Nucleon-nucleon tensor interaction and the triton binding energy

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We have investigated a sensitivity of short range behavior of the nucleon-nucleon tensor interaction on the $^3S_1$-$^3D_1$ mixing parameter and the triton binding energy. It is found that, by varying the short range behavior of a tensor component to repulsive from attractive, we can get an additional triton binding energy of about 0.7 MeV. At the same time, the mixing parameter becomes too small to be acceptable. For a proposed potential to be really "realistic," the mixing parameter should carefully be fitted to experimental values.

Traditionally, interactions among nucleons in low-energy nuclear physics have been assumed to be described by a nonrelativistic nucleon-nucleon (N-N) potential. So far, a number of realistic N-N potentials[1-6] have been proposed, parametrized in a one boson exchange potential (OBEP) or a phenomenological form to reproduce low-energy two-nucleon phenomena. At present, only two- and three-nucleon systems can be solved exactly (in a numerical sense) for a realistic N-N potential with or without a three-nucleon potential.[7-14] Therefore, we apply a proposed potential to these systems and compare theoretical (calculated) values with experimental ones for the purpose of studying to what extent the traditional approach is valid, and/or whether or not we need any further ingredients such as many-nucleon forces, relativistic effects, quark effects, etc.

In Refs. 7–10, we reported some results of our triton bound state calculations with Reid-soft-core (RSC),[1] Argonne (AV),[2] Paris (PARIS),[3] de Tourreil-Rouben-Sprung (TRS),[4] and Bonn (BONN) (Ref. 5) N-N potential models. (BONN, which we used, is the r-space OBEP version whose meson parameters are slightly different from the published version.[5] However, this difference does not affect our discussion.) While the experimental triton binding energy (B_3) is 8.48 MeV, these calculations show that AV, PARIS, and TRS yield at most 7.7 MeV, and as a result, we need a three-nucleon force to eliminate the discrepancy between experimental and theoretical values of about 10%. On the other hand, BONN yields 8.31 MeV for B_3 without a three-nucleon potential.[10] The purpose of this paper is to clarify the origin of this difference.

The values of B_3, referred to above, are calculated by taking into account all partial wave components of the N-N potential with (two-body) total angular momentum $J \leq 4$. When we include only $^1S_0$, $^3S_1$, and $^3D_1$ components, calculated B_3 is about 7.5 MeV for AV, PARIS, and TRS, and 8.23 MeV for BONN. Thus, the difference of the triton binding energy should come from one or some of these components with small angular momentum.

Among these components the $^3S_1$-$^3D_1$ tensor component is known to be very important for nuclear binding. The Bonn group emphasizes that BONN's tensor component is relatively weak. Actually, the deuteron D-state probability (P_D) for BONN is 4.65%, compared with 6.08%, 5.77%, and 5.92% for AV, PARIS, and TRS, respectively. From these numbers, it might be concluded that a potential with a small P_D yields a large B_3. However, it is not actually true. The de Tourreil-Sprung-B (dTS-B) potential yields 4.25% for P_D and 7.74 MeV for B_3. This means that there must exist a qualitative difference between BONN and other potentials. Concerning this, we want to point out an important difference of the $^3S_1$-$^3D_1$ mixing parameter ($\bar{\varepsilon}_1$) calculated by BONN and other potentials: BONN (Ref. 5) predicts almost zero $\bar{\varepsilon}_1$ (Ref. 15), whereas other potentials[1-4,6] predict positive finite values which are in close proximity to experimental (phase shift analysis) values.[16] This difference may be related to the short range behavior of the tensor component. BONN's tensor component is attractive at long range (the contribution from the one-pion-exchange potential), and repulsive at short range ($r < 1$ fm). On the other hand, the tensor components of other potentials are attractive in all regions. The repulsive effect of BONN's tensor component cancels the effect from the outer attractive tensor component. As a result, the mixing between the S and D states becomes weak.

In general, if the tensor component of a two-nucleon potential is weak, the calculated B_3 is large. This is because we must make the central component more attractive to compensate for the reduction of the attractive tensor contribution, so that the deuteron binding energy may be reproduced correctly. In the triton, the increase of the central component affects its binding energy more than in the deuteron, because the size of the triton is smaller than that of the deuteron.

To check the above consideration, we modify the short range part of a realistic potential so that it simulates BONN, and we calculate $\bar{\varepsilon}_1$ and B_3 with this modified potential. As a starting realistic potential, we choose the RSC potential[1] only because of its simplicity.

The RSC potential is written as a sum of Yukawa functions of various ranges (meson mass) for each partial wave. For example, the $^3S_1$-$^3S_1$ component is written as

TABLE I. Adjusted cutoff parameters (W_c) for given sets of a and c as well as P_D and B_3 calculated using the resulting potentials.

Set	a	c (fm^{-1})	W_c (MeV)	P_D (%)	B_3 (MeV)
I	0		9924.3	6.47	7.03
II	0.5	1.0	8873.29	5.82	7.17
III	1.0	1.0	7890.01	4.94	7.30
IV	1.0	0.8	7226.93	3.73	7.71

follows:

$$V_{ss} = -10.463Y(1,x) + 105.468Y(2,x) - 3187.8Y(4,x) + 9924.3Y(6,x) ,$$

$$Y(n,x) = \exp(-nx)/x ,$$

$$x = 0.7r \quad (r \text{ in fm}) .$$

To reduce the tensor component, we multiply some short range cutoff function by the tensor component of RSC. [For a numerical reason, the cutoff procedure is also performed on the spin-orbit (*LS*) component.] Next, to increase the attractive effect of the central component, we reduce the strength of the short-range repulsive part in V_{ss} (W_c of 9924.3 MeV in the original RSC). We adjusted the parameters so that the correct deuteron binding energy (2.225 MeV) is, in any case, obtained.

The cutoff function that we use is

$$1 - a \exp[-(cr)^4] ,$$

where a and c are parameters which adjust the potential value at the origin and the cutoff range, respectively. For the *LS* potential, we use the same cutoff function except that we set $a = 1$.

Typical examples of numerical calculations are shown in Table I and Figs. 1 and 2.

In Table I we show, for some given sets of a and c, the values of W_c which are determined through the above procedure, as well as P_D and B_3 calculated using the resulting potential. Set I is the original RSC.

In Figs. 1 and 2, we show the radial dependence of resulting potentials and calculated $\bar{\varepsilon}_1$, respectively. In these figures, the solid line denotes set I in Table I; the dotted line, set II; the dashed-dotted line, set III; the dashed line, set IV.

From the table and figures, we can see that the reduction of the short range attraction of the tensor component results in three effects: (1) the central component becomes more attractive, (2) $\bar{\varepsilon}_1$ becomes smaller, and (3) B_3 becomes larger.

Besides $\bar{\varepsilon}_1$, there are two parameters which describe the $^3S_1$-$^3D_1$ coupled state: the *S*- and *D*-wave phase shifts ($\bar{\delta}_S$ and $\bar{\delta}_D$, respectively). The above procedure also changes these phase shifts. The change in $\bar{\delta}_D$ might not affect our discussions. At a glance, the change in $\bar{\delta}_S$ seems strange. For example, at the incident nucleon energy $E_{\text{lab}} = 300$ MeV, $\bar{\delta}_S = 3.8$ (deg) for set I (the original RSC), whereas $\bar{\delta}_S = 0.5$ (deg) for set IV. Thus, set IV shows less attractive behavior than set I, contradicting statement (1). However, we can understand this result as indicating that the $\bar{\delta}_S$ is strongly influenced by the coupling to the *D* state.

The reader might think that B_3 of set IV is still below the experimental value. However, our starting potential (RSC) has a rather strong tensor component, and the $\bar{\varepsilon}_1$ predicted by the RSC are somewhat larger than the phase shift analysis and those values calculated by other potentials such as AV, PARIS, or TRS. Also, the calculated B_3 for the RSC is 7.03 MeV, which is smaller than

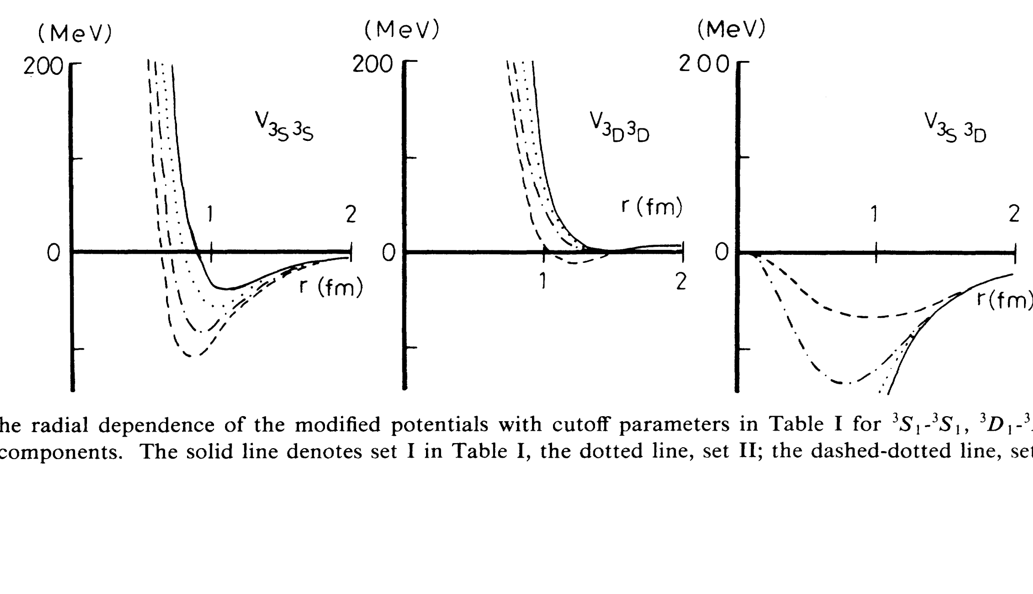

FIG. 1. The radial dependence of the modified potentials with cutoff parameters in Table I for $^3S_1$-$^3S_1$, $^3D_1$-$^3D_1$, and $^3S_1$-$^3D_1$ partial-wave components. The solid line denotes set I in Table I, the dotted line, set II; the dashed-dotted line, set III; the dashed line, set IV.

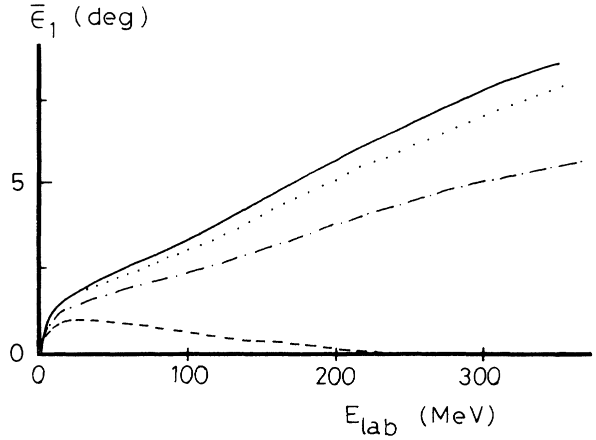

FIG. 2. The $^3S_1$-$^3D_1$ mixing parameters calculated by the modified potentials. For curves, see the caption of Fig. 1.

those given by other potentials. We want to emphasize, however, that the difference between set I and set IV amounts to about 0.7 MeV, which is very close to a value (~ 1 MeV) that should be supplied by a three-nucleon potential in the calculations with usual realistic N-N potentials.[8,9] Therefore, we claim that our "theoretical experiment" stated in this paper is legitimate.

In conclusion, we find that B_3 is very sensitive to $\bar{\varepsilon}_1$ through the short-range radial dependence of the nucleon-nucleon tensor potential: The short range repulsion of a tensor potential makes B_3 large, seemingly approaching the experimental value, but with $\bar{\varepsilon}_1$ too small to be acceptable. BONN is such a case. Before drawing a conclusion as to whether or not a potential predicts the triton binding energy correctly, the two body calculation of the potential must fit $\bar{\varepsilon}_1$.

Recently, a number of works have been published on the N-N interaction based on various quark models. Some of these predicted $\bar{\varepsilon}_1$ values of almost zero.[17,18] In view of our study, these models might have to be improved for their potentials to be realistic.

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