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Quark compound bag model for NN scattering up to 1 GeV

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A quark compound bag model has been constructed to describe NN *s*-wave scattering up to 1 GeV. The model contains a vertex interaction $H_{D\leftrightarrow NN}$ for describing the excitation of a confined six-quark bag state, and a meson-exchange interaction obtained from modifying the phenomenological core of the Paris potential. Explicit formalisms and numerical results are presented to reveal the role of the bag excitation mechanism in determining the relative wave function and the *P* and *S* matrices of NN scattering. We explore the merit as well as the shortcomings of the quark compound bag model developed by the Institute of Theoretical and Experimental Physics group. It is shown that the parameters of the vertex interaction $H_{D\leftrightarrow NN}$ can be more rigorously determined from the data if the concept of the chiral/cloudy bag model is used to justify the presence of the background meson-exchange interaction inside bag excitation region. The application of the model to the study of quark degrees of freedom in nuclei is discussed.

I. INTRODUCTION

The development of quantum chromodynamics (QCD) has motivated many theoretical investigations of quark degrees of freedom in nuclei. In considering the simplest two-nucleon system, the focus has been on the calculation of the short-range part of the nuclear force from various QCD-motivated models of multiquark systems. This has been reasonably successful in either the nonrelativistic quark potential model[1–8] or the relativistic bag model.[9–11] For nuclear studies, we need to combine the resulting short-range quark picture and the well-studied meson-exchange mechanisms to construct a model which can quantitatively describe the NN data. So far the focus has been on the development of models for describing low energy NN data. In this work we report the progress we have made in extending this effort to the intermediate energy region.

Contrary to the situation in the low energy region (below the pion production threshold), the meson-exchange model of nuclear force has encountered difficulties[12–21] in describing the data of intermediate energy NN and πd reactions, particularly the data of polarization observables. The results reported in Ref. 22 have suggested that the problem could be due to the conventional phenomenological parametrization of the short-range part of the baryon-baryon (BB) interaction in terms of meson–baryon-baryon form factors or a convenient local form in coordinate space. Motivated by the success of the MIT bag model[23] and the subsequent application of the model in the *P*-matrix analysis[24–27] of NN scattering, Lee and Matsuyama[22,28] suggested that a possible way to resolve the problem is to describe a part, *but not all*, of the short-range baryon-baryon dynamics by a vertex interaction $H_{D\leftrightarrow BB}$, where BB can be a NN, NΔ, or $\Delta\Delta$ state and D is identified as a six-quark MIT bag state. They have also developed[28] a unitary πNN scattering theory which allows a systematic study of the effect of the bag excitation mechanism $H_{D\leftrightarrow BB}$ on all NN and πd reactions. To explore the dynamical consequence of the vertex interaction $H_{D\rightarrow NN}$, we will carry out a detailed analysis of NN *s*-wave scattering up to 1 GeV. A simplification of this study results from the small inelasticity in this NN channel, hence justifying the neglect of Δ excitation and the associated pion production. In this approximation it is straightforward to see that the operator formulation presented in Sec. IV of Ref. 28 [namely its Eqs. (4.12)] is completely equivalent to the quark compound bag (QCB) model proposed earlier by Simonov.[29] We therefore proceed by first following closely his coordinate-space formulation to explore, in detail, the dynamical content of the QCB model. This study then leads us to develop a new model which is consistent with the general πNN formulation of Ref. 28.

The essence of the QCB model[29] is to postulate that two colliding color singlet three-quark clusters can have a direct transition to a color *singlet* six-quark Bag state when the distance between two clusters is within a narrow range around a distance which roughly characterizes the size of the bag. This notion is formulated by assuming that the bag excitation mechanism $H_{D\leftrightarrow NN}$ is localized at *b* [the simplest form is $\sim\delta(r-b)$], where D is the considered bag state with a mass M_D. As pointed out by Simonov[29] and as will be explicitly shown in this paper, the resulting NN scattering *P* matrix has a pole at $E=M_D$ and hence the theory is consistent with the interpretation by Jaffe and Low.[24] This Hamiltonian formulation of the *P*-matrix interpretation characterizes the essential difference between the QCB model and the other approaches,[30–32] which are also motivated by the bag model. In particular, the usual R-matrix separation of the total wave function into a six-quark component confined within a given radius r_0 and a two-hadron component at $r > r_0$ is not assumed in the QCB model. instead, the relative importance of these two components is determined by the strength of the bag-excitation mechanism $H_{D\leftrightarrow NN}$ relative to the background meson-exchange interaction. It depends strongly on the collision energy, particularly in the energy range where the total collision energy is close to the bag mass. For NN s-wave scattering, considered in this paper, this interesting range is reached at about 600 MeV incident laboratory nucleon energy.

The QCB model has been actively pursued by the ITEP (Institute of Theoretical and Experimental Physics) group[29,33,34] in the last few years. However, as will be demonstrated explicitly in Sec. IV, their fit to the NN data requires a "second" QCB pole which cannot be related unambiguously to the parameters of the bag model or the background meson-exchange mechanism. This uncertainty has also been revealed in a detailed P-matrix analysis of NN data by Bakker *et al.*[27] For nuclear studies it is necessary to introduce more dynamical constraints to resolve this problem. In the language of P-matrix analysis,[24–27] we need to develop a theory for defining the background P matrix, which has been parametrized in terms of "compensation" poles in Refs. 24 and 25, a constant plus an antibound state pole in Ref. 29, a constant plus a distant pole in Ref. 27, and a second QCB pole in Ref. 33. This is achieved in this work.

In Sec. II we give a concise and self-contained presentation of the QCB model. The basic assumptions of the model will be simply stated without recalling their justifications discussed in Ref. 29. We then derive, in Sec. III, formalisms showing analytically how the NN relative wave function and the P and S matrices behave in the intermediate energy region where the total collision energy can be equal to or larger than the bag mass. In Sec. IV we present numerical results to reveal the dynamical content of the QCB model, and show explicitly how the uncertainty arises in the ITEP approach to fit the data. We then show that the parameters of the bag-excitation mechanism $H_{D\leftrightarrow NN}$ can be more rigorously determined from the data if the concept of the chiral-cloudy Bag model is used to allow the presence of the meson-exchange mechanism inside the bag region. Section V is devoted to the discussion of future developments and possible applications of the model in the study of quark degrees of freedom in nuclei.

II. THE QCB MODEL

We start with the assumption that the total wave function of a six-quark system consists of two components

$$|\psi_E\rangle = |\Phi_E\rangle + C_D(E)|D\rangle \ . \tag{2.1}$$

The first component is of the following cluster form in coordinate space:

$$\Phi_E(\mathbf{r}) = \langle \mathbf{r} | \Phi_E \rangle = \chi_E(\mathbf{r}) | \Phi_c(\mathbf{R}_1, \mathbf{R}_2) \rangle \ , \tag{2.2}$$

where

$$|\Phi_c(\mathbf{R}_1,\mathbf{R}_2)\rangle = |\phi(\mathbf{R}_1)\times\phi(\mathbf{R}_2)\rangle \ , \tag{2.3}$$

with

$$\mathbf{R}_1 = \tfrac{1}{3}(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3) \ , \quad \mathbf{R}_2 = \tfrac{1}{3}(\mathbf{r}_4+\mathbf{r}_5+\mathbf{r}_6) \ , \quad \mathbf{r} = \tfrac{1}{2}(\mathbf{R}_1-\mathbf{R}_2) \ . \tag{2.4}$$

Here, r_i denotes the position of each quark, $\phi(\mathbf{R}_i)$ is the internal wave function of a color singlet three-quark system centered at the position $\mathbf{R}_i$, and $\chi_E(\mathbf{r})$ describes the relative motion between two clusters.

The second component $|D\rangle$ of Eq. (2.1) describes a confined six-quark bag state. By projecting the Schrödinger equation onto the cluster component $|\Phi_c\rangle$ and the six-quark bag component $|D\rangle$, we have

$$\langle \Phi_c | (H-E) | \Phi_c \rangle \chi_E(\mathbf{r}) = C_D(E) \langle \Phi_c | (E-H) | D \rangle \tag{2.5a}$$

and

$$\langle D | (H-E) | D \rangle C_D(E) = \langle D | (E-H) | \Phi_c \rangle \chi_E(\mathbf{r}) \ . \tag{2.5b}$$

To proceed, it is necessary to define all of the matrix elements in Eqs. (2.5). We assume that the matrix element of the left-hand side of Eq. (2.5a) is of the form of the usual Schrödinger equation for NN potential[35,36] scattering,

$$\langle \Phi_c | (H-E) | \Phi_c \rangle \chi_E(\mathbf{r}) = \left[-\frac{1}{2\mu}\nabla^2 + V(\mathbf{r},E) - E \right] \chi_E(\mathbf{r}) \ , \tag{2.6}$$

where μ is the reduced mass of two nucleons and $V(\mathbf{r},E)$ is an interaction potential between two clusters. The success of the meson-exchange model of nuclear force suggests that when the distance between two color singlet clusters is larger than a certain length scale d, the interaction potential $V(\mathbf{r},E)$ can be effectively described by meson-exchange mechanisms despite the basic mechanism known to be the QCD quark-gluon processes. We assume that $V(\mathbf{r},E)$ at $r > d$ can be taken from the Paris potential[35] (retaining its original energy dependence). The short-range part of $V(\mathbf{r},E)$ is expected to deviate significantly from the phenomenological core of the Paris potential because of the presence of the excitation of a bag state D in QCB. For simplicity, we assume that

$$V(\mathbf{r},E) = \begin{cases} 0, & r \le d \\ v_{NN}(\mathbf{r},E), & r \ge d \ . \end{cases} \tag{2.7}$$

It is important to note here that the length scale d is a parameter determining the extent to which the basic quark dynamics can be effectively described by the exchange of mesons. In considering the Paris potential, it is justified to set d to be as short as $hc/m_\sigma \simeq 0.5$ fm in order to account for all of the one- and two-pion exchange mechanisms deduced from the chiral πN and $\pi\pi$ dynamics. The choice of d turns out to be crucial in the fit to the data. This will be discussed in detail later.

The matrix element between two six-quark bag states is assumed to be

$$\langle \mathrm{D} \,|\, (H-E) \,|\, \mathrm{D} \rangle = M_\mathrm{D} - E \;. \tag{2.8}$$

The mass M_D is taken from the six-quark bag model calculation in Refs. 37–39. Its value in the ${}^1S_0$ channel is predicted to be about 2200 MeV. It is easy to verify that the uncertainties involved in the bag parameters allow a variation of about $\pm 5\%$. We will use the value 2159 MeV extracted from the P-matrix analysis of Ref. 27, which seems to give the best fit.

The matrix element of $(H-E)$ between a cluster state $|\Phi_c\rangle$ and a confined bag state $|\mathrm{D}\rangle$ is certainly much more difficult to define precisely. In the QCB formulated in Ref. 29, it is assumed to be energy dependent,

$$\begin{aligned} \langle \Phi_c \,|\, H-E \,|\, \mathrm{D} \rangle &= f_\mathrm{D}(E,r) \;, \\ \langle \mathrm{D} \,|\, H-E \,|\, \Phi_c \rangle &= f^*_\mathrm{D}(E,r) \;, \end{aligned} \tag{2.9}$$

with

$$f_\mathrm{D}(E,r)=0, \quad r>b$$

where the parameter b roughly characterizes the size of the bag. Substituting Eqs. (2.6)–(2.9) into Eqs. (2.5) and performing some straightforward algebra, we get

$$\left[-\frac{1}{2\mu}\nabla^2 + V(\mathbf{r},E) - E \right] \chi_E(\mathbf{r}) = -\frac{f^*_\mathrm{D}(\mathbf{r},E)}{E-M_\mathrm{D}} \int f_\mathrm{D}(\mathbf{r}',E)\chi_E(\mathbf{r}')d\mathbf{r}' \tag{2.10}$$

for describing the relative motion of two nucleons. The bag component can then be obtained from the scattering wave function

$$C_\mathrm{D}(E) = \frac{1}{E-M_\mathrm{D}} \int f^*_\mathrm{D}(E,\mathbf{r})\chi_E(\mathbf{r})d\mathbf{r} \;. \tag{2.11}$$

The transition form factor $f_\mathrm{D}(E,r)$ has to be treated phenomenologically since a clear picture of the QCD confinement mechanism is still not available. In the ITEP approach[29] some effort has been made to relate $f_\mathrm{D}(E,r)$ to a resonanting group formulation of a six-quark system. No similar attempt will be made here. It is more useful to simply indicate that their model is designed to generate the following physical properties:

(a) The nonlocal interaction on the right-hand side of Eq. (2.10) should contain a term which has a linear energy dependence of the Paris potential, which defines, via Eqs. (2.7), the background meson-exchange interaction.

(b) At $E=M_\mathrm{D}$ the short-range dynamics must be described only by the quark configuration and, hence, at this energy the NN relative wave function $\chi_E(\mathbf{r})$ must be completely excluded from the bag region $r \le b$ *for any choice of the background potential* characterized by the length scale d in Eqs. (2.7).

(c) Following the interpretation by Jaffe and Low,[24] the resulting scattering P matrix must have a pole at $E=M_\mathrm{D}$.

Property (a) can be obtained by keeping only the leading energy-dependent term in the Taylor expansion of the form factor $f_\mathrm{D}(r,E)$ about the bag energy, M_D,

$$f_\mathrm{D}(r,E) = f_0(r,M_\mathrm{D}) + (E-M_\mathrm{D}) f_1(r,M_\mathrm{D}) \;. \tag{2.12a}$$

As was first pointed out by Simonov[29] and as will be explicitly demonstrated later in this paper, properties (b) and (c) can then be obtained by assuming that the energy independent term f_0 in Eq. (2.12a) is localized at the distance b. To make contact with the ITEP model, we follow their approach and take

$$f_0(r,M_\mathrm{D}) = -c\,\delta(r-b) \;. \tag{2.12b}$$

The energy-dependent term is assumed to be a volume-coupling form

$$f_1(r,M_\mathrm{D}) = x\,\theta(b-r)\sqrt{2/b}\,\sin(\pi r/b) \;. \tag{2.12c}$$

The form Eq. (2.12c) is suggested in a resonant group formulation by Simonov.[29]

Equations (2.7) and (2.10)–(2.12) completely define the QCB model. We now turn to develop a method of solving the NN scattering problem.

III. NN SCATTERING IN THE QCB MODEL

In the partial wave representation, the radial part of the scattering equation (2.10) for an uncoupled NN channel takes the following form (suppress all channel quantum numbers except the relative orbital angular momentum l),

$$\left[\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} - 2\mu V(r,E) + k^2 \right] u_l(kr) = \frac{r f_\mathrm{D}(r,E)}{E-M_\mathrm{D}} \int f_\mathrm{D}(r',E) u_l(kr') r' dr' \;, \tag{3.1}$$

where $E=(1/2\mu)k^2$, and $u_l(r)=r\chi_l(r)$ is the usual radial wave function. The most important feature of this differential-integral equation is the appearance of an energy-dependent nonlocal interaction, which becomes infinite at $E=M_\mathrm{D}$. Clearly, at this energy there will be an infinite potential and the incoming NN wave will be completely reflected at the distance b. This means that the short-range dynamics at $E=M_\mathrm{D}$ is described only by the bag configuration. As the collision energy starts to differ from the bag mass M_D, the incoming wave can penetrate this nonlocal potential and, hence, the cluster component can also exist inside the bag region. This interesting energy dependence plays an important role in describing NN scattering in the energy region where the total collision energy in the c.m. frame can be larger than the bag mass M_D. For $M_\mathrm{D} \simeq 2200$ MeV in the ${}^1S_0$ channel, this interesting energy region is around 600 MeV incident nucleon energy in the laboratory frame. We therefore argue that the QCB parameters can be sensibly determined only when the NN data up to 1 GeV laboratory energy is fitted. The approach of Ref. 11, which also makes use of the QCB model, only considers NN data below 400 MeV.

Following the approach of Simonov,[29] we solve Eq. (3.1) by the Green's function method. The procedure is to first construct a distorted Green's function from the meson-exchange interaction $V(r,E)$,

$$\left[\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}-2\mu V(r,E)+k^2\right] \times G_l(r,r',E)=\delta(r-r') . \quad (3.2)$$

The solution of Eq. (3.2) is

$$G_l(r,r',E)=-2\mu k\,\xi_l^{(1)}(kr_<)\xi_l^{(2)}(kr_>) , \quad (3.3)$$

where $\xi_l^{(1)}$ and $\xi_l^{(2)}$ are, respectively, the regular and irregular solutions of the radial Schrödinger equation

$$\left[\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}-2\mu V(r,E)+k^2\right]\xi_l^{(i)}(k)=0 , \quad (3.4)$$

with the boundary conditions

$$\xi_l^{(1)}(kr)\rightarrow\begin{cases} r^{l+1}, & r\rightarrow 0\\ \frac{1}{2}r[h_l^{(-)}(kr)+e^{2i\bar\delta_l}h_l^{(+)}(kr)], & r\rightarrow\infty\end{cases} \quad (3.5a)$$

$$\xi_\alpha^{(2)}(kr)\rightarrow\begin{cases} r^{-l}, & r\rightarrow 0\\ irh_l^{(+)}(kr), & r\rightarrow\infty\end{cases} \quad (3.5b)$$

and

$$h_l^{(\pm)}(kr)=j_l(kr)\pm in_l(kr) .$$

Here, $\bar\delta_l$ is the phase shift due to the interaction $V(r,E)$; j_l and n_l are, respectively, the regular and irregular modified spherical Bessel functions. By using the property (3.2) and carrying out some algebraic derivations, the scattering solution of Eq. (3.1) with only one bag state can be written as (the corresponding formulation with several bag states is straightforward)

$$u_l(kr)=\xi_l^{(1)}(kr)+\frac{\int G_l(r,r',E)f_{\rm D}(r',E)r'dr'}{E-M_{\rm D}-\Sigma_{\rm D}(E)}X_{l,{\rm D}}(E) , \quad (3.6)$$

and

$$X_{l,{\rm D}}(E)=\int \xi_l^{(1)}(kr)f_{\rm D}(r,E)r\,dr , \quad (3.7)$$

$$\Sigma_{\rm D}(E)=\int f_{\rm D}^*(r',E)G_l(r',r,E)f_{\rm D}(r,E)r\,dr\,r'dr' . \quad (3.8)$$

The quantity $\Sigma_{\rm D}$ is the self-energy of the bag state due to its coupling to the NN channel. The S matrix is then extracted by taking the limit $r\rightarrow\infty$ of Eq. (3.6),

$$\frac{1}{r}u_l \underset{r\rightarrow\alpha}{\rightarrow} \frac{1}{2}[h_l^{(-)}(kr)+S_l(E)h_l^{(+)}(kr)] ,$$

with

$$S_l(E)=e^{2i\bar\delta_l}+\frac{-4\mu kiX_{l,{\rm D}}(E)\tilde X_{l,{\rm D}}(E)}{E-M_{\rm D}-\Sigma_{\rm D}(E)} . \quad (3.9)$$

Here we also define

$$\tilde X_{l,{\rm D}}(E)=\int \xi_l^{(1)}(kr)f_{\rm D}^*(r,E)r\,dr . \quad (3.10)$$

Equation (3.9) shows explicitly that the coupling to a bag state causes a pole in the S matrix. The position of the pole in the complex energy plane is determined by the following nonlinear equation:

$$E_p-M_{\rm D}-\Sigma_{\rm D}(E_p)=0, \quad E_p=E_R+iE_I . \quad (3.11)$$

Clearly, if the imaginary part E_I of the pole position is small, the coupling to the bag state then generates a strong energy dependence in the S matrix and its corresponding scattering observables. In this way the bag state will correspond to the so-called dibaryon resonance. If this is not the case, the role of the bag state is merely to provide a microscopic picture of the short-range mechanism.

Next, we want to show that the solution of Eq. (3.1) exhibits the properties (b) and (c) mentioned in Sec. II. To do this it is sufficient to consider Eq. (3.1) in the energy region near $M_{\rm D}$. Because of the singular nature of $1/(E-M_{\rm D})$, the contribution from the energy-dependent part of the transition form factor $f_{\rm D}(r,E)$ [Eqs. (2.12)] to the right-hand side of Eq. (3.1) can be neglected at the $E\rightarrow M_{\rm D}$ limit. We then have

$$\left[\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}-2\mu V(r,E)+k^2\right]u_l(kr) =2\mu\frac{|c|^2b^2u_l(kb)}{E-M_{\rm D}}\delta(r-b), \quad E\rightarrow M_{\rm D} . \quad (3.12)$$

In the kinematic region near $E=M_{\rm D}$, Eqs. (3.7) and (3.8) become

$$X_\alpha(E)=-cb\,\xi_l^{(1)}(kb) ,$$

$$\Sigma_{\rm D}(E)=|c|^2b^2G_l(b,b,E) ,$$

where

$$G_l(b,b,E)=-2\mu k\,\xi_l^{(1)}(kb)\xi_l^{(2)}(kb) .$$

The wave function (3.6) also takes a simple form,

$$u_l(kr)=\xi_l^{(1)}(kr)+\frac{c^2b^2G(r,b,E)\xi_l^{(1)}kb)}{E-M_{\rm D}-c^2b^2G(b,b,E)} . \quad (3.13)$$

Because of the r dependence of the Green's function (3.3), the wave function inside the region defined by the distance b becomes

$$u_l \underset{r<b}{\rightarrow} \xi_l^{(1)}(kr)+\frac{c^2b^1[-2\mu k\,\xi_l^{(1)}(kr)\xi_l^{(2)}(kb)]\xi_l^{(1)}(kb)}{E-M_{\rm D}-c^2b^2G_l(b,b)}$$

$$=\xi_l^{(1)}(kr)+\frac{c^2b^2\xi_l^{(1)}(kr)G_l(b,b,E)}{E-M_{\rm D}-c^2b^2G_l(b,b,E)}$$

$$=\xi_l^{(1)}(kr)\frac{E-M_{\rm D}}{E-M_{\rm D}-c^2b^2G_l(b,b,E)} . \quad (3.14)$$

Hence, at $E = M_D$, $u_l = 0$ in the entire region $r < b$. It is necessary to stress here that *this is true for any choice of meson-exchange interaction V(r,E) and any form of the volume-coupling term* $f_1(r, M_D)$. This is the desired property (b) listed in Sec. II. It means that the hadronic and quark phases are separated completely only at the energy $E = M_D$. For $E \neq M_D$, the cluster component in QCB is also present in the bag region. This is a reasonable physical picture since we should not expect that the short-range quark dynamics at all energies can be described satisfactorily by the excitation of one or a few low-lying bag states.

To show that our Hamiltonian formulation can yield the P-matrix interpretation of the bag solution, we again consider the scattering in the energy region near the bag energy $E = M_D$. Integrating Eq. (3.12) over a small region $2\epsilon \to 0$ around the point $r = b$ and using the continuity of the wave function.

$$u_l(b+\epsilon) = u_l(b-\epsilon), \quad \epsilon \to 0 \tag{3.15a}$$

we have

$$u_l'(b+\epsilon) - u_l'(b-\epsilon) = \frac{2\mu\,|c|^2 b^2 u_l(b)}{E - M_D}, \quad \epsilon \to 0,\ E \to M_D . \tag{3.15b}$$

The P matrix is defined as

$$P_l(E) = \frac{b u_l'(b+\epsilon)}{u_l(b+\epsilon)}, \quad \epsilon \to 0 . \tag{3.16}$$

By using Eq. (3.15), we have

$$P_l(E) = \frac{b u_l'(b-\epsilon)}{u_l(b-\epsilon)} + \frac{2\mu\,|c|^2 b^2}{E - M_D} . \tag{3.17}$$

By using Eq. (3.14) to evaluate the first term of Eq. (3.17), we obtain

$$P_l(E) = \bar{P}_l(E) + \frac{2\mu\,|c|^2 b^2}{E - M_D} , \tag{3.18}$$

where the background term $\bar{P}_l$ is only determined by the wave function calculated from the background meson-exchange interaction through Eqs. (3.4) and (3.5),

$$\bar{P}_l(E) = \frac{b \xi_l^{(1)\prime}(b)}{\xi_l^{(1)}(b)} . \tag{3.19}$$

We see that for *any* form of the background meson-exchange interaction $V(E,\mathbf{r})$, the P-matrix equation (3.18) has a pole at $E = M_D$. According to the interpretation by Jaffe and Low,[24] we can therefore use the bag model calculation to define the mass M_D of the six-quark state D in our Hamiltonian formulation of the problem. Equation (3.18) was first obtained by Simonov.[29] We want to point out that for an energy-dependent form factor (2.12), Eq. (3.18) is only valid in the energy region very close to the bag mass M_D, where the scattering equation takes the form of Eq. (3.12).

It is interesting to express the S matrix in terms of the P matrix,

$$S_l(E) = -\frac{b\tilde{h}_l^{(-)\prime}(b) - [P_l(E)-1]\tilde{h}_l^{(-)}(b)}{b\tilde{h}_l^{(+)\prime}(b) - [P_l(E)-1]\tilde{h}_l^{(+)}(b)} , \tag{3.20}$$

where $\tilde{h}_l^{(\pm)}$ is the solution of the radial equation (3.4) with the boundary condition

$$\tilde{h}^{(\pm)}(r) \underset{r\to\infty}{\longrightarrow} h_l^{(\pm)}(r) = j_l(kr) \pm i n_l(kr) . \tag{3.21}$$

At $E \to M_D$ the background term $\bar{P}_l$ of Eq. (3.18) can be neglected and we then have

$$P_l(M_D) \simeq \frac{2\mu b^2 |c|^2}{E - M_D}, \quad E \to M_D \tag{3.22}$$

and, hence,

$$S_l(M_D) = -\frac{\tilde{h}_l^{(-)}(b)}{\tilde{h}_l^{(+)}(b)}, \quad E \to M_D$$
$$= e^{2i\delta_l(M_D)} . \tag{3.23}$$

In the absence of any background meson-exchange interaction $V = 0$, we have, for $l = 0$ s-wave scattering,

$$\delta_0(M_D) = -k_D b , \tag{3.24}$$

with

$$M_D = 2m + k_D^2/m . \tag{3.25}$$

Equation (3.24) shows an interesting feature of the QCB. If the background meson-exchange interaction is weak in the energy region near the bag mass M_D, the phase-shift data at $E = M_D$ is directly related to the bag parameters. With $M_D = 2159$ MeV in the considered $^1S_0$ channel, it is found that $b \simeq 1.4$ fm. The effect due to the background interaction with $d \geq 0.6$ fm [Eqs. (2.7)] does not change this value too much. The value $b = 1.4$ fm is not too different from that used in all P-matrix analyses.[24–27] We therefore set $b = 1.4$ fm in all of the calculations presented in this paper.

IV. RESULTS AND DISCUSSIONS

With the choice $M_D = 2159$ MeV and $b = 1.4$ fm, the parameters of the QCB model defined in Sec. II are (i) the cutoff parameter d in Eqs. (2.7) for defining the extent to which a chosen NN potential should be used to describe the meson-exchange mechanism, and (ii) c and x for defining the strength of the transition form factor, Eqs. (2.12). Our task is to examine whether the NN $^1S_0$ phase shift up to 1 GeV laboratory energy can be fitted by varying these three parameters in a χ^2 fit.

It is most desirable to have a model in which the short-range dynamics is entirely described by the excitation of a six-quark bag, and the meson-exchange interaction is excluded completely from the bag region. In our formulation this simplest model can be defined by setting $d = b = 1.4$ fm in defining the background meson-exchange interaction [Eqs. (2.7)]. This model will be called the one-pole QCB, when only the lowest bag state with $M_D = 2159$ MeV is kept.

The best fit by the one-pole QCB is the solid curve shown in Fig. 1. The resulting parameters are listed in Table I. We see that while the data in the high energy region can be fitted, the model cannot provide enough attraction to fit the low energy data. In Fig. 1 we also show the result (dashed curve) with the meson-exchange interaction V turned off. Clearly the one-pion exchange at $r > d = 1.4$ fm contained in V is an important source of the attraction in the low energy region, but it is not enough to fit the data.

To obtain a satisfactory description of the phase shift, we need to include additional attractive mechanisms. The approach taken by the ITEP group[33,34] is to include a coupling with a second bag state with a much higher mass. The fit by this two-pole QCB is the solid curve shown in Fig. 2. In Fig. 2 we also show the effects when the coupling to the second bag state or the meson-exchange interaction V is turned off. Clearly, the attraction provided by the second bag state is indispensable in the fit to the data in the low energy region. However, we find that the fit to the data does not uniquely determine the coupling to the higher mass second bag. In Table II we list four sets of the parameters which give about the same χ^2 value. The parameters c_1 and x_1 for the coupling to the lowest bag state is very well constrained by the data, but the allowed parameters of the higher mass second bag state are rather arbitrary (we set the volume coupling to the higher bag state zero, $x_2 = 0$, for simplicity). In addition, we also find that it is possible to get the same good fit to the data by having more than one higher mass bag state, as far as their masses are larger than about 3000 MeV. This makes the interpretation of those higher mass bag states rather uncertain since we cannot identify them with the well-defined bag model prediction.[37–39] Its implication in the application of the model in nuclear physics calculations, which will be discussed in Sec. V, will be unclear. This uncertainty probably suggests the limitation of the ITEP QCB model.

We now depart from the ITEP approach and propose a model which is consistent with the suggestion of Ref. 28. The model is simply to allow the meson-exchange interaction V defined by the Paris potential to exist also inside the bag region; i.e., setting the cutoff parameter d of Eqs. (2.7) to be less than the distance $b = 1.4$ fm. Admittedly, this procedure makes the physical interpretation of the model less transparent. It remains to be clarified in the future. Perhaps this model can be qualitatively justified by the following arguments. It is now well recognized that the low energy data can be more realistically described by extending the MIT bag model to include the pion cloud. The resulting cloudy-chiral bag model[40] proves to be reasonably successful in describing the properties of the nucleon and Δ, as well as the low energy πN scattering. It is therefore possible that when two nucleons start to overlap there exists a region in which the transition from the hadronic phase to quark-gluon phase is not complete and the interaction can effectively be described by the exchange of pions be-

TABLE I. Parameters of the transition form factor, Eqs. (2.12), in the one-pole QCB model; c is in units of MeV fm$^{1/2}$, x is dimensionless, and M_D is the mass of the bag state.

M_D (MeV)	$d = b$ (fm)	c	x	χ^2 value
2159	1.4	174.06	1.1043	899

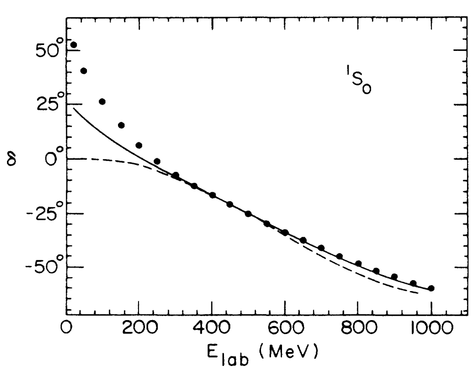

FIG. 1. The solid curve is the best fit to the NN $^1S_0$ phase shift (Ref. 41) within the one-pole QCB model. The dashed curve is obtained when the background meson-exchange interaction V [Eq. (2.17)] is turned off.

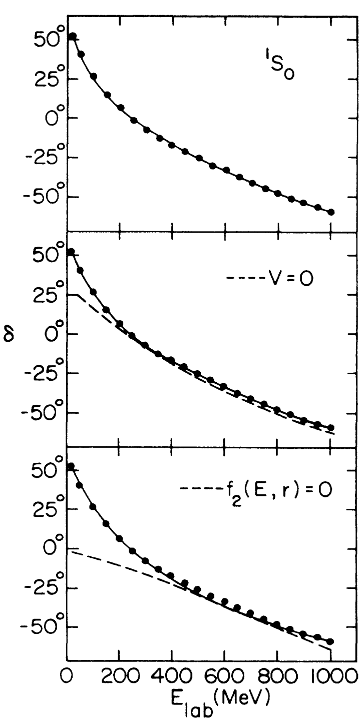

FIG. 2. The solid curve is the best fit to the NN $^1S_0$ phase shift (Ref. 41) within the tow-pole QCB model. The dashed curves are obtained when either the background meson-exchange interaction V [Eq. (2.17)] or the coupling to the higher mass second bag state is set to zero.

TABLE II. Same as Table I, except for the two-pole QCB model.

$d=b$ (fm)	M_{D_1} (MeV)	c_1	x_1	M_{D_2}	c_2	x_2	χ^2
1.4	2159	167.43	1.1754	3376.512	151.18	0	1.97
	2159	167.62	1.1723	3876.512	174.50	0	2.05
	2159	167.74	1.1706	4376.512	195.06	0	2.10
	2159	167.82	1.1695	4876.512	213.66	0	2.13

tween two cloudy bags. It is our assumption that this effect in the region $r \gtrsim 0.6$ fm is already included in the Paris potential.[35] To be more consistent with the Paris potential, we retain its original linear energy dependence in our model. As discussed in Sec. II, this is also the reason why the form of transition form factor, Eqs. (2.12), is chosen. Of course, the included intermediate range meson-exchange interaction has to compete with the bag excitation mechanism, which can happen at a much larger distance, $b=1.4$ fm. This two-mechanism picture is consistent with the πNN formulation of Lee and Matsuyama.[28] At this point it is important to note here that this extended QCB, called the cloudy QCB from now on, still retains all of the properties discussed in Sec. III. In particular, we see from Eq. (3.14) that the clear cut separation of the bag and two-cluster configurations still exists at $E=M_D$, even when the background meson-exchange interaction is now allowed to exist inside the bag region.

To investigate the cloudy QCB, we allow the cutoff parameter d of Eqs. (2.7) along with the bag excitation parameters c and x to vary in our χ^2 fit to the $^1S_0$ phase shift data. We first find that the cloudy QCB cannot fit the data well if the cutoff parameter d is larger than 0.8 fm. As shown in Table III, with $d=0.8$ fm the χ^2 is already very large. With $d=0.65$ fm we can obtain a fit which is as good as the solid curve of Fig. 2. The resulting bag excitation parameters c and x are almost identical to the values of c_1 and x_1 of the two pole QCB model (see Table II). This further establishes the close relationship between the lowest bag state predicted by the theory and the NN phase shift data within the QCB.

We now turn to analyze the dynamical content of the cloudy QCB model in some detail. First, we shown in Fig. 3 that the energy dependent part of the transition form factor (2.12a) is essential in the fit. When x of Eq. (2.12c) is set to zero, the phase shift behaves smoothly only for a very large $c>550$. In this strong δ-function coupling limit the background interaction is completely negligible and the phase shift is determined only by the bag mass M_D, and the radius b, as shown in Eq. (3.24). By decreasing the value of c one can certainly reduce the attraction in the low-energy region and the repulsion in the high-energy region. But the poles of the S matrix [second term of Eq. (3.9)] is then shifted to a position very close to the real axis, and hence the phase shift starts to develop a strong energy dependence. Namely, the model with $x=0$ will generate an unobserved "dibaryon resonance" if we want to reduce the attraction in the low energy region.

The importance of the energy dependent part of the transition form factor is further illustrated in Fig. 4. We

TABLE III. Same as Table I, except for the cloudy QCB model.

M_D (MeV)	d (fm)	b (fm)	c	x	χ^2
2159	0.80	1.4	170.86	1.0459	26.37
2159	0.65	1.4	166.14	1.3064	3.13

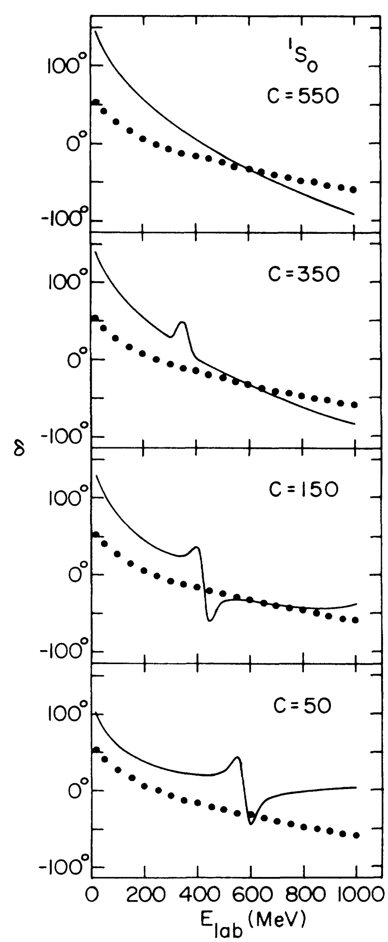

FIG. 3. The dependence of the predicted NN $^1S_0$ phase shift on the strength c of the transition form factor [Eqs. (2.12)]. In this fit to the data, the energy dependent term of Eqs. (2.12) is set equal to zero ($x=0$).

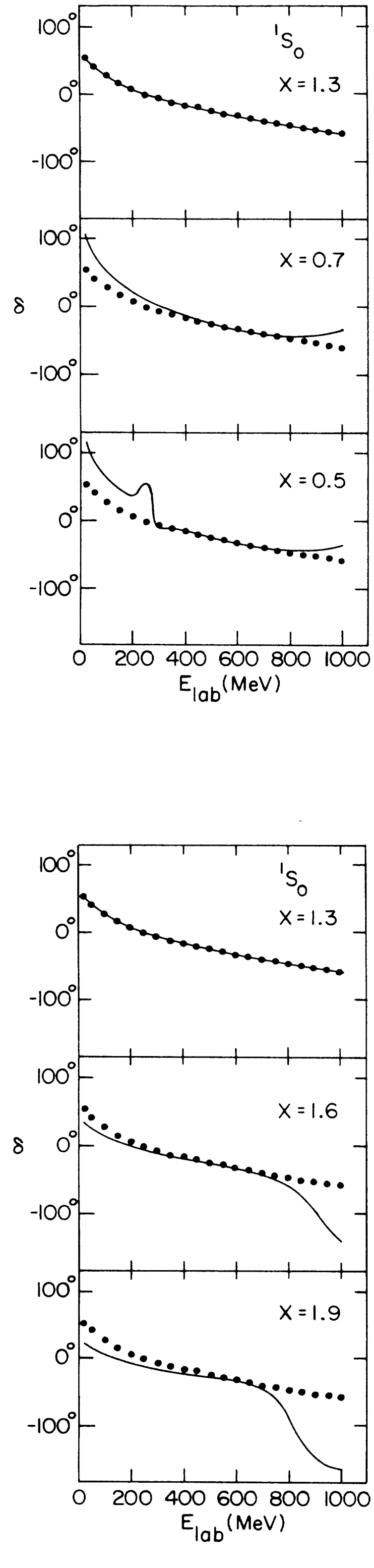

FIG. 4. The dependence of the predicted NN $^1S_0$ phase shift on the strength x of the transition form factor [Eqs. (2.12)].

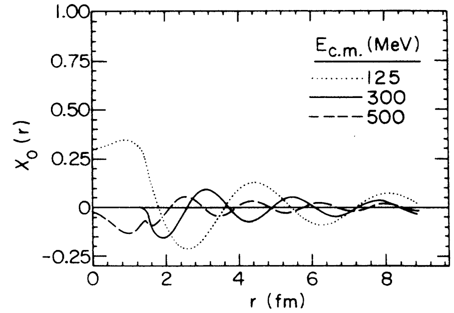

FIG. 5. The relative NN wave functions calculated from the cloudy QCB model in the energy region near the bag mass $M_D = 2159$ MeV (equivalent to $E_{c.m.} = 300$ MeV).

see that if we decrease x from the fitted value $x = 1.3$ to $x = 0.5$ [Fig. 4(a)], the model gradually generates visible "dibaryon resonance," in contradiction with the data. When x is increased to a larger value, the calculated phase shifts become too repulsive [Fig. 4(b)]. It is obvious from Eq. (3.1) that the δ-function coupling generates attraction for $E < M_D$, and repulsion for $E > M_D$. The energy dependent volume coupling f_1 gives a repulsive contribution at all energies. The fit is thus due to a delicate balance between these two different bag excitation mechanisms.

Finally, we want to examine how the bag excitation dynamics determines the NN relative wave function. We see in Fig. 5 that at $E = M_D$ ($E_{c.m.} = 300$ MeV) the wave function (solid curve) inside the bag region $r \leq b$ is completely suppressed, as expected from Eq. (3.14). In other energy regions the short-range dynamics is described by both the two-cluster and bag configurations and hence is not excluded from the bag region. Needless to say, our approach is radically different from the models of Kim[32] and of Kisslinger *et al.*[31]

In Fig. 6 we compare the NN wave function of the cloudy QCB at $E = M_D$ with the wave function calculat-

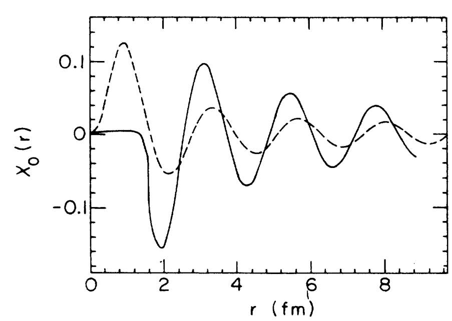

FIG. 6. Comparison of the relative NN wave function (solid) calculated at $E = M_D$ ($E_{lab} = 600$ MeV) from the QCB with that from the wave function (dashed) calculated from the usual potential scattering with the Paris potential.

ed from the usual potential scattering equation with the Paris potential. The suppression of QCB wave functions in the region of $r < b = 1.4$ fm is evident. It is clear that these two models will have very different predictions of any NN reactions which are mainly determined by short-range mechanisms.

V. DISCUSSION

We have explored the dynamical content of the quark compound bag (QCB) model in the intermediate energy region. It has been shown both analytically and numerically that a large part of the short-range NN dynamics can be related to the MIT bag state through a vertex interaction $H_{\mathrm{NN}\rightarrow\mathrm{D}}$ parameterized in the form of Eqs. (2.12). However, a fit to the data cannot be achieved without introducing an additional attractive mechanism. We have verified explicitly that this needed attractive force is generated in the QCB model of ITEP by introducing a second pole, which cannot be identified with the bag model prediction. We have shown that this problem can be resolved by simply allowing the existence of meson-exchange mechanisms in the bag region. We argue that this extension of the QCB model is consistent with the chiral/cloudy bag model and the original construction of the Paris potential. The resulting cloudy QCB model proves to be very successful in describing the data.

The cloudy QCB model is consistent with the πNN formulation of Ref. 28. To explore the extent to which the difficulties encountered in the study of intermediate energy NN and πd reactions can be resolved, we need to follow the unitary scattering theory developed in Ref. 28 to account for the Δ excitation and pion production. In fact, we expect from the coefficients of fractional parentage expansion[38] of the bag wave function that the dominant transition in the $J=2, T=1$ channel is $D \rightleftarrows \mathrm{N}\Delta(^5S_2) \rightleftarrows \pi\mathrm{NN}$.

To end this paper, we would like to point out that the cloudy QCB model can perhaps be used to predict the probability of finding an "off-shell" six-quark subsystem in nuclei. In our approach this prediction is completely determined in the fit to the NN data in the intermediate energy region where this six-quark system is excited "on shell." Clearly our approach is radically different from the model of Kim *et al.*[32] and that of Kisslinger *et al.*[31] The information of intermediate energy NN scattering is never used to constraint the parameters of these models. The second important implication of the work is that our cloudy QCB model can be used to calculate the one-pion exchange interaction between a six-quark subsystem and a nucleon, since the pion coupling with a six-quark bag can be calculated by using the method of Mulder and Thomas.[39] Specifically, we can calculate the three-nucleon force through the excitation of an "off-shell" six-quark bag state, a calculation never attempt before. These two works are in progress and will be published elsewhere. Of course, the extension of the present work to include Δ excitation and pion production is the major challenge in developing an accurate πNN theory for a fundamental description of intermediate energy physics.

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