Effect of high spin states on fusion in heavy ion collisions

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(Received 25 June 1987)

The behavior of very high rotational states of the highly excited compound nuclear systems formed in fusion reactions is studied in the framework of the statistical theory. The very high spin states populated in these reactions results in the observation of phenomena like backbending and yrast traps. The experimental data on the angular momentum limitation on fusion probability are reproduced at higher entropy values. Shape transitions are observed for the systems $^{12}C + ^{12}C$, $^{16}O + ^{16}O$, $^{18}O + ^{18}O$, and $^{40}Ca + ^{40}Ca$. We predict a shift in the yrast minima towards higher angular momentum states in the case of $^{40}Ca + ^{40}Ca$ with the increase in the excitation energy and the entropy of the compound nuclear system. Lines of constant entropy or constant level density are found to be almost equally spaced.

I. INTRODUCTION

One of the main interests in studying very high spin states formed in heavy ion collisions is to estimate the probability of fusion which is inhibited by the very high spin imparted to the final compound nuclear system. This may lead to the formation of a relatively long lived isomeric state at high spin called yrast trap. Calculations for detecting yrast traps have so far been performed in the rare earth region[1-3] and also for the mass range A lying between 75 and 100.[4] The region of low mass numbers has not been studied with the same emphasis as for the heavy nuclei because of the unstable nature of the system in the low mass range. It is found that the same rotational alignment and backbending effects appear in light nuclei as in heavy nuclei and one can look for the presence of yrast traps[5] in these nuclei.

In this work, the excited compound nuclear system is treated microscopically using single particle model within the framework of the statistical theory, and the nuclear level density of the compound system is calculated when the relative angular momentum of the collision partners is transferred to the final system as intrinsic spin of the compound nucleus. Since the inputs for the statistical theory are microscopic single particle levels corresponding to triaxially deformed nuclei, the results exhibit the effect of the shell structure of the system. In fact, certain predictions of the existence of isomeric collective rotational states are possible through the calculations involving the statistical theory.[6] The usual procedure in statistical calculation involves the determination of the grand partition function of the system with the condition that it conserves the energy, the particle number, and the total angular momentum of the system. The single particle level scheme used here involves eigenstates of the triaxially deformed harmonic oscillator for diagonalization.[7] We have performed our calculations for the hot rotating compound systems $^{12}C+^{12}C$, $^{16}O+^{16}O$, $^{18}O+^{18}O$, and $^{40}Ca+^{40}Ca$. Existence of isomeric states along the collective rotational deexcitation of the hot compound system is predicted. In Sec. II, we present the method of obtaining the single particle level spectrum using the triaxially deformed Nilsson harmonic oscillator potential. The renormalization of the total energy is not necessary here, since we are interested only in the energy differences, i.e., the energy difference between the ground and excited states of the system and not in their absolute values. In Sec. III, the statistical theory used in the calculation of nuclear level density for high spin states is presented. In view of the very high excitation energies of the compound nucleus, the statistical theory of hot rotating nuclei initiated by Moretto[6] and modified by us[8] with the inclusion of the dimensionality of phase space is used here. This is an extension of our earlier work on nuclear level density[9-14] for spinless nuclei where we have avoided the use of the usual saddle point method. The results are discussed in Sec. IV and we draw interesting conclusions on the behavior of heavy systems in very high angular momentum states populated in heavy ion reactions. In heavy ion collisions the population of levels in the final compound system with very high spin is an important factor that decides the stability of the resulting system.[15-27]

II. TRIAXIALLY DEFORMED NILSSON OSCILLATOR POTENTIAL

For rotating light nuclei, it has been assumed that the nucleons move in a triaxially deformed Nilsson harmonic oscillator potential with the deformation described by δ and θ. The triaxial Nilsson Hamiltonian is

$$H = p^2/2m + (m/2)(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) + C\boldsymbol{l}\cdot\mathbf{s} + D(l^2 - 2\langle l^2\rangle) . \quad (1)$$

The three oscillator frequencies are given by

$$\omega_x = \omega_0[1-(2\delta/3)\cos(\theta - 2\pi/3)] ,$$
$$\omega_y = \omega_0[1-(2\delta/3)\cos(\theta + 2\pi/3)] , \quad (2)$$
$$\omega_z = \omega_0[1-(2\delta/3)\cos\theta] ,$$

with the constraint that the total volume remains constant such that

$$\omega_x\omega_y\omega_z=\mathring{\omega}_0^3 \text{ a constant.}$$

The deformation parameters δ and θ are varied in the range $\delta=0.0$ to 0.6 with $\Delta\delta=0.1$, and $\theta=-180°$ to $-120°$ with $\Delta\theta=20°$. For the Nilsson parameters κ and μ, and for $\hbar\mathring{\omega}_0$, the following values[7] are chosen:

$$\begin{aligned}&\kappa=0.093\ ,\\&\mu=0.15\ ,\\&\hbar\mathring{\omega}_0=45.3\ \text{MeV}/(A^{1/3}+0.77)\ .\end{aligned}\tag{3}$$

The same values are used for protons and neutrons. The triaxial Nilsson Hamiltonian is diagonalized[28] in the cylindrical representation using the matrix elements given in Ref. 29. The necessity of renormalizing the total energy does not arise here, since we are interested only in the energy differences between the excited state and the ground state of the system and not in the actual magnitude of the energies. The single particle spin projections m_i along the symmetry axis which contribute to the rotation about an axis perpendicular to the symmetry axis are also necessary for the calculations. In this formalism, only the z component M of the total angular momentum is considered. As illustrated by Moretto[6] the laboratory fixed z axis can be made to coincide with the body fixed z' axis and it is possible to identify and substitute M for the total angular momentum I. In the quantum mechanical limit, $M\rightarrow\sqrt{I(I+1)}=I+\frac{1}{2}$, where I is the total angular momentum of the system.

III. THE STATISTICAL THEORY

We start with the grand canonical partition function for a system of N neutrons and Z protons in a state of total angular momentum M along the direction of the rotation axis. The statistical properties of the system are contained in the grand partition function

$$Q(\alpha_z,\alpha_n,\beta,\gamma)=\Sigma\exp(-\beta E_i+\alpha_z Z_i+\alpha_n N_i+\gamma M_i)\ .\tag{4}$$

The Lagrangian multipliers α_z, α_n, and γ conserve the proton number, neutron number, and total angular momentum M along the z axis for a given temperature $T=1/\beta$. The corresponding conservation equations are

$$\begin{aligned}&-\partial\ln Q/\partial\beta=\langle E\rangle\ ,\\&\partial\ln Q/\partial\alpha_z=\langle Z\rangle\ ,\\&\partial\ln Q/\partial\alpha_n=\langle N\rangle\ ,\\&\partial\ln Q/\partial\gamma=\langle M\rangle\ .\end{aligned}\tag{5}$$

The corresponding equations in terms of the single particle energies and the spin projections m_i are given by

$$\begin{aligned}&\langle Z\rangle=\Sigma n_i^z=\Sigma[1+\exp(-\alpha_z+\beta\epsilon_i^z+\gamma m_i^z)]^{-1}\ ,\\&\langle N\rangle=\Sigma n_i^n=\Sigma[1+\exp(-\alpha_n+\beta\epsilon_i^n+\gamma m_i^n)]^{-1}\ ,\\&\langle E\rangle=\Sigma n_i^n\epsilon_i^n+\Sigma n_i^z\epsilon_i^z\ ,\\&\langle M\rangle=\Sigma n_i^n m_i^n+\Sigma n_i^z m_i^z\ .\end{aligned}\tag{6}$$

These equations fix the Lagrangian multipliers α and γ. The entropy of the system is then obtained as

$$\begin{aligned}S&=\partial(T\ln Q)/\partial T\\&=S_z+S_n\ ,\end{aligned}$$

where

$$S_z=-\Sigma[n_i^z\ln n_i^z+(1-n_i^z)\ln(1-n_i^z)]$$

and

$$S_n=-\Sigma[n_i^n\ln n_i^n+(1-n_i^n)\ln(1-n_i^n)]\ .\tag{7}$$

The excitation energy E^* of the system for an angular momentum state M can be expressed as

$$E^*(M,T)=\left[\Sigma n_i^z\epsilon_i^z-\sum_{i=1}^{Z}\epsilon_i^z\right]+\left[\Sigma n_i^n\epsilon_i^n-\sum_{i=1}^{n}\epsilon_i^n\right]\tag{8}$$

The level density[16] of the system $\rho(M,E^*)$ as obtained in Ref. 8 is

$$\rho(M,E^*)=\beta e^{S(M,E^*)}/S_{\max}\ .\tag{9}$$

The normalization factor $S_{\max}$ depends on the dimensionality of phase space which is the number of eigenstates used.

$$S_{\max}=\ln\{N_0!/[N!(N_0-N)!]\}+\ln\{N_0!/[Z!(N_0-Z)!]\}\ .\tag{10}$$

When $N/N_0\ll1$, the denominator in ρ varies as $1/N_0$. It is obvious that the dependence of the normalization factor on N_0 is weak compared to its dependence on N or Z as should be expected. But the entropy $S(M,E^*)$ is very sensitive to the variation of the value of N_0 especially at very high temperatures.

In our calculations, we vary the temperature from 0 to 3 MeV, and for each temperature the excitation energy $E^*(M,T)$ and the entropy $S(M,E^*)$ are computed as a function of the deformation parameters δ and θ. The total energy $\langle E\rangle$ is then minimized with respect to δ and θ. For a given spin M the entropy is different for different temperatures. Constant entropy lines are then drawn in the plane of E^* and M. The constant entropy lines are equivalent to drawing constant level density lines. These lines are depicted in Figs. 1–4 for the systems $^{12}C+^{12}C$, $^{16}O+^{16}O$, $^{18}O+^{18}O$, and $^{40}Ca+^{40}Ca$.

IV. RESULTS AND CONCLUSIONS

The calculations and the results presented here are for hot rotating compound nuclear systems and should not be confused with those for the molecular resonance spectra. In Fig. 1, the excitation energy E^* for the system $^{12}C+^{12}C$ is plotted against the z projection M of the spin for a constant entropy of the system. These constant en-

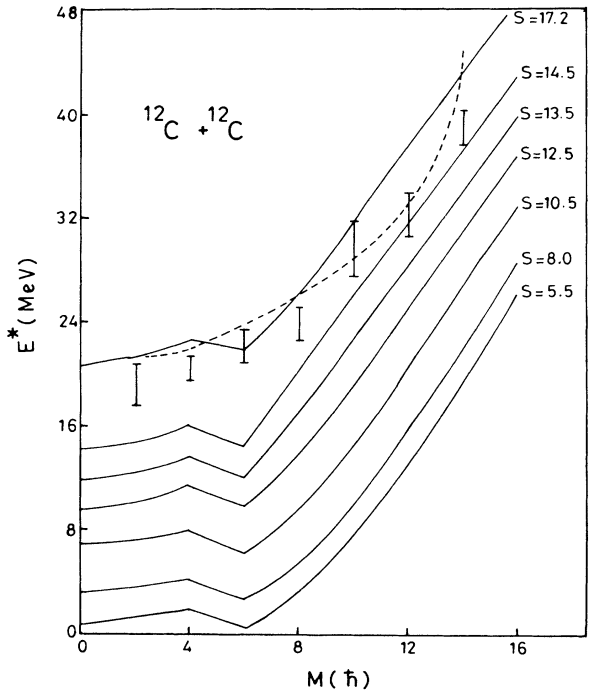

FIG. 1. Constant entropy lines for the compound nuclear system $^{12}C+^{12}C$ formed in fusion. The lowest curve corresponding to the temperature $T\rightarrow 0$ is identified as the yrast line. The dashed line represents an entrance model fit (Ref. 32) to σ_{fus} and gives the limiting angular momenta for fusion of $^{12}C+^{12}C$ from Refs. 30 and 31. The vertical bars give position and width of molecular resonances in $^{12}C+^{12}C$.

tropy lines are useful in obtaining the available region for fusion for a given final excitation energy when the system has lost all its spin. These lines for various values of the entropy show a systematic dip at $M=6\hbar$. This sudden drop in excitation energy at $M=6\hbar$ indicates a structural change in the rotating system which may be associated with a shape transition. The energy minimum shifts from $\theta=-120°$ to $-180°$, i.e., from prolate to oblate as the spin increases. We predict that the system may well be trapped in this pocket in the process of deexcitation along the constant entropy line. Consequently a sudden change in the moment of inertia of the system occurs around this value of M and leads to the familiar backbending phenomenon. The lowest entropy line corresponds to the yrast line. The experimental points (dashed lines) are also plotted and we find that the entropy line $S=17.2$ corresponds to the experimental observation. It is predicted that the system should ultimately end up with an excitation energy $E^*\simeq 20$ MeV at $M=0$.

Figures 2 and 3 show similar lines for the two systems $^{16}O+^{16}O$ and $^{18}O+^{18}O$, respectively. The effect of backbending is found to be less in these two systems when compared to $^{12}C+^{12}C$. For a temperature below 0.8 MeV the compound nuclear system $^{16}O+^{16}O$ changes its shape as the spin increases. However, at higher temperatures, almost for all spins, the shape remains oblate.

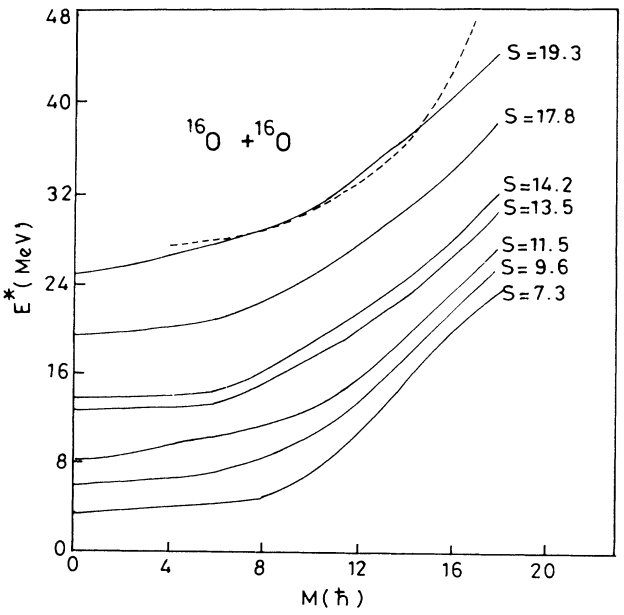

FIG. 2. Constant entropy lines for the compound nuclear system $^{16}O+^{16}O$. The dashed line gives the limiting angular momenta for fusion of $^{16}O+^{16}O$ from Ref. 33.

From Fig. 2 we find that for the system $^{16}O+^{16}O$, the entropy line $S=19.3$ corresponds to the experimental observation (dashed line) and it is expected that the system should finally end up with an excitation energy $E^*\simeq 25$ MeV at $M=0$. The compound nuclear system ^{36}S formed in the fusion of $^{18}O+^{18}O$ seems to be almost spherical for $M<6$ and acquires oblate symmetry for $M>6$ at all temperatures.

In the case of $^{40}Ca+^{40}Ca$ interesting results are obtained. The difference in behavior of this system from the previous two systems is obvious from Fig. 4. The pockets of minima of the excitation energy is found to shift from $M=8\hbar$ to $M=18\hbar$ for various excitations of the system as the entropy increases from $S\simeq 6.8$ to 32.

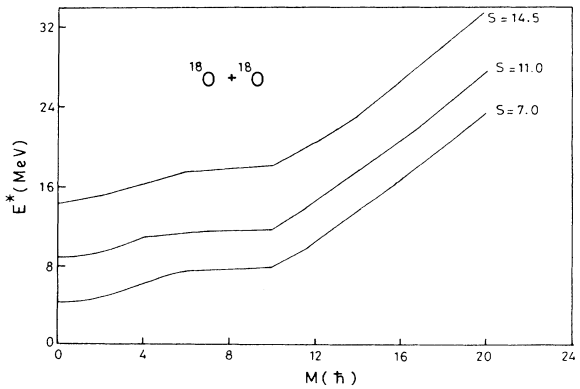

FIG. 3. Constant entropy lines for the compound nuclear system $^{18}O+^{18}O$.

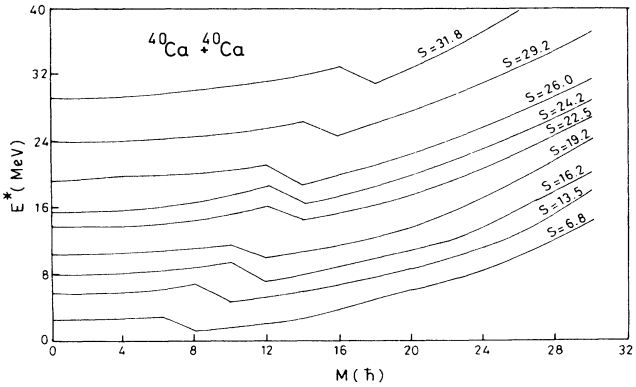

FIG. 4. Constant entropy lines for the compound nuclear system $^{40}Ca + ^{40}Ca$.

This type of peculiar behavior has not been hitherto observed. As the excitation energy and consequently the entropy increases, the energy minimum in the δ-θ plane shifts towards larger values, causing the minima to occur at higher angular momentum states. This is an interesting observation which reflects the interplay between the effects of temperature, spin and deformation. This interplay between the excitation of the system due to the temperature T which causes the particle to occupy higher levels $\epsilon_i > \epsilon_F$ and that due to the spin M of the system is via the Lagrangian multipliers α and γ. The excitation energy E^* when $M=0$ is the excitation energy for the nonrotating nuclei and is purely due to the temperature of the system.

Comparing the present set of curves with those of Diebel *et al.*[5,30–33] we may conclude that, for cold systems, there is no yrast line limitation of fusion but, for hot systems, the constant entropy lines may have an effect on the fusion of colliding partners since the phase space available for the hot compound system is determined by the entropy S.[34] These lines are found to be roughly at constant energy above the yrast line as stated by Newton.[35] Pairing correlations do not play any role in the highly excited compound system. The superfluid to normal transition takes place even at $T \simeq 0.6$ MeV. The effects of neck formation[36–38] have to be included in the study to throw more light in understanding the complex phenomenon of nuclear fusion.

ACKNOWLEDGMENTS

This investigation has been suggested by Professor Walter Greiner and it is a pleasure to thank him for stimulating discussions on this subject. Fruitful discussions with Dr. G. Shanmugam are also acknowledged with thanks. This work is partially supported by the University Grants Commission, India under the program of the Committee on Strengthening the Infrastructure of Science and Technology.

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