Spatial development of inelastic scattering and particle transfer cross sections in very heavy ion reactions

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(Received 24 June 1987)

A quantal method is presented to investigate the spatial development of inelastic scattering cross sections in heavy ion reactions. Application is made to the case of 1120 MeV $^{208}\text{Pb}+{}^{238}\text{U}(0^+,2^+,\ldots,40^+)$.

The abstract of a recent Letter[1] promises to demonstrate that "direct one- or two-particle transfer reactions are capable of probing nuclear structure in the $I\simeq 20\ \hbar$ region for rare earth nuclei and 30 $\hbar$ for actinide nuclei." In this paper we develop a theoretical method to investigate the angular momentum development of the wave functions in the specific spatial region where particle transfer takes place. We find that the $^{208}\text{Pb}+{}^{238}\text{U}$ at the Coulomb barrier does not probe nuclear structure in the $I\approx 30\ \hbar$ region via one and two particle transfer to any significant degree. But to make our demonstration we must first discuss how the inelastic cross section develops spatially in a quantal coupled channels calculation.

For the sake of simplicity we consider only the orbital angular momentum partial wave $l=0$. This allows us to compute the quantal cross sections for the backward scattering trajectories in question. Our method can be generalized for forward scattering angles by the semiclassical identification

$$\theta=2\arctan\left[\frac{\eta}{l+\frac{1}{2}}\right],$$

but we will not consider that generalization in this paper.

Consider our case of a spherical projectile scattering on a deformed target, e.g., $^{208}\text{Pb}+{}^{238}\text{U}(0^+,2^+,\ldots,40^+)$. We only consider the strongest excitations: those of the ground state band of the target. Then in a converged solution of the coupled equations one has a set of scattering wave functions indexed by the state of the target nucleus, $\psi^I_{l=0}(r)$. The scattering amplitude, a^I_{out}, in a given channel may then be obtained by matching the wave function solution with the incoming and outgoing (Coulomb) wave functions, H, outside the range of the coupling,

$$\psi^I_{l=0}(r\to\infty)=\delta_{I,0}H_I^-(r\to\infty)-a^I_{\text{out}}H_I^+(r\to\infty)\ . \tag{1}$$

Note that the outgoing scattering state orbital angular momentum must be equal to the particular bound state angular momentum I. Thus the $l=0$ case has the simplifying feature that there is only one channel per bound state in the coupled channels calculation.

In the case where the semiclassical idea of a trajectory has validity, we may then simply write the cross section at 180° (in units of the Rutherford scattering cross section) as

$$\sigma^I(180^\circ)=|\,a^I_{\text{out}}\,|^2\ . \tag{2}$$

Note that we use a quantal definition for the quantities a^I_{out}. We also assume that much of the essential quantum nature of the calculation will not be destroyed by the $l=0\to\theta=180^\circ$ identification.[3]

To investigate how the $\sigma^I(180^\circ)$ develops along a trajectory, we propose the following ansatz: Define quantities $a^I_{\text{out}}(r)$ and $a^I_{\text{in}}(r)$ at every radius from the classical turning point outward,

$$\psi^I_{l=0}(r)=a^I_{\text{in}}(r)H_I^-(r)-a^I_{\text{out}}(r)H_I^+(r)\ . \tag{3}$$

Then in a semiclassical sense we may look at the development of the cross section as it comes in toward the turning point,

$$\sigma^I_{\text{in}}(r)=|\,a^I_{\text{in}}(r)\,|^2\ , \tag{4}$$

and then back out,

$$\sigma^I_{\text{out}}(r)=|\,a^I_{\text{out}}(r)\,|^2\ . \tag{5}$$

Of course, if the ansatz has complete validity, then at the turning point, r_t, we should find

$$\sigma^I_{\text{in}}(r_t)\approx\sigma^I_{\text{out}}(r_t) \tag{6}$$

as the flux in each channel reflects from the barrier.

We have implemented solutions for $a^I_{\text{in}}(r)$ and $a^I_{\text{out}}(r)$ in the quantum mechanical coupled channels code QUICC.[4] Inside the matching radius at which nuclear effects become negligible, Eq. (3) and its derivative are matched numerically to obtain $a^I_{\text{in}}(r)$ and $a^I_{\text{out}}(r)$. Outside the matching radius (in the region where only Coulomb excitation takes place) we note that the cross sections may be conveniently written in terms of the radially varying coefficients actually used in the code,[4] $C_I(r)\,C_I^+(r)$:

$$\psi^I_{l=0}(r)=C_I(r)f_I(r)-C_I^+(r)h_I^+(r)\ . \quad (7)$$

Outside the matching radius the regular and outgoing homogeneous optical model wave functions may be written in terms of the Coulomb wave functions and the homogeneous scattering amplitude η_I,

$$f_I(r)=\frac{i}{2}[H_I^-(r)-\eta_I H_I^+(r)]\ , \quad (8)$$

$$h_I^+(r)=H_I^+(r)\ , \quad (9)$$

so that

$$\psi^I_{l=0}(r)=\frac{i}{2}C_I(r)H_I^-(r) - \left[C_I^+(r)+\frac{i}{2}\eta_I C_I(r)\right]H^+(r)\ , \quad (10)$$

and

$$\sigma^I_{\rm in}(r)=\tfrac{1}{4}\,|\,C_I(r)\,|^2\ , \quad (11)$$

$$\sigma^I_{\rm out}(r)=\left|C_I^+(r)+\frac{i}{2}\eta_I C_I(r)\right|^2\ , \quad (12)$$

in terms of quantities already available in the computer program.

Application is made now to the case calculated classically by Guidry *et al.*[1] which is near the upper limit of possible high spin excitation via inelastic heavy ion reactions: ^{208}Pb+ ^{238}U at a bombarding energy near the Coulomb barrier. The bombarding energy of 1120 MeV corresponds to a distance of closest approach of $1.5(A_p^{1/3}+A_t^{1/3})$ as assumed in Ref. 1, which in this case equals 18.18 fm. The effects of nuclear distortion and deformation are included in addition to Coulomb excitation. Optical model parameters, reduced Coulomb excitation transition elements, and nuclear deformation parameters are taken to be the same as those of Guidry and collaborators.[1,5]

Figure 1 shows the development of the cross section probabilities in each state along the $l=0$ trajectory. Of course, at $r=\infty$ the incoming cross section $\sigma^I_{\rm in}=\delta_{I0}$, corresponding to only incoming flux in the elastic channel. Coming in to 8 fm outside the turning point the probability of the ^{238}U nucleus being in the 2^+ state, $\sigma^{2+}_{\rm in}$ is greater than 0.5, but no significant flux is in states higher than 6^+. As one follows the distribution of $\sigma^I_{\rm in}$ inward, one sees constant movement of flux to higher states. Most of the excitation in this case is Coulomb induced: The sum of the $\sigma^I_{\rm out}$ at $r=\infty$ is 0.92, indicating the small effect played by the complex nuclear optical potential. Note that the wave function does approximately reflect at the turning point, with $\sigma^I_{\rm in}(r_t)\approx\sigma^I_{\rm out}(r_t)$. Note also the large amount of Coulomb excitation that takes place in the region from the turning point to $\frac{1}{2}$ fm outside it. The most likely state goes from 10^+ at $\frac{1}{2}$ fm coming in, to 14^+ at the turning point, and to 18^+ at $\frac{1}{2}$ fm going out.

A similar calculation at 1200 MeV (corresponding to a distance of closest approach of 16.96 fm) exhibits less of

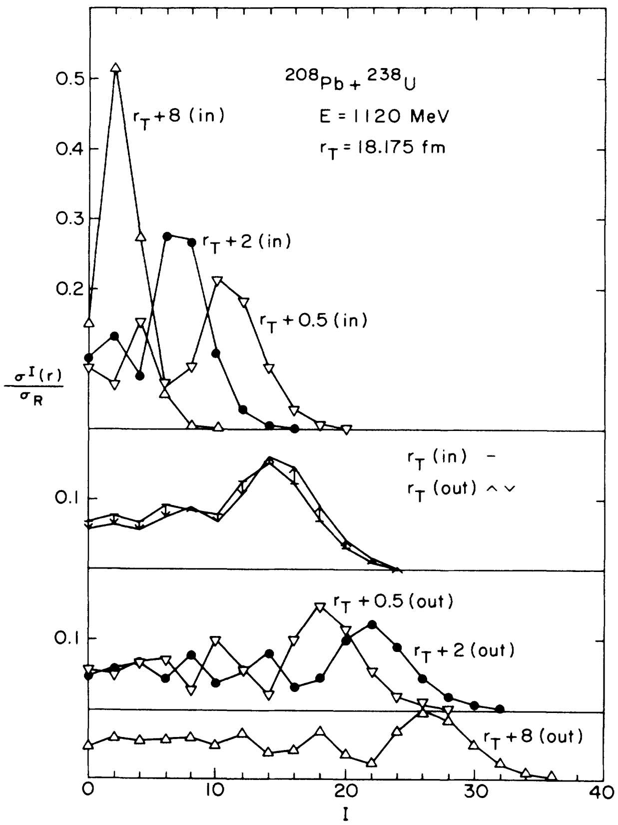

FIG. 1. Development of the cross section for each state along the $l=0$ trajectory for 1120 MeV ^{208}Pb+ ^{238}U. σ_R is the Rutherford cross section. At the turning point there is little difference between $\sigma^I_{\rm in}$ and $\sigma^I_{\rm out}$, indicating the dominance of reflection. The mark for $\sigma^I_{\rm out}$ points away from that for $\sigma^I_{\rm in}$. This emphasizes that inside the turning point low spin states lose a little cross section while high spin states gain a little.

a simple reflection of the wave function at the turning point. At this energy the total incoming flux at the turning point is 0.76, while the outgoing flux is 0.62 (the asymptotic total outgoing flux is 0.51 as compared to 0.92 for 1120 MeV). Thus, non-negligible absorption (and rearrangement) of flux takes place under the barrier at this slightly higher energy.

At the top of Fig. 2 the asymptotic cross section is shown for the various final states at 1120 MeV. Note that quite a bit of cross section remains in the low spin states. But the more relevant calculation is at the bottom. To facilitate comparison with the classical calculation of Guidry *et al.*, we have weighted the cross sections along the trajectory (as has been done in Ref. 1) with a transferlike factor "proportional to the square of a form factor dropping exponentially to $\frac{1}{2}$ of its value at the turning point when the ions are 1 fm outside the turning point." This weighted integration of flux or cross sections provides a measure of which states will be probed by one or two nucleon transfer reactions. Effects of changing the angular momentum from initial to final state by the transfer process itself are ignored in both

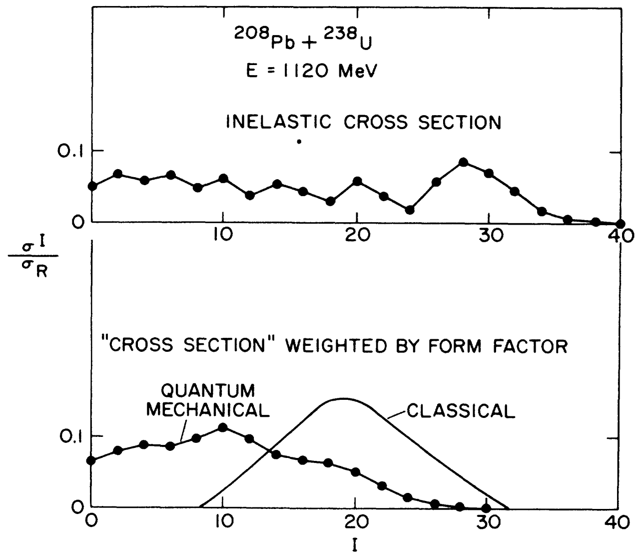

FIG. 2. Asymptotic cross section at 180° from the $l=0$ trajectory for 1120 MeV ^{208}Pb+ ^{238}U (top). Comparison of quantal and classical calculations for the same case in which both have a particle transfer form factor weighting (bottom).

the purely classical calculation and the present calculation; indeed, under sub-Coulomb conditions it is expected[6] that with similar interacting nuclei little change in angular momentum from initial to final state would occur—at least for nucleon transfers. Of course, for lighter heavy ions, changes in angular momentum due to particle transfer have been well understood for some time.[7] However, at present there has been no experimental evidence of either systematic increase or decrease of the average angular momentum in very heavy ion reactions due to particle transfer. In the bottom of Fig. 2 is shown a comparison of the present quantal calculation with the corresponding classical calculation. The quantal calculation shows a large amount of flux in the low spin states, with the 10^+ state being the most likely. The classical calculation has no flux in the lowest spin states, with the most likely state being 18^+ or 20^+. The lack of agreement of the classical calculations (both quantitatively and qualitatively) with our parallel quantal one calls into question the utility of such a classical approach.

As has been previously pointed out, the most favorable situation for observing particle transfer between the highest spin states occurs close to the Coulomb barrier.[2] At lower energies the Coulomb excitation is weaker and the closest distance between the two ions is too large to maximize particle transfer. At higher energies, the grazing angle (at which particle transfer takes place) moves forward, corresponding to a lower maximum spin state excited by Coulomb excitation. Furthermore, the effect of the nuclear inelastic interaction, which comes more into play above the barrier, is to further suppress the excitation of high spin states. Thus, the beam energy we have considered for the ^{208}Pb+ ^{238}U case is near optimal for probing high spin states in particle transfer. The results of this near optimal case presented at the bottom of Fig. 2 are clear: We find that, contrary to the classical calculation, the quantal calculation based on commonly accepted parameters shows that the ^{208}Pb+ ^{238}U reaction cannot probe nuclear structure in the $I=30\ \hbar$ region via one and two particle transfer reactions.

This research was carried out under the auspices of the U.S. Department of Energy under Contract No. DE-AC02-76CH-00016.

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