Current conservation and interaction currents in relativistic meson theories

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The relation between the nucleon-nucleon interaction and exchange currents needed for current conservation are derived for the Bethe-Salpeter formalism, and for the approach in which the spectator particle is restricted to its mass shell. For both approaches, it is shown how to achieve current conservation for a completely general isospin dependent, energy dependent interaction with arbitrary phenomenological electromagnetic form factors for the nucleon and mesons, and with strong form factors at the meson-nucleon vertices. Contrary to what has often been stated in the literature, the development shows that current conservation places no restrictions on the use of different electromagnetic form factors for mesons and nucleons, and that phenomenological meson-nucleon form factors can be introduced in a way which is consistent with current conservation. The longitudinal part of the exchange current is uniquely determined by current conservation, and for the common case of an interaction that only depends on the invariant momentum transfer variable an explicit expression for this longitudinal exchange current is given. The transverse part, which contains all electromagnetic form factors, is unconstrained by current conservation.

I. INTRODUCTION

It is well known that non-nucleonic degrees of freedom may reveal themselves in the form of interaction current contributions to nuclear electromagnetic observables even at low energies.[1] When such interaction currents contain longitudinal components they may be determined by the nuclear interaction through current conservation. Exchange currents that are associated in this way with the interaction, and which have to be included in consistent calculations of bound state matrix elements, are usually referred to as model independent exchange currents. Existing empirical indications are that such model independent exchange currents are the ones of main importance for the electromagnetic properties of the few-nucleon systems.[2]

The model independent exchange currents are either of relativistic origin and associated with intermediate antinucleon states,[3–5] due to the energy dependence of the interaction, or finally due to the isospin dependence of the interaction.[6,7] It is the aim of this paper to elaborate on this distinction by deriving the proper form of the two-body interaction currents that have to be included in relativistic calculations of matrix elements of the nuclear electromagnetic current operator.

The key to the development is the generalized Ward-Takahashi (WT) identity for the divergence of the current of an off-shell particle.[8] One principal result of this paper is that the electromagnetic interactions of any two-body system described by a relativistic two-body equation (such as the Bethe-Salpeter equation or an equation in which one particle is restricted to its mass shell) will always conserve current provided the following three conditions are met: (1) the electromagnetic currents for the interacting off-shell nucleons and mesons satisfy the appropriate WT identity, (2) the interacting incoming and outgoing two body system satisfy the same two body relativistic equation (with the same interaction kernel), and (3) the exchange (or interaction) current is built up from the relativistic kernel by coupling the virtual photon to all possible places in the kernel. Conditions (1) and (2) are possible to satisfy provided calculations are done consistently and with care, but condition (3) becomes cumbersome if the two body interaction includes higher order kernels such as crossed box diagrams.

A second principal result of this paper is that, contrary to results obtained by some previous investigators,[9–12] current conservation places *no constraint* on the use of electromagnetic form factors for the hadrons. This means that we are free to use *different* phenomenological form factors for the nucleon and mesons (pion and rho). We have the freedom to choose these form factors to describe on-shell data (when it exists) and may even allow the q^2 dependence of these form factors to vary with the virtual mass of the interacting nucleon or meson. This freedom can only be constrained by fits to experimental data, or by microscopic calculations of the form factors based on the underlying quark structure of the hadrons.

This result comes about because the WT identities only constrain the longitudinal current and it is possible to construct currents in which all of the electromagnetic form factors occur in transverse terms only. Since the on-shell currents are purely transverse, this can be done in a way which is consistent with the experimental results. For the case in which the interaction kernel depends solely on the invariant four-momentum transfer variable we give an explicit expression for the longitudinal exchange current that is needed for current conservation.

A third principal result of this paper is that phenomenological form factors can be introduced at the strong meson-nucleon vertices without violating current conservation. (We will sometimes refer to these as the "strong" form factors to distinguish them from electromagnetic form factors.) To accomplish this, the strong form factors are reinterpreted as phenomenological self-energies (which means that they must depend only on the off-shell mass of the hadron). This modifies the propagator and, through the WT identity, leads to modifications in the currents. If the currents are modified as required, current conservation is unaffected.

This paper is divided into seven sections. In Sec. II we discuss the two body current for point particles in the framework of the Bethe-Salpeter equation[13] in the one pion exchange ladder approximation. We show how conditions (1)–(3) cooperate to ensure current conservation in this simple case, and derive the longitudinal exchange current for the case of an interaction that depends only on momentum transfer. In Sec. III we extend the method for achieving current conservation to a general interaction. In Sec. IV we discuss current conservation in the quasipotential framework developed by one of us.[14] In Sec. V we illustrate the results for the cases of single pion and ρ-meson exchange interactions with derivative couplings, and in Sec. VI we show how both electromagnetic and strong hadronic form factors may be introduced without violation of current conservation. Section VII contains a concluding discussion.

II. THE BETHE-SALPETER THEORY IN THE LADDER APPROXIMATION

We turn now to an explicit construction of the current in the Bethe-Salpeter (BS) theory. We begin with a treatment of the ladder approximation, and extend the discussion in the next section to the most general case.

In the ladder approximation the scattering amplitude M is assumed to satisfy the following equation:

$$M(p,p';P)=V(p,p';P)+i\int\frac{d^4k}{(2\pi)^4}V(p,k;P)\times S_1(\tfrac{1}{2}P+k)S_2(\tfrac{1}{2}P-k)\times M(k,p';P)\ , \tag{2.1}$$

where the notation is illustrated in Fig. 1; p, p', and k are relative 4-momenta, P is the total 4-momentum (which is constant), and V is the irreducible kernel. S is the free "nucleon" propagator, defined as

$$S_i^{-1}(p)=p\cdot\gamma_i-M\ , \tag{2.2}$$

M being the nucleon mass. In the ladder approximation,

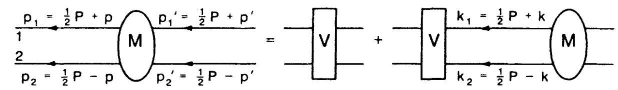

FIG. 1. Diagrammatic representation of the BS equation (2.1) for the scattering amplitude, showing the notation used for the various 4-momenta.

the irreducible kernel V depends only on $p-k$ in addition to isospin.

We may project states of definite total angular momentum, helicity, and parity from (2.1) by introducing a suitable projection operator. For our purposes it is convenient to specialize the initial state so that both particles are on shell, in which case in the center of mass of the pair,

$$P=(W,\mathbf{0})\ ,\quad p_0'=0\ ,\quad W=2E(p')\ , \tag{2.3}$$

where $E(p)\equiv(M^2+p^2)^{1/2}$. In this case the projection operator takes the general form

$$P_{\rm op}\{F\}=\int d\Omega_{\hat{\mathbf{p}}'}F(\mathbf{p}')u_1^{(r)}(\mathbf{p}')u_2^{(s)}(-\mathbf{p}')0_{rs}(\hat{\mathbf{p}}')\ . \tag{2.4}$$

In this paper we will suppress all reference to the quantum numbers of the initial state. Using the operator (2.4) we introduce a relativistic BS scattering wave function according to

$$\psi(p,P)=P_{\rm op}\{(2\pi)^4\delta^4(p-p')+iS_1(\tfrac{1}{2}P+p)S_2(\tfrac{1}{2}P-p)M(p,p';P)\}\ . \tag{2.5}$$

Substituting (2.5) into (2.1) gives us the equation we seek:

$$S_1^{-1}(\tfrac{1}{2}P+p)S_2^{-1}(\tfrac{1}{2}P-p)\psi(p,P)=i\int\frac{d^4k}{(2\pi)^4}V(p,k;P)\psi(k,P)\ . \tag{2.6}$$

The equation for the bound state wave function is obtained in a somewhat different way. The presence of a bound state implies a pole in the M matrix, which for a spin zero bound state takes the form

$$M(p,p';P)=-\frac{\Gamma(p,P)\Gamma(p',P)}{M_B^2-P^2}+R(p,p';P)\ , \tag{2.7}$$

where R is regular at $P^2=M_B^2$. Substituting (2.7) into (2.1), and approaching the bound state pole gives the homogeneous equation

$$\Gamma(p,P)=i\int\frac{d^4k}{(2\pi)^4}V(p,k;P)S_1(\tfrac{1}{2}P+k)\times S_2(\tfrac{1}{2}P-k)\Gamma(k,P)\ . \tag{2.8}$$

Defining the bound state wave function

$$\psi_B(p,P)=iS_1(\tfrac{1}{2}P+p)S_2(\tfrac{1}{2}P-p)\Gamma(p,P)\ , \tag{2.9}$$

gives the same equation, (2.6), for ψ_B.

We now turn to the central question of specifying the current operator which should be used with the relativistic states which satisfy (2.6). The answer will depend on the specific form of V. In this section we will discuss the ladder sum for the case in which V has the simple form

$$V(p,k;P)=-g^2\tau_1\cdot\tau_2\gamma_1^5\gamma_2^5\Delta(p-k)\ , \qquad (2.10)$$

with the meson propagator

$$\Delta^{-1}(k)=k^2-\mu^2\ . \qquad (2.11)$$

This interaction, which corresponds to exchange of single pseudoscalar pions, will be sufficient to illustrate the approach. Some other examples will be considered in Sec. V, and the completely general result will be given in Sec. III.

The general form for the matrix element of the electromagnetic current between two relativistic states described by the wave functions ψ_i and ψ_f is, in the BS formalism,

$$\langle J^\mu\rangle=\int \frac{d^4p\, d^4p'}{(2\pi)^8}\bar{\psi}_f(p',D')J^\mu(p'D',pD)\psi_i(p,D)\ , \qquad (2.12)$$

where the notation is given in Fig. 2. The general form for the current operator is

$$\begin{aligned}J^\mu(p'D',pD)=&-i(2\pi)^4S_2^{-1}(p_2)\delta^4(p_2-p_2')\mathrm{j}_1^\mu(p_1',p_1)\\&-i(2\pi)^4S_1^{-1}(p_1)\delta^4(p_1-p_1')\mathrm{j}_2^\mu(p_2',p_2)\\&+J_V^\mu(p'D',pD)\ . \qquad (2.13)\end{aligned}$$

Here the single nucleon current operator for pointlike nucleons is

$$\mathrm{j}_i^\mu(p_i',p_i)=\gamma_i^\mu\tfrac{1}{2}[1+\tau_i^3]\ , \qquad (2.14)$$

and J_V^μ is an interaction or exchange current, the structure of which depends on the form of V. If V is given by (2.10), which corresponds to the ladder sum, the exchange current is

$$\begin{aligned}J_V^\mu(p'D',pD)&=J_L^\mu(p'D',pD)\\&=-g^2\tau_1^i\tau_2^j\gamma_1^5\gamma_2^5 j_\pi^{ij\mu}(p_1'-p_1,p_2-p_2')\\&\quad\times\Delta(p_1'-p_1)\Delta(p_2-p_2')\ , \qquad (2.15)\end{aligned}$$

where, for pointlike pions,

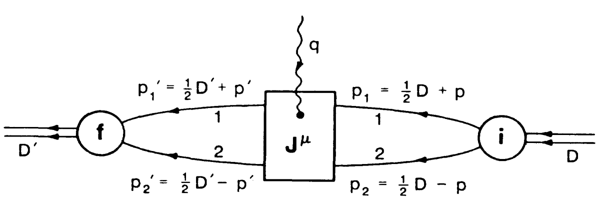

FIG. 2. Diagrammatic representation of the matrix element (2.12) of the two body current in the BS formalism between initial (i) and final (f) relativistic two body states.

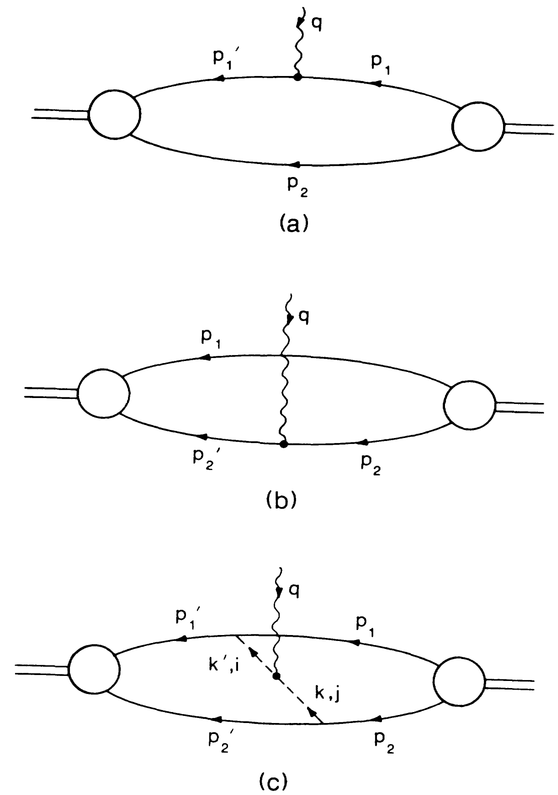

FIG. 3. Diagrammatic representation of the three terms in Eq. (2.13) for the BS equation in the ladder approximation.

$$j_\pi^{ij\mu}(k',k)=-i(k'+k)^\mu\epsilon^{ij3}\ . \qquad (2.16)$$

In (2.16), i and j are the isospin indices of the outgoing and incoming pions, respectively. The three terms in (2.13) are illustrated diagrammatically in Fig. 3.

The form of the operator (2.13) follows naturally if one imagines the photon coupling to all possible places in the ladder sum, and uses the BS equation to collect all contributions into the three distinct classes which occur. This intuitive argument also shows that (2.13) should conserve current. The remainder of this section will be devoted to a proof of this result.

The WT identities[8] are crucial to the proof, and will also be the key to generalizing the results to include nucleon and pion structure, which will be carried out in Sec. VI. In our notation, these are

$$\begin{aligned}&q_\mu \mathrm{j}_i^\mu(p_i',p_i)=[S_i^{-1}(p_i')-S_i^{-1}(p_i)]\tfrac{1}{2}[1+\tau_i^3]\ ,\\&q_\mu j_\pi^{ij\mu}(k',k)=-i[\Delta^{-1}(k')-\Delta^{-1}(k)]\epsilon^{ij3}\ .\end{aligned} \qquad (2.17)$$

The proof depends only on the validity of (2.17) and the BS equation (2.6).

Begin by using the nucleon WT identity on the first two terms of the current operator (2.13) to obtain

$$q_\mu\langle J^\mu_{1+2}\rangle=-i\int\frac{d^4p}{(2\pi)^4}\{\bar\psi_f(p+\tfrac{1}{2}q,D')S_2^{-1}(p_2)[S_1^{-1}(p_1')-S_1^{-1}(p_1)]\tfrac{1}{2}[1+\tau_1^3]\psi_i(p,D)$$
$$+\bar\psi_f(p-\tfrac{1}{2}q,D')S_1^{-1}(p_1)[S_2^{-1}(p_2')-S_2^{-1}(p_2)]\tfrac{1}{2}[1+\tau_2^3]\psi_i(p,D)\}\ . \tag{2.18}$$

Using the BS equation (2.6) this reduces to

$$q_\mu\langle J^\mu_{1+2}\rangle=\tfrac{1}{2}\int\frac{d^4p\,d^4p'}{(2\pi)^8}\{\bar\psi_f(p',D')[\,V(p',p+\tfrac{1}{2}q;D')[1+\tau_1^3]-[1+\tau_1^3]V(p'-\tfrac{1}{2}q,p;D)$$
$$+V(p',p-\tfrac{1}{2}q;D')[1+\tau_2^3]-[1+\tau_2^3]V(p'+\tfrac{1}{2}q,p;D)]\psi_i(p,D)\}\ . \tag{2.19}$$

Note that (2.19) would be zero if the potential were to depend only on momentum transfer and were independent of isospin. For the ladder sum, the potential does depend only on momentum transfer, and hence current conservation would be proved for isoscalar exchange, in which case the exchanged meson would be neutral and there would be no meson exchange term in the current operator. In our case the isospin dependence of the pion spoils the cancellation, giving

$$[\boldsymbol\tau_1\cdot\boldsymbol\tau_2,\tau_1^3]=2i[\boldsymbol\tau_1\times\boldsymbol\tau_2]^3\ , \tag{2.20}$$

and hence (2.19) reduces to

$$q_\mu\langle J^\mu_{1+2}\rangle=-ig^2\int\frac{d^4p\,d^4p'}{(2\pi)^8}\bar\psi_f(p',D')[\boldsymbol\tau_1\times\boldsymbol\tau_2]^3\gamma_1^5\gamma_2^5[\Delta(p'-p-\tfrac{1}{2}q)-\Delta(p'-p+\tfrac{1}{2}q)]\psi_i(p,D)\ . \tag{2.21}$$

The divergence of the meson exchange term is precisely what is necessary to cancel (2.21). Using the pion WT identity on this term gives

$$q_\mu\langle J^\mu_L\rangle=+ig^2\int\frac{d^4p\,d^4p'}{(2\pi)^8}\bar\psi_f(p',D')[\boldsymbol\tau_1\times\boldsymbol\tau_2]^3\gamma_1^5\gamma_2^5[\Delta(p_2-p_2')-\Delta(p_1'-p_1)]\psi_i(p,D)\ , \tag{2.22}$$

which cancels (2.21).

Equation (2.22) suggests an alternative form for the exchange current (2.15) which explicitly displays its role in conserving current. Writing the interaction (2.10) in the general form

$$V=v(p-k)\boldsymbol\tau_1\cdot\boldsymbol\tau_2\ , \tag{2.23}$$

it can be readily shown that (2.15) can be written

$$J^\mu_I(p'D',pD)=i(\boldsymbol\tau_1\times\boldsymbol\tau_2)^3\frac{(k'+k)^\mu}{(k'^2-k^2)}\times[v(k')-v(k)]+J^\mu_{TV} \tag{2.24}$$

where $k'=p_1'-p_1$ and $k=p_2-p_2'$ are the momenta of the outgoing and incoming pions, respectively [see Fig. 3(c)], and J^μ_{TV} is a purely transverse contribution which in this case is zero.

We will show in Secs. V and VI below that the form (2.24) holds for the *longitudinal* part of the exchange current, even in the presence of contact terms generated by derivative couplings, strong hadronic form factors at the meson-nucleon vertices, and electromagnetic form factors for the exchanged meson. In these more general cases, the additional *transverse* terms, J^μ_{TV}, which cannot be determined by current conservation, are not zero. It is interesting to note that (2.24) reduces to exactly the same form as the nonrelativistic continuity equation[4,7] in a reference frame in which $q_0=0$.

III. THE BETHE-SALPETER EQUATION FOR THE GENERAL CASE

We now consider the general case where V is a sum of irreducible kernels, shown up to sixth order in Fig. 4. We will show that the correct operator in this case has the form of (2.13), where the interaction current J^μ_I is constructed from the irreducible kernels which make up

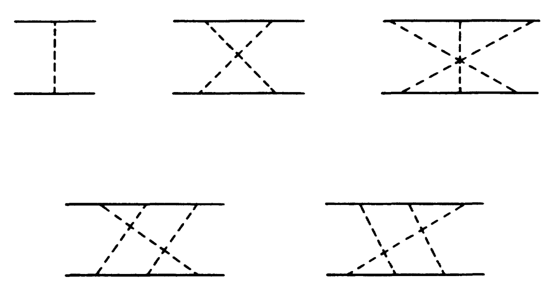

FIG. 4. The irreducible kernel for the BS equation up to sixth order, for the case when all ladders and crossed ladders are summed.

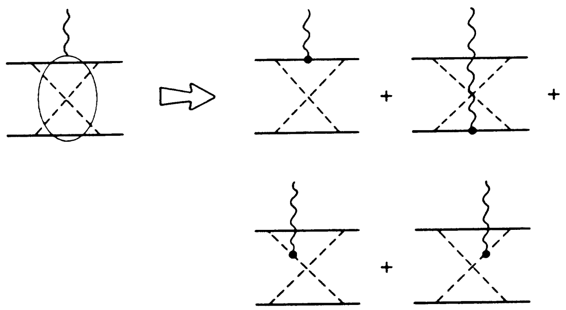

FIG. 5. The interaction current generated by the crossed box diagram in the BS theory. The virtual photon is coupled to *all* particles *inside* the diagram (as enclosed by the oval) but is not coupled to the external nucleons.

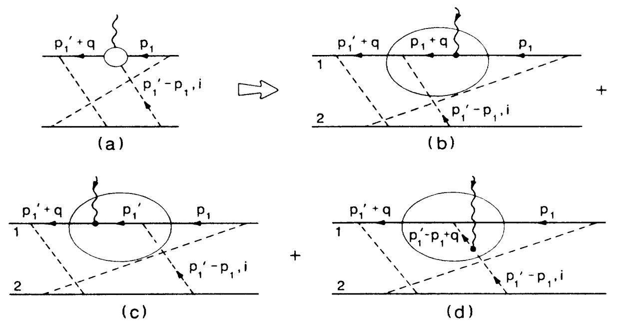

FIG. 6. Illustration of the three terms given in Eq. (3.1) which arise from the coupling of the virtual photon to all particles entering (or leaving) a single vertex [indicated with a small circle in (a)] in the interior of a higher order diagram (sixth order in this case).

V by inserting the virtual photon by minimal coupling at *all possible places inside the diagrams.* Note that our result for the second order case (the ladder sum) satisfies this rule. The diagrams generated in the fourth order by the crossed box are shown in Fig. 5. In general, each irreducible $(2n)$th order diagram involving the exchange of n mesons will generate $3n-2$ diagrams with couplings to nucleons and mesons. In addition, there will be $2n$ diagrams with contact terms if these are generated by the elementary meson-nucleon-nucleon interaction, as is the case for pions with pseudovector $(\gamma^5\gamma^\mu)$ coupling and ρ mesons with tensor coupling (see Sec. V). For the moment we will assume that no contact terms are present—they may be added easily once the general proof has been outlined.

In developing the argument, it is convenient to first focus on the three diagrams which have photon couplings to the three lines entering any particular interior vertex, as shown schematically in Fig. 6 for one of the sixth order diagrams. Writing only the propagators and couplings for the lines included in the oval shown on each graph, we have

$$\begin{aligned}\langle J_V^\mu\rangle = {} & I_1 S_1(p_1'+q)\{\mathrm{j}_1^\mu(p_1'+q,p_1')S(p_1')\gamma_1^5\tau_1^i+\gamma_1^5\tau_1^i S(p_1+q)\mathrm{j}_1^\mu(p_1+q,p_1)\\ & -i\gamma_1^5\tau_1^j\Delta(p_1'-p_1+q)j_\pi^{ji3}(p_1'-p_1+q,p_1'-p)\}S(p_1)\Delta(p_1'-p_1)I_2\,,\end{aligned} \tag{3.1}$$

where I_1 and I_2 represent the rest of the factors in each diagram, which are the same for all three. Applying the WT identities to these three terms will generate six contributions. Keeping only those terms in which one of the internal propagators [those in the curly braces in (3.1)] is annihilated, gives

$$q_\mu\langle J_V^\mu\rangle \simeq I_1S_1(p_1'+q)\gamma_1^5\{-\tfrac{1}{2}(1+\tau_1^3)\tau_1^i+\tau_1^i\tfrac{1}{2}(1+\tau_1^3)-i\tau_1^j\epsilon^{ji3}\}S(p_1)\Delta(p_1'-p_1)I_2=0\,. \tag{3.2}$$

Hence the contributions from the three terms which annihilate internal propagators cancel. Furthermore, the three terms which annihilate propagators *external* to the curly braces, which we have not yet considered, can be grouped with terms from other diagrams to make new triplets of *internal* terms for surrounding, neighboring vertices, and when considered together these terms also cancel in the way described above. This argument shows that all terms cancel, *except* for the exceptional cases of vertices which are at one of the corners of the diagram. To complete the internal cancellation for these exceptional vertices, precisely four terms are needed, which include V with one of the external momenta shifted by q (or $-q$), as shown in Fig. 7. If we add these four terms to all of the others we obtain a perfect cancellation, which can be written

$$\begin{aligned}& q_\mu J_V^\mu-\tfrac{1}{2}(1+\tau_1^3)V(p'-\tfrac{1}{2}q,p;D)-\tfrac{1}{2}(1+\tau_2^3)V(p'+\tfrac{1}{2}q,p;D)\\ & \qquad +V(p',p+\tfrac{1}{2}q;D')\tfrac{1}{2}(1+\tau_1^3)+V(p',p-\tfrac{1}{2}q;D')\tfrac{1}{2}(1+\tau_2^3)=0\,.\end{aligned} \tag{3.3}$$

Finally, using this relation together with the result obtained from the single nucleon currents, (2.19), proves that the operator (2.13), as defined at the beginning of this section, satisfies current conservation in the general case.

It is now clear how to generalize this result to apply to all mesons and other forms of meson-nucleon couplings, including those which generate contact terms. As long as the elementary tree diagrams for meson electroproduction from nucleons conserve current, as illustrated in Fig. 8, the WT identities (2.17) are satisfied (including those for other mesons), and the BS equation

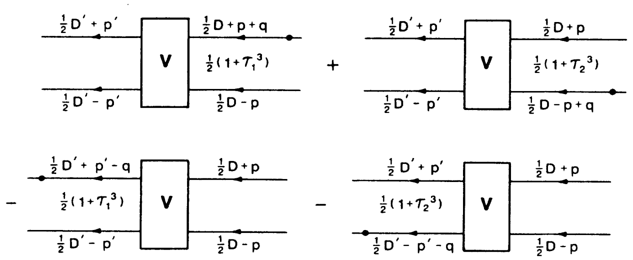

FIG. 7. Diagrammatic representation of the last four terms in Eq. (3.3). In each diagram, q *enters* at the location of the black dot, and the momenta are labeled so that before (for the top row) or after (for the bottom row) the 4-momenta of each nucleon have the canonical values introduced in Fig. 2.

(2.6) applies to both the initial and final state wave functions, the BS matrix elements (2.12) and (2.13) will satisfy current conservation.

IV. CURRENT CONSERVATION WITH THE SPECTATOR NUCLEON ON SHELL

In practical calculations of matrix elements of the nuclear em current operator it is often convenient to use a relativistic formalism which has a smooth nonrelativistic limit. For this purpose one can employ some three-dimensional reduction of the BS equation that has the Schrödinger equation as the nonrelativistic limit. The most direct quasipotential framework, at least from the point of the relativistic formulation, is the method developed by one of us,[14] in which the spectator particle in the relativistic impulse approximation is taken to be on its mass shell.

It will not be necessary in this section to start with the ladder approximation; because the spectator formalism can be so easily tied to Feynman diagrams, we will rely on the general results obtained in Sec. III to discuss the general case immediately.

The equation for the scattering amplitude when one particle (No. 2) is restricted to its mass shell is

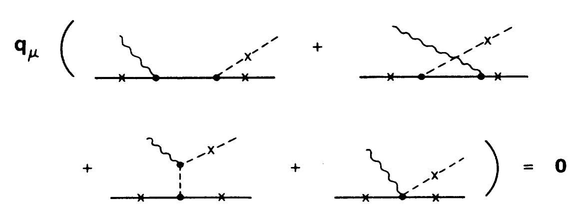

FIG. 8. The tree diagrams for electroproduction of mesons, including a possible contact term. The small $\times$'s on the external lines indicate that the particles are on their mass shell. If these diagrams conserve current, then the interaction current as constructed in the text will satisfy Eq. (3.3), and the full two body current (2.13) will also conserve current.

$$\overline{M}(p,p';P)=\overline{V}(p,p';P)+\int\frac{d^3k}{(2\pi)^3}\frac{M}{E(k_2)}\overline{V}(p,k;P)\times S_1(\tfrac{1}{2}P+k)\overline{M}(k,p';P)\,, \tag{4.1}$$

where the mass shell condition means that, in the center of mass of the pair,

$$p_0=\tfrac{1}{2}W-E(p)\,, \tag{4.2}$$

and similarly for p_0' and k_0, and the mass shell condition requires us to limit ourselves to the positive energy subspace of particle 2, so that the matrix elements

$$\begin{aligned}\overline{M}(p,p';P)&\equiv\overline{u}_2(p_2)M(p,p';P)u_2(p_2')\,,\\ \overline{V}(p,p';P)&\equiv\overline{u}_2(p_2)V(p,p';P)u_2(p_2')\end{aligned} \tag{4.3}$$

are sufficient for our purposes (two component spin indices of particle 2 will normally be suppressed). The nucleon propagators are as defined in Sec. II.

The scattering wave function is defined by

$$\phi(p,P)=P'_{\rm op}\left[\frac{E(p)}{M}(2\pi)^3\delta^3(p-p')+S_1(-\tfrac{1}{2}P+p)\overline{M}(p,p';P)\right], \tag{4.4}$$

where the projection operator $P'_{\rm op}$ is similar to that defined in Eq. (2.4),

$$P'_{\rm op}\{F\}=\int d\Omega_{\hat{p}'}F_s(p')u_1^{(r)}(\mathbf{p}')0_{rs}(\hat{\mathbf{p}}')\,. \tag{4.5}$$

Here the sum over the spin s of the initial particle 2 is shown explicitly for clarity, but u_2 is not needed in the definition of $P'_{\rm op}$ (as it was for $P_{\rm op}$) because the u_2 matrix element has been taken throughout [Eq. (4.3)]. Substituting (4.4) into (4.1) gives the equation for ϕ,

$$S_1^{-1}(\tfrac{1}{2}P+p)\phi(p,P)=\int\frac{d^3k}{(2\pi)^3}\frac{M}{E(k)}\overline{V}(p,k;P)\phi(k,P)\,. \tag{4.6}$$

Note that this is manifestly covariant.

Similarly, the bound state wave function can be defined by

$$\begin{aligned}\phi_B(p,P)&=S_1(\tfrac{1}{2}P+p)\overline{u}_2(\tfrac{1}{2}P-p)\Gamma C\\ &=S_1(\tfrac{1}{2}P+p)\Gamma C\,\overline{u}\,{}_2^T(\tfrac{1}{2}P-p)\,,\end{aligned} \tag{4.7}$$

where Γ was introduced in Sec. II, and the introduction of C permits us to construct Γ from the usual bilinear covariants.[15] The bound state wave function (4.7) also satisfies Eq. (4.6).

The general form for the relativistic current can now be written in this formalism as

$$\langle J^\mu\rangle=\int\frac{d^3p\,d^3p'}{(2\pi)^6}\frac{M^2}{E(p_2)E(p_2')}\overline{\phi}_f(p',D')\times\tilde{J}^{\,\mu}(p'D',pD)\phi_i(p,D)\,, \tag{4.8}$$

where the kinematic variables are as defined in Fig. 2, except that $p_2^2=p_2'^2=M^2$, and

$$\tilde{J}^{\mu}(p'D',pD)=\bar{u}_2(p_2')\hat{J}^{\mu}(p'D',pD)u_2(p_2)\ , \qquad (4.9)$$

and

$$\hat{J}^{\mu}(p'D',pD)=(2\pi)^3\frac{E(p_2)}{M}\delta^3(p_2-p_2')\mathbf{j}_1^{\mu}(p_1',p_1) \qquad (4.10a)$$

$$+\mathbf{j}_2^{\mu}(p_2',p_2'-q)S_2(p_2'-q)V(p'+\tfrac{1}{2}q,p;D) \qquad (4.10b)$$

$$+V(p',p-\tfrac{1}{2}q;D')S_2(p_2+q)\mathbf{j}_2^{\mu}(p_2+q,p_2) \qquad (4.10c)$$

$$+J_V^{\mu}(p'D',pD)\ , \qquad (4.10d)$$

where the $\mathbf{j}_i^{\mu}$ and J_V^{μ} were defined in Sec. II. The four diagrams [(a)–(d)] which make up (4.10) are shown in Fig. 9. Diagrams (b) and (c) arise when the virtual photon is coupled to particle 2. Since it is impossible for both the initial and final nucleon to be on shell, two diagrams must be included corresponding to the two terms which arise when the integration contour over the relative energy is closed around the positive energy poles of particle 2. The way in which these terms originate is shown schematically in Fig. 10.

There are special cases where it can be shown that the two terms (b) and (c) in (4.10) do not arise, or are not necessary for current conservation. For example, if one particle is chargeless and does not couple to the photon (even for $q^2\neq 0$), we may choose this for the on-shell particle, and terms (b) and (c) will not contribute. For identical particles, there may also be methods of getting along without these terms. In this case, it would seem that it should be possible to eliminate these terms in favor of counting the first term (4.10a) twice. The issue is of practical importance, since each of these terms has a singularity (which, however, cancels in the sum), and they spoil some of the simplicity of the spectator nucleon approach. A general discussion of the simplest way to define the current operator in the spectator nucleon approach will be taken up elsewhere. What we will show here is that the two terms (b) and (c) are sufficient to ensure current conservation in the general case.

Before presenting the algebraic proof, it is useful to see in general how current conservation works in this formalism. First, note that the relation (3.3) still holds because this is unaffected by whether or not any of the external particles are on shell. Second, note that when a particle is on shell, application of the operator $S^{-1}(p)$ which arises from the WT identity gives zero. This means that when q_{μ} is contracted into terms like those of Figs. 9(b) and (c), only contributions from the off-shell propagation of particle 2 will survive, and these terms are cancelled by two of the four terms which arise from Eq. (3.3) [Fig. 9(d)]. The result is that only terms coming from propagators connected directly to particle 1 survive; the elimination of these terms will finally require the use of the wave equation.

Algebraically, the last three terms of (4.10) give

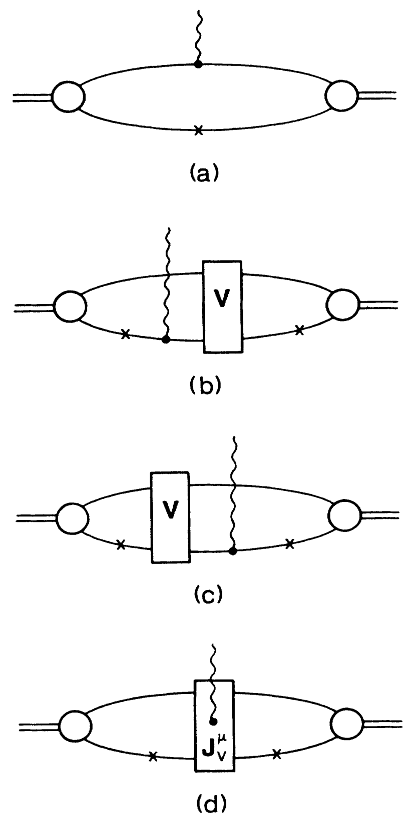

FIG. 9. Diagrammatic representation of the four terms in Eq. (4.10). The small $\times$'s mean that the particle is on the mass shell; the internal integrations are therefore over 3-momenta only and propagations of mass-shell particles are replaced by two component spin sums over matrix elements constructed from mass-shell positive energy spinnors, as in (4.3).

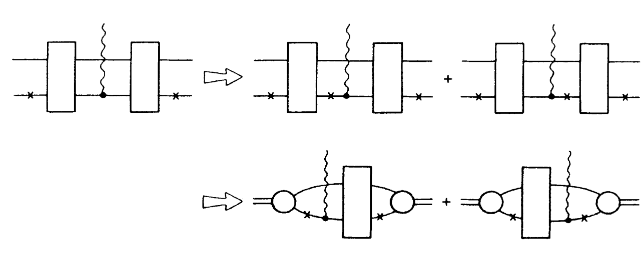

FIG. 10. The figure shows the origin of the two terms (b) and (c) in Eq. (4.10) and illustrated in Figs. 9(b) and (c).

$$q_\mu \langle J^\mu \rangle_{(b)-(d)} = \int \frac{d^3p\, d^3p'}{(2\pi)^6} \frac{M^2}{E(p_2)E(p_2')} \bar{\phi}_f(p',D')\bar{u}_2(p_2')$$
$$\times \{ + [S_2^{-1}(p_2') - S_2^{-1}(p_2'-q)] S_2(p_2'-q) \tfrac{1}{2}[1+\tau_2^3] V(p'+\tfrac{1}{2}q, p; D)$$
$$+ V(p', p-\tfrac{1}{2}q; D')\tfrac{1}{2}[1+\tau_2^3] S_2(p_2+q)[S_2^{-1}(p_2+q) - S_2^{-1}(p_2)] + q_\mu J_V^\mu \} u_2(p_2)\phi_i(p,D)$$
$$= \int \frac{d^3p\, d^3p'}{(2\pi)^6} \frac{M^2}{E(p_2)E(p_2')} \bar{\phi}_f(p',D')\bar{u}_2(p_2')$$
$$\times \{ + \tfrac{1}{2}[1+\tau_1^3] V(p'-\tfrac{1}{2}q, p; D) - V(p', p+\tfrac{1}{2}q; D')\tfrac{1}{2}[1+\tau_1^3] \} u_2(p_2)\phi_i(p,D)$$
$$= \int \frac{d^3p\, d^3p'}{(2\pi)^6} \frac{M^2}{E(p_2)E(p_2')} \phi_f(p',D') \{ + \tfrac{1}{2}[1+\tau_1^3] \bar{V}(p'-\tfrac{1}{2}q, p; D) - \bar{V}(p', p+\tfrac{1}{2}q; D')\tfrac{1}{2}[1+\tau_1^3] \} \phi_i(p,D) \,, \tag{4.11}$$

where the result (3.3) was used to evaluate $q_\mu J_V^\mu$ in the general case; note that the $[1+\tau_2^3]$ terms cancel. Finally, the first term in (4.10) simplifies using the wave equation (4.6),

$$q_\mu \langle J^\mu \rangle_{(a)} = \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(p_2)} \bar{\phi}_f(p',D')[S_1^{-1}(p_1+q) - S_1^{-1}(p_1)]\tfrac{1}{2}[1+\tau_1^3]\phi_i(p,D)$$
$$= \int \frac{d^3p\, d^3p'}{(2\pi)^6} \frac{M^2}{E(p_2)E(p_2')} \bar{\phi}_f(p',D') \{ \bar{V}(p', p+\tfrac{1}{2}q; D')\tfrac{1}{2}[1+\tau_1^3] - \tfrac{1}{2}[1+\tau_1^3]\bar{V}(p'-\tfrac{1}{2}q, p; D) \} \phi_i(p,D) \,. \tag{4.12}$$

This term, when added to (4.11), gives zero, proving that current is conserved.

We remind the reader that the algebraic proof of conservation first presented holds for an arbitrary interaction, providing Eq. (3.3) holds for J_V^μ. While we proved this for irreducible kernels in the BS formalism, the irreducible kernel in the one-particle on-shell formalism has extra terms, which have been classified and discussed extensively elsewhere.[16] It is straightforward to see that these extra terms also satisfy (3.3); they differ only in having some of the internal particles on shell, and the use of the WT identities is not affected by this restriction (any extra terms generated are zero). Hence the proof of current conservation also holds for a general irreducible kernel in this quasipotential method. It is not clear to use whether the proof can be given as easily for other quasipotential approaches.[17]

V. EXCHANGE CURRENTS WITH CONTACT TERMS

The general form for the exchange current operator in the ladder approximation (2.24) applies even when the interaction involves derivative couplings, and when there is hadronic structure (form factors). Whenever there are derivative couplings in the meson-nucleon interactions, there will be additional purely transverse terms J_{TV}^μ in the exchange current operator which depend on the interaction Lagrangian and which are not zero. While these cannot be determined by the continuity equation, it is nevertheless quite straightforward to construct them from the contact (or seagull) current operators obtained from minimal substitution of the em field operator in the derivative terms in the Lagrangian. We shall illustrate this by considering single pion exchange with pseudovector coupling, which satisfies the condition of chiral invariance, and single ρ-meson exchange with tensor coupling. Modifications required by hadronic structure will be discussed in the next section.

The pseudovector πNN coupling Lagrangian is

$$L = \frac{g}{2M}\bar{\psi}\gamma^\nu\gamma^5\partial_\nu\boldsymbol{\phi}\cdot\boldsymbol{\tau}\psi \,, \tag{5.1}$$

where $\psi, \bar{\psi}$ are the nucleon fields and $\boldsymbol{\phi}$ the isovector pion field. By coupling the em field minimally through the derivative in this Lagrangian one obtains the point $\gamma\pi$NN coupling[2]

$$L' = i\frac{eg}{2M}\bar{\psi}\gamma^\nu\gamma^5\tfrac{1}{2}[\tau^3, \boldsymbol{\phi}\cdot\boldsymbol{\tau}]\psi A_\nu \tag{5.2a}$$

where A_ν is the em field vector. This point coupling Lagrangian will generate a contact term of the form

$$j_{\pi\mathrm{cont}}^{i\mu} = \frac{g}{2M}\epsilon^{ij3}\tau^j\gamma^\mu\gamma^5 \,. \tag{5.2b}$$

The total exchange current operator will, in this case, have the form (2.24) with the interaction having the form

$$v(k) = \left[\frac{g}{2M}\right]^2 [(\gamma\cdot k)\gamma^5]_1[(\gamma\cdot k)\gamma^5]_2\Delta(k) \,, \tag{5.3}$$

with the additional transverse current operator [in the notation of Fig. 3(c)]

$$J_{T\pi}^\mu = i(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^3 \left[\frac{g}{2M}\right]^2 \left\{ g^{\mu\nu} - \frac{K^\mu q^\nu}{K\cdot q} \right\}$$
$$\times \{ (k'\cdot\gamma\gamma^5)_1(\gamma_\nu\gamma^5)_2\Delta(k')$$
$$+ (k\cdot\gamma\gamma^5)_2(\gamma_\nu\gamma^5)_1\Delta(k) \} \,. \tag{5.4}$$

As a final example we consider the ρ-meson exchange interaction, taking the ρNN coupling to be

$$L_{\rho NN}=-g_\rho\bar{\psi}\left\{\gamma^\mu-\frac{\kappa}{2M}\sigma^{\mu\nu}\partial_\nu\right\}\rho_\mu\cdot\tau\psi\ , \tag{5.5}$$

where g_ρ is the vector and κ the tensor coupling constant and ρ_μ is the isovector vector meson field. The corresponding ρ-meson exchange interaction is, in the notation of (2.23),

$$v(k)=-g_\rho^2\Delta_\rho(k)\left\{\gamma_1\cdot\gamma_2-\frac{1}{m_\rho^2}\gamma_1\cdot k\gamma_2\cdot k+i\frac{\kappa}{2M}(\sigma_1^{\mu\nu}\gamma_{2\mu}k_\nu-\sigma_2^{\mu\nu}\gamma_{1\mu}k_\nu)+\frac{\kappa^2}{4M^2}\sigma_1^{\mu\alpha}k_\alpha\sigma_{2\mu\beta}k^\beta\right\}\ . \tag{5.6}$$

Here the propagator is defined as

$$\Delta_\rho^{\mu\nu}(k)=P^{\mu\nu}(k)\Delta_\rho(k)\ , \tag{5.7a}$$

with

$$\Delta_\rho^{-1}(k)=k^2-m_\rho^2\ , \tag{5.7b}$$

$$P^{\mu\nu}(k)=g^{\mu\nu}-\frac{k^\mu k^\nu}{m_\rho^2}\ , \tag{5.7c}$$

with m_ρ being the mass of the ρ meson. Note that the term proportional to m_ρ^{-2} in the interaction vanishes for on shell nucleons.

The electromagnetic couplings that are necessary for the gauge invariant construction of the ρ-meson exchange current operator are obtained by minimal substitution of the em field A_μ in the derivative tensor coupling term in (5.5) and in the free ρ-meson Lagrangian

$$L_\rho=-\tfrac{1}{4}\mathbf{F}_{\mu\nu}\cdot\mathbf{F}^{\mu\nu}-\tfrac{1}{2}m_\rho^2\rho^\mu\cdot\rho_\mu\ . \tag{5.8}$$

Here the ρ-meson field tensor is defined as

$$\mathbf{F}^{\mu\nu}=\partial^\nu\rho^\mu-\partial^\mu\rho^\nu\ . \tag{5.9}$$

Minimal substitution in the ρN coupling (5.5) yields the contact $\rho\gamma$NN coupling

$$L_\rho'=ie\frac{g_\rho\kappa}{2M}\bar{\psi}\sigma^{\mu\nu}A_\nu\tfrac{1}{2}[\tau^3,\rho_\mu\cdot\tau]\psi\ , \tag{5.10}$$

and, in the free ρ-meson Lagrangian (5.8), the $\gamma\rho$-coupling interaction

$$L_{\gamma\rho}'=eA_\mu\epsilon^{ij3}\rho_\nu^i(\partial^\mu\rho^{j\nu}-\partial^\nu\rho^{j\mu})\ . \tag{5.11}$$

In the notation of Fig. 3(c) with $K=k+k'$ this leads to the following point $\gamma\rho\rho$ vertex:

$$\begin{aligned}&j_\rho^{ij,\alpha\beta,\mu}(k',k)=+i\epsilon^{ij3}\Gamma_\rho^{\alpha\beta,\mu}(k',k)\ ,\\&\Gamma_\rho^{\alpha\beta,\mu}(k',k)=g^{\alpha\beta}K^\mu-k^\alpha g^{\mu\beta}-g^{\alpha\mu}k'^\beta\ ,\end{aligned} \tag{5.12}$$

where i,α are isospin index and 4-vector spin index of the outgoing ρ, and j,β are the corresponding quantities for the incoming ρ. Γ_ρ satisfies the WT identity

$$q_\mu\Gamma_\rho^{\alpha\beta,\mu}(k',k)=R^{\alpha\beta}(k')-R^{\alpha\beta}(k)\ , \tag{5.13}$$

where

$$R^{\alpha\beta}(k)=[\Delta_\rho^{\alpha\beta}(k)]^{-1}=g^{\alpha\beta}(k^2-m_\rho^2)-k^\alpha k^\beta\ . \tag{5.14}$$

With these definitions, it can be shown that the total ρ-meson exchange current takes the form (2.24) with the interaction (5.6) and an additional transverse term $J_{T\rho}^\mu$ of the form

$$J_{T\rho}^\mu=i(\tau_1\times\tau_2)^3g_\rho^2\left[I_1^\mu+\frac{\kappa}{2M}I_2^\mu+\left[\frac{\kappa}{2M}\right]^2I_3^\mu\right]\ , \tag{5.15}$$

where

$$\begin{aligned}I_1^\mu=&-\Delta_\rho(k)\Delta_\rho(k')\left\{\gamma_1^\mu[\gamma_2\cdot k'-m_\rho^{-2}(k\cdot k')\gamma_2\cdot k]+m_\rho^{-2}k'^\mu\gamma_1^\alpha\gamma_2^\beta(k_\alpha'k_\beta'-k_\alpha k_\beta)\frac{\Delta^{-1}(k)}{k\cdot q}\right\}+(1\leftrightarrow2\text{ and }k'\leftrightarrow k)\ ,\\
I_2^\mu=&+i\Delta_\rho(k)\left[\frac{K^\mu}{K\cdot q}\sigma_1^{\alpha\beta}q_\beta-\sigma_1^{\alpha\mu}\right]\gamma_{2\alpha}+i\Delta_\rho(k)\Delta_\rho(k')\{\sigma_1^{\alpha\beta}q_\alpha k_\beta'[\gamma_2^\mu+m_\rho^{-2}k'^\mu\gamma_2\cdot k]-i\sigma_1^{\mu\beta}k_\beta'[\gamma_2\cdot q+m_\rho^{-2}(q\cdot k')\gamma_2\cdot k]\\
&+i\Delta_\rho(k)\sigma_1^{\mu\beta}q_\beta m_\rho^{-2}(\gamma_2\cdot k)\}-(1\leftrightarrow2\text{ and }k'\leftrightarrow k)\ ,\\
I_3^\mu=&\Delta_\rho(k)\left[\sigma_1^{\mu\alpha}-\sigma_1^{\beta\alpha}q_\beta\frac{K^\mu}{K\cdot q}\right]\sigma_{2\alpha\beta}k^\beta-\Delta_\rho(k)\Delta_\rho(k')\sigma_1^{\mu\alpha}k_\alpha'\sigma_2^{\beta\nu}k_\beta'k_\nu+(1\leftrightarrow2\text{ and }k'\leftrightarrow k)\ .\end{aligned} \tag{5.16}$$

We leave it to the reader to verify that $q_\mu I_i^\mu=0$.

Finally, we note that it is not difficult to relax the restrictions on the magnetic and quadrupole moments of the ρ implied by (5.12). If two purely transverse terms are added to (5.12),

$$\Gamma_\rho^{*\alpha\beta,\mu}(k',k)=\Gamma_\rho^{\alpha\beta,\mu}(k',k)-(\mu^*-1)[(g^{\alpha\mu}q^\beta-q^\alpha g^{\mu\beta})]-\frac{2}{m_\rho^2}(Q^*+\mu^*-1)[q^\alpha q^\beta K^\mu-\tfrac{1}{2}(K\cdot q)(q^\alpha g^{\mu\beta}+g^{\alpha\mu}q^\beta)]\ , \tag{5.17}$$

it can be shown[18] that μ^* and Q^* are related to the magnetic moment, μ_ρ, and quadrupole moment Q_ρ of the ρ^+,

$$\mu^* = m_\rho \mu_\rho ,$$
$$Q^* = \tfrac{1}{4} m_\rho^2 Q_\rho . \tag{5.18}$$

(Note that $\mu^*=1$ and $Q^*=0$ is implied if these terms are absent.) Since these are purely transverse, they contribute only to the transverse part of the current.

The relaxation of the conditions $\mu^*=1$ and $Q^*=0$ is associated with internal structure of the ρ meson, and consistent treatment of this structure requires the discussion in the next section.

VI. CURRENT CONSERVATION AND EXCHANGE CURRENTS IN THE PRESENCE OF HADRONIC STRUCTURE

Our discussion so far has been limited to the case where the meson-nucleon and photon-hadron vertices are all pointlike. In this section we will describe how these results can be generalized to the case where electromagnetic form factors are inserted at the photon-hadron vertices, and where strong form factors are used at the meson-nucleon vertices. Our approach will be very general, and we will show that (1) phenomenological strong form factors can be inserted at the meson-hadron vertices without spoiling the general results of the previous sections, provided modifications dictated by the WT identities in the off-shell current operators are made and (2) empirical electromagnetic form factors appropriate for each particle may be used in the current operators. In particular, the latter result means that the form factors used in pion exchange currents can be different from those for the nucleon without violating current conservation. This is important in practical calculations of meson-exchange-current (MEC) effects.

The key to our approach is illustrated in Fig. 11. Here the meson (pion in this example) exchange potential is regularized by a phenomenological form factor $f_\pi(k^2)$, where k is the 4-momentum carried by the meson, and $f_\pi(\mu^2)=1$. In our treatment, we will regard this form factor as a phenomenological self-energy correction, as illustrated in Fig. 11(b), so that the meson propagators used in the previous sections of the paper are modified:

$$\Delta(k) = \frac{f_\pi^2(k^2)}{k^2-\mu^2} = [k^2-\mu^2+\Pi(k^2)]^{-1} , \tag{6.1}$$

where Π is introduced for convenience only, and

$$\Pi(k^2) = \left[\frac{1}{f_\pi^2(k^2)} - 1 \right] (k^2-\mu^2) . \tag{6.2}$$

The normalization of f_π ensures that

$$\Pi(\mu^2) = \frac{\partial}{\partial k^2} \Pi(k^2) \bigg|_{k^2=\mu^2} = 0 , \tag{6.3}$$

so that Δ is suitably renormalized.

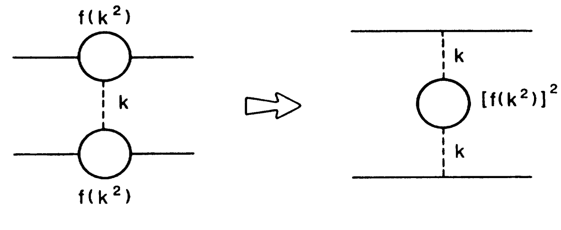

FIG. 11. Two equivalent ways of viewing the strong form factors at the meson-NN vertices. (a) Form factor $f(k^2)$ at each vertex with bare propagators, and (b) point interactions with a phenomenological self energy.

The current operators for the meson will then be modified in two ways. First, we will introduce phenomenological electromagnetic form factor(s), and second, we will require the WT identity to hold with the new propagators Δ given in Eq. (6.1). Using the pion as an example of the general approach, we introduce structure into the meson electromagnetic vertex,

$$j_\pi^{ij\mu}(k',k) = -i\Gamma_\pi^\mu(k',k)\epsilon^{ij3} , \tag{6.4a}$$

and define a reduced vertex function Γ_R by

$$\Gamma_\pi^\mu(k',k) = f_\pi f'_\pi \Gamma_{\pi R}^\mu(k',k) , \tag{6.4b}$$

where $f_\pi = f_\pi(k^2)$ and $f'_\pi = f_\pi(k'^2)$. It is the reduced vertex function which satisfies the WT identity with the modified propagator (6.1) as illustrated in Fig. 12. For the pion, the most general form such a vertex can take is

$$\Gamma_{\pi R}^\mu(k',k) = A(q^2,k'^2,k^2) \left[K^\mu - \frac{K\cdot q q^\mu}{q^2} \right] + B(q^2,k'^2,k^2) K^\mu , \tag{6.5}$$

where, to avoid kinematic singularities, we require that at $q^2=0$

$$A(0,k'^2,k^2) = 0 , \tag{6.6a}$$

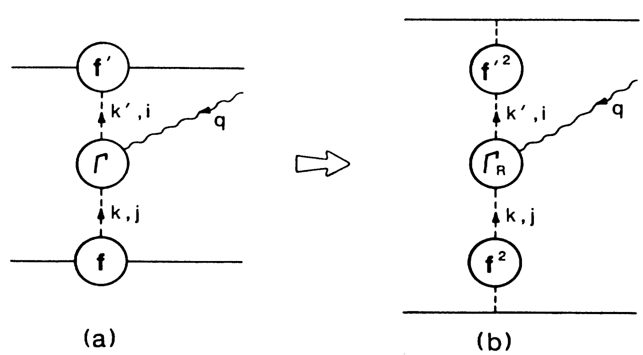

FIG. 12. (a) The normal current operator with form factors at the meson-NN vertices. (b) Reduced current operator with phenomenological self energies and point meson-NN vertices. The reduced current operator is defined so that both expressions are identical.

and to reproduce the empirical pion form factor,

$$A(q^2,\mu^2,\mu^2)+B(q^2,\mu^2,\mu^2)=F_\pi(q^2)\ . \tag{6.6b}$$

B can be uniquely determined from the WT identity (2.17), and a simple calculation gives

$$B(q^2,k'^2,k^2)=B(k'^2,k^2)=1+\frac{\Pi(k'^2)-\Pi(k^2)}{k'^2-k^2}\ , \tag{6.7}$$

which is finite at $k'^2=k^2$ and satisfies

$$B(\mu^2,\mu^2)=1\ . \tag{6.8}$$

Hence, from (6.6b)

$$A(q^2,\mu^2,\mu^2)=F_\pi(q^2)-1\ , \tag{6.9}$$

which is consistent with (6.6a). Except for this condition A is arbitrary and the most general form of A can be written

$$A(q^2,k'^2,k^2)=[F_\pi(q^2)-1]\gamma(q^2,k'^2,k^2)\ , \tag{6.10}$$

where γ is any function free of kinematic singularities and is symmetric in k^2 and k'^2, which satisfies

$$\gamma(q^2,\mu^2,\mu^2)=1\ . \tag{6.11a}$$

The fact that the transverse part of the current operator cannot be uniquely determined is not a surprise, but it has been customary in MEC calculations to make the simplest assumptions about the form of the current operators. The importance of MEC, and the possibility that the underlying quark structure of mesons and nucleons could well lead to modifications of the current operators when the mesons are off shell, suggests that this arbitrariness should be taken into account in future studies, and that either data or quark calculations are needed to fix the off-shell function γ.

In order to parametrize γ realistically, we note that it is the combination of $A+B$ which is seen in electron scattering (since the q_μ term is zero when contracted into the conserved electron current) and since B does not depend on q^2 it must be cancelled if the total current operator is to go to zero at large q^2. This requires that γ approach B at large q^2

$$\gamma(q^2,k'^2,k^2)\to B(k'^2,k^2),\quad q^2\to\infty\ . \tag{6.11b}$$

A simple form which satisfied both of these conditions (6.11) is

$$\gamma(q^2k'^2,k^2)=B(k'^2,k^2)\left[\frac{1-F_0(q^2)}{1-F_\pi(q^2)}\right]+1-\frac{1-F_0(q^2)}{1-F_\pi(q^2)}\ , \tag{6.12}$$

where F_0 approaches zero as q^2 approaches infinity and is normalized to unity at $q^2=0$, but is otherwise arbitrary. With this choice, $\Gamma^\mu_{\pi R}$ becomes

$$\Gamma^\mu_{\pi R}(q^2,k'^2,k^2)=\left\{F_\pi(q^2)-F_0(q^2)+F_0(q^2)\left[\frac{\Delta^{-1}(k')-\Delta^{-1}(k)}{k'^2-k^2}\right]\right\}\left[K^\mu-\frac{K\cdot qq^\mu}{q^2}\right]+[\Delta^{-1}(k')-\Delta^{-1}(k)]\frac{q^\mu}{q^2}\ . \tag{6.13}$$

Note that this form gives $F_\pi(q^2)$ when both mesons are on shell, and that, if $k'^2=k^2$,

$$\Gamma^\mu_{\pi R}(q^2,k^2,k^2)=\left\{F_\pi(q^2)+F_0(q^2)\left[\frac{d\Delta^{-1}(k)}{dk^2}-1\right]\right\}K^\mu+q^\mu\ \text{terms}\ . \tag{6.14}$$

However, with the strong form factors

$$\frac{d\Delta^{-1}(k)}{dk^2}-1<0\ , \tag{6.15}$$

if $k^2<\mu^2$ (which is the normal case in nuclei), and hence F_0 subtracts from F_π, making the effective pion form factor decrease more rapidly with q^2. This is what would be expected if the pion were to become larger in the nuclear medium. Note that in this model the size of this effect depends on how far the pions are off mass shell.

Using (6.13) we can cast our new pion MEC into the form (2.24) with $v(k)$ defined by Eqs. (2.10), (2.23), and (6.1) and with the transverse term given by

$$J^\mu_{T\pi}=i(\tau_1\times\tau_2)^3g^2\gamma_1^5\gamma_2^5\left[K^\mu-\frac{K\cdot qq^\mu}{q^2}\right]\times\left\{[F_\pi(q^2)-F_0(q^2)]\Delta(k)\Delta(k')+[F_0(q^2)-1]\frac{\Delta(k)-\Delta(k')}{k'^2-k^2}\right\}\ . \tag{6.16}$$

Note that the pion form factors occur only in the purely transverse part of the exchange current, and hence are in no way constrained by the requirements of current conservation.

Structure can also be included in the case of pseudovector coupling. Here a new complication arises—how should structure be included in the $\gamma\pi$NN contact term? One way, which is probably not unique, is to generalize (5.2b) and introduce a reduced contact term

$$j^{i\mu}_{\pi\mathrm{cont}}(k)=\epsilon^{ij3}\tau^j f_\pi(k)\Gamma^\mu_{\pi c}(q) \tag{6.17}$$

where k is the 4-momentum of the incoming pion, q is the 4-momentum of the incoming virtual photon, and

$$\Gamma^\mu_{\pi c}(q)=\frac{g}{2M}\left\{\gamma^\mu\gamma^5+[F_c(q^2)-1]\gamma_c \times\left[\gamma^\mu-\frac{\gamma\cdot q}{q^2}q^\mu\right]\gamma^5\right\}. \tag{6.18}$$

When used with a conserved current, the q^μ term in the transverse part vanishes, and if $\gamma_c=1$ as $q^2\to\infty$, the entire term goes to zero as $F_c(q^2)$, the arbitrary form factor of the contact term. The first term will play the same role as it did in the case without structure which was treated in Sec. V; the second term will contribute an additional transverse term to (6.16).

Next we turn to the ρ-meson exchange current. Here the same techniques can be used, and the additional degrees of freedom associated with the spin 1 nature of the ρ offer more choices for construction of the current operators, suggesting that the ρ exchange current will be less effectively constrained by current conservation.

If a single strong form factor is introduced at the ρNN vertex, $f_\rho(k^2)$, leading to a phenomenological self-energy, the longitudinal part of the $\gamma\rho\rho$ vertex will be modified through the WT identity. Defining the reduced $\gamma\rho\rho$ vertex,

$$\Gamma^{\alpha\beta,\mu}_\rho(k',k)=f_\rho f'_\rho\Gamma^{\alpha\beta,\mu}_{R\rho}(k',k) , \tag{6.19}$$

we find

$$\begin{aligned}\Gamma^{\alpha\beta,\mu}_{R\rho}(k',k)=&\ g^{\alpha\beta}K^\mu B_\rho(k',k)\\ &-[g^{\alpha\mu}k'^\beta+k^\alpha g^{\mu\beta}]C^+_\rho(k',k)\\ &-[k'^\alpha k'^\beta+k^\alpha k^\beta]C^-_\rho(k',k)\frac{K^\mu}{K\cdot q} ,\end{aligned} \tag{6.20}$$

where

$$\begin{aligned}B_\rho(k'k)&=1+\frac{\Pi_\rho(k'^2)-\Pi_\rho(k^2)}{k'^2-k^2} ,\\ C^\pm_\rho(k',k)&=\frac{1}{2}\left[\frac{1}{f'^2}\pm\frac{1}{f^2}\right] .\end{aligned} \tag{6.21}$$

Up to arbitrary purely transverse terms, this result seems to be unique if terms with kinematic singularities are excluded.

Transverse terms which contain electromagnetic form factors can now be added. One form, which satisfies all of the restrictions discussed in the beginning of this section, is

$$\begin{aligned}\Gamma^{*\alpha\beta,\mu}_{R\rho}(k',k)=&\ \Gamma^{\alpha\beta,\mu}_{R\rho}(k',k)+[F_1(q^2)-1]\gamma_1 g^{\alpha\beta}\left[K^\mu-\frac{K\cdot q}{q^2}q^\mu\right]\\ &-[F_2(q^2)-1]\gamma_+\left[k^\alpha g^{\mu\beta}+g^{\alpha\mu}k'^\beta+\frac{q^\mu}{q^2}(k^\alpha k^\beta-k'^\alpha k'^\beta)\right]\\ &-F_2(q^2)(\mu^*-1)[g^{\alpha\mu}q^\beta-q^\alpha g^{\mu\beta}]-[F_4(q^2)-1]\gamma_-(k'^\alpha k'^\beta+k^\alpha k^\beta)\left[\frac{K^\mu}{K\cdot q}-\frac{q^\mu}{q^2}\right]\\ &-\frac{2F_3(q^2)}{m^2_\rho}[q^\alpha q^\beta K^\mu-\tfrac{1}{2}(K\cdot q)(q^\alpha g^{\mu\beta}+g^{\alpha\mu}q^\beta)] ,\end{aligned} \tag{6.22}$$

where γ_1 and $\gamma_\pm$ are arbitrary functions which are unity on shell and which approach as $q^2\to\infty$

$$\begin{aligned}&\gamma_1\to B_\rho ,\\ &\gamma_\pm\to C^\pm_\rho .\end{aligned} \tag{6.23}$$

If these conditions are satisfied, $\Gamma^{*\mu}_{R\rho}\to 0$ as $q^2\to\infty$, and when both ρ's are on shell, it reduces to

$$\begin{aligned}\xi'^*_\alpha\Gamma^{*\alpha\beta,\mu}_{R\rho}\xi_\beta=&\ F_1(q^2)\xi'^*\cdot\xi K^\mu\\ &+\mu^*F_2(q^2)[(\xi'^*\cdot q)\xi^\mu-\xi'^{*\mu}(\xi\cdot q)]\\ &-\frac{2F_3(q^2)}{m^2_\rho}(\xi'^*\cdot q)(\xi\cdot q)K^\mu ,\end{aligned} \tag{6.24}$$

which is the correct form provided $F_1(0)=F_2(0)=1$ and $F_3(0)=Q^*+\mu^*-1$.

The reduced ρ contact term, defined as for the pion in Eq. (6.17), can be taken to be

$$\Gamma^{\nu\mu}_{\rho c}(q)=\frac{g_\rho\kappa}{2M}\left\{\sigma^{\nu\mu}+[F_c(q^2)-1]\left[\sigma^{\nu\mu}-\sigma^{\nu\alpha}\frac{q_\alpha q^\mu}{q^2}\right]\right\} , \tag{6.25}$$

where ν and μ are the ρ and γ vector indices, respectively.

We leave it to the reader to confirm (2.24) for this ρ exchange current and to obtain the explicit form for the transverse current.

We conclude this section by discussing the off-shell nucleon current briefly. Clearly the method developed above can be applied to this case also. The current operator for a single nucleon with internal structure is usually written in the form

$$\mathbf{j}^\mu(p',p)=\mathbf{F}_1\gamma^\mu+i\frac{\sigma^{\mu\nu}}{2M}q_\nu\mathbf{F}_2 . \tag{6.26}$$

The isospin structure of the two form factors F_1 and F_2 is

$$\mathbf{F}_{1,2}=\tfrac{1}{2}[F_{1,2}^S(q)+F_{1,2}^V(q)\tau_3] \ . \tag{6.27}$$

The current operator (6.26) does not satisfy the WT identity (2.17). A simple generalization of (6.26) which does satisfy the identity is

$$\mathbf{j}^\mu(p',p)=\tfrac{1}{2}\{F_1^S(q)-1+[F_1^V(q)-1]\tau_3\}\left[\gamma^\mu-\frac{q^\mu(\gamma\cdot q)}{q^2}\right] + \frac{1+\tau_3}{2}\gamma^\mu-\frac{i}{2M}\sigma^{\mu\nu}q_\nu\mathbf{F}_2 \ . \tag{6.28}$$

With this generalization the terms that involve the form factors drop out from the current divergence (2.17) and hence the continuity equation conditions derived for the exchange current in Secs. II and III still apply. Furthermore, on shell (and even off shell when used with the conserved electron current so that the q^μ term vanishes) this reduces to the usual form (6.26).

VII. SUMMARY AND DISCUSSION

There are three principal results of this paper.

(1) A general method for constructing matrix elements of the electromagnetic current of a relativistic two body system, which ensures that the current is conserved, is given. This method requires that:

(a) All relativistic one body current operators satisfy the appropriate Wark-Takahashi identity,

(b) the initial and final interacting two body system satisfy the same relativistic equation with the same interaction kernel, which is assumed to be a known sum of relativistic Feynman diagrams, and

(c) the current operator includes, in addition to the one body current operators, interaction currents built up from the interaction kernels by coupling the virtual photon to all possible nucleon and meson lines *inside* the kernel.

We have demonstrated that this procedure works for the Bethe-Salpeter formalism, where our principle results are Eqs. (2.12), (2.13), and (3.3), and for the formalism where the spectator is on shell, Eqs. (4.8) and (4.10).

(2) The method described above places *no constraint* on the introduction of phenomenological electromagnetic form factors for the hadrons. In particular, different form factors can be used for the nucleon, pion, and ρ meson, and arbitrary magnetic and quadrupole form factors can be used in ρ-meson exchange currents. The result, displayed in Eqs. (2.24), (6.16), and (6.22) comes from the fact that one body currents can be constructed in which all electromagnetic form factors occur in purely transverse terms, which are unconstrained by current conservation.

(3) The general method also permits the introduction of strong hadronic form factors at all meson-NN vertices. Such form factors require modification of the current operators through constraints imposed by the WT identities, but our discussion shows that this can be done in all cases of interest.

The overall conclusion is that it is possible to carry out consistent relativistic calculations of electromagnetic interactions of two nucleon systems which conserve current and which include necessary phenomenological form factors due to the internal hadronic structure. We have found a way to separate the problem of internal hadronic structure from the problem of the dynamics of relativistic meson theory. The hadronic structure can be calculated from the underlying quark degrees of freedom, and the resulting form factors and coupling constants inserted in a relativistic meson theory which conserves current. We make no claim that this procedure will give correct answers, only that it can be carried out consistently. This result is of importance to the program planned for the Continuous Electron Beam Accelerator Facility.

The role of WT identities in obtaining conserved currents for relativistic two body systems has been previously pointed out by Bentz,[19] who obtains two body ward identities for the two body system directly from field theory. This derivation, which is complementary to our diagrammatic approach, leads to relations similar to (2.19) and (3.3). [These are Eqs. (2.19b) and (2.29) in Ref. 19.] Bentz also discusses PCAC, but does not discuss the introduction of hadronic structure. The presence of ambiguities in the definition of off shell currents has been previously emphasized by deForest,[20] who also pointed out that, if one is not careful, kinematic singularities can arise from any q^μ terms added to current operators to preserve current conservation. In our work we have been careful to eliminate these kinematic singularities.

It should be possible, by taking the nonrelativistic limit, to use the results of this paper to resolve the outstanding issue as to whether G_E or F_1 is the correct form factor to use in pion exchange calculations. We have not done this, but we note that our results show that different form factors are permitted for the pion and nucleon, and $\gamma\pi$NN contact term, so that the usual nonrelativistic pion exchange current, which is a sum of these different pieces, is probably not correctly described by a single form factor. Furthermore, the freedom to adjust the off-shell form factors, as discussed in Sec. VI, shows that a unique answer within the framework of relativistic meson theory (where hadronic structure is treated phenomenologically) is not possible. We are inclined to think that the question of F_1 versus G_E is transcended by the ambiguities and inconsistencies inherent in nonrelativistic calculations which treat the NN dynamics and meson interaction currents as completely independent quantities. What is needed is a fully relativistic calculation of both the NN interaction and the two nucleon electromagnetic current which uses the *same* strong meson-NN form factors and hadron electromagnetic form factors *consistently* throughout. Such a program is currently being developed.

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