

## Potential model analysis of low energy ${}^2\text{H}(d,\gamma){}^4\text{He}$ fusion data

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Previously experimental data on the  ${}^2\text{H}(d,\gamma){}^4\text{He}$  reaction have been analyzed within a potential model of structureless particles to derive quantitative information about the  ${}^4\text{He}$  ground state. We have tested the applicability of the potential model by comparing cross sections calculated for a wide range of Woods-Saxon parametrizations covering those of previous studies with recent low energy  ${}^2\text{H}(d,\gamma){}^4\text{He}$  fusion data. We find that the results for the capture cross sections from the  $d+d$   $S$ -wave into the  $D$ -state component of the  ${}^4\text{He}$  ground state, which dominate the fusion data at  $E \leq 200$  keV, are very sensitive to the parametrization of the nucleus-nucleus potential. Hence simple potential model studies seem not to be an appropriate tool for quantitative analysis of low energy  ${}^2\text{H}(d,\gamma){}^4\text{He}$  fusion data.

### I. INTRODUCTION

Recent measurements of the tensor analyzing powers of the  ${}^2\text{H}(d,\gamma){}^4\text{He}$  reaction indicate the existence of a  $D$ -state admixture in the  ${}^4\text{He}$  ground state.<sup>1,2</sup> A straightforward analysis of these measurements is hampered by the fact that the data, taken at c.m. energies in the  $d+d$  channel of roughly  $E \approx 5$  MeV, are a superposition of several electromagnetic multipolarities and that fragments other than the entrance channel configuration are also expected to contribute. However, to obtain quantitative statements about the strength of the  $D$ -state component the data have been analyzed on the basis of a phenomenological  $d+d$  potential model with structureless particles resulting in predictions about the  $D$ -state admixture in the range of 3–6% (Refs. 1 and 3).

The existence of a  $D$ -state component in the  ${}^4\text{He}$  ground state has recently been convincingly confirmed in measurements of the  ${}^2\text{H}(d,\gamma){}^4\text{He}$  fusion reaction at low energies ( $E \approx 0.04$ – $1$  keV, Refs. 4 and 5). Applying penetrability arguments in a model of structureless fragments these fusion cross sections at energies far below the Coulomb barrier arise from  $E2$  transitions from  $d+d$  scattering states with orbital angular momentum  $L=0$  and total spin  $S=2$  into the  $(L=2, S=2)$  component of the  ${}^4\text{He}$  ground state ( $D$  state). This assumption has been experimentally confirmed<sup>5</sup> by the energy dependence of the astrophysical  $S$  factor exhibiting the typical pattern for nonresonant  $S$  wave capture as well as by the angular distributions measured at  $E=75$  keV showing a nearly isotropic behavior, as expected for an  $E2$  transition from  $(L=0, S=2)$  scattering states into a  $(L=2, S=2)$  bound state. This trend towards isotropy in the angular distributions has also been observed in Ref. 3.

It is therefore natural that the low energy  ${}^2\text{H}(d,\gamma){}^4\text{He}$  fusion data have also been used to derive quantitative predictions about the  $D$ -state admixture in the  ${}^4\text{He}$  ground state. Based on a potential model of structureless fragments, Weller *et al.*<sup>3</sup> deduced a  $D$ -state strength of 3–6% from fusion data above  $E=300$  keV. Barnes *et al.*<sup>5</sup> em-

ploying similar assumptions within a standard direct capture formalism (which is also based on a potential model of structureless particles) analyzed a  $D$ -state component of  $1.4 \pm 0.6\%$  for the low energy data at  $E \approx 40$ – $200$  keV.

Recognizing that the  ${}^2\text{H}(d,\gamma){}^4\text{He}$  cross sections at  $E \leq 200$  keV are the most unambiguous experimental information about the  $D$ -state admixture in the  ${}^4\text{He}$  ground state presently at hand, we have critically studied these data within a potential model of structureless particles, in this perspective following the basic assumptions of Refs. 1, 3, and 5–7. However, we are quite aware of the fact that a potential model of structureless fragments might not be an appropriate tool to study the low energy fusion cross section or the tensor analyzing power of the  ${}^2\text{H}(d,\gamma){}^4\text{He}$  reaction as the following occur.

(i) The  ${}^4\text{He}$  ground state does not have predominantly a  $d+d$  cluster structure if the two fragments are described by wave functions appropriate for free deuterons.<sup>8,9</sup>

(ii) The contribution arising from the internal quadrupole moment of the deuteron, allowing for a transition from the  $d+d$   $S$  waves into the dominant  $S$ -wave component of the  ${}^4\text{He}$  ground state, might well yield a non-negligible component to these cross sections.

Hence, the aim of our potential model studies is not to derive quantitative statements about the  $D$ -state admixture in the  ${}^4\text{He}$  ground state, but to demonstrate the following.

(i) The apparent reproduction of the low energy fusion data reported in Ref. 5 and the discussion of these data as given in Ref. 3 are based on questionable assumptions.

(ii) The potential parametrizations used in Refs. 1, 6, and 7 to study the tensor analyzing power data at  $E \approx 5$  MeV are not consistent with the low energy fusion data.

(iii) Small variations in the potential parametrization of the  ${}^4\text{He}$  ground state can lead to drastic changes in the predictions about the magnitude of the  $D$ -state admixture, even if, as this is suggested in Refs. 6 and 7, the uncertainty in the potential model studies is reduced by deriving the scattering states from potentials which are phase equivalent to the resonating group method (RGM) potentials of Ref. 10, which are known to give a good descrip-

tion to  $d + d$  scattering at low energies.

Our paper is organized as follows. In Sec. II we present a brief description of the potential model used in our studies. The  ${}^2\text{H}(d,\gamma){}^4\text{He}$  cross sections calculated by use of the potential parametrizations adopted from Refs. 1, 3, and 5–7 are presented in Sec. III. Finally, we discuss these results regarding the applicability of simple potential model studies of low energy  ${}^2\text{H}(d,\gamma){}^4\text{He}$  fusion cross sections in Sec. IV.

## II. THEORETICAL BACKGROUND OF THE POTENTIAL MODEL

In our potential model studies we have calculated the  ${}^4\text{H}(d,\gamma){}^4\text{He}$  fusion cross sections at energies  $E \leq 3$  MeV assuming that in this energy region the capture cross sections are dominated by  $E2$  radiation.<sup>3</sup> As is indicated by the microscopic study of Ref. 18, this assumption is certainly a very good approximation at very low energies ( $E \leq 500$  keV) where, due to penetrability arguments, the fusion reaction will mainly occur via  $E2$  radiation from  $d + d$   $S$ -wave scattering states into the  ${}^4\text{He}$  ground state. Since this transition yields information about the  $D$ -state admixture in the  ${}^4\text{He}$  ground state, within potential models of structureless particles the limitation of the present study to  $E2$  radiation only does not represent any restriction regarding the aims of this paper.

Assuming that the  $E2$  radiation is dominated by the contributions arising from the spin-independent part of the transition operator the initial and the final nuclear states have the same total spin quantum numbers. The  $E2$  capture cross section from a  $d + d$   $J=2$  scattering state at energy  $E$  with orbital angular momentum  $L$  and spin  $S$ , denoted by  $g_{LS}$  into the  ${}^4\text{He}$  ground state described as a normalized bound state of the  $d + d$  system with quantum numbers  $J=0$ ,  $L'$  and  $S$  ( $g_{L'S}$ ) is then given within a potential model of structureless deuterons by<sup>11</sup>

$$\sigma(E) = \sum_{L,L',S} \sigma_{LL'}^S(E), \quad (1)$$

$$\sigma_{LL'}^S(E) = \frac{e^2\pi}{45} \left[ \frac{E_\gamma}{\hbar c} \right]^5 \frac{1}{\hbar v} (2L'+1) \times (L'020 | L0)^2 \begin{vmatrix} 2 & L' & L \\ S & 2 & 0 \end{vmatrix}^2 |I_2|^2. \quad (2)$$

Here the radial matrix element is defined as

$$I_2 = \int_0^\infty dr g_{LS}(r) r^2 g_{L'S}(r) \quad (3)$$

and the radial scattering wave functions are normalized as

$$g_{LS} \rightarrow \frac{1}{kr} [\cos(\delta_L) F_L(kr) + \sin(\delta_L) G_L(kr)] \quad (4)$$

to guarantee unit flux in the entrance channel. In (2) and (4),  $k$  and  $v$  are the wave number and the relative velocity in the entrance channel,  $\delta_L$  is the nuclear phase shift, and  $F_L, G_L$  denote the regular and irregular Coulomb functions. Finally,  $E_\gamma$  is the energy of the emitted photon ( $E_\gamma = E + 23.84$  MeV).

Following the intention of the paper we have calculated the nuclear wave functions from a Schrödinger equation

of relative motion. According to Refs. 1, 3, and 5–7 a Woods-Saxon form factor

$$V(x) = -V_0 \left[ 1 + \exp \left( \frac{r - R_0}{a} \right) \right]^{-1} \quad (5)$$

has been adopted for the nucleus-nucleus potential. The Coulomb potential was approximated by that of a homogeneously charged sphere whose radius was set equal to the radius parameter of the Woods-Saxon (WS) potential.

Since it is not our intention to derive at any quantitative statements about the  $D$  state admixture in the  ${}^4\text{He}$  ground state within the presently used potential model, we will present our calculated partial cross sections  $\sigma_{LL'}^S$  at full strength in the following rather than fitting them to the experimental data. For the same reason we have not introduced any spectroscopic factors  $S_L$  in our definition of the cross sections as this has been done in several previous studies to account at least partly for the fact that the  ${}^4\text{He}$  ground state is apparently not a pure bound state of the  $d + d$  system. However, we will give some remarks on this procedure later in this paper.

For convenience we will discuss the capture cross sections in terms of the astrophysical  $S$  factor

$$S(E) = \sigma(E) E \exp\{2\pi\eta\}, \quad (6)$$

where  $\eta$  is the Sommerfeld parameter.

All values concerning the magnitude of the  $D$ -state admixture in the  ${}^4\text{He}$  ground state  $P_D$  given in this paper are calculated by comparing the experimental and the theoretical  $S$  factor in the low energy region following Ref. 5. Assuming full spectroscopic strength, the  $D$ -state admixture is then given by

$$P_D = \frac{S_{\text{expt}}(E)}{S_{\text{calc}}(E)} \quad (7)$$

for  $E \leq 100$  keV. Note that realistic values of the spectroscopic factor  $S_L$  are smaller than unity, so Eq. (7) gives only a lower limit of the  $D$ -state admixture in the  ${}^4\text{He}$  ground state.

## III. RESULTS

At first we have calculated the low energy  ${}^2\text{H}(d,\gamma){}^4\text{He}$  cross sections using the potential parametrization as given by Weller *et al.*<sup>1</sup> Adopting a heuristic model they analyzed the tensor analyzing power of the  ${}^2\text{H}(d,\gamma){}^4\text{He}$  reaction at  $E = 4.85$  MeV assuming that the scattering states with angular momentum  $L$  are generated from the same potential as the component of the ground state with the same angular momentum. They adopted the parameters  $R_0 = 2.52$  fm and  $a = 1.0$  fm for their WS potential and adjusted the depth of the potential for both components individually ( $V_0 = 54.61$  MeV for  $L = 0$  and 120.8 MeV for  $L = 2$ ) requiring bound states at  $E = 23.84$  MeV, the experimental binding energy of the  ${}^4\text{He}$  ground state. As can be seen from Fig. 1 the used potential parametrization agrees with the experimental  ${}^2\text{H}(d,\gamma){}^4\text{He}$  cross sections at around  $E = 5$  MeV, but it does not reproduce the energy dependence of the data at lower energies. In fact, the potential parametrization of Ref. 1 yields a  $D$ -wave reso-

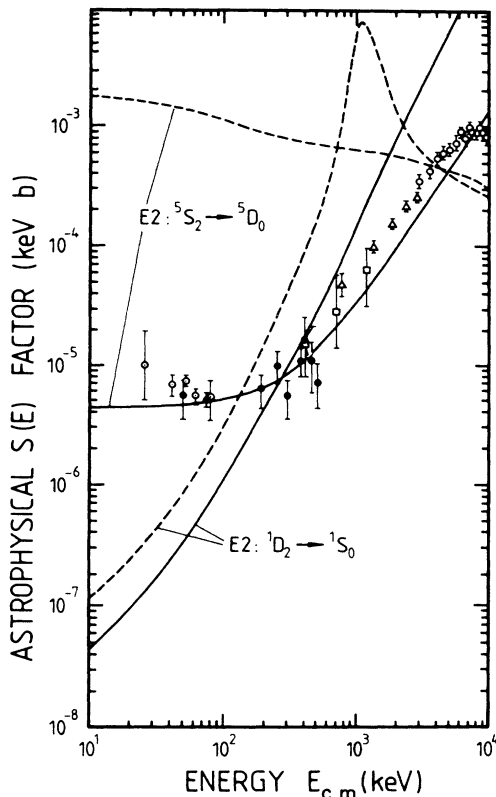


FIG. 1. Excitation functions calculated using the potential parameters of Santos (Ref. 13) ( $R_0=3.78$  fm,  $a=0.5$  fm; solid lines) and Weller (Ref. 1) ( $R_0=2.52$  fm,  $a=1.0$  fm; dashed lines). The parameter set of Ref. 13 has also been used in the studies of Refs. 3 and 6. Experimental data are taken from Refs. 4, 5, and 19–23.

nance at around  $E \approx 1$  MeV and two bound  $J=2$  states. Both facts are not in agreement with our present knowledge about the  ${}^4\text{He}$  system.<sup>12</sup>

We have tested the idea of Ref. 1 that the two components of the  ${}^4\text{He}$  ground state as well as the  $d+d$   $S$ -wave and  $D$ -wave scattering states can be generated by the same WS potentials by varying the diffuseness and the radius of the Woods-Saxon potential over a wide range of parameters. But we failed to determine such a set of two WS potentials which can simultaneously describe the level structure in  ${}^4\text{He}$  as well as the low energy  ${}^2\text{H}(d,\gamma){}^4\text{He}$  fusion cross sections.

Coming to a similar conclusion, Santos *et al.*<sup>6</sup> suggested to derive the ground state components and the  $d+d$  scattering states from different sets of potentials. In order to reduce the uncertainty in the fit procedure for  $d(d,\gamma){}^4\text{He}$  cross section they suggested furthermore to adjust the  $d+d$  scattering states to independent elastic  $d+d$  scattering data. Due to the lack of knowledge of experimental  $d+d$  phase shifts such an adjustment has to be performed using the phase shifts of a microscopic RGM calculation which, however, gives a good account of the experimental elastic scattering differential cross section data and the total  ${}^4\text{He}(\gamma,d){}^2\text{H}$  photo disintegration

cross section measurements. We report now about potential model calculations in which we follow the suggestion of Ref. 6. For that reason we have adjusted two Woods-Saxon potentials to reproduce the RGM phase shifts in the partial waves ( $L=0, S=2$ ) and ( $L=2, S=0$ ). We find that the RGM phase shifts of Ref. 10 at  $E \leq 10$  MeV can be reproduced with an accuracy better than  $1.5^\circ$ , if the parameters in the WS potential are adopted as  $V_0=15.5$  MeV,  $R_0=3.59$  fm, and  $a=0.81$  fm in the partial wave ( $L=0, S=2$ ) and  $V_0=13.5$  MeV,  $R_0=3.39$  fm, and  $a=0.79$  fm in the partial wave ( $L=2, S=0$ ). Our calculated phase shifts are compared to the RGM phase shifts in Fig. 2. Note that our potential in the ( $L=0, S=2$ ) partial wave exhibits one bound state to simulate the Pauli forbidden state which, within a microscopically backed picture of the two-deuteron system with the two fragments in their ground states, accounts for the fact that the configuration with the four nucleons in  $S$  orbitals and their spins coupled to  $S=2$  is forbidden by the Pauli principle. There is no bound state in our ( $L=2, S=0$ ) potential as there exists no Pauli forbidden state in this partial wave.

In Refs. 3 and 5 the low energy  ${}^2\text{H}(d,\gamma){}^4\text{He}$  cross sections have apparently been reproduced within a potential model. The calculation presented in Ref. 3 followed the line as suggested by Santos *et al.* and generated the scattering states from a potential phase equivalent to the RGM phase shifts, while the ground state components

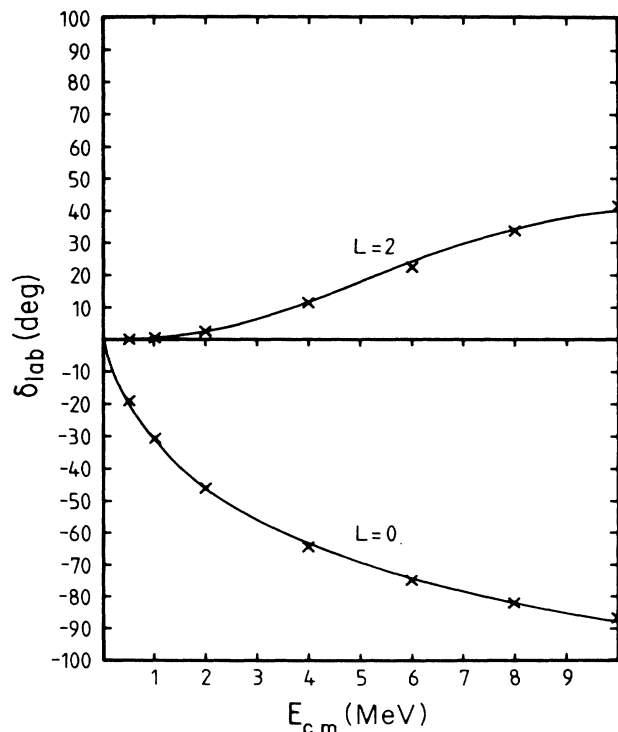


FIG. 2. Fit of the RGM phase shifts of Ref. 10 (crosses) in the partial waves ( $L=0, S=2$ ) and ( $L=2, S=0$ ) by the presently derived Woods-Saxon potentials. The potential parameters are given in the text.

have been derived from WS potentials whose parameters have been given in Ref. 13 ( $R_0=3.78$  fm,  $a=0.5$  fm). Note that in the calculation of Ref. 13 apparently no Coulomb interaction has been considered. In Ref. 3 the effect of the Coulomb interaction has been qualitatively taken into account for scattering states by multiplying the various  $E2$  amplitudes calculated without the consideration of a Coulomb interaction by an approximate energy dependent Coulomb penetration factor. We have repeated the potential model calculation of Weller *et al.*, however, including the Coulomb interaction as defined above. Our results are shown in Fig. 1. We find that the potential parametrization gives a fair account of the fusion data at  $E \approx 0.4$ – $3$  MeV assuming a spectroscopic factor for the  $d + d$   $S$  component of roughly 0.4. However, the calculation predicts fusion cross sections at  $E \leq 200$  keV dominated by the  $E2$  transition into the  $L=2$  component of the  ${}^4\text{He}$  ground state which are roughly in agreement with the experimental data if the  $D$ -state admixture in the  ${}^4\text{He}$  ground state is 100%.

Barnes *et al.*<sup>5</sup> deduced a  $D$ -state admixture of 1.4% from a reproduction of the  ${}^2\text{H}(d,\gamma){}^4\text{He}$  fusion data within the direct capture potential model of Rolfs.<sup>15</sup> Furthermore, they found that their results are approximately stable against modest variations of the potential parameters from which they calculated their ground state wave functions. In fact, they deduced an error of  $\pm 0.6\%$  in their  $D$ -state admixture which they derived from adjusting the low energy fusion data with WS potentials, varying the radius and diffuseness parameters in the range  $R_0=2.27$ – $3.53$  fm and  $a=0.7$ – $1.0$  fm. However, the model of Ref. 15 is based on the assumption that the scattering states can be derived from pure hard-sphere scattering setting the radius of the hardsphere equal to the radius parameter of the nucleus-nucleus potential from which the nuclear bound states are determined. Consequently, the direct capture model does not consider any contributions to the capture process arising from the nuclear interior region  $r \leq R_0$ . Such a simplification has been shown to be justified for nonresonant capture into slightly bound states, but it is certainly not valid for the capture into a state as tightly bound as the  ${}^4\text{He}$  ground state. In fact we find in our potential model studies that a significant contribution to the  $E2$  transition matrix elements arises from the nuclear interior region. Due to penetrability arguments this is especially true for the  $E2$  capture from the  $d + d$   $S$  wave and changes noticeably the results about the strength of the  $D$ -state admixture as derived in Ref. 5.

To be more quantitative about our reservations regarding the analysis of Ref. 5 we have performed potential model calculations of the low energy  ${}^2\text{H}(d,\gamma){}^4\text{He}$  cross sections varying the parameters in the WS potential over the same range as in Ref. 5. We have exemplified our results in Fig. 3 for the extreme choices of parameters considered by Barnes *et al.*,  $R_0=2.27$  fm;  $3.53$  fm and  $a=0.7$  fm;  $1.0$  fm.

In all calculations using parameter sets of Ref. 5 we find similar predictions about the  $E2$  cross sections for the capture from the  $d + d$   $L=2$  scattering states into the  ${}^4\text{He}$  ground state. Furthermore, these results might be in-

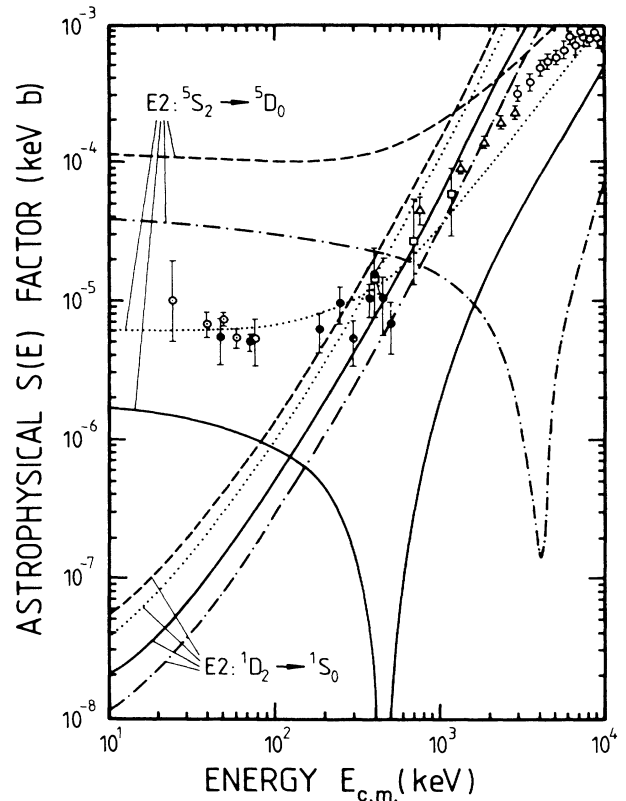


FIG. 3. Excitation functions calculated with the extreme choices of Woods-Saxon parameters given by Barnes *et al.* (Ref. 5):  $R_0=2.27$  fm,  $a=1.0$  fm (solid lines);  $R_0=3.53$  fm,  $a=1.0$  fm (dashed lines);  $R_0=3.53$  fm,  $a=0.7$  fm (dotted lines);  $R_0=2.27$  fm,  $a=0.7$  fm (dashed-dotted lines). Experimental data are taken from Refs. 4, 5, and 19–23.

terpreted as compatible with the experimental data in the energy range  $E \approx 0.2$ – $2$  MeV assuming a spectroscopic factor of  $\approx 0.3$ – $0.5$  for the  $S$ -state admixture in the  ${}^4\text{He}$  ground state. In contrast, the calculations based on the same range of potential parameters predict cross sections for the  $E2$  capture into the  ${}^4\text{He}$   $D$ -state component which differ from each other by up to two orders of magnitude for energies  $E \leq 200$  keV. This result indicates that a rather small variation in the parameters of the WS potential can lead to drastic changes in the predicted strength of the  $D$ -state component. Furthermore, using the parameter set  $R_0=2.27$  fm and  $a=1.0$  fm the magnitude of the  $D$ -state admixture determined by Eq. (7) is roughly 500%; this of course is an absurd prediction. Note that the pole in the cross sections at  $E \approx 450$  keV calculated with this parameter set is caused by a change of sign in the transition matrix element.

In Ref. 6 a  $D$ -state component of the  ${}^4\text{He}$  ground state has been extracted from tensor analyzing data of the  ${}^2\text{H}(d,\gamma){}^4\text{He}$  reaction on the basis of potential model analyses. In this calculation the scattering states are derived from potentials adjusted to reproduce the microscopic RGM phase shifts of Ref. 10, while the ground state components are derived from the WS potential of Ref. 13.

Since this potential is the same as used in Ref. 3 the respective potential model cross sections are identical to those already shown in Fig. 1.

We have also performed a potential model calculation using the potential parametrization for the  ${}^4\text{He}$  ground state as very recently given by Tostevin.<sup>7</sup> These parameters are derived from the sophisticated microscopically calculated  ${}^4\text{He}$  ground state wave function of Schiavilla, Pandharipande, and Wiringa,<sup>14</sup> however, forcing their results into the straight jacket of a bound state of a WS potential. Note that apparently no Coulomb interaction has been considered in Ref. 7. As is shown in Fig. 4 we calculate cross sections for  $E2$  capture from the  $d + d$   $L = 0$  scattering states which are obviously not in agreement with the experimental data.

Finally we have evaluated the low energy  ${}^2\text{H}(d,\gamma){}^4\text{He}$  cross sections within our potential model deriving the  ${}^4\text{He}$  ground state components from WS potentials as given in Refs. 1 and 16. Note that in Ref. 16 the WS parametrization of the  ${}^4\text{He}$  ground state was used in a distorted-wave Born approximation analysis of tensor analyzing powers of the  ${}^{89}\text{Y}(d,\alpha_0){}^{87}\text{Sr}$  reaction resulting in  $D$ -state component of the  ${}^4\text{He}$  ground state of 7%. The results of our

potential model studies based on the WS potentials of Refs. 1 and 16 are shown in Fig. 4. Obviously both parametrizations are not compatible with the  ${}^2\text{H}(d,\gamma){}^4\text{He}$  fusion data at  $E \leq 200$  keV.

#### IV. CONCLUDING REMARKS

In conclusion, we have studied the  ${}^2\text{H}(d,\gamma){}^4\text{He}$  cross sections at low energies  $E \leq 2$  MeV within a  $d + d$  potential model assuming the domination of an electromagnetic capture process of  $E2$  multipolarity. Following previous studies of the  ${}^2\text{H}(d,\gamma){}^4\text{He}$  reaction the scattering states in the entrance channel as well as the  ${}^4\text{He}$  ground state have been derived from WS potentials. The parametrization of these nucleus-nucleus potentials has been chosen in accordance with the previous studies, hence covering a wide range of choices for the radius ( $R_0 = 2.27 - 3.78$  fm) and the diffuseness parameter ( $a = 0.5 - 1.0$  fm). The depth of the potentials has been adjusted to reproduce the experimental binding energy of the  ${}^4\text{He}$  ground state in all cases. Except for a calculation based on the parametrization of Ref. 1, the scattering states have been determined from a WS potential which reproduces the phase shifts found in a microscopic RGM calculation which in turn gives a good description of low energy  $d + d$  scattering. This procedure which follows a suggestion of Refs. 3, 6, and 7 is aimed to reduce the uncertainties within our potential model calculations.

Regarding the intention of this paper the two following results are of interest.

(1) The  ${}^2\text{H}(d,\gamma){}^4\text{He}$  cross sections in the energy range  $E \approx 0.2 - 2$  MeV are compatible with the potential model predictions for an  $E2$  transition process from the  $d + d$   $L = 2$  scattering states into the  ${}^4\text{He}$  ground state if a spectroscopic factor of  $\approx 0.3 - 0.5$  is introduced for the  $S$ -state component of the ground state. This result is in agreement with the generally accepted understanding of the  ${}^2\text{H}(d,\gamma){}^4\text{He}$  process before the low energy fusion data for  $E \leq 200$  keV became available.<sup>17</sup>

(2) An analysis of the low energy fusion data within a simple potential model is not reasonable, especially if this study is aimed at deriving quantitative statements about the  ${}^4\text{He}$  ground state. We have shown that slight variations of the WS parameters can lead to drastic changes in the calculated cross sections for the  $E2$  transition from the  $d + d$   $S$  waves into the  ${}^4\text{He}$  ground state. This sensitivity on the WS parameters is then transferred into the quantitative statements about the  $D$ -state component in the  ${}^4\text{He}$  ground state, if the strength of the  $D$ -state component is determined by adjusting the calculated cross sections to the experimental fusion data at  $E \leq 200$  keV. In particular, we have found that such a procedure might even result in an absurd  $D$ -state component of more than 100%, although reasonable WS parameters have been used in the calculation.

The fact that the potential model can describe the  $E2$  capture from the  $d + d$   $L = 2$  scattering states into the  ${}^4\text{He}$  ground state, but apparently not the capture from the  $L = 0$  scattering states, can be understood in terms of penetrability arguments. Due to the strong centrifugal barrier the  $L = 2$  scattering states in the energy range

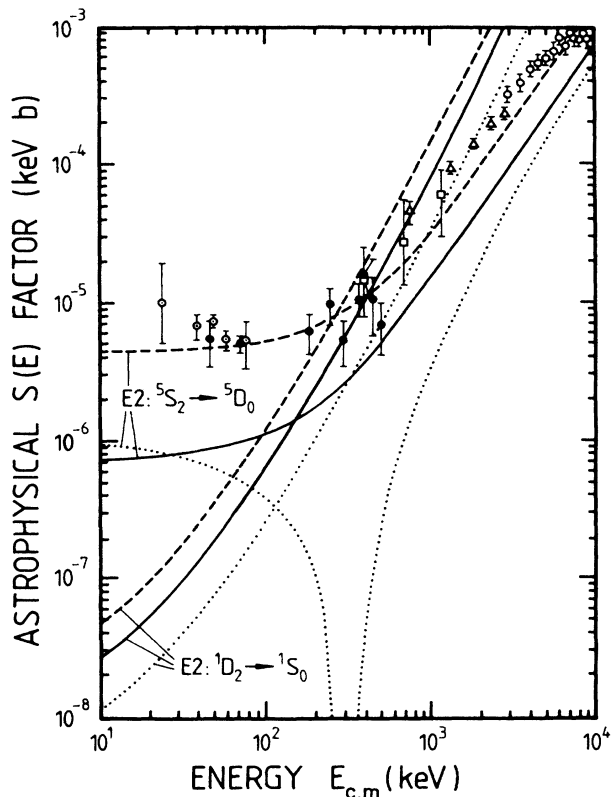


FIG. 4. Excitation functions calculated using the potential parameters of Weller *et al.* (Ref. 1):  $R_0 = 2.52$  fm,  $a = 1.0$  fm (solid lines); Karp *et al.* (Ref. 16):  $R_0 = 3.78$  fm,  $a = 0.5$  fm (dashed lines); Tostevin (Ref. 7):  $R_0 = 2.11$  fm,  $a = 0.75$  fm for  $L = 0, S = 2$  and  $R_0 = 2.65$  fm,  $a = 0.9$  fm for  $L = 2, S = 0$  (dotted lines). Experimental data are taken from Refs. 4, 5, and 19–23.

$E \leq 2$  MeV are strongly suppressed in the nuclear interior region and hence the transition matrix elements are only sensitive to the asymptotic region of the initial and final nuclear wave functions. The situation is obviously different for the  $d + d$   $S$  waves which, as we have already mentioned in the discussion of the validity of the hard-sphere direct capture model, are very sensitive to the nuclear wave functions in the interior region.

Summarizing these results, an analysis of the low energy fusion data within a simple potential model seems not to be a reasonable tool, in particular, if this analysis is aimed at deriving quantitative statements about the  ${}^4\text{He}$  ground state. The applicability of a potential model to study low energy  ${}^2\text{H}(d,\gamma){}^4\text{He}$  fusion can also not be justified by the introduction of  $d + d$  spectroscopic factors for the  ${}^4\text{He}$  ground state components (as suggested in Refs. 3, 5, and 7) or by deducing information about the  $D$ -state component from a "ratio of ratios" by (Ref. 5) comparing experimental and calculated cross sections at two different energies, since the results are sensitive to the potential parametrizations. Often the strength of the  $(d + d)$   $D$ -state admixture in the  ${}^4\text{He}$  ground state is discussed relative to the strength of the  $(d + d)$   $S$ -state component in terms of the asymptotic  $D$ -to- $S$  state ratio (for a definition and a discussion of this quantity see Refs. 13 and 3). It is believed that an extraction of this quantity from experimental data is less model dependent than a

direct analysis of the  $D$ -state amplitude.<sup>3</sup> However, due to penetrability arguments an analysis of the low energy fusion data at  $E \leq 200$  keV in terms of a potential model of structureless particles does yield direct informations about the  $D$ -state amplitude. Hence, a discussion of its strength in terms of the asymptotic  $D$ -to- $S$  state ratio seems not to be necessary.

We believe that our calculations have confirmed the finding stated previously (Ref. 9) that the  ${}^4\text{He}$  ground state can not simply be treated as a bound state of the two-deuteron system and requires a more sophisticated microscopic four-nucleon description. The low energy  ${}^2\text{H}(d,\gamma){}^4\text{He}$  fusion data of Refs. 4 and 5 are then a well suited test case for these microscopic wave functions and for the underlying nucleon-nucleon interaction. A first step into the direction of such a consistent microscopic description of the  ${}^2\text{H}(d,\gamma){}^4\text{He}$  reaction has been already taken in Ref. 18 in which the data at  $E \leq 200$  keV were found to be compatible with a  $D$ -state admixture in the  ${}^4\text{He}$  ground state of 5–7%. However, this microscopic calculation lacks the inclusion of possible  $n + {}^3\text{He}$  and  $p + t$  fragmentations and neglects the internal quadrupole moment of the deuteron. Hence further improved microscopic studies (i.e., incorporating the sophisticated many-body wave functions of Ref. 14 for the  ${}^4\text{He}$  ground state) are desirable.

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