Rotational model and shell model pictures of magnetic dipole excitations

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A comparison is made of the rotational model and the large shell model for the calculation of the magnetic dipole excitations in selected nuclei in the *s-d* and *f-p* shells. Focus is given on both the relatively low lying strong states and higher "spin flip" excitations. It is found that the two models agree fairly well for summed $B(M1)$ strength, but for the deformations that were used give different results for orbital and spin decomposition. The rotational model yields too much low lying strength as compared with the shell model. A technique for eliminating spurious contributions to the summed strength in the rotational model is developed. It is shown that in the asymptotic limit in some nuclei the magnetic dipole excitations are purely orbital. The implications of these calculations to results in heavier deformed nuclei for "scissor mode" excitations is discussed.

INTRODUCTION

We have previously studied magnetic dipole excitations in the single j shell approximation in the titanium isotopes with several motives.[1,2] We wished to draw an analogy with the recently discovered "scissor" modes that were found in the deformed region, especially ^{156}Gd (at 3.1 MeV).[3] We gratified that such states were looked for in ^{46}Ti and ^{48}Ti and that indeed strong excitations were found at energies that, allowing for a reasonable A dependence, were systematically similar to those in ^{156}Gd (4.3 MeV in ^{46}Ti and 3.8 MeV in ^{48}Ti).[4] We wished to show that such states could be described in shell model terms and indeed that even the single j shell truncation had in it enough degrees of freedom to make sensible predictions for these low lying excitations. Furthermore, the shell model results could be put into a simple correspondence with interacting boson model calculations done in heavier nuclei (this point was especially emphasized by Talmi).[5] We also studied the effective interaction dependence of the energies and strengths of these modes,[6] showing, for example, that using at one extreme quadrupole-quadrupole interaction these collective modes are very strongly excited, but at another extreme, with a pairing interaction (as shown analytically by Halse[7]) all the strength went to higher isospin states.

A two state rotational model was devised to describe these low lying excitations.[8] It was found to give too much strength, as compared with the shell model, probably because of the neglect of pairing correlations. We studied the isospin nature of these models (in the single j shell these excitations are rigorously isovector) and the spin and orbital ratio of the $B(M1)$ strength [in the single j shell approximation for $j=l+\frac{1}{2}$ the ratio of orbit to spin matrix elements is $l/(\mu_p-\mu_n)$]; in the deformed region the excitation is evidently mainly orbital.

Now we wish to look at larger spaces in both the shell and rotational models. While the single j shell model may be good enough to qualitatively describe the low lying states in various nuclei, there are higher excitations of interest which it cannot describe, such as spin flip excitations from $f_{7/2}$ to $d_{5/2}$. Also it is not clear *a priori* whether the low lying strong excitations resulting from single j shell calculations will or will not be completely fragmented when larger spaces are used. Also larger spaces are needed to get the general strength distributions. These spaces are also of interest in answering questions that have come up in the heavier deformed region—for example, does the 3.1 MeV excitation in ^{156}Gd exhaust all the orbital strength or is there more strength still unseen at higher energies?

There have been extensive studies of magnetic transitions in the *s-d* shell using shell model programs. These include works of Preedom and Wildenthal,[9] Brown and Wildenthal,[10] Chavez and Poves,[11] and some others.[12]

The use of special distribution methods for studying magnetic dipole strength distributions in the *s-d* shell was pioneered by Halemane and French.[13] In the ^{156}Gd region itself, preliminary results of a projected Hartree-Fock calculation by Moya de Guerra and Dieperink[14] were reported at the Santonder course. These results indicate that the total calculated strength is significantly larger than what is found experimentally in the 3.1 MeV region.

ROTATIONAL MODEL FOR THE TOTAL MAGNETIC DIPOLE STRENGTH: ELIMINATION OF THE SPURIOUS STATE CONTRIBUTION

The expression in the rotational model for a transition from the $K=0, J=0$ ground state of an even-even nucleus to a $K=1, J=1$ excitation state is

$$B(M1)=|M|^2\ ,$$
$$M=\sqrt{3/4\pi}\left\langle K=1\left|\sum_i (g_l\, l_+ + g_s\, s_+)_i\right|K=0\right\rangle\ ,$$

where l_+ and s_+ are the angular momentum raising operators for orbit and spin and the sum is over nucleons. Bare values of the g factors are used:

proton: $g_l=1,\quad g_s=5.586$,

neutron: $g_l=0,\quad g_s=-3.826$.

We form $K=1$ basis states by exciting particles from occupied Nilsson orbits to unoccupied ones. The single particle Nilsson orbit is denoted by $|K,m,\pi\rangle$ and $|K,m,\nu\rangle$ for the proton and neutron, respectively, where m corresponds to the mth state (counting from low energy to high energy) with quantum number K. For example, the state $|\frac{1}{2},1,\pi\rangle$ is the state which in the weak deformation limit would become $d_{5/2,1/2}$; $|\frac{1}{2},2,\pi\rangle$ would become $s_{1/2,1/2}$ and $|\frac{1}{2},3,\pi\rangle$ would become $d_{3/2,1/2}$, etc. The time reversal state of $|K,m,\pi\rangle$ is expressed as $\overline{|K,m,\pi\rangle}$, which has quantum number $-K$. We write $K=1$ basis state as $|K_p,m_p,K_h,m_h,\pi\rangle$ corresponding to proton excitation from $|K_h,m_h,\pi\rangle$ to $|K_p,m_p,\pi\rangle$. Neutron excitations are denoted in the same way. We use the abbreviated notation $|\alpha\rangle=|K_p,m_p,K_h,m_h,\pi\rangle$ for these basis states.

To make things more concrete we give some details of the calculation in ^{22}Ne. We here list the various $|K=1\rangle$ states and the value of $|M|^2$, where

$$M=\langle K_p,m_p,K_h,m_h,\pi \text{ (or } \nu)\,|\,M(1)\,|\,\text{g.s.}\rangle$$

for each of these:

Particle state	Hole state	$\lvert M\rvert^2$ (μ_N^2)
$\lvert\frac{3}{2},1,\pi\rangle$	$\lvert\frac{1}{2},1,\pi\rangle$	2.869
$\lvert\frac{3}{2},2,\pi\rangle$	$\lvert\frac{1}{2},1,\pi\rangle$	0.010
$\lvert\frac{1}{2},2,\pi\rangle$	$\overline{\lvert\frac{1}{2},1,\pi\rangle}$	0.046
$\lvert\frac{1}{2},3,\pi\rangle$	$\overline{\lvert\frac{1}{2},1,\pi\rangle}$	0.014
$\lvert\frac{3}{2},2,\nu\rangle$	$\lvert\frac{1}{2},1\nu\rangle$	0.010
$\lvert\frac{1}{2},2,\nu\rangle$	$\overline{\lvert\frac{1}{2},1,\nu\rangle}$	0.205
$\lvert\frac{1}{2},3,\nu\rangle$	$\overline{\lvert\frac{1}{2},1,\nu\rangle}$	0.017
$\lvert\frac{5}{2},1,\nu\rangle$	$\lvert\frac{3}{2},1,\nu\rangle$	0.150
$\overline{\lvert\frac{1}{2},2,\nu\rangle}$	$\overline{\lvert\frac{3}{2},1,\nu\rangle}$	2.857
$\overline{\lvert\frac{1}{2},3,\nu\rangle}$	$\overline{\lvert\frac{3}{2},1,\nu\rangle}$	0.241

The simple sum $\sum_\alpha B(M1)_\alpha$ will contain spurious components. We can form the spurious state $|K_s\rangle$ by acting with the operator $J_+=\sum_i j_+(i)$ on the ground state $|K_s\rangle=NJ_+|K=0\rangle$, where N is a normalization factor. The spurious state can be expected as a combination of the basis states $|\alpha\rangle$ $|K_s\rangle=\sum_\alpha d^\alpha|\alpha\rangle$. In more detail,

$$|K_s\rangle=\sum_\alpha d(K_p,K_h,\nu)\,|K_p,K_h,\nu\rangle + d(K_p,K_h,\pi)\,|K_p,K_h,\pi\rangle\ .$$

The total strength, from which the spurious contribution is reduced, is equal to

$$\sum{}' B(M1)=\sum_\alpha |M_\alpha|^2-M(K_s)^2\ ,$$

where $M(K_s)$, the spurious contribution is given by

$$M(K_s)=\sqrt{3/4\pi}\left\langle K_s\left|\sum(g_l l_+ + g_s s_+)_i\right|0\right\rangle\ .$$

We then see that the correct summed strength is

$$\sum{}' B(M1)=\sum_\alpha M_\alpha M'_\alpha\ ,$$

where $M'_\alpha=M_\alpha-\sum_\beta d^\alpha d^\beta M_\beta$. It is easy to show further that $\sum_\alpha M_\alpha M'_\alpha=\sum_\alpha|M'_\alpha|^2$.

We can look at this from a slightly different point of view. M'_α can be regarded as the matrix element of a new operator,

$$\sum_i (g_l l_+ + g_s s_+)_i - g_R J_+\ ,$$

where g_R is obtained by requiring that

$$\left\langle K_s\left|\sum_i (g_l l_+ + g_s s_+)_i - g_R J_+\right|0\right\rangle=0\ ,$$

i.e., the operator will not connect the ground state with the spurious state. One can show that $g_R=N\sum_\alpha d^\alpha M_\alpha$. In effect, then, to remove the spurious component from the summed strength, we perform the calculation the same way we would otherwise do provided we replace g_l and g_s by $g'_l=g_l-g_R$ and $g'_s=g_s-g_R$.

IMPORTANCE OF REMOVING SPURIOUS STATE COMPONENTS FROM THE SUMMED STRENGTH

To see how important it is to remove the spurious components from the summed magnetic dipole strength, we make a comparison of $\sum MM$ with $\sum M'M'$. The later also being equal to $\sum MM'$. Let us consider ^{46}Ti as an example. We find the following results:

$$\text{Orbit: } \sum(MM)_o=2.89,\quad \sum(M'M')_o=1.56\ ,$$

$$\text{Spin: } \sum(MM)_s=6.64,\quad \sum(M'M')_s=6.61\ ,$$

$$B(M1)\text{: } \sum(MM)=11.16,\quad \sum(M'M')=9.40\ .$$

We see that the removal of spurious strength is most important for the orbital part. This is true for the other nuclei.

In doing the calculation we used different values of g_R for evaluation $\sum(\text{orbit})^2$, $\sum(\text{spin})^2$, and $B(M1)$. These values are listed in Table I. In other words, g_R is a function of g_l and g_s. In evaluating $\sum(\text{orbit}),^2$ we used the condition

$$\left\langle K_s\left|\sum_i (g_l l_+)_i - g_R J_+\right|0\right\rangle=0\ ,$$

etc.

COMPARISON OF THE SHELL MODEL AND THE ROTATIONAL MODEL FOR THE TOTAL STRENGTHS

The shell model calculations were done with OXBASH code of Brown, Etchegoyen, and Rae.[15] The nuclei considered are ^{20}Ne, ^{22}Ne, ^{44}Ti, ^{46}Ti, and ^{48}Ti. In the *sd* shell all possible configurations were allowed. All possible configurations in the *fp* shell are allowed for the nucleus ^{44}Ti, but for ^{46}Ti and ^{48}Ti only two particles are allowed to leave the $f_{7/2}$ shell.

The interaction used in the *s-d* shell was Wildenthal's $A=17-39$ "USD" interaction.[16] For the *f-p* shell nu-

TABLE I. Values of g_R used to eliminate spurious state contributions.

Nucleus	Orbit	Spin	Total
^{20}Ne	0.487	0.023	0.510
^{22}Ne	0.434	−0.121	0.313
^{44}Ti	0.462	0.066	0.528
^{46}Ti	0.439	0.065	0.504
^{48}Ti	0.504	0.155	0.659

clei the FPY interaction is used.[15] The results are shown in Fig. 1.

Note that in the shell model calculations the main strength for $N=Z$ nuclei (^{20}Ne, ^{44}Ti) goes to higher isospin states, but for $Z \neq N$ nuclei most of the strength is to states with the same isospin as the ground state. This is readily understood from the fact that for $T=0$ to $T=0$ transitions only the isoscalar part of the magnetic multipole operator enters. Since the sign of the magnetic moment of the neutron is the opposite of that of the proton the isovector transitions are strongly suppressed.

For $N \neq Z$ nuclei with ground state isospins T we can get isovector contributions to all excited 1^+ states. This is because $|\mathbf{T}+\mathbf{1}| = T-1$, T, or $T+1$.

We see that the overall $B(M1)$ strengths in the two models are in surprisingly good agreement. However, the breakdown into orbit and spin is less impressive. For the titanium isotopes the rotational model gives too much orbital strength and too little spin strength as compared with the shell model.

The deformation parameter used for neon isotopes is $\eta=6$, while $\eta=4$ was used for the titanium isotopes. The Nilsson wave functions were obtained from Nilsson's original paper.[17]

In general, the $B(M1)$ strengths in the rotational model depend on the choice of the deformation parameter. To show this we consider the case of ^{20}Ne in two extreme limits—zero deformation and asymptotic limit $\eta \rightarrow \infty$.

The intrinsic state for one $K=\frac{1}{2}$ nucleon in the zero deformation limit is $d_{5/2,1/2}$. In the asymptotic limit it is $(\sqrt{2}d_0 - s_0)/\sqrt{3}$ with spin up. In the asymptotic limit it is easy to see that the only state the $M1$ operator can connect to is d_1 with spin up, and this can occur only via the orbital term. There is no spin excitation in this limit! Thus we find

$$\text{asymptotic: } \sum(\text{orbit})^2 = 3/\pi = 0.95\ \mu_N^2, \quad \sum(\text{spin})^2 = 0.0, \quad \sum B(M1) = 0.95\ \mu_N^2 ,$$

$$\text{zero deformation: } \sum(\text{orbit})^2 = 0.65, \quad \sum(\text{spin})^2 = 6.77, \quad \sum B(M1) = 9.58\ \mu_N^2 .$$

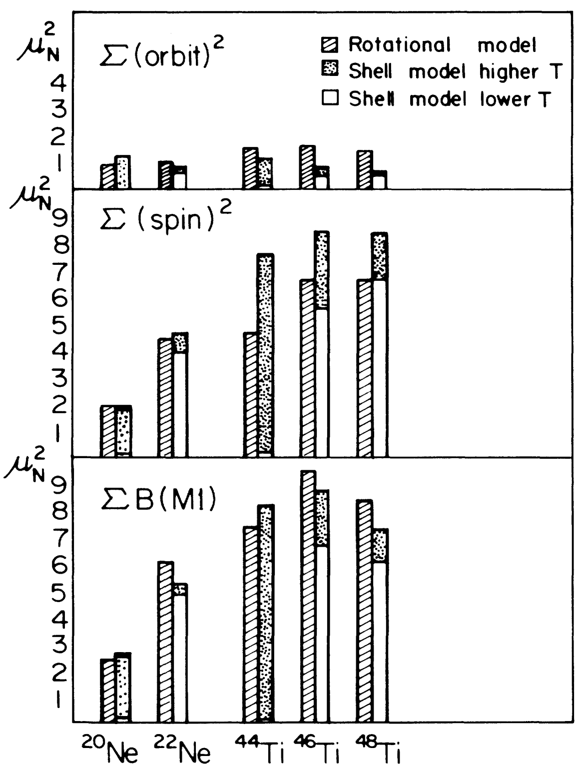

FIG. 1. A comparison of the summed $M1$ strength in the rotational and shell models. The dashed rectangles correspond to the rotational model results, the open rectangles to lower T, and the dotted rectangles to higher T excitations in the shell model.

Note that the value of $B(M1)$ varies rapidly with deformation. Therefore it is essential to have the correct deformation (or in the case of shell model calculation the correct effective interaction) before one can make any meaningful statement concerning whether or not the magnetic transitions are quenched.

COMMENTS ON THE ENERGY DISTRIBUTION OF THE LOW LYING STRENGTH IN THE SHELL MODEL

Note that for $N=Z$ nuclei the energies of the low lying strong states is considerably higher than for $N \neq Z$ nuclei. Thus in ^{20}Ne the energy is 11.2 MeV for the strong $T=1$ state, whereas in ^{22}Ne it drops to 6.48 MeV (we have taken the centroid of the lowest two states). In a recent publication Chavez and Poves[11] have questioned why one of the present authors got the "scissor model" so low in the titanium isotopes, e.g., 4.0 MeV in ^{48}Ti in a single j shell calculation, whereas they got the mode at very high energy in ^{20}Ne—about 11.2 MeV. This analysis answers the question. Part of the answer is, of course, the A dependence and the other part is due to the fact that these authors only focused on $N=Z$ nuclei. The sudden drop in energy in ^{22}Ne, as compared with ^{20}Ne, makes it much less surprising that in ^{46}Ti the state should come at 4.3 MeV.

Note that there is also a drop in the energy of the low lying strength when one compares ^{46}Ti and ^{48}Ti with the $N=Z$ nucleus ^{44}Ti. This was also noted in the previous single j shell work.[1,2]

It should be noted that Chavez and Poves[11] make the "scissor mode" interpretation of the 11.2 MeV state in ^{20}Ne plausible by cleverly showing that the intrinsic state quadrupole moments for the ground state band ($J=0^+,2^+,4^+,\ldots$) and for the $|K=1\rangle$ band, of which the 11.2 MeV 1^+ state is a member, are fairly close to each other.

COMPARISON OF ORBITAL STRENGTH OF LOW LYING STRONG STATES TO THE TOTAL ORBITAL STRENGTH

This section is motivated by ongoing debates concerning the nature of collective magnetic excitations in heavier deformed regions, especially the classic nucleus for the scissor mode ^{156}Gd. Here Richter's group, using the 50 MeV electron machine at Darmstadt, found a substantial bump at 3.1 MeV excitation.[3] This is a magnetic dipole excitation. The fact that this bump was not seen with protons lent credence to this excitation being mainly an orbital excitation. An orbital "scissor" or "wobble" mode had been previously predicted by LoIudice and Palumbo,[18] in which the excitation consisted of the neutron symmetry axis vibrating against the proton symmetry axis. However, whereas experimentally this mode [now seen with finer resolution to be five separate states in (γ,γ') (Ref. 19)] has a strength of $2.1\pm0.3\ \mu_N^2$, the original prediction was 17 μ_N^2 and the mode was predicted to be at an excitation energy of about 10 MeV. In a proton-neutron interacting-boson-model (IBM-2) calculation, Dieperink and Iachello[20,21] modified the above picture, allowing only valence nucleons to participate in the collective motion. Except for the fact that IBM-2 predicts only one state rather than five, their calculation of the $B(M1)$ and energy are in fairly good agreement with the experiment.

However, others feel that the 3.1 MeV orbital strength represents only a splinter of the total orbital strength and the latter occurs at higher energies, perhaps strongly fragmented over many states.

Aside from their own intrinsic interest, we can view the light nuclei in the *s-d* and *f-p* shells as playing fields in which the ideas proposed for the heavier nuclei can be put to the test. We can take advantage of the fact that in these light nuclei large shell model calculations can be and have been carried out.

In Table II we show the results for states in ^{20}Ne, ^{22}Ne, ^{44}Ti, ^{46}Ti, and ^{48}Ti. The first thing to note is that

TABLE II. The percent of orbital strength in selected low lying strong states in the shell model.

Nucleus	Energy (MeV)	$(\text{orbit})^2/\sum(\text{orbit})^2$ (%)
^{20}Ne	11.2	50.66
^{22}Ne	5.45	17.95
	6.66	22.19
^{44}Ti	6.54	60.50
^{46}Ti	4.75	45.90
	6.14	7.03
^{48}Ti	4.45	24.91
	6.57	2.34

the low lying "strong" states do carry considerable orbital strength, and in this sense bear some analogy to the states in ^{156}Gd. The states do not exhaust all the orbital strength, however. For nuclei with $N\neq Z$, one obvious source of fragmentation of the strength is the distribution into two isospin channels. For example, in ^{22}Ne 65.64% of the orbital strength is in $T=1$ states and 34.36% in $T=2$ states. In ^{46}Ti the corresponding numbers are 70.44% and 29.56%.

It is interesting to note that in the shell model calculation some of the orbital strength gets hidden in the sense that the orbital and spin add destructively to get a very small value of $B(M1)$. For example, in ^{20}Ne there is a calculated state at 17.2 MeV which contains 22% of the total strength, but for which $B(M1)$ is equal to 0.003 μ_N^2, which is truly negligible.

Looking at Table II we see that there is more fragmentation of orbital strength for $N=Z$ nuclei than for $N\neq Z$ nuclei. In the titanium isotopes, for example, the strongest low lying state has 60.5% of the strength for ^{44}Ti, 45.9% for ^{46}Ti, but only 24.9% for ^{48}Ti.

THE OVERALL STRENGTH DISTRIBUTIONS IN THE ROTATIONAL AND SHELL MODELS

In the rotational model relatively few $K=1$ states are strongly excited. In Table III we give such states, using the somewhat arbitrary criterion that the value of $B(M1)$ has to be greatest than 0.5 μ_N^2 to make the list.

We see that by this criterion only the low lying excitations in the $N=Z$ nuclei ^{20}Ne and ^{44}Ti carry significant strength in the rotational model corresponding to transitions from the last occupied orbit K to the first unoccupied orbit $K+1$, i.e., $|\frac{1}{2},1,\pi\rangle\rightarrow|\frac{3}{2},1,\pi\rangle$ and $|\frac{1}{2},1,\nu\rangle\rightarrow|\frac{3}{2},1,\nu\rangle$.

For ^{22}Ne, ^{46}Ti, and ^{48}Ti one other transition is important, from $|\frac{3}{2},1,\nu\rangle$ to $\overline{|\frac{1}{2},2,\nu\rangle}$. This is also a low lying transition. In fact, for ^{22}Ne this transition has a lower Nilsson energy splitting than does the proton transition $|\frac{1}{2},1,\pi\rangle\rightarrow|\frac{3}{2},1,\pi\rangle$. It is interesting to note that in the zero deformation limit this transition would vanish. In the *s-d* shell it would correspond to a transition from $d_{5/2,-3/2}$ to $s_{1/2,-1/2}$, and in the *f-p* shell from $f_{7/2,-3/2}$ to $p_{3/2,-1/2}$. However, at the other extreme $(\eta\rightarrow\infty)$ the transition in the *s-d* shell is from $d_{-1}\downarrow$ to $d_{-1}\uparrow$, a strong spin transition.

Surprisingly, the transitions to the stats which in the zero deformation limit would correspond to a spin flip transition, i.e., $d_{5/2}\rightarrow d_{3/2}$ or $f_{7/2}\rightarrow f_{5/2}$, have very little strength at the deformation that are used here.

We see that one problem with the rotational Nilsson model is that although the summed $B(M1)$ strength comes out fairly good, the energy distribution is not well reproduced. Most of the strength, if we use the Nilsson single particle energy splittings as a criterion, comes much too low in energy. Indeed, all the "strong" states are at relatively low energies. In more realistic models we can expect some of the strength to be pushed up from the fact that one has to break two $J=0$ pairs to form a 1^+ state and from the fact that there is a repulsive particle-hole interaction.

TABLE III. Strength to states in the rotational model with $B(M1) > 0.5\ \mu_N^2$.

Nucleus	Particle state	Hole state	$B(M1)$ (μ_N^2)	
			($\eta=6$)	
^{20}Ne	$\vert\frac{3}{2},1,\pi\rangle$	$\vert\frac{1}{2},1,\pi\rangle$	1.901	
	$\vert\frac{3}{2},1,\nu\rangle$	$\vert\frac{1}{2},1,\nu\rangle$	0.502	
^{22}Ne	$\vert\frac{3}{2},1,\pi\rangle$	$\vert\frac{1}{2},1,\pi\rangle$	2.275	
	$\overline{\vert\frac{1}{2},2,\nu\rangle}$	$\overline{\vert\frac{3}{2},1,\nu\rangle}$	3.175	
			($\eta=2$)	($\eta=4$)
^{44}Ti	$\vert\frac{3}{2},1,\pi\rangle$	$\vert\frac{1}{2},1,\pi\rangle$	6.050	4.833
	$\vert\frac{1}{2},3,\pi\rangle$	$\overline{\vert\frac{1}{2},1,\pi\rangle}$	0.652	0.220
	$\vert\frac{3}{2},1,\nu\rangle$	$\vert\frac{1}{2},1,\nu\rangle$	1.969	1.472
^{46}Ti	$\vert\frac{3}{2},1,\pi\rangle$	$\vert\frac{1}{2},1,\pi\rangle$	5.620	4.993
	$\vert\frac{1}{2},3,\pi\rangle$	$\overline{\vert\frac{1}{2},1,\pi\rangle}$	0.652	0.221
	$\vert\frac{5}{2},1,\nu\rangle$	$\vert\frac{3}{2},1,\nu\rangle$	1.471	0.983
	$\overline{\vert\frac{1}{2},2,\nu\rangle}$	$\overline{\vert\frac{3}{2},1,\nu\rangle}$	0.973	2.144
	$\overline{\vert\frac{1}{2},3,\nu\rangle}$	$\overline{\vert\frac{3}{2},1,\nu\rangle}$	0.825	0.298
^{48}Ti	$\vert\frac{3}{2},1,\pi\rangle$	$\vert\frac{1}{2},1,\pi\rangle$	4.443	4.290
	$\vert\frac{1}{2},3,\pi\rangle$	$\overline{\vert\frac{1}{2},1,\pi\rangle}$	0.650	0.218
	$\vert\frac{7}{2},1,\nu\rangle$	$\vert\frac{5}{2},1,\nu\rangle$	0.924	0.603
	$\overline{\vert\frac{1}{2},2,\nu\rangle}$	$\overline{\vert\frac{3}{2},1,\nu\rangle}$	1.038	2.245
	$\overline{\vert\frac{1}{2},3,\nu\rangle}$	$\overline{\vert\frac{3}{2},1,\nu\rangle}$	0.773	0.244

In Table IV and in Fig. 2 we give the shell model results obtained from the OXBASH code. We list the isospins, energies, the values of (orbit)^2, (spin)^2, and $B(M1)$ for all states in ^{20}Ne, ^{22}Ne, ^{44}Ti, and ^{46}Ti which have $B(M1)$ values greater than 0.2 μ_N^2, and also a few other states of interest [including the 17.2 MeV state in ^{20}Ne, for which the orbit and spin terms are relatively large, but interfere destructively to give a very small value of $B(M1)$].

In ^{20}Ne, as noted by Chavez and Poves[11] most of the $B(M1)$ strength is concentrated in one state. In ^{22}Ne there is more fragmentation. This is also a feature of the rotational model.

Note, however, that most of the orbital strength in ^{22}Ne is in the first two states, at 5.43 and 6.66 MeV. The state at 9.19 MeV has a value of (orbit)^2=0.001. This is negligible compared with the value of $B(M1)$, 1.95 μ_N^2.

In the $Z \neq N$ nucleus ^{46}Ti there are three regions of significant strength. The first is the lowest state at 4.79 MeV with $B(M1)=1.38\ \mu_N^2$ and the second region is at 9.82 and 9.96 MeV with $B(M1)$ strengths of 1.16 and 0.67 μ_N^2, respectively. these may correspond to spin flip excitations with the same isospin as the ground state. Lastly, at 14.55 and 14.68 MeV the strength are 0.70 and 0.38 μ_N^2. These states have isospin $T=2$.

In ^{48}Ti the strength seems more fragmented. However, one common feature of all the calculations is that there still is a low lying, relatively well isolated state (ex-

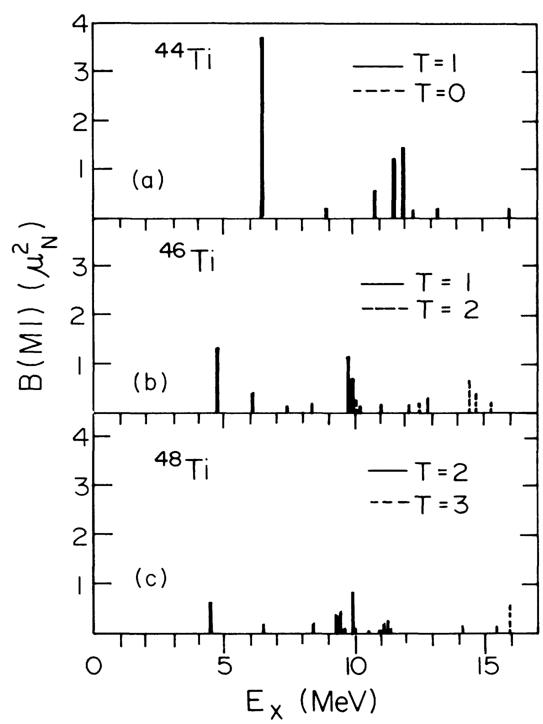

FIG. 2. Shell model predictions of the values of $B(M1)$ in titanium isotopes using the FPY interactions.

TABLE IV. Calculated $M1$ strength in the shell model to states with $B(M1)>0.2\ \mu_N^2$ and other selected states.

Nucleus	Isospin	Energy (MeV)	(orbit)^2 (μ_N^2)	(spin)^2 (μ_N^2)	$B(M1)$ (μ_N^2)
^{20}Ne	1	11.2	0.62	0.38	1.95
	1	17.2	0.21	0.34	0.004
^{22}Ne	1	5.4	0.15	0.02	0.27
	1	6.7	0.19	0.37	1.08
	1	9.2	0.001	1.88	1.95
	1	10.0	0.001	0.19	0.22
	1	11.1	0.003	0.31	0.37
	1	11.8	0.001	0.48	0.43
	2	16.2	0.01	0.11	0.21
^{44}Ti	1	6.5	0.64	1.30	3.76
	1	9.0	0.02	0.10	0.21
	1	12.0	0.09	1.07	0.53
	1	12.5	0.02	1.57	1.25
	1	12.9	0.05	2.08	1.47
	1	13.4	0.001	0.22	0.19
	1	14.2	0.00	0.16	0.18
^{46}Ti	1	4.8	0.32	0.37	1.38
	1	6.1	0.05	0.18	0.41
	1	8.4	0.000	0.20	0.21
	1	9.8	0.002	1.25	1.16
	1	10.0	0.002	0.58	0.67
	1	10.3	0.000	0.19	0.19
	1	11.0	0.004	0.26	0.20
	2	14.5	0.05	1.14	0.70
	2	14.7	0.04	0.65	0.38
	2	15.2	0.002	0.25	0.21
^{48}Ti	2	4.5	0.14	0.16	0.60
	2	6.6	0.01	0.13	0.21
	2	8.4	0.000	0.20	0.21
	2	9.2	0.003	0.42	0.40
	2	9.4	0.001	0.52	0.49
	2	10.0	0.005	0.97	0.84
	3	14.8	0.01	0.22	0.12
	3	15.4	0.02	0.24	0.13
	3	15.9	0.05	0.97	0.59

cept for ^{22}Ne, where there are two nearby states) which carries a significant amount of strength, and which—relative to higher energy strong excitations—carries a large amount of orbital strength.

It should be noted that the results in Table IV will be modified if spin and orbit renormalizations are introduced, in general making the spin contributions smaller and the orbital ones bigger.[8]

CLOSING REMARKS

In this work we have compared the rotational model and shell model applied to the problem of magnetic dipole excitations in open shell light nuclei. We see that the simple rotational model is not good enough to give a quantitative description of the magnetic dipole distribution, but it can provide some qualitative insights concerning some general features. For example, as previously mentioned, in the asymptotic limit ($\eta \rightarrow \infty$) the magnetic excitation in ^{20}Ne would be purely orbital, and indeed in the shell model calculation the strongest state in ^{20}Ne has a large orbital part in it. In the rotational model for ^{22}Ne there is an additional contribution due to a neutron excitation. This is a spin contribution. This is also borne out by the shell model—the spin contribution in ^{22}Ne is much larger than in ^{20}Ne. Both the rotational and shell models predict strong, relatively low lying states, but the rotational model overpredicts the strength by a significant amount. The summed strength, however, is fairly close in both models.

We find that the low lying strong states carry a significant amount of orbital strength, although by no means all of such strength. Much of the orbital strength not in the lowest states is strongly fragmented over many states, including states of higher isospin. It would therefore not be surprising if, in ^{156}Gd, the "scissor" mode at 3.1 MeV did not contain *all* the orbital strength and that some strength would be favored, albeit highly fragmented, at higher energies.

On the other hand, the behavior in the lighter nuclei is in many ways analogous to that in the deformed region. The low lying states do carry considerable $M1$ strength and also considerable orbital strength. The transitions are mainly isovector and in the single j shell approximation they are rigorously so. To form these states one therefore needs both open shell neutrons and protons. For these reasons one has a right to call these states collective modes both in the light nuclei considered here and in the deformed region around ^{156}Gd.

Note added. We note the appearance of work by Moya de Gurra *et al.*[22] concerning scissor model excitation in ^{46}Ti and light nuclei. They performed a deformed Hartree-Fock calculation with pairing, including several major shells. They claim that their calculation supports a "scissor mode" interpretation for the first excited 1^+ state in ^{46}Ti and that the ratio of orbit to spin is much larger than what one obtains in a single major shell calculation.

Very recently papers closely related to this one have been published by the Tokyo Institute of Technology,[23,24] using shell model methods. One should also note group theoretical approaches by the Sussex collaboration.[25,26] A critical comparison of these different works in the near future would be in order.

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