Charge-symmetry breaking in neutron-proton elastic scattering

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The effects of charge-symmetry breaking of nuclear forces can be observed in neutron-proton elastic scattering. We present and apply a formalism used to compute the effects. The major contributions arise from the neutron-proton mass difference in one-pion and one-rho exchanges, from the neutron anomalous magnetic moment in one-photon exchange, and from rho-omega meson mixing. Two-pion exchange and quark effects were found to be very small. We find our results in agreement with the one existing measurement.

I. INTRODUCTION

Charge symmetry implies the invariance of a system under a transformation which reverses the sign of the third component of the isospin for all of its components, (e.g., p$\rightarrow$n and n$\rightarrow$p). The significance of charge-symmetry breaking (CSB) has been discussed previously in a number of reviews.[1-3] Here we are interested in the phenomenon of neutron (n) -proton (p) elastic scattering. This is of special interest because Coulomb forces are absent and hence it is possible to establish the existence of CSB unambiguously. A number of theoretical treatments of CSB in the n-p system have appeared previously.[1,4-7] A recent TRIUMF experiment[8] has found evidence of CSB in this system. In a recent, brief publication[9] we reported on a calculation which agreed with the TRIUMF result. The purpose of the present work is to provide a complete formalism (including relativistic kinematics) that will allow more detailed dynamics to be included in future calculations, to present some new results for CSB potentials, and to extend our earlier calculations to higher energies.

We begin this introduction with a brief discussion of the TRIUMF measurement[8] of the difference in neutron-proton analyzing powers in n-p elastic scattering at a laboratory energy of 477 MeV. Their nonzero result is direct evidence of CSB. The difference in n and p analyzing powers is $\Delta A(\theta)\equiv A_n(\theta)-A_p(\theta)$, where $A_n(\theta)$ and $A_p(\theta)$ are the neutron and proton analyzing powers, respectively. $\Delta A(\theta)$ is expected to be small, i.e., of the order of the fine structure constant. For this reason a zero-crossing measurement was performed, i.e., a measurement of the angle at which the analyzing power goes through zero. The actual measurement was of the difference in laboratory zero-crossing angles for the n and p analyzing powers, which is converted to the difference in analyzing powers at the zero-crossing angle (ΔA) by multiplying by the slope of the analyzing power as a function of the laboratory scattering angle. The difference in zero-crossing angles was $\Delta\theta\equiv\theta_n-\theta_p =0.13°\pm0.06°(\pm0.03°)$ (for the neutron scattering angle), where $A_n(\theta_n)$ and $A_p(\theta_p)$ are zero by definition. In terms of center-of-mass (c.m.) angles $(\theta_n)_{c.m.}\simeq(\theta_p)_{c.m.}\simeq70°$. The quoted result at 477 MeV (determined from neutron laboratory scattering angles) is $\Delta A=[37\pm17$ (stat.)±8 (syst.)] $\times10^{-4}$, which is significant since CSB had not previously been established unambiguously.[1-3,9] An experiment at 350 MeV is planned at TRIUMF, while another at 188 MeV is proceeding at the Indiana University Cyclotron Laboratory.[8] Similar measurements at energies up to 800 MeV are possible at LAMPF.[10]

For completeness we present a few well-known definitions. Charge independence results when the system Hamiltonian H commutes with the total isospin operator $\mathbf{T}$, i.e., $[H,\mathbf{T}]=[H,T^2]=0$. Charge symmetry is a weaker symmetry, which requires only that H by invariant under reflection in the x-y plane is isospin, if the z axis is the T_3 (i.e., the charge) axis. With our conventions the charge operator for a system of protons and neutrons is $Q\equiv e(B/2+T_3)$, where B measures baryon number and $e>0$ is the charge of the proton. Charge symmetry requires $(P_{cs},H)=0$, where $P_{cs}\equiv\exp(i\pi T_2)$. Charge symmetry not only implies equality of the nn and pp forces, but also has consequences for the np system.

Following Henley and Miller,[1] potentials in the NN system can be divided into four classes. Class I potentials are isospin (i.e., charge) independent, while class II potentials are charge dependent but maintain charge symmetry. Both class III and IV potentials violate charge independence and charge symmetry. Class III potentials have the general form

$$V^{\mathrm{III}}=D(\tau_1+\tau_2)_3\ . \tag{1.1}$$

While class III potentials differentiate between nn and pp systems, they clearly can not lead to CSB in the np system since for $T_3=0$ systems $[V^{\mathrm{III}},T^2]\propto[T_3,T^2]=0$ which implies that no isospin mixing is possible. Only class IV potentials can contribute to CSB (and hence to ΔA) in n-p elastic scattering and they have the general form

$$V^{\mathrm{IV}}=E(\tau_1-\tau_2)_3+F(\tau_1\times\tau_2)_3 \ , \tag{1.2}$$

where $[V^{\mathrm{IV}},T^2]\neq 0$. Assuming both parity conservation and time-reversal invariance, the simplest class IV potentials are

$$V^{\mathrm{IV}}=(\tau_1-\tau_2)_3(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{L}v(r) \ , \tag{1.3a}$$

$$V^{\mathrm{IV}}=(\tau_1\times\tau_2)_3(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\mathbf{L}w(r) \ , \tag{1.3b}$$

where $\mathbf{L}$ is the orbital angular momentum operator in the center of mass frame and $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ is the internucleon separation.

CSB in n-p elastic scattering implies mixing of isospin $T=0$ and $T=1$ states. Isospin mixing requires that there must also be spin singlet-triplet mixing, i.e., of $S=0$ and $S=1$ states, since parity conservation implies that L remains unchanged. Thus only states with $J=L$ can contribute to CSB in n-p elastic scattering and hence to ΔA.

Our aim here is to provide a complete formalism to allow computation of CSB effects. Earlier work in Refs. 4 and 7 included effects of strong distortion but did not include enough partial waves to handle the electromagnetic (em) contribution at all angles. Conversely, sufficient partial waves were included in Refs. 5 and 6, but no account was taken of the effects of nuclear distortion on the n and p radial wave functions. Here we include both effects and try to present an account complete enough to allow more extensive calculations. In addition, we attempt to understand the nonzero result for ΔA in terms of em interactions, one-boson exchange potentials (OBEP), and the two-pion exchange potential (TPEP). Careful attention is paid to relativistic kinematics. Quark effects will also be considered. Detailed comparisons between different theories are made.

A brief report on some of the results to be presented here has already been made.[9] We shall see that the dominant contribution to ΔA arises from the n-p mass difference in the one-pion exchange potential (OPEP). Although the conventional OPEP is well known, in this process we are isolating a new spin-transition matrix element that has never been measured before. In this sense we are making a novel test of the meson exchange theory of nuclear forces. The other contribution from a long-range force arises from the effect of the neutron anomalous magnetic moment on the motion of the proton. Other shorter-ranged effects are associated with rho exchange, rho-omega mixing, two-pion exchange, and quark interactions, the last two of these giving very small contributions. The diagrams contributing to CSB are shown in Fig. 1 (except quark effects). The total of these contributions to ΔA give a result in agreement with experiment.

The remainder of this paper is organized as follows.

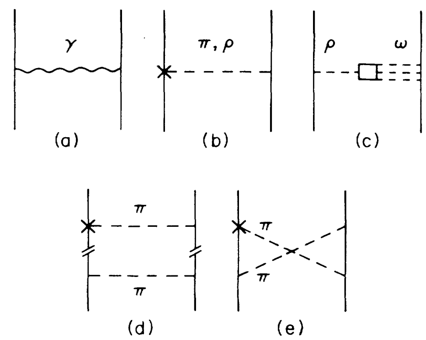

FIG. 1. The charge-symmetry breaking processes considered in this work. Shown are (a) one-photon, (b) one-pion or one-rho, (c) mixed rho-omega, and (d) uncrossed and (e) crossed two-pion exchanges. No diagram depicting quark effects has been included. The crosses indicate the CSB vertex function arising from the neutron-proton mass difference. The cross hatching refers to the usual subtraction procedure.

In Sec. II we introduce the formalism (with relativistic kinematics) necessary to describe the CSB amplitudes, while the various CSB contributions and their origins are discussed in Sec. III. The details of our calculations and the numerical results are presented in Sec. IV, where our previous calculations[9] are extended to higher energies.

II. FORMALISM

Assuming only Lorentz invariance, parity conservation, and time-reversal invariance the scattering matrix for the elastic scattering of two spin $\frac{1}{2}$ particles can always be specified in terms of six invariant amplitudes.[6,11] These amplitudes are complex functions of the total center-of-mass (c.m.) energy E_T and the c.m. scattering angle θ. Using invariant amplitudes a, b, c, d, e, and f we can write

$$\begin{aligned}\hat{M}=\tfrac{1}{2}[&(a+b)+(a-b)(\boldsymbol{\sigma}_1\cdot\mathbf{n})(\boldsymbol{\sigma}_2\cdot\mathbf{n})\\&+(c+d)(\boldsymbol{\sigma}_1\cdot\mathbf{m})(\boldsymbol{\sigma}_2\cdot\mathbf{m})+(c-d)(\boldsymbol{\sigma}_1\cdot\boldsymbol{l})(\boldsymbol{\sigma}_2\cdot\boldsymbol{l})\\&+e(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{n}+f(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{n}] \ .\end{aligned} \tag{2.1}$$

where $\mathbf{k}_i$ and $\mathbf{k}_f$ are the initial and final c.m. momenta for the scattered particle (particle 1) and where

$$\boldsymbol{l}\equiv\frac{\mathbf{k}_f+\mathbf{k}_i}{|\mathbf{k}_f+\mathbf{k}_i|}, \quad \mathbf{m}\equiv\frac{\mathbf{k}_f-\mathbf{k}_i}{|\mathbf{k}_f-\mathbf{k}_i|}, \quad \mathbf{n}\equiv\frac{\mathbf{k}_i\times\mathbf{k}_f}{|\mathbf{k}_i\times\mathbf{k}_f|} \ . \tag{2.2}$$

In terms of the scattering matrix spin operator $\hat{M}$, the differential cross section for unpolarized particles in the c.m. system is

$$\begin{aligned}\left[\frac{d\sigma}{d\Omega}\right]_{\mathrm{unpol}}&\equiv\sigma_0=\tfrac{1}{4}\mathrm{Tr}(\hat{M}\hat{M}^{\dagger})\\&=\tfrac{1}{2}(|a|^2+|b|^2+|c|^2\\&\quad+|d|^2+|e|^2+|f|^2) \ .\end{aligned} \tag{2.3}$$

The most general form for the c.m. differential cross section in a polarization experiment is[11]

$$\left[\frac{d\sigma}{d\Omega}\right]_{\rm pol} \equiv \sigma_0 X_{pqik}$$
$$= \tfrac{1}{4}\mathrm{Tr}[(\boldsymbol{\sigma}_1\cdot\mathbf{p})(\boldsymbol{\sigma}_2\cdot\mathbf{q})\hat{M}(\boldsymbol{\sigma}_1\cdot\mathbf{i})(\boldsymbol{\sigma}_2\cdot\mathbf{k})\hat{M}^{\dagger}] , \tag{2.4}$$

where the four arbitrary unit vectors **p**, **q**, **i**, and **k** specify the polarization directions of the scattered, recoil, beam, and target particles, respectively.

The experiment of interest to us here[8] is that of a neutron beam (particle 1) impinging on a proton (i.e., hydrogen) target (particle 2), where either the beam or target are initially polarized perpendicular to the scattering plane [i.e., with polarization in the direction **n** of Eq. (2.2)]. The ratio of the resulting cross sections to the unpolarized cross section (σ_0) gives the neutron and proton analyzing powers $A_{\rm n}(\theta)$ and $A_{\rm p}(\theta)$, where[11]

$$A_{\rm n}(\theta) \equiv X_{00n0} = \tfrac{1}{4}\mathrm{Tr}[\hat{M}(\boldsymbol{\sigma}_1\cdot\mathbf{n})\hat{M}^{\dagger}] = \mathrm{Re}(a^*e + b^*f)/\sigma_0 , \tag{2.5}$$
$$A_{\rm p}(\theta) \equiv X_{000n} = \tfrac{1}{4}\mathrm{Tr}[\hat{M}(\boldsymbol{\sigma}_2\cdot\mathbf{n})\hat{M}^{\dagger}] = \mathrm{Re}(a^*e - b^*f)/\sigma_0 .$$

Hence we have for the difference in analyzing powers

$$\Delta A(\theta) \equiv A_{\rm n}(\theta) - A_{\rm p}(\theta) = 2\,\mathrm{Re}(b^*f)/\sigma_0 . \tag{2.6}$$

CSB can be observed in n-p elastic scattering if it is accompanied by spin singlet-triplet mixing, which can only occur if $f \neq 0$ in Eq. (2.1).

We now wish to relate the class IV CSB potentials of Eq. (1.3) to $f(k,\theta)$ in Eq. (2.1) ($k \equiv |\mathbf{k}_i| = |\mathbf{k}_f|$). We will maintain relativistic kinematics throughout this work. It is possible to generalize the bar phase shift L-S representation of N-N scattering[12] to include a new parameter $\bar{\gamma}_J$ which is the mixing angle of the spin singlet-triplet transition.[5,6,11] Using the definition

$$S(J) \equiv 1 + 2iT(J) , \tag{2.7}$$

then the matrix element $\langle L'S' | T(J) | LS \rangle$ for the singlet-triplet transition is

$$\langle J1 | T(J) | J0 \rangle = -\tfrac{1}{2}\sin(2\bar{\gamma}_J)\exp(i\bar{\delta}_J + i\bar{\delta}_{JJ}) , \tag{2.8}$$

where $\bar{\delta}_J$ and $\bar{\delta}_{JJ}$ are the singlet and uncoupled triplet bar phase shifts, respectively. The bar phase shift L-S representation of the scattering matrix can be related to the six-amplitude representation of Eq. (2.1) and in particular we have[6]

$$f(k,\theta) = \frac{i}{2k}\sum_{J=1}^{\infty}(2J+1)\sin(2\bar{\gamma}_J) \times \exp(i\bar{\delta}_J + i\bar{\delta}_{JJ})d^J_{10}(\theta) , \tag{2.9}$$

where the d^J_{10} are Wigner functions. Since the CSB potentials are very small it is a good approximation to treat the potentials $V^{\rm IV}$ of Eq. (1.3) in the distorted wave Born approximation (DWBA). This then allows a calculation of the singlet-triplet transition amplitude $f(k,\theta)(\boldsymbol{\sigma}_{\rm n} - \boldsymbol{\sigma}_{\rm p})\cdot\mathbf{n}$ in terms of the $V^{\rm IV}$ potentials. Using the results for $f(k,\theta)$ and approximating $\sin(2\bar{\gamma}_J) \simeq 2\bar{\gamma}_J$ (since $\bar{\gamma}_J$ is very small), we find

$$\bar{\gamma}_J = -2E_T k\sqrt{J(J+1)}\int_0^{\infty} dr\, r^2 R_J(r) g(r) R_{JJ}(r) , \tag{2.10}$$

where $E_T \equiv$(total) c.m. energy $\equiv 2E$ with E the energy per nucleon in the c.m. frame and where

$$g(r) \equiv v(r) \text{ and } (-1)^J w(r) \tag{2.11}$$

for Eqs. (1.3a) and (1.3b), respectively. The distorting effects of the strong interaction are included through the radial wave functions $R_J(r)$ and $R_{JJ}(r)$, both of which are normalized such that

$$R(r) \underset{r\to\infty}{\longrightarrow} \sin(kr - \tfrac{1}{2}J\pi + \bar{\delta})/kr , \tag{2.12}$$

where $\bar{\delta} \equiv \bar{\delta}_J$ and $\bar{\delta}_{JJ}$ for $R_J(r)$ and $R_{JJ}(r)$, respectively. In our calculations $R(r)$ are solutions of the Schrödinger equation for the Reid soft-core potential,[13] which adequately describes the relevant ($J = L$) experimental phase shifts.[14] This point will be discussed in more detail in Sec. IV. In Appendix A we give some conventions and describe how to relate invariant amplitudes[15] of OBEP's to CSB potentials.

III. CLASS IV CONTRIBUTIONS

In this section we discuss the various class IV contributions, beginning with the two of longest range, i.e., those due to one-photon and one-pion exchange. The first of these requires special treatment since Eq. (2.9) is a divergent sum due to the infinite range of the em interaction. The shorter range contributions due to one-rho exchange, rho-omega mixing, two-pion exchange, and quark effects are then treated.

Before proceeding to the details of the class IV CSB potentials, we include a few general remarks. Note that, while the n-p mass difference does give rise to class IV potentials (see Sec. III), no such forces arise due to the appearance of the n and p masses in kinematic factors [e.g., Eq. (A2)] and so to an excellent approximation we can use the average nucleon mass (M) in place of $M_{\rm n}$ and $M_{\rm p}$ in such factors. Similarly, in the c.m. frame we replace the n and p energies, $E_{\rm n}$ and $E_{\rm p}$, by $E \equiv \frac{1}{2}E_T$ in any factors not contributing to class IV CSB potentials.

A. One-photon exchange

The class IV part of the em spin-orbit force arises from one-photon exchange between the proton charge and the anomalous magnetic moment of the neutron. The photon-nucleon interaction Hamiltonian density, with the anomalous magnetic moments included, is given by

$$\mathcal{H}_{\gamma\mathrm{NN}}=e\bar{\psi}\left[(F_1^S+F_1^V\tau_3)\gamma^\mu A_\mu -(F_2^S+F_2^V\tau_3)\frac{1}{2M}\sigma^{\mu\nu}\partial_\nu A_\mu\right]\psi\ , \quad (3.1)$$

where F_i^S and F_i^V are isoscalar and isovector nucleon electromagnetic form factors, respectively, and $e>0$ is the absolute value of the electronic charge. If we were to neglect the $\mathbf{q}^2$ dependence of the form factors and take them to have their $\mathbf{q}=0$ values, where

$$\mathbf{q}\equiv\mathbf{k}_f-\mathbf{k}_i\ , \quad (3.2)$$

then we would find V^{IV} has the form of Eq. (1.3a) with

$$v_\gamma(r)=-\left[\frac{e^2}{4\pi}\right]\frac{\kappa_\mathrm{n}}{4M^2}\left[\frac{M}{E}\right]\frac{1}{r}\frac{d}{dr}\left[\frac{1}{r}\right] \quad (3.3)$$

since the neutron anomalous magnetic moment (κ_n) is given by

$$\kappa_\mathrm{n}=-1.91\equiv 2[F_1^V(0)F_2^S(0)-F_1^S(0)F_2^V(0)]\ . \quad (3.4)$$

The $\mathbf{q}^2$ dependence of the form factors can be included by replacing κ_n in the momentum-space form of Eq. (3.3) with

$$2[F_1^V(\mathbf{q}^2)F_2^S(\mathbf{q}^2)-F_1^S(\mathbf{q}^2)F_2^V(\mathbf{q}^2)]\ .$$

For our calculations a simpler form factor was chosen, which consisted of the momentum-space replacement

$$\frac{1}{\mathbf{q}^2}\to\frac{1}{\mathbf{q}^2}\left[\frac{\Lambda^2}{\Lambda^2+\mathbf{q}^2}\right]\ , \quad (3.5)$$

with $\Lambda=412$ MeV fixed to reproduce the proton rms charge radius. We found the results insensitive to changes in the form factor. Equation (3.3) contains a factor (M/E), which is a relativistic correction.

We first address the problem of how to sum Eq. (2.9) for the one-photon exchange contribution, since the infinite range of the em interaction causes large J contributions to be important. In practical calculations we wish to calculate $\bar{\gamma}_J^{\mathrm{em}}$ using Eq. (2.10) for only a finite number of J's. For large J the nuclear distortion effects can be neglected and $\bar{\gamma}_J^{\mathrm{em}}$ can be well approximated by replacing $R_J(r)$ and $R_{JJ}(r)$ with the spherical Bessel function $j_J(kr)$. We then find the results[5,6]

$$\bar{\gamma}_J^{\mathrm{em}\text{-}B}=\left[\frac{e^2}{4\pi}\right]\frac{|\kappa_\mathrm{n}|k}{2M[J(J+1)]^{1/2}} \quad (3.6)$$

and from Eq. (2.9) using $\sin(2\bar{\gamma}_J)\simeq 2\bar{\gamma}_J$ and $\bar{\delta}_J=\bar{\delta}_{JJ}=0$,

$$f^{\mathrm{em}\text{-}B}(k,\theta)=-i\left[\frac{e^2}{4\pi}\right]\frac{|\kappa_\mathrm{n}|\sin\theta}{2M(1-\cos\theta)}\ , \quad (3.7)$$

where the superscript B implies that these results are in first Born approximation only (i.e., no distortion). If $\bar{\gamma}_J^N$ denotes the sum of *all* CSB contributions *except* $\bar{\gamma}_J^{\mathrm{em}}$ then we can subtract some $\bar{\gamma}_J^{\mathrm{em}}$ terms and add $f^{\mathrm{em}\text{-}B}$ to Eq. (2.9) to obtain[5]

$$f(k,\theta)=\frac{i}{2k}\sum_{J=1}^{J_{\max}}(2J+1)[\sin 2(\bar{\gamma}_J^N+\bar{\gamma}_J^{\mathrm{em}})\exp(i\bar{\delta}_J+i\bar{\delta}_{JJ}) -2\bar{\gamma}_J^{\mathrm{em}\text{-}B}]d_{10}^J(\theta)+f^{\mathrm{em}\text{-}B}(k,\theta)\ , \quad (3.8)$$

where $J_{\max}$ is the largest J included in the calculations. We typically include all partial waves $J=1,\ldots,5$. Having established this result we can now turn to the various strong interaction CSB effects which constitute $\bar{\gamma}_J^N$.

B. One-pion exchange

The neutron-proton mass difference leads to a class IV CSB potential when it is explicitly included in the one-pion exchange term.[4,5] To calculate V^{IV} for this case it is sufficient to consider pseudoscalar coupling for the pion-nucleon vertex, since in using Eqs. (A2)–(A4) of Appendix A to obtain V^{IV} the nucleons are always on mass shell and so the results are identical to those obtained with pseudovector coupling. The pion-nucleon interaction Hamiltonian density we use is then

$$\mathcal{H}_{\pi\mathrm{NN}}=-g_\pi\bar{\psi}i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\phi}\psi\ , \quad (3.9)$$

where $g_\pi\equiv\sqrt{4\pi}2Mf_{\pi\mathrm{NN}}/m_\pi$ is the pseudoscalar coupling constant and where $(g_\pi^2/4\pi)=14.4$ and m_π is the π^+ mass.

We keep the first relativistic correction and the first order term in the n-p mass difference and find a CSB potential with the form of Eq. (1.3b) where

$$w_\pi(r)=-\left[\frac{g_\pi^2}{4\pi}\right]\frac{1}{2M^2}\left[\frac{M}{E}\right]^3 \times\left[\frac{M_\mathrm{n}-M_\mathrm{p}}{2M}\right]\frac{1}{r}\frac{d}{dr}\left[\frac{1}{r}e^{-m_\pi r}\right] \quad (3.10)$$

where $(M/E)^3$ is a relativistic correction. No form factor is shown, but in calculations a form factor F_π was applied to the vertices,[16,17] where

$$F_\pi(\mathbf{q}^2)=\left[\frac{\Lambda_\pi^2-m_\pi^2}{\Lambda_\pi^2-\mathbf{q}^2}\right] \quad (3.11)$$

and where $\Lambda_\pi=1300$ MeV.

C. One-rho exchange

The n-p mass difference also leads to a class IV CSB potential in the one-rho exchange term. The rho-nucleon interaction is given by[16–18]

$$\mathcal{H}_{\rho\mathrm{NN}}=-\bar{\psi}\left[g_\rho\gamma^\mu\boldsymbol{\rho}_\mu+\frac{f_\rho}{2M}\sigma^{\mu\nu}\partial_\mu\boldsymbol{\rho}_\nu\right]\cdot\boldsymbol{\tau}\psi\ , \quad (3.12)$$

where g_ρ and f_ρ are the vector and tensor rho-nucleon coupling constants, respectively. The coupling constants that we use are $g_\rho^2/4\pi=0.55\pm0.06$ and $f_\rho/g_\rho=6.1\pm0.6$, which are typical values extracted from low energy analyses.[18] The uncertainty in the coupling constants, particularly f_ρ/g_ρ, leads to considerable leeway in the prediction of this CSB contribution.

We have found contributions of both types (1.3a) and (1.3b). The contribution of type (1.3a) arises from the vector-vector coupling and has the form

$$v_\rho(r) = -\left[\frac{g_\rho^2}{4\pi}\right]\frac{1}{4M^2}\left[\frac{M}{E}\right]^3 \times\left[\frac{M_{\rm n}-M_{\rm p}}{2M}\right]\frac{1}{r}\frac{d}{dr}\left[\frac{1}{r}e^{-m_\rho r}\right], \quad (3.13)$$

where $m_\rho = 770\pm 3$ MeV is the rho-meson mass. The Eq. (1.3b) contribution has vector-vector (g_ρ^2), vector-tensor ($f_\rho g_\rho$), and tensor-tensor (f_ρ^2) pieces and is given by

$$w_\rho(r) = -\left[\frac{g_\rho^2}{4\pi}\right]\left[1+\frac{E+3M}{2M}\frac{f_\rho}{g_\rho}+\frac{E+M}{2M}\left[\frac{f_\rho}{g_\rho}\right]^2\right] \times\frac{1}{2M^2}\left[\frac{M}{E}\right]^3\left[\frac{M_{\rm n}-M_{\rm p}}{2M}\right]\frac{1}{r}\frac{d}{dr}\left[\frac{1}{r}e^{-m_\rho r}\right]. \quad (3.14)$$

Relativistic corrections have been included (and vanish if we take $E\to M$). No form factor is shown, but again in calculations a form factor was applied to the vertices,[16,17] where

$$F_\rho(\mathbf{q}^2) = \left[\frac{\Lambda_\rho^2-m_\rho^2}{\Lambda_\rho^2+\mathbf{q}^2}\right], \quad (3.15)$$

and where $\Lambda_\rho = 1400$ MeV. These results [Eqs. (3.13) and (3.14)] differ from those reported by Gersten.[5] From the values of the coupling constants it is clear that this contribution is dominated by the tensor-tensor piece, i.e., the $(f_\rho/g_\rho)^2$ term in Eq. (3.14).

D. Mixed rho-omega exchange

The ρ^0-ω mixing part of the OBEP also has a class IV CSB contribution, the omega-nucleon interaction is the isoscalar equivalent to the isovector rho-nucleon interaction, i.e.,

$$\mathcal{H}_{\omega\rm NN} = -\bar\psi\left[g_\omega\gamma^\mu\omega_\mu+\frac{f_\omega}{2M}\sigma^{\mu\nu}\partial_\mu\omega_\nu\right]\psi\,. \quad (3.16)$$

We use the coupling constants[19] $g_\omega^2/4\pi = 8.1\pm 1.5$ and $f_\omega/g_\omega = 0.14\pm 0.2$. These coupling constants are not at all well known.[18] The form factor for the omega-nucleon vertex[16,17] is F_ω, which is also given by Eq. (3.15), but with $m_\omega \equiv$ omega meson mass $=783$ MeV and $\Lambda_\omega = 1500$ MeV replacing m_ρ and Λ_ρ.

The resulting class IV CSB is of the Eq. (1.3a) type with[1,4]

$$v_{\rho\omega}(r) = -\frac{f_\rho g_\omega}{4\pi}\frac{1}{2M^2}\frac{\langle\omega|H|\rho^0\rangle}{m_\omega^2-m_\rho^2} \times\frac{1}{r}\frac{d}{dr}\left[\frac{e^{-m_\rho r}-e^{-m_\omega r}}{r}\right], \quad (3.17)$$

where $\langle\omega|H|\rho^0\rangle$ is the ρ^0-ω mixing matrix element. Again we have not explicitly shown the form factors in Eq. (3.17). No relativistic correction has been included in the above, so for consistency we put $E_T\to 2M$ in Eq. (2.10) for the ρ-ω contribution. The mixing matrix element $\langle\omega|H|\rho^0\rangle$ has been extracted from data by a number of groups[1,20] with predictions ranging from -3400 to -6000 MeV^2. We adopt the former, more conservative value here, i.e., $\langle\omega|H|\rho^0\rangle = 3400$ MeV^2. The uncertainty in this effect is *at least* 30%. Note that π^0-η mixing does not give rise to a class IV force.

E. Two-pion exchange

The class IV CSB contribution from the two-pion exchange potential (TPEP) has its origins in the n-p mass difference. The size of the effect is smaller than might otherwise be expected since the crossed and uncrossed two-pion exchange diagrams tend to cancel.[21] There are at least two ways of estimating the effect. One approach consists of evaluating directly the CSB contribution from both the crossed and uncrossed pion diagrams using the approach of Partovi and Lomon (PL).[22] An alternative approach is to study the exchange of the delta, which is a scalar-isovector meson which plays the phenomenological role of two-pion exchange in OBEP model calculations.[17] These approaches give similar results.[9]

We begin with an evaluation of the two-pion exchange graphs of Figs. 1(d) and (e). The charge symmetry breaking can arise from the n-p mass difference, or in principle from possible charge dependence of the coupling constants.

Allowing the magnitude of the πNN coupling to vary with the nucleon charge does not change the form of the vertex operator, and thus produces no class IV force. [For example, a γ_5 acting at each vertex of Figs. 1(d) and (e) gives no charge-asymmetric $(\boldsymbol\sigma_1-\boldsymbol\sigma_2)$-type term.]

Thus we consider this influence of the n-p mass difference. The masses appear in the evaluation of the initial and final spinors and in the intermediate state Feynman propagators. We shall show below that the latter effect produces no class IV force. Thus we are left with the mass dependence of the n and p spinors.

The approach of PL is employed to obtain this first estimate of the influence of two-pion exchanges. In PL the γ_5 form of the πNN vertex function is used. The problem involving anomalously large s-wave pion-nucleon scattering does not arise here, since we shall be examining isovector terms.

The evaluation of Figs. 1(d) and (e) proceeds by applying the procedure of PL with one modification. The spinors representing the initial and final states are charge dependent:

$$u(p,M)=u(p,\overline{M})-\frac{\tau_3}{2}(M_n-M_p)\frac{\partial u}{\partial M}(p,M)\bigg|_{M=\overline{M}} .$$

Thus we need only state the results. The class IV part of the TPEP, $V_{\rm TPE}^{\rm IV}$, has contributions of both types given in Eq. (1.3). Define the radial function of the term of type (1.3a) as $v_{\rm TPE}(r)$ and that of type (1.3b) as $w_{\rm TPE}(r)$. Then

$$v_{\rm TPE}(r)=\frac{1}{8\pi}g_\pi^4\frac{(M_n-M_p)}{M^3}[4I_1(r)+5I_2(r)-10I_5(r)+4MrI_7(r)] , \quad (3.18a)$$

$$w_{\rm TPE}(r)=\frac{1}{8\pi}g_\pi^4\frac{(M_n-M_p)}{M^3}\{4I_1(r)+4[I_4(r)-I_3(r)]\} , \quad (3.18b)$$

where $I_n(r)$ with $n=1,\dots,7$ are integrals defined by

$$\begin{aligned}
I_1(r)&=\int_0^1\int_0^1 d\zeta\,\zeta\,d\beta\,D[1+rE(\zeta^2)]e^{-E(\zeta^2)r} ,\\
I_2(r)&=\int_0^1\int_0^1 d\zeta\,\zeta^{-1/2}d\beta\,D[1+rE(\zeta)]e^{-E(\zeta)r}(1-2\zeta) ,\\
I_3(r)&=\int_0^1\int_0^1 d\zeta\,\zeta\,d\beta\int_1^\infty d\xi\,D[1+rE(\zeta^2\xi^2)]e^{-E(\zeta^2\xi^2)r} ,\\
I_4(r)&=\int_0^1\int_0^1 d\zeta\,d\beta\,\zeta^{1/2}\int_1^\infty d\xi\,D[1+rE(\zeta\xi^2)]\times e^{-E(\zeta\xi^2)r} ,\\
I_5(r)&=\int_0^1\int_0^1 d\zeta\,d\beta\,\zeta^{-1/2}\int_1^\infty\frac{d\xi}{\xi^2}D[1+rE(\zeta\xi^2)]\times e^{-E(\zeta\xi^2)r}(1-\zeta) ,\\
I_7(r)&=\int_0^1\int_0^1 d\zeta\,d\beta\frac{\zeta^3D^2}{(1-\zeta)}[1+rE(\zeta^2)]e^{-rE(\zeta^2)} ,
\end{aligned} \quad (3.19)$$

where $I_6(r)$ is not needed here and where

$$D\equiv D(\zeta,\beta)=(1-\zeta)^{-1}(1-\beta^2)^{-1} , \quad (3.20)$$

$$E(\alpha)=2D^{1/2}\left[\mu^2+\frac{\alpha m^2}{1-\zeta}\right]^{-1/2} . \quad (3.21)$$

These have been evaluated and the results presented in Ref. 9. Since the effects are small we do not include TPEP terms in Sec. IV.

Next consider the influence of CSB terms arising from the charge dependence of the intermediate propagators. We claim that the mass dependence of the propagator serves to moderate the strength of the graph, but not to modify the spin dependence. This may be verified by considering Figs. 1(d) and (e) directly. The relevant amplitude for Fig. 1(d), $\mathcal{M}_d$, can be written as

$$\mathcal{M}_d\sim\tau_1\cdot\tau_2(\tau_1-\tau_2)_3(\not k_1-\not k_2)\tau_1\cdot\tau_2 , \quad (3.22)$$

where k is the 4-momentum of the virtual meson and $\not k=k^\mu\gamma_\mu$. Equation (3.22) is to be evaluated between spinors $\bar u(\mathbf{p}')$ and $u(\mathbf{p}')$ for nucleon 1 and $\bar u(-\mathbf{p}')$ and $u(-\mathbf{p})$ for nucleon 2. It is straightforward to show that $\bar u(\mathbf{p}')\gamma^\mu u(\mathbf{p})-\bar u(-\mathbf{p}')\gamma^\mu u(-\mathbf{p})=0$, so there is no class IV contribution from Fig. 1(d). Similarly the term of Fig. 1(e) behaves as

$$\mathcal{M}_e\sim\sum_{n,m}(\tau_1)_n(\tau_2)_m(\tau_1-\tau_2)_3(\tau_1)_n(\tau_2)_m(\not k_1-\not k_2) \quad (3.23)$$

and has no class IV piece. Thus the charge dependence arising from the dependence on intermediate nucleon masses is not relevant here.

It should be noted that the influence of intermediate Δ's is not included here. These are absent in the treatment of PL, which nevertheless gives a good accounting of the relevant $\tau_1\cdot\tau_2$ terms of the N-N force. A more detailed evaluation using a more modern force would include the influence of deltas. However, the small nature of our TPEP results[9]—combined with the knowledge that the $\tau_1\cdot\tau_2$ terms of PL are reasonably well treated—leads one to suspect that any TPEP model constrained by the phenomenology of the $\tau_1\cdot\tau_2$ term would have similar results.

F. Short-range quark effects

The one gluon exchange potential (OGEP) between quarks acquires charge dependence via the mass differences between quarks of different flavor. Here we are concerned only with the possible influence of the up-down (u-d) mass difference.[23] A straightforward evaluation of the OGEP gives a charge-dependent term $H_{\rm OGE}^{\rm IV}$ of the form

$$H_{\rm OGE}^{\rm IV}=\frac{\alpha_s}{2\overline{m}^2r^3}\frac{\Delta m}{\overline{m}}\lambda_1\cdot\lambda_2\,\mathbf{\Sigma}\cdot(\mathbf{r}\times\mathbf{p})t_3 , \quad (3.24)$$

where $\overline{m}=(m_u+m_d)/2$, $\Delta m=(m_u-m_d)/2$, $\mathbf{\Sigma}=(\mathbf{s}_1-\mathbf{s}_2)$, and $t_3=(\tau_1-\tau_2)_3$. Note that $H_{\rm OGE}^{\rm IV}$ is not the entire one gluon exchange term, but is only the term that yields a class IV force at the nucleon level. The term of (3.24) may be compared with the conventional hyperfine interaction, $H_{\rm OGE}^{\rm hyp}$, with

$$H_{\rm OGE}^{\rm hyp}=\frac{8\pi}{3}\alpha_s\lambda_1\cdot\lambda_2\frac{\mathbf{s}_1\cdot\mathbf{s}_2}{m_1m_2}\delta(\mathbf{r}) . \quad (3.25)$$

There is an additional relevant term which arises from the well-known Thomas precession effect. Suppose the confining potential is of the form $V_{\rm conf}$:

$$V_{\rm conf}=\frac{\beta r^2}{2}\lambda_1\cdot\lambda_2 . \quad (3.26)$$

A spin-orbit force, which depends on the quark mass, arises from the acceleration induced by $V_{\rm conf}$. The term leading to a class IV NN interaction is

$$H_{\rm th}^{\rm IV}=\frac{\beta}{4\overline{m}^2}\frac{\Delta m}{\overline{m}}\lambda_1\cdot\lambda_2\mathbf{\Sigma}\cdot(\mathbf{r}\times\mathbf{p})t_3 . \quad (3.27)$$

Typical values of the various parameters may be obtained from the literature. In particular m_d-m_u is constrained by the n-p mass difference.[24]

The influence of the quark interactions terms (3.24) and (3.27) can be evaluated in models which incorporate quark effects in computations of the short range N-N force. Here we employ a prescription in the literature[25] in which one replaces the short-distance NN wave function by a six quark state. The six quark states are given by

$$|\,{}^3P_{M_J}\rangle_{6q}=\eta_1\sum_{i=1}^{6}(\mathbf{r}_i\times\mathbf{s}_i)\,|\,{}^1S_0\rangle_{6q}\ ,$$

$$|\,{}^1P_0\rangle_{6q}=\eta_0\sum_{i=1}^{6}\mathbf{r}_i\cdot\mathbf{s}_i\,|\,{}^3S_1\rangle_{6q}\ ,$$

where $|\,{}^1S_0\rangle_{6q}$ and $|\,{}^3S_1\rangle_{6q}$ consist of six quarks confined in a single harmonic oscillator potential. The spatial wave function is completely symmetric: [6]. The constants η_0 and η_1 are introduced so that the states are normalized to unity. The components $|\,{}^3P_{M_J}\rangle_{6q}$ and $|\,{}^1P_0\rangle_{6q}$ appear with amplitudes constrained[25] by conventional NN wave functions evaluated at longer range. The resulting value of γ_1 is at most $\frac{1}{15}$ of the dominant OPEP term. A more elaborate computation[26] using the resonating group method gives results that are even smaller.

G. Additional effects

To complete this section we briefly address other CSB effects. The simultaneous exchange of a photon and a pion will lead to a CSB contribution. While it is not expected to be important, since it should be of the same order as the CSB part of TPEP, it should be estimated in the future. The charge dependence of the exchange of a pion and a rho (and other meson pairs) is of very short range and hence is expected to be even smaller. The influence of pion production is also expected to be unimportant since the imaginary parts of the phase shifts are relatively small at our energies.

There is no class IV CSB from charge dependence of meson-nucleon coupling constants (at least in the OPEP), since one needs different operators on the two nucleon lines to get such a potential (see, e.g., Fig. 1). There is no CSB contribution from η-π^0 mixing since only a class III potential results. Similarly, the one-photon exchange coupling to the nucleon magnetic moments on both nucleons gives rise to no class IV CSB.

IV. NUMERICAL RESULTS

Now that the relevant class IV CSB potentials have been obtained, we calculate the various contributions to the spin singlet-triplet mixing angles $\bar{\gamma}_J$. These are obtained using Eq. (2.10) for $1\le J\le(J_{\max}=5)$ with the appropriate form factors included.

In our calculations of $\bar{\gamma}_J$ the distorted radial wave functions R [see Eq. (2.10)], are solutions of the Schrödinger equation for the Reid soft core potential. For $J\ge 3$ it is a good approximation to use plane waves in Eq. (2.10) for $E\lesssim 800$ MeV. For the $J=L=1,2$ partial waves needed, the potential yields real phase shifts that agree with those of Ref. 14 to typically 10–15 % up to 477 MeV. The agreement is still reasonable at 600 MeV, but less so at 800 MeV. Inelastic effects have been neglected here (see below). Some uncertainty is introduced by making a particular choice of distorting potential. There have been some recent calculations of $\bar{\gamma}_J$ and $\Delta A(\theta)$ by Ge and Svenne,[7] who used the Paris potential.[27] The sensitivity to the potential of the long-range pion and photon terms should be considerably smaller than that for the short-range terms. A detailed study of sensitivity to the distorting potential has yet to be carried out.

Once the mixing angles ($\bar{\gamma}_J$) have been determined it remains to calculate $\Delta A(\theta)$ and hence ΔA at the energies of interest. To calculate $\Delta A(\theta)$ we used a program provided by Knutson after making modifications to allow the inclusion of em effects for all J according to Eq. (3.8). CSB contributions with the form of Eq. (1.3a) are generally weaker than those of Eq. (1.3b), since the strong phase shifts are repulsive for $J=1$ and attractive for $J=2$. As a result there is cancellation for the former potentials and addition for the latter [due to the additional factor of $(-1)^J$ in $\bar{\gamma}_J$]. This is part of the reason

TABLE I. The spin singlet-triplet mixing angle parameters $\bar{\gamma}_J$ are given (in deg) for $L=J=1,\ldots,5$ at 477 MeV. The π, γ, ρ, and ρ-ω contributions and their total are given. In each case the upper results were calculated with form factors and nuclear distortion as described in the main text. These numbers were used to calculate ΔA. The lower numbers are exact analytic results calculated according to Appendix B and include no form factors and no distortion.

	π	γ	ρ	ρ-ω	Total
$\bar{\gamma}_1$	3.72×10^{-2}	3.27×10^{-2}	5.97×10^{-3}	2.74×10^{-2}	1.03×10^{-1}
	1.41×10^{-1}	1.43×10^{-1}	9.66×10^{-2}	2.72×10^{-1}	6.53×10^{-1}
$\bar{\gamma}_2$	-7.02×10^{-2}	6.19×10^{-2}	-1.12×10^{-2}	5.11×10^{-2}	3.16×10^{-2}
	-6.82×10^{-2}	8.23×10^{-2}	-1.54×10^{-2}	8.45×10^{-2}	8.32×10^{-2}
$\bar{\gamma}_3$	3.90×10^{-2}	4.98×10^{-2}	2.21×10^{-3}	1.43×10^{-2}	1.05×10^{-1}
	3.95×10^{-2}	5.82×10^{-2}	2.75×10^{-3}	2.29×10^{-2}	1.23×10^{-1}
$\bar{\gamma}_4$	-2.46×10^{-2}	4.17×10^{-2}	-4.90×10^{-4}	4.10×10^{-3}	2.07×10^{-2}
	-2.46×10^{-2}	4.51×10^{-2}	-5.44×10^{-4}	5.89×10^{-3}	2.58×10^{-2}
$\bar{\gamma}_5$	1.60×10^{-2}	3.51×10^{-2}	1.02×10^{-4}	1.10×10^{-3}	5.23×10^{-2}
	1.60×10^{-2}	3.68×10^{-2}	1.08×10^{-4}	1.48×10^{-3}	5.44×10^{-2}

TABLE II. Listed are the laboratory kinetic energy E_{lab}, the c.m. angle (in deg) at which the analyzing power goes to zero $\theta_{\text{c.m.}}$, and the separate contributions from π, γ, ρ, and ρ-ω exchanges to ΔA ($\times 10^4$). Also shown are the total theoretical predictions together with the one experimental result. The small and somewhat uncertain two-pion exchange and quark contributions are not included.

E_{lab} (MeV)	$\theta_{\text{c.m.}}$ (deg)	π	γ	ρ	ρ-ω	Total	Expt.
188	96	7	8	1	5	21	
350	72	42	3	6	−3	48	
477	70	43	5	8	−6	50	37±17±8
600	70	34	9	9	−5	47	
800	70	32	14	11	1	58	

that the em contribution is not large compared with the shorter range strong interaction effects.

In Table I we give the calculated values for the π, γ, ρ, and ρ-ω contributions to $\bar{\gamma}_J$ for $J=L=1,\ldots,5$ for the case of 477 MeV. The upper results include the form factors as described in Sec. III and for $J=L=1,2$ nuclear distortions were included using the Reid soft-core potential as has already been discussed. These upper numerical results were the ones used in the calculation of $\Delta A(\theta)$ and hence ΔA. The lower results in the table have been included for comparison and do not include any form factors or distortion. These were calculated using the exact analytic expressions for the $\bar{\gamma}_J$'s in Appendix B. The results of Appendix B have been used to perform a partial check on our numerical calculation of the $\bar{\gamma}_J$'s in the limit of no distortion and no form factors. The agreement was typically better than one percent. As is evident from results of Table I the effects of form factors and distortion are most pronounced for $L=J=1$ and become progressively less important for the higher partial waves.

Our results for ΔA ($\times 10^4$) (i.e., the difference in n-p analyzing powers at the zero-crossing angle $\theta_{\text{c.m.}}$) from OPEP, em one-rho, and rho-omega mixing terms are shown in Table II for various energies of interest. Also shown are the total predictions for ΔA ($\times 10^4$) and the TRIUMF measurement[8] of ΔA at 477 MeV. We find good agreement between our result and the experimental value. The em spin-orbit contribution (one-photon exchange) was found to have a fairly small effect at the analyzing power zero-crossing angle $\theta_{\text{c.m.}}$ when realistic form factors and distortions are included. The remaining long-range CSB contribution from the OPEP is found to play the dominant role. The shorter range ρ and ρ-ω exchange contributions tend to cancel and are less important. The $\rho\omega$ results differ slightly from those in our Ref. 9 due to an error in that work. The very small and somewhat uncertain TPEP and quark contributions have not been included. In Fig. 2 the various contributions to $\Delta A(\theta)$ are shown for the various energies of interest. From these curves it is apparent that the photon and ρ-ω mixing contributions change sign quite near the zero-crossing angle in all but the 188 MeV case, which suppresses these effects.

There are a number of important sources of uncertainty in the predictions. The most obvious of these are the uncertainties in the ρ and ω coupling constants and in the ρ-ω mixing matrix element. The uncertainties in the ρ and ρ-ω mixing terms are at least 30%. Fortunately, ΔA is dominated by the relatively well understood OPEP term. There is a small dependence of our predictions on the exact nature of the form factors used. For example, if the ρ-vertex form factor is $\Lambda_\rho = 1300$ MeV as in the full Bonn model (cf. 1400 MeV in the OBEP model),[16,17] then this causes a variation of -1×10^4 in ΔA from ρ, and $+0.5\times 10^4$ in ΔA from ρ-ω at 477 MeV. Another source of uncertainty is in the exact form of the nuclear distorting potential used, as was previously discussed.

The choice of the phase shifts used to calculate $A(\theta)$ and $\Delta A(\theta)$ can also cause some variation in the prediction for ΔA. In our calculations we used the phase shifts of Arndt *et al.*[14] and made linear extrapolations to the energies of interest. In particular, for 188 and 350 MeV the 1983 n-p energy-dependent phase shifts were used, while for the higher energies at 477, 600, and 800 MeV the more recent 1987 n-p single-energy analysis phase shifts were used since new high energy data were included in these. At 477 MeV when we use the 1983 phase shifts rather than 1987 we find essentially identical results, i.e., the changes are $+2$, -1, $+0.3$, and -1 for π, γ, ρ, and ρ-ω respectively. The changes were somewhat larger when 1977 phase shifts were used. The γ and ρ-ω predictions are sensitive to changes in the analyzing power zero-crossing angle, since these contributions also change sign close to this angle.

The results at 600 and 800 MeV have additional uncertainty over lower-energy results since only first-order relativistic corrections have been included in the CSB potentials (and not at all for ρ-ω mixing), and since inelasticities have been neglected in this work. For our calculations the inelasticity in the ${}^1D_2$ partial wave is expected to be most important since it is relatively large at high energies and since it enters into the $\bar{\gamma}_2$ contribution to $f(k,\theta)$ through Eq. (2.9). Following Ref. 14 (1983) we have $\exp(i\bar{\delta}_2)\to\sqrt{\eta}\exp(i\bar{\delta}_2)$, where $\eta<1$ when inelasticity is included. We find for ${}^1D_2$ that $\sqrt{\eta}=0.95$, 0.86, and 0.84 at 477, 600, and 800 MeV, respectively, using the 1987 phase shifts.[14] From Eq. (2.9) we see that we can write $\bar{\gamma}_2\to\sqrt{\eta}\bar{\gamma}_2$ since the $\bar{\gamma}_J$ are small. Thus we can test the sensitivity to this inelastic effect by modifying $\bar{\gamma}_2$. We find changes to the prediction for ΔA due to the OPEP (for example) of -1.5, -4, and -5, respectively. This then is an indication that inelastic effects are significant above about 500 MeV, but that they are probably not too large.

We study the influence of strong distortion and scattering phase shifts in Fig. 3. We show three curves for the pion contribution to $\Delta A(\theta)$ at each of three different energies. The first curves (1) for each energy are the pion results already given in Fig. 2, while for the curves (2) we have performed the identical calculation with no strong distortion of the radial wave functions [i.e., spherical Bessel functions are used in Eq. (2.10) for $\bar{\gamma}_J$]. The third set of curves (3) are calculations of $\Delta A(\theta)$ which include neither radial distortion nor any phase shifts in Eq. (2.9) for $f(k,\theta)$ (which is then purely imaginary). A comparison of curves (1) and (2) for each

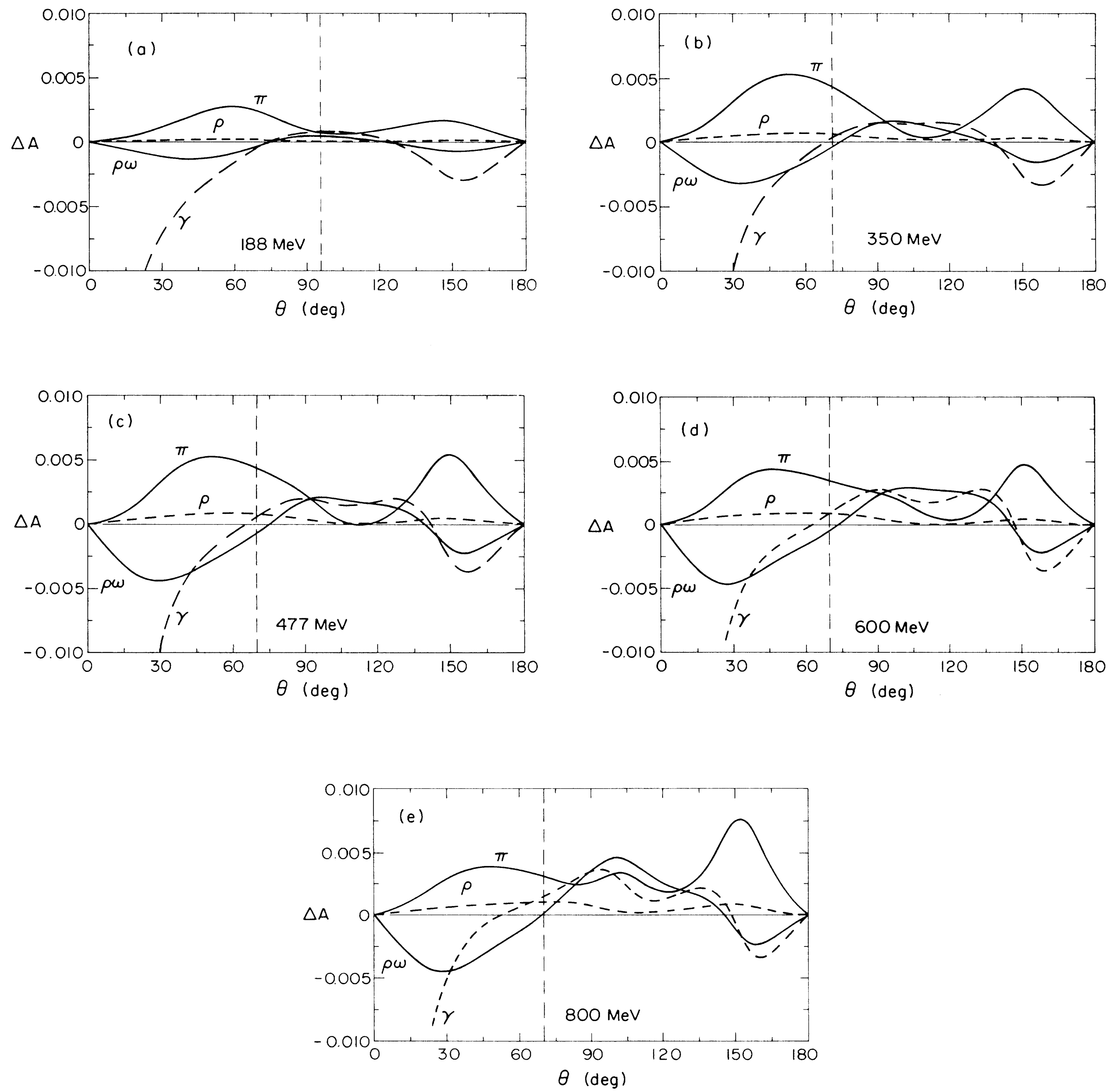

FIG. 2. We show the contributions to $\Delta A(\theta)$ at the various energies of interest from π, γ, ρ, and ρ-ω exchanges as a function of the c.m. scattering angle θ. The vertical lines show the angle for which the analyzing power vanishes.

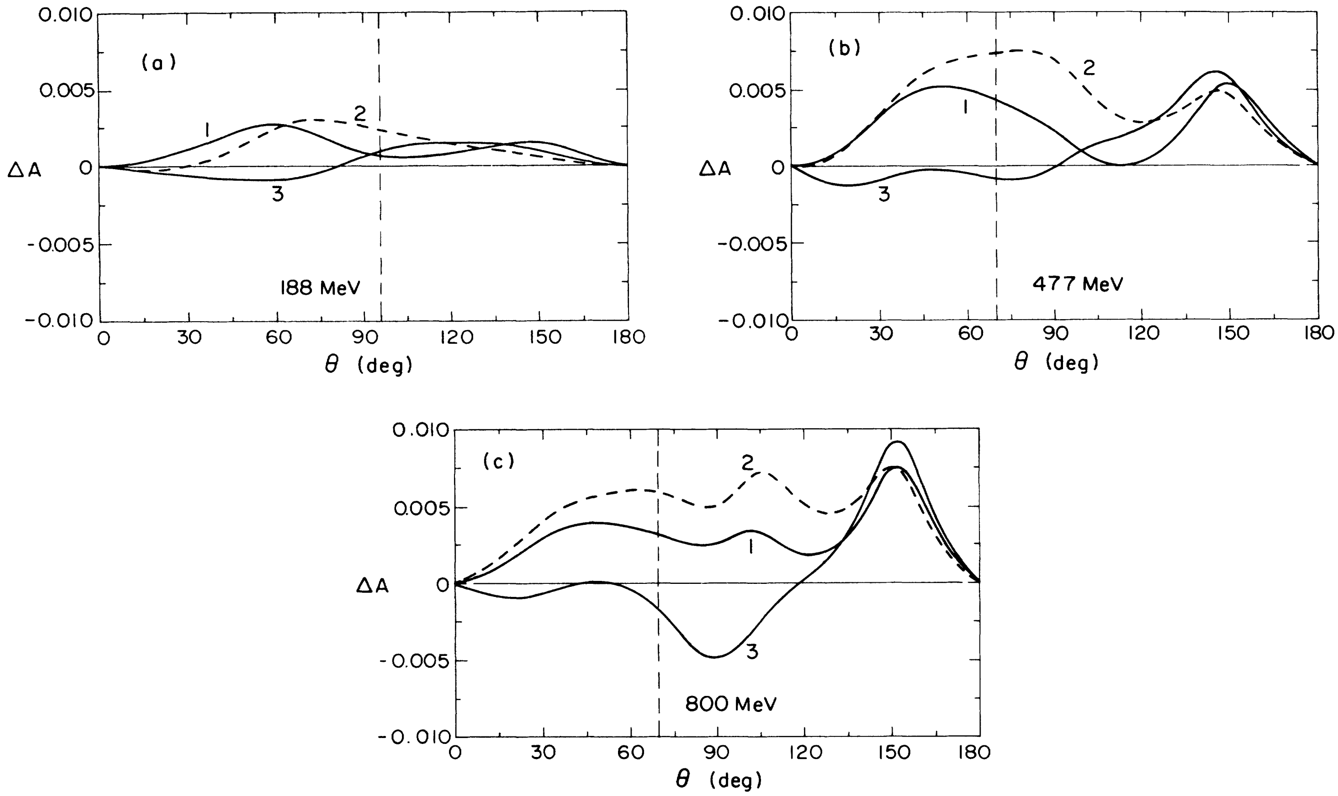

FIG. 3. Three different cases for the π contribution to $\Delta A(\theta)$ are shown for each of three energies: (1) is the calculation already shown in Fig. 2 for π and includes the effects of both strong distortion of the radial wave functions and of phase shifts, (2) has no strong distortion of the radial wave functions but does include phase shifts in the calculation of $f(k,\theta)$, and (3) is a pure plane-wave calculation of $f(k,\theta)$ which includes neither.

energy shows that the omission of the radial distortion has had a significant effect, but that the curves are qualitatively similar. When phase shifts in $f(k,\theta)$ are then neglected, we see from a comparison of curves (2) and (3) that the predictions are completely different. Thus we see that the phase shifts play a very important role as to a lesser extent does the radial distortion. Plane-wave treatments appear unreliable.

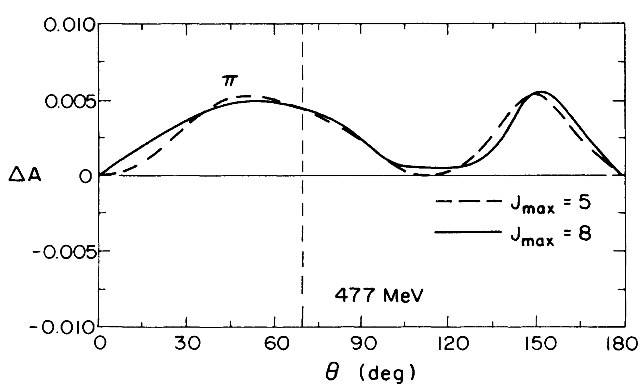

FIG. 4. The π contribution to $\Delta A(\theta)$ at 477 MeV for $J_{max}=8$ and $J_{max}=5$ in the partial wave expansion.

Finally, we have examined the question of whether or not it was sufficient to set $J_{max}=5$ in the partial wave expansion for the π, ρ, and ρ-ω contributions. We really need only consider the π contribution, since the other two arise from much shorter-range forces. In Fig. 4 we show the change in the π contribution to $\Delta A(\theta)$ at 477 MeV when we use $J_{max}=8$ rather than $J_{max}=5$. It is seen that the difference is fairly small everywhere, and is almost negligible at the analyzing power zero-crossing angle. At 477, 600, and 800 MeV the changes to the π contribution to $\Delta A(\theta)$ at the zero-crossing angle (i.e., changes to the OPEP part of ΔA) are 1, $+1$ and $+2$ ($\times 10^{-4}$), respectively. At 800 MeV the changes in the OPEP contribution at some angles is $\simeq 10$ ($\times 10^{-4}$), indicating a need for $J_{max}>5$ at this energy. A conservative approach would be to use $J_{max}>5$ for energies above say 400 MeV.

As a result of inelastic effects, the possible influence of relativistic dynamics, and due to an increased need for the inclusion of higher partial waves at higher energies, we estimate a further uncertainty of the order of 25% for our 600 and 800 MeV results in addition to the uncertainty in our lower energy results.

V. CONCLUSION

We have given a reasonably complete description of a formalism which allows the computation of CSB effects in neutron-proton elastic scattering. In particular, we have shown that it is possible to understand the recent nonzero measurement of the n-p analyzing power difference at TRIUMF (Ref. 8) in terms of electromagnetic interactions, one-boson exchange potentials, and the two-pion exchange potential. Quark effects were also considered and found to be very small. While the agreement between prediction and experiment is rather satisfying, it should be remembered that the TRIUMF result is only two standard deviations from a null effect. Clearly there is a need for more accurate experiments to be carried out in the future. It is also important to test our understanding at both lower and higher energies against measurements from ongoing and planned experiments.

The work reported here has been largely exploratory and there are potential improvements in a number of areas. Desirable future theoretical developments include a detailed study of the effects of using different nuclear distorting potentials, an evaluation of the π-γ CSB contribution, a fully relativistic extension of the formalism described here, and a means of including inelastic effects at higher energies.

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APPENDIX A

The potential $V^{\rm IV}$ from a particular boson exchange Feynman diagram in N-N scattering is obtained by taking the appropriate invariant amplitude[15] $\mathcal{M}_{fi}$ and making a suitable nonrelativistic reduction. The c.m. cross section is given by

$$\left[\frac{d\sigma}{d\Omega}\right]_{fi} = |f_{fi}|^2 , \tag{A1}$$

where $i \equiv$ initial state, $f \equiv$ final state, $M \equiv$ average nucleon mass,

$$f_{fi} = \frac{1}{2\pi}\frac{M^2}{E_T}\mathcal{M}_{fi} = -(2\pi)^2\frac{E_T}{4}T_{fi} , \tag{A2}$$

and $E_T \equiv$(total) c.m. energy$=2E$ with $E \equiv$c.m. energy per nucleon. The reduced T matrix is related to the reduced potential $V(\mathbf{k}_f,\mathbf{k}_i)$ in first Born approximation by

$$\chi^\dagger_{T_f}\chi^\dagger_{S_f}V(\mathbf{k}_f,\mathbf{k}_i)\chi_{S_i}\chi_{T_i} = T_{fi} , \tag{A3}$$

where χ_T and χ_S are Pauli spinors for isospin and spin, respectively. The coordinate space reduced potential operator $V(\mathbf{r})$ is defined by

$$V(\mathbf{k}_f,\mathbf{k}_i) \equiv \frac{1}{(2\pi)^3}\int d^3r\, e^{-i\mathbf{k}_f\cdot\mathbf{r}}V(\mathbf{r})e^{i\mathbf{k}_i\cdot\mathbf{r}} . \tag{A4}$$

We are only interested here in the class IV contributions to $V(\mathbf{r})$, i.e., those that have the form of Eq. (1.3).

Equations (A2)–(A4) are used to obtain the $V^{\rm IV}(\mathbf{r})$ from a particular Feynman diagram. The contribution to T_{fi} [as defined in Eq. (A2)] arising from some $V^{\rm IV}(\mathbf{r})$ is given in the DWBA by

$$\delta T_{fi} = \int d^3r\, \psi_f^{(-)*}(\mathbf{r})V^{\rm IV}(\mathbf{r})\psi_i^{(+)}(\mathbf{r}) , \tag{A5}$$

where the distorted waves ψ contain spin and isospin information. They are determined by the radial wave functions $R(r)$ of Eq. (2.10) and the experimental bar phase shifts.[14]

APPENDIX B

As a partial check on numerical calculations it is useful to compare analytic and numerical calculations of the mixing angles ($\bar{\gamma}_J$) in the limit where nuclear distortions of the radial wave functions are neglected. In this limit the radial wave functions R_J and R_{JJ} of Eq. (2.10) are replaced by the spherical Bessel functions j_J. We will use the identity

$$2\sqrt{J(J+1)}\int_0^\infty dr\, r^2 j_J^2(kr)\frac{1}{r}\frac{d}{dr}\left[\frac{1}{r}e^{-mr}\right] = A_J(x_m) , \tag{B1}$$

where

$$x_m \equiv 1+\frac{m^2}{2k^2} , \tag{B2}$$

$$A_J(x) \equiv \left[\frac{J}{J+1}\right]^{1/2}[xQ_J(x)-Q_{J-1}(x)] , \tag{B3}$$

and where Q_J are Legendre functions of the second kind. For simplicity no form factors are included in the following results.

The result for one-photon exchange has already been given in Eq. (3.6). For one-pion exchange we find

$$\bar{\gamma}_J = (-1)^J\left[\frac{g_\pi^2}{4\pi}\right]\left[\frac{M}{E}\right]^2\frac{k}{M}\left[\frac{M_{\rm n}-M_{\rm p}}{2M}\right]A_J(x_\pi) , \tag{B4}$$

where $x_\pi = 1+m_\pi^2/(2k^2)$. This result follows directly from using Eqs. (B1), (2.10), and (3.10). Similarly the result for one-rho exchange is

$$\bar{\gamma}_J = \left[\frac{g_\rho^2}{4\pi}\right]\left\{(-1)^J\left[1+\frac{E+3M}{2M}\left[\frac{f_\rho}{g_\rho}\right] + \frac{E+M}{2M}\left[\frac{f_\rho}{g_\rho}\right]^2\right]+\tfrac{1}{2}\right\} \times\frac{k}{M}\left[\frac{M}{E}\right]^2\left[\frac{M_{\rm n}-M_{\rm p}}{2M}\right]A_J(x_\rho) . \tag{B5}$$

Finally, for the ρ-ω mixing contribution we find the nonrelativistic result (i.e., $E \to M$)

$$\bar{\gamma}_J = \left[\frac{f_\rho g_\omega}{4\pi}\right]\frac{k}{M}\frac{\langle\omega|H|\rho^0\rangle}{m_\omega^2-m_\rho^2}[A_J(x_\rho)-A_J(x_\omega)] . \tag{B6}$$

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