Application of quantum theory of particle collisions to $^{16}O + {}^{16}O$ reactions

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(Received 29 December 1986)

An extended time-dependent Hartree-Fock theory which incorporates the effects of nucleon-nucleon collisions into the mean field is presented and applied to head-on collisions of $^{16}O + {}^{16}O$ at low incident energies. Particularly investigated is the influence of nucleon-nucleon collisions on the low angular momentum limit for fusion which is predicted by time-dependent Hartree-Fock calculations. It is found that the threshold energy for the low angular momentum limit is dramatically increased because of nucleon-nucleon collision effects.

I. INTRODUCTION

The time-dependent Hartree-Fock (TDHF) theory has extensively been used to simulate low energy heavy-ion collisions, e.g., deep-inelastic collisions and fusion reactions.[1,2] On the other hand, it has been pointed out that TDHF theory has limitations inherent in the mean field theory. Here we are interested in nucleon-nucleon (NN) collision effects. At low incident energies the Pauli exclusion principle is effective in suppressing NN collisions, making TDHF theory a good description for the dynamics of heavy-ion collisions. NN collisions are expected to become increasingly important with increasing incident energy. At intermediate energies where new data are being accumulated, they play as important a role in the dynamics of heavy-ion collisions as the mean field.

Although much work has been devoted to the derivation of the NN collision theory,[3] its applications to realistic nucleus-nucleus collisions are scarce except for numerical simulations based on a semiclassical model.[4] The aim of this paper is to apply the quantum theory of NN collisions to nucleus-nucleus collisions. The theory used here, which determines the time evolution of the one-body density matrix, was derived in a previous paper[5] from the Kadanoff-Baym equation for the one-body Green's function.[6] Wong and Tang[7] and Orland and Schaeffer[8] have also used this equation in the derivation of a collision term. In contrast to their works, however, our model contains memory effects in the collision term and keeps nondiagonal elements of the occupation matrix. In an application of our model to one-dimensional systems,[5] we demonstrated that these ingredients of the theory are indispensable for a correct description of dissipative phenomena in finite systems.

In Sec. II we describe our model and discuss conservation laws and the time reversal property of the theory. In numerical applications we focus on low energy reactions, where TDHF calculations predict a low angular momentum (low-L) limit for fusion above a certain incident energy.[1,2] Several experiments have been performed to observe directly this low-L limit. However, there has been no experimental evidence that proves its existence.[9] We will demonstrate that the threshold energy for the low-L limit is dramatically increased due to NN collision effects. The same conclusion has been obtained in our previous work[10] based on a phenomenological model[11] for NN collision effects in which the one-body density matrix is assumed to approach thermal equilibrium one.

Numerical calculations are performed for head-on collisions of $^{16}O + {}^{16}O$. Details of the calculations are explained in Sec. III. Results are presented in Sec. IV. Section V is devoted to summary.

II. THE MODEL

A. Equation of motion for one-body density matrix

Since the derivation of a collision term was presented in a previous paper,[5] we briefly summarize the main result in this subsection. [The theory is called the time-dependent density matrix (TDDM) method hereafter.] We study the time evolution of the one-body density matrix, which can be expanded in terms of single-particle (s.p.) wave functions ψ_λ and the occupation matrix $n_{\lambda\lambda'}$ as

$$\rho(\mathbf{r},\mathbf{r}';t)=\sum_{\lambda\lambda'} n_{\lambda\lambda'}(t)\psi_\lambda(\mathbf{r},t)\psi^*_{\lambda'}(\mathbf{r}'t) \ . \tag{1}$$

The s.p. wave functions are chosen to satisfy the TDHF-like equation

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\psi_\lambda(\mathbf{r},t) &= h\psi_\lambda(\mathbf{r},t) \\ &= [-\hbar^2\nabla^2/2M + U_{\rm HF}(\rho)]\psi_\lambda(\mathbf{r},t) \ , \end{aligned} \tag{2}$$

where $U_{\rm HF}$ is the self-consistent mean potential. Since ρ includes NN collision effects through $n_{\lambda\lambda'}$, $U_{\rm HF}(\rho)$ in the TDDM method becomes different from that in the TDHF theory as NN collisions proceed. The above representation of the s.p. basis allows us to express the time derivative of $n_{\lambda\lambda'}$ in terms of the correlated part of the two-body Green's function,[7,8] which is treated in the Born approximation. The two-body Green's function in this approximation is given by products of four one-body Green's functions. Then the time derivative of $n_{\lambda\lambda'}$ is given as[5]

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$$\frac{d}{dt}n_{\lambda\lambda'}(t)=-(1/\hbar^2)[F_{\lambda\lambda'}(t)+F^*_{\lambda'\lambda}(t)] , \tag{3}$$

where

$$F_{\lambda\lambda'}(t)=\sum_{\alpha\alpha'\beta\beta'\gamma\delta\delta'}\langle\lambda\delta'\,|\,v\,|\,\alpha\beta\rangle\int_{-\infty}^{t}dt'\langle\alpha'\beta'\,|\,v\,|\,\gamma\delta\rangle_A\,|_{t'}\{[\delta_{\alpha\alpha'}-n_{\alpha\alpha'}(t')][\delta_{\beta\beta'}-n_{\beta\beta'}(t')]n_{\gamma\lambda'}(t')n_{\delta\delta'}(t')$$
$$-n_{\alpha\alpha'}(t')n_{\beta\beta'}(t')[\delta_{\gamma\lambda'}-n_{\gamma\lambda'}(t')][\delta_{\delta\delta'}-n_{\delta\delta'}(t')]\} . \tag{4}$$

Here, v is the residual interaction and the subscript A means that the matrix element is antisymmetrized. In the derivation of Eqs. (3) and (4) the one-body Green's functions in the two-body Green's function are parametrized by ψ_λ and $n_{\lambda\lambda'}$ to close the equation motion for ρ. The one-body Green's functions thus parametrized satisfy the same equation as for ψ_λ (see Ref. 5). This property of the one-body Green's function guarantees the total energy conservation, as was discussed by Grangé *et al.*[12] Equation (3) is similar to Eq. (B7) of Ref. 7, where a simple time dependence of ψ_λ and $n_{\lambda\lambda'}$ was assumed and the time integration was carried out.

There are two important memory effects in the t' integration in Eq. (4). Single-particle wave functions in a time-dependent mean field, in general, do not have a sharp energy distribution. The spreading effect of the s.p. spectra in the time-dependent mean field is included correctly in the t' integration, which makes the transition of nucleons possible even if there is no crossing in the mean values of the s.p. energies defined by $\epsilon_{\lambda\lambda}=\langle\lambda\,|\,h\,|\,\lambda\rangle$. The other memory effect is from the damping of s.p. states due to NN collisions which also makes s.p. spectra broad. This effect is included through the decay of $n_{\lambda\lambda}$ and $1-n_{\lambda\lambda}$ in the integrand of the t' integration.

As was studied in previous papers,[5,10] the nondiagonal elements of $n_{\lambda\lambda'}$ are essential for a correct description of available phase space for NN collisions when the time-evolved TDHF-like states ψ_λ [see Eq. (2)] are used as a basis. In fact, there is no additional damping when nondiagonal elements of $n_{\lambda\lambda'}$ are ignored.[5,10]

In the following, we discuss conservation laws and symmetry of Eqs. (2)–(4).

B. Conservation of the total number of particles

Equations (3) and (4) conserve the total number of particles. The time derivative of the total number of particles, $N=\sum_\lambda n_{\lambda\lambda}$, is identical to zero, which may be seen as follows:

$$\frac{dN}{dt}=\sum_\lambda\dot{n}_{\lambda\lambda}=-\frac{1}{\hbar^2}\sum_\lambda(F_{\lambda\lambda}+F^*_{\lambda\lambda})$$
$$=-\frac{1}{\hbar^2}\left[\sum_\lambda F_{\lambda\lambda}-\sum_\lambda F_{\lambda\lambda}\right]=0 . \tag{5}$$

C. Total energy conservation

The total energy $E_{\rm tot}$ is defined as the expectation value of the total Hamiltonian which consists of the kinetic energy term and the two-body interaction term. The expectation value of the latter is written in terms of the two-body Green's function. Since the two-body Green's function is treated in the Hartree-Fock (HF) + Born approximation, $E_{\rm tot}$ consists of the HF energy $E_{\rm HF}$ and the correlation energy $E_{\rm cor}$:[7]

$$E_{\rm tot}=E_{\rm HF}+E_{\rm cor} , \tag{6}$$

where

$$E_{\rm HF}=\sum_{\lambda_1\lambda_2}\left\langle\lambda_1\left|-\frac{\hbar^2\nabla^2}{2M}\right|\lambda_2\right\rangle n_{\lambda_2\lambda_1}$$
$$+\tfrac{1}{2}\sum_{\lambda_1\lambda_2\lambda_3\lambda_4}\langle\lambda_1\lambda_2\,|\,V\,|\,\lambda_3\lambda_4\rangle_A n_{\lambda_3\lambda_1}n_{\lambda_4\lambda_2} , \tag{7}$$

and $E_{\rm cor}$ is given as

$$E_{\rm cor}=-\frac{i}{2\hbar}\sum_\lambda F_{\lambda\lambda} . \tag{8}$$

This originates from the Born term of the two-body Green's function. The force V in Eq. (7) is the effective interaction used to construct the mean potential, which, in general, is different from the residual interaction v. Since the s.p. states satisfy Eq. (2), the time derivative of $E_{\rm HF}$ can be expressed in terms of $\dot{n}_{\lambda\lambda'}$,

$$\frac{dE_{\rm HF}}{dt}=\sum_{\lambda\lambda'}\epsilon_{\lambda\lambda'}\dot{n}_{\lambda'\lambda} , \tag{9}$$

where $\epsilon_{\lambda\lambda'}$ is defined as $\epsilon_{\lambda\lambda'}=\langle\lambda\,|\,h\,|\,\lambda'\rangle$. The time derivative of $E_{\rm cor}$ consists of two terms: one has the time derivative of the first matrix element on the right-hand side of Eq. (4) and the other the time derivative of the t' integral. The latter vanishes, as can be easily seen from Eq. (4). The time derivative of the matrix element is rewritten with the use of Eq. (2) as follows:

$$\frac{d}{dt}\langle\alpha\beta\,|\,v\,|\,\gamma\delta\rangle=\langle\dot{\alpha}\beta\,|\,v\,|\,\gamma\delta\rangle+\langle\alpha\dot{\beta}\,|\,v\,|\,\gamma\delta\rangle+\langle\alpha\beta\,|\,v\,|\,\dot{\gamma}\delta\rangle+\langle\alpha\beta\,|\,v\,|\,\gamma\dot{\delta}\rangle$$
$$=\frac{i}{\hbar}[\langle(h\alpha)\beta\,|\,v\,|\,\gamma\delta\rangle+\langle\alpha(h\beta)\,|\,v\,|\,\gamma\delta\rangle-\langle\alpha\beta\,|\,v\,|\,(h\gamma)\delta\rangle-\langle\alpha\beta\,|\,v\,|\,\gamma(h\delta)\rangle]$$
$$=\frac{i}{\hbar}\sum_\lambda[\epsilon_{\alpha\lambda}\langle\lambda\beta\,|\,v\,|\,\gamma\delta\rangle+\epsilon_{\beta\lambda}\langle\beta\lambda\,|\,v\,|\,\gamma\delta\rangle-\langle\alpha\beta\,|\,v\,|\,\lambda\delta\rangle\epsilon_{\lambda\gamma}-\langle\alpha\beta\,|\,v\,|\,\gamma\lambda\rangle\epsilon_{\lambda\delta}] . \tag{10}$$

Using this relation and Eqs. (3) and (4), we can show that $dE_{\rm cor}/dt$ becomes

$$-\sum_{\lambda\lambda'}\epsilon_{\lambda\lambda'}\dot{n}_{\lambda'\lambda}=-\left[\frac{dE_{\rm HF}}{dt}\right] .$$

Thus the total energy is conserved. However, the relation (10) holds true only if a complete set of the s.p. basis is used. Truncation of s.p. basis leads to a small violation of the total energy conservation, as will be discussed in Sec. IV.

D. Total momentum conservation

The conservation of the total momentum can be proved in a similar way if the interaction has Galilean invariance. The total momentum is defined as

$$\mathbf{P}=\sum_{\lambda\lambda'}\langle\lambda\,|\,\mathbf{p}\,|\,\lambda'\rangle n_{\lambda'\lambda} . \tag{11}$$

The time derivative of $\mathbf{P}$ is given as

$$\frac{d\mathbf{P}}{dt}=\frac{i}{\hbar}\sum_{\lambda\lambda'}\langle\lambda\,|\,[h,\mathbf{p}]\,|\,\lambda'\rangle n_{\lambda'\lambda}+\sum_{\lambda\lambda'}\langle\lambda\,|\,\mathbf{p}\,|\,\lambda'\rangle\dot{n}_{\lambda'\lambda} , \tag{12}$$

where Eq. (2) is used for the time derivative of the s.p. states. For simplicity, we neglect the exchange term in $U_{\rm HF}$ and assume that the effective interaction is a functional of $|\mathbf{r}-\mathbf{r}'|$. Then, $U_{\rm HF}$ is given by $U_{\rm HF}=\int d\mathbf{r}'V(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}')$. The first term on the right-hand side of Eq. (12) becomes

$$\frac{i}{\hbar}\sum_{\lambda\lambda'}\langle\lambda\,|\,[h,\mathbf{p}]\,|\,\lambda'\rangle n_{\lambda'\lambda}=-\int d\mathbf{r}\,d\mathbf{r}'\rho(\mathbf{r})[\nabla_{\mathbf{r}}V(\mathbf{r}-\mathbf{r}')]\rho(\mathbf{r}')=0 , \tag{13}$$

where we use $\mathbf{p}=-i\hbar\nabla$.

The second term on the right-hand side of Eq. (12) is rewritten with the use of Eq. (4),

$$\sum_{\lambda\lambda'}\langle\lambda\,|\,\mathbf{p}\,|\,\lambda'\rangle\dot{n}_{\lambda'\lambda}=-(1/\hbar^2)\sum_{\lambda\alpha\alpha'\beta\beta'\gamma\delta\delta'}\langle\lambda\delta'\,|\,[\mathbf{p},v]\,|\,\alpha\beta\rangle\int_{-\infty}^{t}dt'\langle\alpha'\beta'\,|\,v\,|\,\gamma\delta\rangle_{A}\,|_{t'}$$
$$\times\{[\delta_{\alpha\alpha'}-n_{\alpha\alpha'}(t')][\delta_{\beta\beta'}-n_{\beta\beta'}(t')]n_{\gamma\lambda}(t')n_{\delta\delta'}(t')$$
$$-n_{\alpha\alpha'}(t')n_{\beta\beta'}(t')[\delta_{\gamma\lambda}-n_{\gamma\lambda}(t')][\delta_{\delta\delta'}-n_{\delta\delta'}(t')]\} . \tag{14}$$

The first matrix element in the above equation becomes

$$\langle\lambda\delta'\,|\,[\mathbf{p},v]\,|\,\alpha\beta\rangle=-i\hbar\langle\lambda\delta'\,|\,\nabla_{\mathbf{r}}v(\mathbf{r}-\mathbf{r}')\,|\,\alpha\beta\rangle , \tag{15}$$

where we assume that the residual interaction depends only on the relative coordinate. In the sum of Eq. (14) the matrix element appears in the following combinations:

$$\langle\lambda\delta'\,|\,\nabla_{\mathbf{r}}v(\mathbf{r}-\mathbf{r}')\,|\,\alpha\beta\rangle+\langle\delta'\lambda\,|\,\nabla_{\mathbf{r}}v(\mathbf{r}-\mathbf{r}')\,|\,\beta\alpha\rangle \tag{16}$$

or

$$\langle\lambda\lambda\,|\,\nabla_{\mathbf{r}}v(\mathbf{r}-\mathbf{r}')\,|\,\alpha\alpha\rangle . \tag{17}$$

These combinations of the matrix elements are identical to zero because the interaction depends only on $|\mathbf{r}-\mathbf{r}'|$. Thus the total momentum is also conserved.

E. Time reversal property

We discuss a time reversal property of the equation of motion. We show that the time reversal of the density matrix $\rho(\mathbf{r},\mathbf{r}';t)$ defined as

$$\rho_T(\mathbf{r},\mathbf{r}';t)=\rho(\mathbf{r},\mathbf{r}';-t)^* \tag{18}$$

satisfies the same equation of motion as $\rho(\mathbf{r},\mathbf{r}';t)$. Since the complex conjugate of Eq. (2) is

$$-i\hbar\frac{\partial}{\partial t}\psi^*(\mathbf{r},t)=h(\rho^*)\psi^*(\mathbf{r},t) , \tag{19}$$

the time reversal of the s.p. wave function, $\psi_T(\mathbf{r},t)=\psi^*(\mathbf{r},-t)$, satisfies the same equation of motion as Eq. (2):

$$i\hbar\frac{\partial}{\partial t}\psi_T(\mathbf{r},t)=h(\rho_T)\psi_T(\mathbf{r},t) . \tag{20}$$

The time reversal of $n_{\lambda\lambda'}$ is defined in the same way as ρ_T,

$$n^T_{\lambda\lambda'}(t)=n^*_{\lambda\lambda'}(-t) . \tag{21}$$

Using the equation of motion for $n_{\lambda\lambda'}$, we can show that the time derivative of $n^T_{\lambda\lambda'}(t)$ is given as

$$\frac{d}{dt}n^T_{\lambda\lambda'}(t)=-(1/\hbar^2)[G_{\lambda\lambda'}(\psi_T(t),n^T_{\lambda\lambda'}(t))+G^*_{\lambda'\lambda}(\psi_T(t),n^T_{\lambda\lambda'}(t))] , \tag{22}$$

where $G_{\lambda\lambda'}$ is the same functional as $F_{\lambda\lambda'}$ of ψ_λ and $n_{\lambda\lambda'}$, except for the interval of the t' integration. The t' integration in $G_{\lambda\lambda'}$ is from ∞ to t. Equation (22) is apparently different from the equation of motion for $n_{\lambda\lambda'}$, Eq. (3). However, the starting time of the t' integration does not have special meaning; if we start the t' integration from 0, the equation of motion for $n^T_{\lambda\lambda'}$ is the same as that for $n_{\lambda\lambda'}$. In this sense the equation of motion for ρ is time reversal invariant.

III. CALCULATIONAL DETAILS

We apply the model to head-on collisions of $^{16}O+{}^{16}O$. The TDHF code with axial symmetry[13] is used. The mean potential is obtained from the Bonche-Koonin-Negele force[14] with the Coulomb interaction. The s.p. states are taken up to the $2s$-$1d$ shell. Each state can be occupied by four nucleons due to spin-isospin degeneracy. The $1s$ and $1p$ states are assumed to be initially completely occupied and the $2s$-$1d$ states totally empty. The s.p. states are labeled by absolute azimuthal quantum number $|m|$ and z parity since the system is axially symmetric

along the z direction (beam direction) and has reflection symmetry with respect to the plane perpendicular to the beam. The nondiagonal elements of $n_{\lambda\lambda'}$ between s.p. states with different $|m|$ and z parity vanishes.

We use a residual interaction of the δ function form $v=v_0\delta^3(r-r')$, with $v_0=-300$ MeV fm^3. The residual interaction gives the NN cross section of about 40 mb in the Born approximation. In the evaluation of the t' integration in Eq. (4), we assume that the residual interaction vanishes as $t'\to-\infty$ and neglect the ground-state correlation effect on $n_{\lambda\lambda'}$ before the two nuclei interact. For simplicity we neglect the exchange part of the matrix element of the right-hand side of Eq. (4).

The mesh size is 0.4 fm both in the beam direction and in the perpendicular direction. The time step ΔT is 0.25×10^{-23} s. The time derivative of $n_{\lambda\lambda'}$ is evaluated at the midpoint between T and $T+\Delta T$, and the predictor-corrector method is used to get $n_{\lambda\lambda'}(T+\Delta T)$ from $n_{\lambda\lambda'}(T)$ as is used for ψ_λ in TDHF. The computation time for one time step in the TDDM method is a factor of 70 larger than that in TDHF theory.

IV. RESULTS

We show the results for $E_{\rm lab}=100$ MeV. At this energy the low-L window is already open in TDHF theory. The fusion threshold in TDHF theory is $E_{\rm lab}=54$ MeV for the force of Ref. 14. In Fig. 1 the time evolution of the diagonal elements of $n_{\lambda\lambda'}$ as well as that of the s.p. energies are shown for four states near the initial Fermi level. The occupation probabilities of these states change most significantly during the collision. The s.p. energies $\epsilon_{\lambda\lambda}$ are shown in the lower part of Fig. 1. There is no crossing in the s.p. energies. Nevertheless, the relaxation of $n_{\lambda\lambda}$ starts after the significant change in the s.p. energies. This is due to the spreading of the s.p. spectrum during the collision, which is taken into account by the t' integration in Eq. (4).

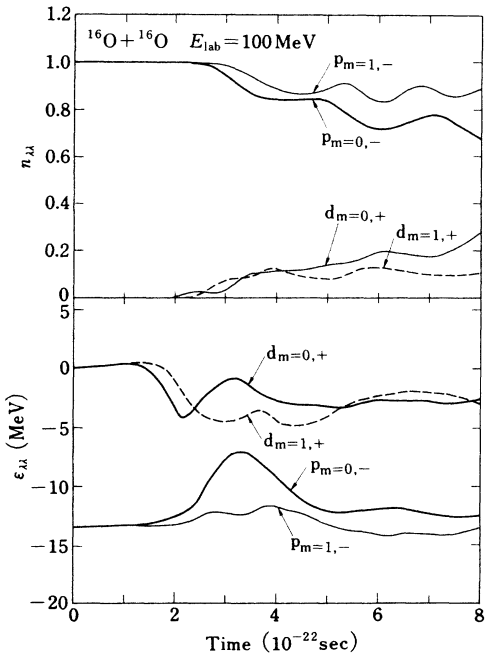

FIG. 1. Time evolution of the diagonal elements of the occupation matrix (upper part) and that of the s.p. energies (lower part) defined by $\epsilon_\lambda=\langle\lambda|h|\lambda\rangle$ in the head-on collision of ^{16}O + ^{16}O at $E_{\rm lab}=100$ MeV. The s.p. states are labeled by the asymptotic quantum numbers, i.e., orbital angular momentum, azimuthal quantum number, and z parity.

It may be useful to see the equilibration process with the entropy given by the eigenvalues n_α of $n_{\lambda\lambda'}$,

$$S/k_B=-\sum_\alpha[n_\alpha\ln n_\alpha+(1-n_\alpha)\ln(1-n_\alpha)]\ , \tag{23}$$

where k_B is the Boltzmann constant. The entropy is shown in Fig. 2 as a function of time. The entropy increases rapidly in the early stage of the collision and seems to saturate in the later stage. The time during which the entropy approaches its saturated value is about 2×10^{-22} s. (See Fig. 2.) This is near the value estimated by Toepffer and Wong[15] in the Fermi gas model. The entropy oscillates in the later stage and is, therefore, not an increasing function. In general, the entropy defined by Eq. (23) does not always increase for the quantum system consisting of a small number of particles such as that considered here.[16]

In the following we discuss the energy conservation. Although the theory conserves the total energy, the numerical calculation shows a small deviation of the total energy as indicated in Fig. 3, where the time evolution of the total energy $E_{\rm tot}$, the Hartree-Fock energy $E_{\rm HF}$, and the correlation energy $E_{\rm cor}$ are shown. The total energy is increased by about 20 MeV during the collision. The origin of the violation of the total energy conservation was studied in a previous paper for one-dimensional systems.[5] It was found that the violation is due not to numerical inaccuracy of the calculation, but to the truncation of the s.p. basis. As was discussed in Sec. II, the time derivative of $E_{\rm cor}$ [Eq. (8)] is equal to that of $E_{\rm HF}$ unless the s.p. basis is truncated.

The distance between the center of mass of the right-hand fragment and that of the left-hand fragment is shown in Fig. 4 as a function of time. The solid curve is

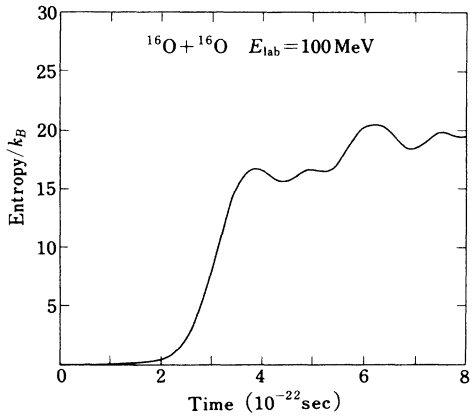

FIG. 2. Time evolution of the entropy in the head-on collision of ^{16}O + ^{16}O at $E_{\rm lab}=100$ MeV.

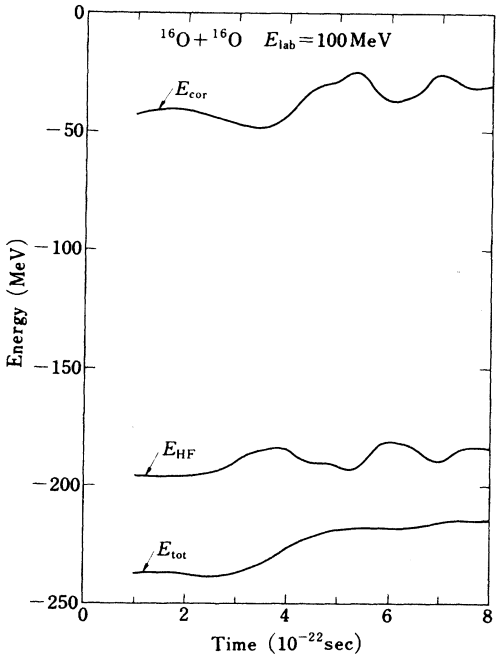

FIG. 3. Time evolution of the total energy E_{tot}, the Hartree-Fock energy E_{HF}, and the correlation energy E_{cor} in the head-on collision of $^{16}O + ^{16}O$ at $E_{lab} = 100$ MeV.

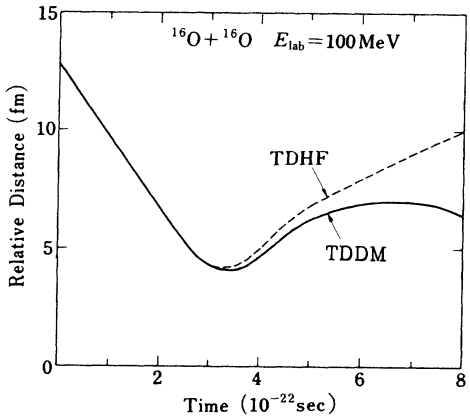

FIG. 4. Time evolution of the relative distance between the right-hand fragment and the left-hand fragment in the head-on collision of $^{16}O + ^{16}O$ at $E_{lab} = 100$ MeV. The solid curve denotes the TDDM result and the dashed curve the TDHF one.

the TDDM result and the dashed curve the TDHF result. In the TDDM model the system survives the first separation phase. This event can be regarded as fusion, according to the criterion of fusion given in Ref. 17.

The effects of NN collisions on the dissipation of the kinetic energy of the relative motion is very small at this incident energy, as might be understood from the time evolution of the occupation numbers in Fig. 1. In fact, about 75% of the kinetic energy of the relative motion is dissipated in TDHF theory. However, NN collisions play a crucial role in determining whether the system fuses or not. This is because the final kinetic energy of the relative motion in TDHF theory is only several MeV above the Coulomb barrier.

In order to determine the threshold energy for the opening of inelastic scatterings, we performed calculations at higher energies and found the threshold to lie between 140 and 150 MeV. (Since the numerical calculation was time consuming, we did not try to find a more accurate value of the threshold energy.) The threshold will become even higher if we use more s.p. states in the calculation.

V. SUMMARY

We presented a model for the extended TDHF theory which incorporates the nucleon-nucleon collision effects into the mean field in a quantum mechanical way. It was shown that the model conserves particle number, total momentum, and total energy. The time reversal property of the equation of motion was also discussed. The model was applied to head-on collisions of $^{16}O + ^{16}O$ to study the controversial problem of the low-L limit for fusion which is predicted by TDHF calculations. It was found that the threshold energy for the low-L limit is increased from $E_{lab} = 54$ MeV to $E_{lab} \approx 145$ MeV by nucleon-nucleon collisions effects. This is consistent with experiments.

The nucleon-nucleon collision is not a unique effect which increases the threshold energy for the low-L limit. There is an attempt to resolve the problem of the low-L window by improving the TDHF calculations. Umar *et al.*[18] included within the framework of TDHF theory the spin-orbit force, which has so far been neglected in TDHF calculations, and found that the threshold energy above which the low-L window opens is increased by a factor of 2.

ACKNOWLEDGMENTS

The author would like to thank Prof. G. Bertsch for a critical reading of the manuscript. He is indebted to Dr. K. T. R. Davies for providing him with the TDHF code. The generous hospitality of the Cyclotron Laboratory at RIKEN is also acknowledged. This work was supported in part by the Japan Society for the Promotion of Science Fellowships for Japanese Young Scientists and in part by the National Science Foundation. The numerical calculation was carried out with the FACOM-M380 at RIKEN.

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