Low-energy ^{2}H(d,γ)^{4}He reaction and the *D*-state admixture in the ^{4}He ground state

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We have studied the low-energy ^{2}H(d,γ)^{4}He reaction based on a microscopic description of the nuclear wave functions. Our study reproduces the experimental reaction cross sections at $E \leq 3$ MeV; it allows for an extrapolation of the data to energies relevant for big-bang nucleosynthesis resulting in a cross section about 35 times higher than given in present compilations. Our results indicate a *D*-state admixture in the ^{4}He ground state of $\approx 5-7\%$.

It is well known that the ground state of ^{4}He (total angular momentum and parity $J^\pi = 0^+$) has a dominant component in which the spins of the four nucleons are coupled to $S=0$. Recent theoretical studies[1,2] based on various nucleon-nucleon interactions indicate that the ^{4}He ground state has also a non-negligible four-particle component with total spin $S=2$ and hence total orbital angular momentum $L=2$. Depending on the nucleon-nucleon interaction used, the calculations, however, make different predictions for the strength of this *D*-state admixture, ranging from 5.4% for the Paris potential[2] to 13% for local soft-core potentials.[1] Experimentally, the existence of this *D*-state component has been indicated by measurements of the tensor analyzing power in the ^{2}H(d,γ)^{4}He reaction at $E_{\text{c.m.}} = 4.85$ MeV (Ref. 3) and in the ^{89}Y(d,α_0)^{87}Sr reaction (Refs. 4 and 5). Heuristic model calculations which derived the two components of the ^{4}He ground state as (uncoupled) bound states of Woods-Saxon potentials and adjusted their amplitudes to these experimental tensor analyzing power data resulted in *D*-state admixtures of 4.8% and 7%, respectively. However, Mellema *et al.*[6] recently reported evidence that the radiation at $E_{\text{c.m.}} = 4.85$ MeV in the ^{2}H(d,γ)^{4}He reaction includes other multipoles in addition to the dominant $E2$ component, contradicting the assumption of a solely $E2$ transition made by Weller *et al.*

Very recently, new experimental data became available which undoubtedly indicate the existence of a *D*-state component in the ^{4}He ground state. Wilkinson and Cecil (Ref. 7) and Barnes *et al.* (Ref. 8) have measured the ^{2}H(d,γ)^{4}He reaction at very low energies ($E_{\text{c.m.}} \approx 25-300$ keV) and found the cross sections strongly enhanced compared with what is expected for an $E2$ transition from the $L=2$ scattering states in the d + d system into the $(L=0, S=0)$ component of the ^{4}He ground state. Furthermore, by analyzing angular distributions in this energy range, Barnes *et al.*[8] determined the ^{2}H(d,γ)^{4}He cross sections at $E_{\text{c.m.}} \leq 100$ keV to be due to $E2$ transitions from $(L=0, S=2)$ scattering states in the d + d system to the $(L=2, S=2)$ component in the ^{4}He ground state. Consequently, the new data on the ^{2}H(d,γ)^{4}He reaction finding cross sections at $E_{\text{c.m.}} < 200$ keV larger than previously accepted[9] will not only have a strong influence on big-bang astrophysics, but they can also be considered the best presently available tool to determine the *D*-state admixture in the ^{4}He ground state.

Until now low energy ^{2}H(d,γ)^{4}He data have been exclusively analyzed on the basis of a d + d potential model treating both deuterons as structureless particles. However, such a procedure has recently been seriously criticized and has been shown to be inappropriate if one aims to derive at quantitative statements about the ^{4}He ground state.[10] It has therefore been concluded that a reasonable study of the low energy ^{2}H(d,γ)^{4}He fusion reaction has to be performed on the basis of a microscopic many-body theory. In this paper we report about the first microscopic study of the ^{2}H(d,γ)^{4}He reaction at low energies deriving the $(L=0, S=0)$ and $(L=2, S=2)$ components of the ^{4}He ground state consistently from antisymmetrized four-nucleon wave functions and a nucleon-nucleon interaction containing a central, a spin-orbit, and a tensor component. In detail, we make the following ansatz for the ^{4}He ground state wave function,

$$\Psi_{\text{g.s.}} = \alpha\Psi_{\text{SM}}^{S=0,\, L=0} + \beta\Psi_{\text{SM}}^{S=2,\, L=2} + \Psi_{\text{dd}}^{S=0,\, L=0} + \Psi_{\text{dd}}^{S=2,\, L=2} \,, \tag{1}$$

where

$$\Psi_{\text{dd}}^{S,L} = \mathcal{A}\{[[\Phi_{\text{d}}^{I=1}\Phi_{\text{d}}^{I=1}]_S Y_L(\hat{\mathbf{r}})]_J g_{JL}(r)\} \tag{2}$$

is an antisymmetrized d + d cluster wave function in which the channel spin S of the two deuterons and their relative orbital angular momentum L is coupled to J. For the internal wave functions of the deuterons we adopt the three-Gaussian ansatz of Ref. 11. The $\psi_{\text{SM}}^{S,L}$ in (1) are the lowest ^{4}He harmonic oscillator shell model wave functions with spin S and orbital angular momentum L. The explicit consideration of the shell model components in (1) accounts for the fact that the ^{4}He ground state is apparently not a pure d + d cluster state.[12]

As the nucleon-nucleon interaction we adopt an effective interaction containing central,[11] spin-orbit,[13] and tensor[14] components which have been previously used in microscopic studies of the $A=4$ and 5 nucleon systems. This interaction reproduces the binding energy and the rms radius of the deuteron for the deuteron wave functions used in the present study. Our choice of the oscillator parameter ($b=1.39$ fm) minimizes the binding energy of the ^{4}He shell model ground state. The unknown quantities in (1)—the coefficients α,β and the relative wave functions g_{JL}—were determined by solving the four-particle Schrödinger equation which, for our model space, results in a system of four coupled equations of relative motion. These equations have been solved by the variational method developed in Ref. 15. In our study we find a binding energy of the ^{4}He nucleus of $E_B=30.5$ MeV, which is slightly higher than the experimental value $E_B=28.4$ MeV (Ref. 16). In the ground state wave function both shell model components dominate over the d + d cluster states. The two $(L=2, S=2)$ configurations in our ground state wave function add up to a total D-state admixture of 4.5%

In our study of the ^{2}H(d,γ)^{4}He reaction at low energy, the scattering states are assumed to be antisymmetrized d + d cluster wave functions (2), neglecting the explicit consideration of shell model components. This is justified since the scattering states relevant for the present study are nonresonant and highly collective. The relative wave functions g_{JL} are calculated from the Schrödinger equation of relative motion applying the technique of Ref. 15 and using the nucleon-nucleon interaction as defined above. Note that for the $(L=2, S=0)$ and $(L=0, S=2)$ scattering states, which as we discuss below are the entrance channel wave functions important for the present study, the spin-orbit interaction does not contribute, while the tensor interaction only yields a nonvanishing matrix element for the coupling between the two states. Motivated by the experimental observation that the tensor analyzing power in d + d elastic scattering is very small,[17] we neglect the tensor component and correspondingly the coupling of $S=0$ and 2 scattering states in the entrance channel. Note that this neglect is expected to have only a very weak influence on the low energy ^{2}H(d,γ)^{4}He cross section and hence on our determination of the D-state admixture in the ^{4}He ground state, as one can see by studying the influence of the tensor coupling term on the $(L=0, S=2)$ state in perturbation theory and by applying penetrability arguments. It has been shown that elastic d − d scattering at low energies is well described under the present assumptions.[11]

The ^{2}H(d,γ)^{4}He cross sections at low energies can be calculated in perturbation theory using the long-wavelength approximation for the many-body electromagnetic transition operator. In agreement with experiment,[8] we assume the γ radiation at $E_{\text{c.m.}} \le 3$ MeV to be of $E2$ multipolarity; hence,

$$Q^E_{2\mu}=\frac{e}{2}\sum_i |\mathbf{r}_i-\mathbf{r}_{\text{c.m.}}|^2 Y_{2\mu}(\hat{\mathbf{r}}_i-\hat{\mathbf{r}}_{\text{c.m.}})(1-\tau_{iz}) \ , \tag{3}$$

where the sum is over all particles and τ_{iz} is the z component of the isospin. Since $[Q^E_{2\mu},\mathbf{S}]=0$, the $E2$ transitions into the $(L=0, S=0)$ component of the ^{4}He ground state can only occur from the $(L=2, S=0)$ d + d scattering states, while $E2$ radiation into the D-state admixture of the ground state is possible from the $(L=0, S=2)$, $(L=2, S=2)$, and $(L=4, S=2)$ d + d scattering states. Due to penetrability arguments, the two latter configurations can be safely neglected when calculating the ^{2}H(d,γ)^{4}He cross sections at low energies, which is consequently given by

$$\sigma(E_{\text{c.m.}})=\sum_{S=0,2}\frac{4\pi}{75\hbar}\left[\frac{E_\gamma}{\hbar c}\right]^5\frac{2S+1}{45\hbar} \times |\langle \Psi_{\text{g.s.}}\|Q^E_2\|\Psi^{S,L}_{\text{dd}}(E_{\text{c.m.}})\rangle|^2 \ , \tag{4}$$

where $\Psi^{S,L}_{\text{dd}}(E_{\text{c.m.}})$ denotes the d + d scattering state with quantum numbers (L,S) at the energy $E_{\text{c.m.}}$ which we assume to be normalized to unit flux. In (4), E_γ is the energy of the emitted photon, which is connected with the ^{4}He binding energy E_B via $E_\gamma=E_{\text{c.m.}}+E_B-2E_{\text{d}}$, where $E_{\text{d}}=2.2$ MeV is the deuteron binding energy.

In accordance with the angular distribution analyses of Refs. 8 and 18, we expect our assumptions about the model space and the radiation multipolarity to be adequate for the ^{2}H(d,γ)^{4}He reaction at energies $E_{\text{c.m.}} \le 3$ MeV. At higher energies the results of Ref. 6 suggest multipolarities and fragmentations other than those considered in this study to be present in the experimental data.

Our results for the low-energy ^{2}H(d,γ)^{4}He cross sections are shown in Fig. 1 in terms of the astrophysical S factor,

$$S(E_{\text{c.m.}})=\sigma(E_{\text{c.m.}})E_{\text{c.m.}}\exp(2\pi\eta) \ , \tag{5}$$

with $2\pi\eta=31.4E_{\text{c.m.}}^{-1/2}$ ($E_{\text{c.m.}}$ in keV). The calculation reproduces the slope of the experimental data qualitatively

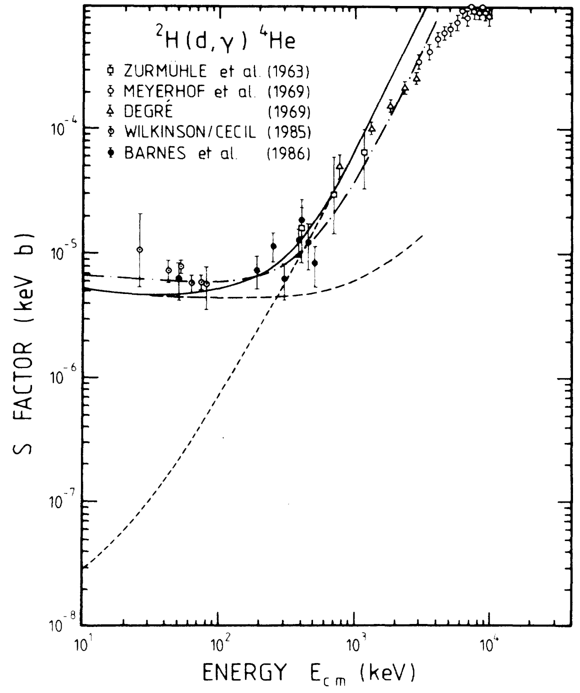

FIG. 1. Comparison of our microscopically calculated excitation function (solid line) for the reaction ^{2}H(d,γ)^{4}He in form of the astrophysical S factor with experimental data of Refs. 7, 8, and 18–22. This total rate is split into the components corresponding to capture into the $(L=0, S=0)$ and $(L=2, S=2)$ configurations of the ^{4}He ground state which are shown as long- and short-dashed lines, respectively. The astrophysical S factor as determined in our semimicroscopic calculation is shown by the dashed-dotted line.

well for energies $E_{\text{c.m.}} < 3$ MeV. We find that in the energy range $E_{\text{c.m.}} = 0.5-3$ MeV the capture cross sections are dominantly caused by $E2$ capture from the $(L=2, S=0)$ d + d scattering states, while at energies $E_{\text{c.m.}} < 200$ keV the S factor is given by the $E2$ transition from the $(L=0, S=2)$ d + d scattering states into the *D*-wave component of the ^{4}He ground state. These results confirm the assumptions of Ref. 8.

Although our parameter-free microscopic calculation reproduces the low energy ^{2}H(d,γ)^{4}He cross sections qualitatively well, the quantitative agreement with the experimental data is not good enough in detail to derive at definite quantitative conclusions about the ^{4}He ground state or to allow for a meaningful extrapolation of the experimental fusion data into the energy regime of astrophysical interest $(E \approx 0)$. In particular, the calculated cross sections are lower than the experimental data at energies $E_{\text{c.m.}} < 50$ keV. Furthermore, due to the slight overbinding of the ^{4}He ground state energy, the E_γ^5 factor used in our calculation of the cross sections is too large by roughly a factor of 1.5. Consequently, we believe that the *D*-state admixture in the ^{4}He ground state wave function is probably larger than the 4.5% predicted in our microscopic study.

This supposition was confirmed within a subsequent study of the low energy ^{2}H(d,γ)^{4}He cross sections which reproduces the experimental data at energies $E_{\text{c.m.}} < 3$ MeV. This calculation differed from the microscopic study discussed above in two details: (a) The strength of the effective tensor interaction was treated as a parameter to allow for adjustment to the experimental cross section. (b) In calculating the energy of the emitted photon, the ^{4}He binding energy was set to the experimental value $E_B = 28.4$ MeV, rather than to its calculated value. This adjustment, which might be viewed as inconsistent with the requirements of a pure microscopic study, guarantees, however, that the phase space factor in (4) is correct and therefore allows for a direct analysis of the nuclear matrix elements from the low energy ^{2}H(d,γ)^{4}He data, provided the latter are reproduced within the calculation.

If the strength of the tensor interaction is increased by a factor of 1.4, this semimicroscopic calculation reproduces the experimental ^{2}H(d,γ)^{4}He cross sections for energies $E_{\text{c.m.}} < 3$ MeV consistently (Fig. 1). Hence we might use our present calculation for extrapolating the experimentally observed ^{2}H(d,γ)^{4}He fusion cross section into the energy regime of astrophysical interest. For energies $E \leq 100$ keV, covering the energy range which is important for the ^{4}He nucleosynthesis during the big bang, our S factor is nearly linear in energy: $S(E) \approx S_0 + S_1 E$ and can be well approximated using the coefficients $S_0 = 7.3 \times 10^{-5}$ keV b and $S_1 = -4 \times 10^{-7}$ b. Note that our astrophysical S factor is about 35 times larger than the presently recommended value. This strong enhancement of the ^{2}H(d,γ)^{4}He rate might have interesting consequences on the big-bang nucleosynthesis. Detailed studies are asked for.

Our semimicroscopic calculation also confirms the qualitative conclusions about the behavior of the low energy ^{2}H(d,γ)^{4}He reaction and of the ^{4}He ground state wave function as found in the microscopic calculation. Motivated by the consistent reproduction of the experimental data, we feel justified to perform a quantitative analysis of the ^{4}He ground state wave function as determined in the semimicroscopic calculation. We find a total *D*-state admixture which is the sum of the two $(L=2, S=2)$ configurations in the ground state wave function of 6.8%. We have tested the dependence of our results on the assumptions about the adjustment procedure of the tensor interaction. This has been done by adopting one of the two Gaussians as given in Ref. 14 and adjusting the strength of the other to the experimental data. In both cases we are able to reproduce the experimental ^{2}H(d,γ)^{4}He data at $E_{\text{c.m.}} < 3$ MeV. The strength of the *D*-state admixture is only slightly affected within this variation. Interpreting the individual adjustment of the long- and short-ranged component in the tensor interaction of Ref. 14 to the experimental data as extreme cases, our semimicroscopic calculation indicates a *D*-state admixture of $(6.8 \pm 0.4)\%$ in the ^{4}He ground state.

Combining the results of our microscopic and semimicroscopic studies, the low energy ^{2}H(d,γ)^{4}He cross sections are consistent with a *D*-state admixture in the ^{4}He ground state wave function of $\approx 5-7\%$. This result agrees rather well with the theoretical predictions using the Paris potential (5.4%, Ref. 2) and the Gogny–Pires–de Tourreill potential (8%, Ref. 1), but it is noticeably smaller than found for super-soft-core potentials (13%, Ref. 1). Our result also agrees with that of the heuristic model analyses of tensor analyzing powers yielding *D*-state admixtures of 7% (Refs. 4 and 5) and 4.8% (Ref. 3), but we do not confirm the low value (1.4%) as suggested by a phenomenological direct capture model calculation (Ref. 8). We also found that the potential parametrization as adopted in Ref. 3 does not reproduce the energy dependence of the experimental ^{2}H(d,γ)^{4}He cross sections.

In conclusion, we have presented the first microscopic study of the ^{2}H(d,γ)^{4}He reaction at low energies. We have shown that the experimental data at energies $E_{\text{c.m.}} < 3$ MeV can be reproduced within a microscopic four-nucleon calculation on the basis of a *D*-state admixture of $\approx 5-7\%$ in the ^{4}He ground state. We believe that the present study represents a very reasonable description of the ^{2}H(d,γ)^{4}He reaction at low energies and is certainly superior to the heuristic model analyses.[3–5,8] However, one may think of refinements of our calculation which should include the consideration of (a) one of the "realistic" nucleon-nucleon interactions, (b) (n) + ^{3}He and p + t configurations in the model wave functions, (c) the tensor component in the nucleon-nucleon interaction in the calculation of the scattering states, and (d) the internal quadrupole moment of the deuteron. An improved calculation taking account of the points (a)–(d) is in progress.

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