single-particle model, such as the Nilsson model, the prevailing belief in light of much experience is that only some slight modifications will be required. How the modifications should be made evidently depends very much on what is known experimentally for comparison. In this context and in view of the existence of a 0_4^+ band, it is only natural and desirable to initiate a calculation using the same approach as Lin *et al.*[2] with some revision of the single-particle parameters.

Let us write the Nilsson-BCS wave functions for the ground state Φ_0 and its 0^+ $(\alpha\bar{\alpha})$ excited state Φ in the usual form as[17]

$$|\Phi_0\rangle=\prod(u_k+v_k b_k^\dagger b_{\bar{k}}^\dagger)|0\rangle \tag{1}$$

and

$$|\Phi\rangle=(u'_\alpha b_\alpha^\dagger b_{\bar{\alpha}}^\dagger - v'_\alpha)\prod_{k\neq\alpha}(u'_k+v'_k b_k^\dagger b_{\bar{k}}^\dagger)|0\rangle . \tag{2}$$

The number equations are, respectively,

$$F(v)=2\sum v_k^2=N \tag{3}$$

and

$$2u'^2_\alpha+2\sum_{k\neq\alpha}v'^2_k=2\sum v'^2_k+\frac{2(\varepsilon_\alpha-\lambda)}{E_\alpha}=N . \tag{4}$$

Equation (4) can be written as

$$F(v')=2\sum v'^2_k=F(v)+\delta F=N+\delta N=N-\frac{2(\varepsilon_\alpha-\lambda)}{E_\alpha} . \tag{5}$$

The change of Fermi energy $\delta\lambda$ for the 0^+ excited state can be shown to be

$$\delta\lambda=\left[\frac{\partial\lambda}{\partial N}\right]\delta N=-\frac{2(\varepsilon_\alpha-\lambda)}{E_\alpha}\Big/\left[\Delta^2\sum E_\alpha^{-3}\right] . \tag{6}$$

Therefore $\delta\lambda>0$ if $\varepsilon_\alpha<\lambda$, and $\delta\lambda>0$ if $\varepsilon_\alpha>\lambda$. That is to say that $|\varepsilon_\alpha-\lambda|$ always gets larger for the 0^+ $(\alpha\bar{\alpha})$ state. If α is the Nilsson level nearest to the Fermi energy in a heavy deformed nucleus, calculations show that $|\delta\lambda|$ is about 300 keV. In the BCS ground state of a heavy deformed nucleus, the position of λ is usually such that $|\varepsilon_+-\lambda|\simeq|\varepsilon_--\lambda|$, where ε_+ and ε_- are the Nilsson energies of the two states nearest to λ. The band head with excited two-quasiparticles can be written as

$$E_k=[(\varepsilon_k-\lambda)^2+\Delta^2]^{1/2}+\eta_k . \tag{7}$$

For the present problem, η_k is usually about 120 ± 50 keV, where the $\pm$ sign corresponds to ε_- and ε_+, respectively. Besides, the Δ for excited bands will usually be reduced by 150–250 keV.

For ^{168}Er, we find that the separation between ε_+ and ε_- is about 400 keV for neutrons when we look it up in the Nilsson single-particle diagram.[3,18] From the above arguments and the value of Δ from Ref. 2, we will have the band heads of the excited 0_2^+ and 0_3^+ at about 1220 and 1420 keV, in excellent agreement with experimental values. However, we also find $|\varepsilon_+-\varepsilon_-|$ for protons is about 1 MeV, and we should have the lowest 0^+ state for protons at about 2.5 MeV. With the estimated reduction of Δ_p, we will still have about 2.3 MeV. Unless there may be collective states, there should not be other low-lying 0^+ states. The possibility of the existence of a strongly collective 0^+ state can be ruled out by an argument of Bohr and Mottelson[19] according to which a strongly collective state can exist only if the associated single-particle level density is high enough. States with a small collectivity cannot be pulled down in energy by such a large amount using a reasonable residu-

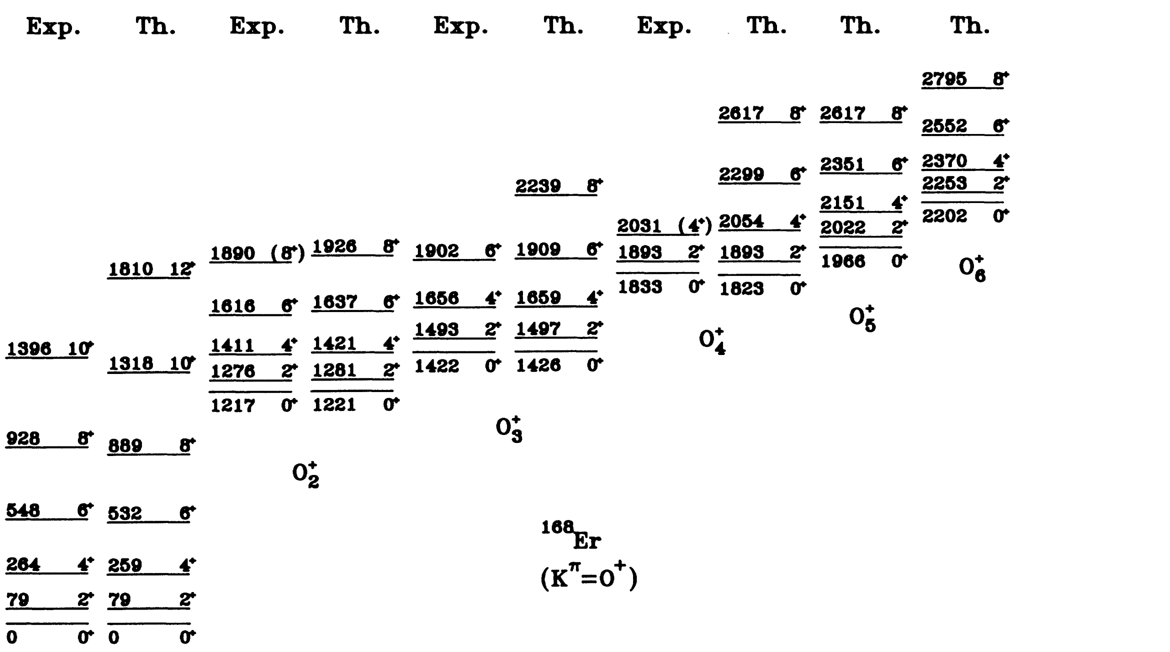

FIG. 1. Comparison of the calculated energies with experiment for the ground state band and $K^\pi=0^+$ excited bands in ^{168}Er. All energies are in units of keV.

al interaction. Therefore, we came to the conclusion as stated in Ref. 2.

Given the existence of the $K^{\pi}=0_4^+$ band, it is reasonable to assume that some of the adjustable parameters in the Nilsson model which are fitted for calculations using wave functions without good angular momentum may have to be modified slightly for our purpose. Mostly by speculation, we feel that the l^2-depressing parameter may be slightly too large for protons in this region of nuclei ($\mu_p=0.6$, $\mu_n=0.42$). If we reduce μ_p by 50% of the difference between μ_p and μ_n so that μ_p is down to 0.51, we find the proton-($d_{3/2,\pm 1/2}$) two-quasiparticle $K^{\pi}=0_4^+$ band head to be at 1823 keV with $\Delta_n=642$ and $\Delta_p=675$ keV, and that of the proton-($h_{11/2,\pm 7/2}$) $K^{\pi}=0_5^+$ at 1966 keV with $\Delta_n=634$ and $\Delta_p=705$ keV. There is also a proton-($h_{9/2,\pm 1/2}$) $K^{\pi}=0_6^+$ at 2202 keV with the latter set of Δ_n and Δ_p. Numerical calculations were performed in exactly the same way as done in Ref. 2. However, in order to cut down computer time and have the calculation done more easily, we have used instead an equivalent approximation method[16] with which the calculated position of the band head was found to be off by an amount which was only a few keV for the $J=2$ excited band and increased with J to about 30 keV for the $J=10$ excited band. All parameters were taken exactly the same as in Ref. 16 except that the Nilsson μ_p was set to be 0.51 as mentioned above. This same μ_p is expected to be appropriate for all nuclei in the same major shell in very much the same way as that of the original Nilsson model. In Fig. 1, we show the numerical results for $K^{\pi}=0_1^+$, 0_2^+, 0_3^+, 0_4^+, and 0_5^+ together with available experimental values for comparison.

From the above analysis and numerical results, we are inclined to make two preliminary conclusions. Firstly, it is quite clear that there should be at least one more $K^{\pi}=0^+$ proton two-quasiparticle band 0_5^+. However, the exact position of this 0_5^+, being within about ± 230 keV of the 0_4^+, is not given with utmost certainty at present as we cannot be very definite about the exact reduction of μ_p. More experimental data are evidently needed if a least-squares fitting procedure is to be implemented for determining the μ_p. Secondly, the experimental moment of inertia of the 0_4^+ band fits better with that of the theoretical value for the 0_5^+ band. We suggest that there should be admixture from both proton-($d_{3/2,\pm 7/2}$) and proton-($h_{11/2,\pm 7/2}$) bands in the experimental 0_4^+ band. Further investigations of the problem will be worthwhile both theoretically and experimentally.

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