Microscopic optical-potential analysis of charge-symmetry violation in $\pi^{\pm}$ elastic scattering from ^{3}H and ^{3}He

Kr. T. Kim and Y. E. Kim
Department of Physics, Purdue University, West Lafayette, Indiana 47907

R. H. Landau
Department of Physics, Oregon State University, Corvallis, Oregon 97331
(Received 30 March 1987)

Charge symmetry violation due to the Coulomb force in the elastic scattering of $\pi^{\pm}$ from ^{3}H and ^{3}He is studied using a theoretical optical potential with spin. Our calculated results compare favorably with the experimental differential cross sections of Källne *et al.* and of Nefkens *et al.*, indicating that the observed violation of the charge symmetry can be due to direct Coulomb effects.

Recently Nefkens *et al.*[1] claimed to have observed charge symmetry violation in $\pi^{\pm}$ elastic scattering from ^{3}H and ^{3}He. They measured the differential cross sections for 180 MeV pions scattered between 45° and 96° in the c.m. system, formed the ratios

$$r_1=[d\sigma/d\Omega(\pi^+\,{}^3\mathrm{H})]/[d\sigma/d\Omega(\pi^-\,{}^3\mathrm{He})]\ , \quad (1)$$

$$r_2=[d\sigma/d\Omega(\pi^-\,{}^3\mathrm{H})]/[d\sigma/d\Omega(\pi^+\,{}^3\mathrm{He})]\ , \quad (2)$$

$$R=r_1r_2\ , \quad (3)$$

and found them to vary significantly from 1 with scattering angle; e.g., $R=1.31\pm0.09$ at $\theta_{\mathrm{c.m.}}=65°$. If the entire interaction were charge symmetric, the individual ratios, r_1 and r_2, and the super-ratio R would all identically be equal to 1. The authors of Ref. 1 concluded, after a series of indirect arguments, that the simple addition of the Coulomb force to the trinucleon strong force is not enough of a perturbation to cause the large deviations, and consequently charge symmetry in the (^{3}He,^{3}H) structure or strong interaction is violated.

There have been several theoretical attempts to explain the deviation of the super-ratio R from 1. Barshay and Sehgal[2] used a geometrical model for the trinucleon structure to ascribe the deviation to short range three-nucleon correlations. One of the present authors (Y.E.K.) has postulated that the observed deviation may be due to multiquark compound resonances,[3] and suggested a direct experimental test of the importance of these resonances. In addition, Kim, Krell, and Tiator,[4] while not able to reproduce the data quantitatively, have used a local optical potential to show that the direct Coulomb force could produce large charge symmetry violations in this reaction.

The present work takes the view that no new mechanisms should be introduced until the well-known and precise techniques for calculating pion scattering with the combined Coulomb plus nuclear force have been tried. In particular, since the denominators in the ratios r_1 and r_2, (1) and (2), can approach zero at the minima of $d\sigma/d\Omega$, the large deviations of the ratios from 1 may be simply a reflection of how "small" effects can affect the depths of minima. In the present case the minima are filled largely by spin-flip scattering; that may be a small effect for other nuclei, but in ^{3}He and ^{3}H with $\frac{1}{3}$ of the nucleons having unsaturated spin, it makes an important contribution. Spin-flip scattering, which is essential to describing the minima, was not present in Ref. 4 but is now included with some precision. This leads to better agreement between theoretical predictions and experimental differential cross sections than previously, and consequently more reliable conclusions regarding the ratio of cross sections.

The present calculation uses a momentum space, nonlocal potential, LPOTT,[5] with theoretical parameters derived over a good number of years.[6–8] The previous calculation used a phenomenological potential with parameters fit to heavy-nuclei, pionic-atomic data. There really is no comparison between the two on theoretical grounds,[8] with the theoretical potential developed with a realistic description of the nuclear structure of the three-nucleon system. Specifically, this improves upon the calculation of Ref. 4 by providing an improved description of off-energy-shell, recoil, binding, and kinematic effects; spin-flip scattering arising from realistic nuclear structure; and several "exact" treatments of the nuclear force. These effects are important here since they tend to fill in the minima in $d\sigma/d\Omega$—and thus directly affect the ratios. Yet it is not meaningful to isolate any one effect (particularly at lower energies) since a number of them are needed before an accurate picture develops, and since a number of them nearly cancel.

The basic assumption made here is that the total π-nucleus potential U is the sum of nuclear plus Coulomb potentials,

$$U(\mathbf{k}',\mathbf{k};E)=U^{\mathrm{nucl}}(\mathbf{k}',\mathbf{k};E)+U^{\mathrm{Coul}}(\mathbf{k}',\mathbf{k})\ , \quad (4)$$

and that the π-nucleus scattering amplitude is obtained by solving the Lippmann-Schwinger equation. This is the simplest and most basic inclusion of the Coulomb force, and consequently the first place to look for an understanding of the experimental charge symmetry violation. We have also used LPOTT to examine Coulomb effects at an even more microscopic level where they modify (by several MeV) the subenergy of the π-nucleon T matrix contained in $U^{\rm nucl}$, the overall energy dependence of the optical potential, the nuclear mass, and the wave equation.[8] However, since these latter effects are less direct than (4) (which produces 20–30 % effects), are small here ($\approx 1\%$), and since they tend to influence mainly the energy dependences of the integrated cross sections, we present here only the simplest calculation.

For the scattering of a spin 0 particle ($\mathbf{k}\to\mathbf{k}'$) from a spin $\frac{1}{2}$ nucleus, the optical potential (and the nuclear T matrix) have central plus spin-orbit (or flip) terms:

$$U^{\rm nucl}(\mathbf{k}',\mathbf{k};E)=\langle \mathbf{k}' \mid U^c \mid \mathbf{k}\rangle + i\sigma\cdot\mathbf{n}\langle \mathbf{k}' \mid U^{\rm sp} \mid \mathbf{k}\rangle \ . \quad (5)$$

In the impulse and factorization approximations, U^c and $U^{\rm sp}$ are the sums of products of π-nucleon (spin-flip and nonflip) T matrices and nuclear (matter and spin) form factors:[6,7]

$$\langle \mathbf{k}' \mid U^c \mid \mathbf{k}\rangle = \langle f \mid T^{\pi p}_{\rm nf} \mid i\rangle Z\rho^{\rm p}_{\rm matter} + \langle f \mid T^{\pi n}_{\rm nf} \mid i\rangle N\rho^{\rm n}_{\rm matter} \quad (6)$$

and

$$\langle \mathbf{k}' \mid U^{\rm sp} \mid \mathbf{k}\rangle = \langle f \mid T^{\pi p}_{f} \mid i\rangle Z\rho^{\rm p}_{\rm spin} + \langle f \mid T^{\pi n}_{f} \mid i\rangle N\rho^{\rm n}_{\rm spin} \ , \quad (7)$$

where

$$\mid i\rangle = \mid \mathbf{k},\mathbf{p}_0\rangle, \quad \mid f\rangle = \mid \mathbf{k}',\mathbf{p}_0-\mathbf{q}\rangle \ , \qquad \mathbf{q}=\mathbf{k}'-\mathbf{k}, \quad \mathbf{n}=\mathbf{k}'\times\mathbf{k} \ , \quad (8)$$

and

$$\mathbf{p}_0 = -\mathbf{k}/A + \mathbf{q}(A-1)/2A \ . \quad (9)$$

$\mathbf{p}_0$ is the momentum of the struck nucleon which optimizes the factorization approximation.[7,8] [Although not indicated in (6) and (7), we also include "rho squared" terms in our potential to account for pion annihilation.]

The distribution of matter and spin for the ^{3}He nucleus can be described with the form factors[16,17]

$$\rho^{\rm p}_{\rm matter}(q)=F_{\rm ch}(^3{\rm He})/f^{\rm p}_{\rm ch} \ , \quad (10)$$

$$\rho^{\rm n}_{\rm matter}(q)=F_{\rm ch}(^3{\rm H})/f^{\rm p}_{\rm ch} \ , \quad (11)$$

$$\rho^{\rm n}_{\rm spin}(q)=[\mu_{\rm p}^2F_m(^3{\rm H})-\mu_{\rm n}^2F_m(^3{\rm He})]/[f^{\rm p}_{\rm ch}(\mu_{\rm p}^2-\mu_{\rm n}^2)] \ , \quad (12)$$

$$\rho^{\rm p}_{\rm spin}(q)=\mu_{\rm p}\mu_{\rm n}[F_m(^3{\rm He})-F_m(^3{\rm H})]/[2f^{\rm p}_{\rm ch}(\mu_{\rm p}^2-\mu_{\rm n}^2)] \ , \quad (13)$$

where $\mu_{\rm p,n}=(2.793,-1.913)$, and $f^{\rm p}_c(q)$ is the proton charge form factor. In general, we need four form factors to describe ^{3}He and an additional four to describe ^{3}H. We assume isospin symmetry is good at the nuclear structure level, so the four form factors for ^{3}H are related to those for ^{3}He by the interchange p$\leftrightarrow$n on the left-hand sides (lhs's) of (10)–(13). Note that even with this assumption, the charge and magnetic form factors for ^{3}He and ^{3}H are still independent functions, and indeed we use four different functions to describe the nuclear structure [effectively $F_{{\rm ch},m}(^3{\rm He},^3{\rm H})$]. We have also experimented with including isospin breaking at the nuclear-structure level, but have found that the major effect on the experimental R arises from effects at the pion-nucleus level, i.e., (4); only the dominant physics is discussed in this Brief Report.

We use the most recent electron scattering determination of the nuclear form factors: Juster *et al.*[9] for the charge and magnetic form factors of ^{3}H, McCarthy *et al.*[10] for the charge form factor of ^{3}He, and Dunn *et al.*[11] for the magnetic form factor of ^{3}He. The previous large uncertainties in magnetic form factor of ^{3}He were reflected as large uncertainties in the predicted π-^{3}He scattering[7] minima, and are now removed.[11] Uncertainties arising from lack of knowledge of the electromagnetic form factors at very large momentum transfers are negligible for the momentum transfers important in the present pion calculations ($q^2 < 11$ fm^{-2}). Likewise, while meson exchange currents should be removed from these form factors before they are used in strong interaction calculations,[17] with the small momentum transfers involved here the effect on R is small. (We have further checked this point by using the pure nucleonic form factors of Hadjimichael *et al.*[18] to calculate R.)

The off-energy-shell πN T matrices in $U(\mathbf{k}',\mathbf{k};E)$ are determined from the Almehed-Lovelace phase shifts[12] (on-shell behavior) and from a separable potential model[13] (off-shell behavior). The T matrices are transformed to the π-nucleus c.m. system with a covariant, optimal impulse approximation that preserves the full angular and nonlocal nature of the kinematic, binding, and recoil effects.

Since the Coulomb force is the ultimate source of charge symmetry violation, we have tried four different procedures for its inclusion at the π-nucleus level in order to gauge the model dependence of our conclusions. Two are "exact" and employ the Vincent-Phatak subtraction[14] to remove the $q=0$ singularity of the Coulomb potential in momentum space:

$$U^{\rm Coul}(\mathbf{k}',\mathbf{k})=Z_\pi Z_A e^2[\rho(q)-\cos(qR_{\rm cut})]/q^2 \ . \quad (14)$$

Here $\rho(q)$ is the nuclear charge form factor, and $R_{\rm cut}$ is the radius at which the Coulomb potential is set to zero (it is also the radius at which we match the "inner" phase shift, $\delta_L^{\rm NC}$, arising from the nuclear and extended charge Coulomb force, to the "outer" one, σ_L, arising from the pure, point charge Coulomb force). In one exact procedure we use the empirical charge form factor in (14), and in the second we assume $\rho(q)$ is due to a uniform sphere with the same rms radius as the realistic form factor.

In the exact methods the scattering amplitude is given by

$$f(\theta)=f^{\rm Coul}(\theta)+\sum_{L=0}^{\infty}(2L+1)e^{2i\sigma_L}(e^{2i\delta_L^{\rm NC}}-1) \times P_L(\cos\theta)/2ik_0 \ , \quad (15)$$

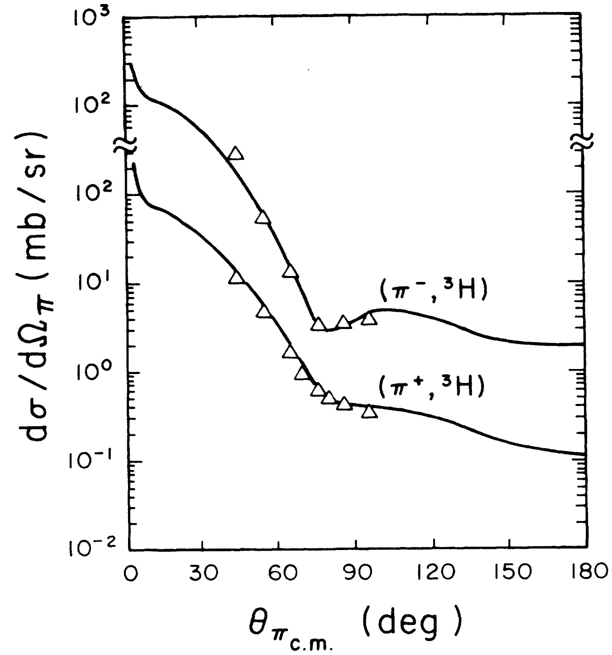

FIG. 1. Comparison of the calculated elastic differential cross sections for $\pi^\pm + {}^3H$ at the pion laboratory kinetic energy 180 MeV with the experimental data of Nefkens *et al.* (Ref. 1).

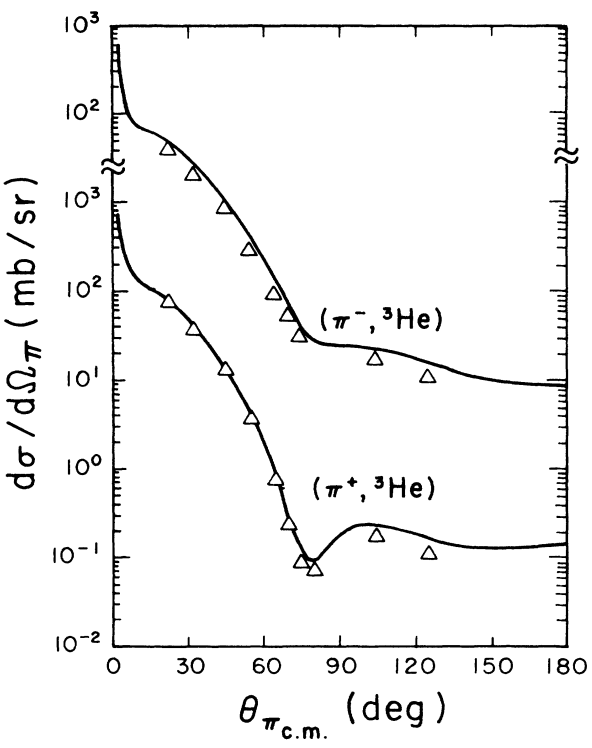

FIG. 2. Comparison of the calculated elastic differential cross sections for $\pi^\pm + {}^3He$ at the pion laboratory kinetic energy 200 MeV with the experimental data of Källne *et al.* (Ref. 15).

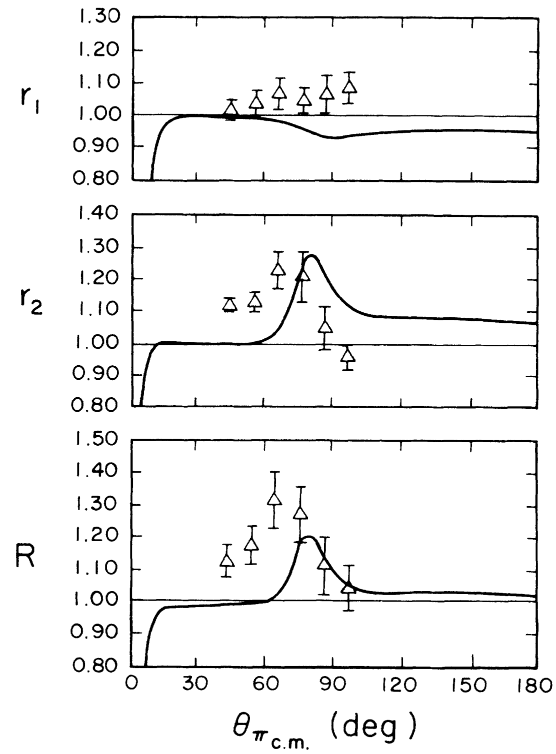

FIG. 3. Comparison of the ratios r_1, r_2, and R at the pion laboratory kinetic energy 180 MeV with the experimental data of Nefkens *et al.* (Ref. 1).

where $f^{\mathrm{Coul}}(\theta)$ is the point Coulomb scattering amplitude. In the approximate inclusions of the Coulomb force, the finite size of the charge distribution is included by multiplying the point amplitude by the nuclear charge form factor

$$f^{\mathrm{Coul}} = f^{\mathrm{Coul}}_{\mathrm{pt}} \rho(q) , \tag{16}$$

and setting δ_L^{NC} equal to the phase shift due to the pure nuclear potential U^{nucl}. Two variations of this approximate procedure involve evaluating the sum in (15) with and without the Coulomb phase σ_L. We have verified that all four procedures produce equivalent predictions for the 180 MeV data. This indicates that for scattering at intermediate energies from a nucleus as light as ${}^3He$, the Coulomb distortion of the wave function upon which the nuclear force acts is not large enough to depend upon higher order mixing of the Coulomb and nuclear forces.

In order to obtain a meaningful comparison of the super-ratio R with the data, it is necessary to have good qualitative agreement for the individual differential cross sections. For this purpose we show a comparison of our 180 MeV parameter-free predictions with the π^+-${}^3H$ differential cross section data of Nefkens *et al.*[1] in Fig. 1, and our 200 MeV calculations with the $\pi^\pm$-${}^3He$ data of Källne *et al.*[16] in Fig. 2. Since the neutron (proton) in ${}^3He$ (${}^3H$) has its spin unpaired, and since the isospin $\frac{3}{2}$ wave resonates at this energy, there should be[7] more spin flip scattering in π^--${}^3He$ (π^+-${}^3H$) scattering than when the pion has the opposite charge. Yet since spin-flip scattering is largest near 90°, we expect the minima in π^--^{3}He (π^+-^{3}H) scattering to be filled in more than when the pion has the opposite charge. These trends are confirmed by the data, and we consider the detailed agreement with experiment rather good for an unnormalized, parameter-free lowest order, microscopic theory.

In Fig. 3 we compare the experimental ratios r_1, r_2 and R, (1)–(3), at 180 MeV with the results of our calculation. As can be seen in the top of Fig. 3, the theory predicts a rather flat value for r_1, with a slight dip near 80°. This is a consequence of "filled in" minima for both these charge symmetric reactions. Indeed, this appears to be the trend of the experimental data. As can be seen in the middle part of Fig. 3, the cross sections that go into forming r_2 both have relatively "unfilled" minima—but not precisely of the same shape and at the same angle—and so when we form the ratio of cross sections, the division by a small number produces a larger peak in r_2 than is present in r_1. This trend is also present in the data. Finally we note in the lower part of Fig. 3 that the super-ratio is also predicted to have a peak of a magnitude and shape comparable to experiment, although the absolute magnitude is not in very good agreement.

In examining the figures, it is valuable to keep in mind that we consider the improvement in our calculation not to be an improvement in the fit to R, but rather in the validity of our theoretical framework and in the agreement with the individual cross sections before the ratios are formed. And while we would not normally consider the prediction of the depths of minima as a strong point for a theory which does not include all higher-order terms, in this case the minima are filled mainly by first-order, spin-flip scattering, and only slightly by annihilation terms. What is unequivocal is that the difference in character for the cross sections that form the theoretical r_1 and r_2 is caused *solely* by the direct addition of the pion-nucleus, Coulomb, and nuclear potential; more subtle charge-symmetry breaking effects, e.g., at the nuclear structure, or pion-nucleon levels, appear to be a small correction to our macroscopic violation. Consequently, we view the charge-symmetry violation reported by Nefkens *et al.*[1] as a real effect, the dominant part of which arises from addition of the pion-nucleus, Coulomb, and nuclear forces.

This work was supported in part by the National Science Foundation and in part by the U.S. Department of Energy.

[1]B. M. Nefkens *et al.*, Phys. Rev. Lett. **52**, 735 (1984).

[2]S. Barshay and L. M. Sehgal, Phys. Rev. C **31**, 2133 (1985).

[3]Y. E. Kim, Phys. Rev. Lett. **53**, 1508 (1984).

[4]Y. E. Kim, M. Krell, and L. Tiator, Phys. Lett. B **172**, 287 (1986).

[5]R. H. Landau, Comput. Phys. Commun. **28**, 109 (1982).

[6]R. H. Landau, S. C. Phatak, and F. Tabakin, Ann. Phys. (N.Y.) **78**, 299 (1973).

[7]R. H. Landau, Ann. Phys. (N.Y.) **92**, 205 (1975); Phys. Rev. C **15**, 2127 (1977).

[8]R. H. Landau and A. W. Thomas, Phys. Lett. **88B**, 226 (1979); **58**, 121 (1980).

[9]F.-P. Juster *et al.*, Phys. Rev. Lett. **55**, 2261 (1985).

[10]J. S. McCarthy *et al.*, Phys. Rev. C **15**, 1396 (1977).

[11]P. C. Dunn *et al.*, Phys. Rev. C **27**, 71 (1983).

[12]S. Almehed and C. Lovelace, Nucl. Phys. **B40**, 157 (1972).

[13]C. Coronis and R. H. Landau, Phys. Rev. C **24**, 605 (1981).

[14]C. M. Vincent and S. C. Phatak, Nucl. Phys. **C10**, 391 (1974).

[15]J. Källne *et al.*, Phys. Lett. **B103**, 13 (1981).

[16]B. F. Gibson and L. I. Schiff, Phys. Rev. **138**, B26 (1965); B. F. Gibson, *ibid.* **139**, B1153 (1965).

[17]R. H. Landau and M. Sagen, Phys. Rev. **33**, 447 (1986).

[18]E. Hadjimichael, B. Goulard, and R. Bornais, Phys. Rev. C **27**, 831 (1983).