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Off-shell effects in the nucleon-nucleon-alpha system

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Off-shell properties of individual partial waves of the nucleon-alpha potential are systematically investigated by nucleon-nucleon-alpha Faddeev calculations. For this purpose, the ground state energies of ^{6}Li and ^{6}He and the observables of the d-α scattering system at $E_\alpha = 15$ and 24 MeV (lab) are calculated and compared with experiment. From inspection of the nucleon-alpha wave functions, a suitable short range behavior can be proposed.

I. INTRODUCTION

Compared to the three-nucleon (N) system, the Faddeev calculations for systems with composed nuclei are in a beginning stage. The neutron-proton-^{4}He (n-p-α) system is one of the few systems where several extensive calculations exist.[1–4] Also in the case of nuclei the study of the off shell effects is interesting because these effects reveal information about the short range behavior of the subsystem wave function, and thereby the off shell effects contain, beyond the information about the dynamics, also information about the inner structure of the nuclei and the Pauli principle. A first trial to fix the off shell behavior of the n-p-α system by the resonating group model and to compare the results with experimental data was made in Ref. 4. Now in this study we want to take a complementary (phenomenological) point of view and try to find out where measurable n-p-α observables can give guidelines for the short range properties of the N-α wave functions (or equivalently, for the off shell properties of the N-α t matrix, or of its interaction). As the microscopic derivation of the N-α interaction is not free from doubtful assumptions,[4,5] such guidelines can be helpful in the derivation of a reasonable N-α potential.

In the following we will propose a suitable off shell behavior for the N-α system by studying a few examples. We will investigate the ground state energies of ^{6}Li and ^{6}He, and the elastic scattering system deuteron-^{4}He (d-α) at the energies $E_\alpha = 15$ and 24 MeV (lab). The difficult problem of an analytical analysis of the relations between the observables and the short range behavior of the subsystem wave functions was not considered.

II. THE NUCLEON-α SUBSYSTEM

For our Faddeev calculations we used two sets of potentials for the N-α subsystem; Coulomb forces were not included. The N-N interactions were not investigated, and therefore, except in one example, not varied. One N-α potential set was taken from Charnomordic, Fayard, and Lamot;[1] they use separable potentials with rank = 1 in the partial waves $^2S_{1/2}$, $^2P_{1/2}$, and $^2P_{3/2}$. We call these CFL potentials. The other set, called R, is derived from the resonating group model. They are higher rank separable potentials which are defined explicitly in Ref. 4 [see therein Eq. (7) and Table III].

The quality of the agreement in the phase shifts for the two potential sets up to 18 MeV (c.m.) is shown in Fig. 3 of Ref. 4; the potential set R was adapted by a mean square fitting routine to the phase shifts of CFL. The maximal deviation of the two phase shifts in this energy region occurs in the $^2P_{3/2}$ partial wave; it was not possible to perfectly fit the sharp phase shift maximum, $\delta_{\rm CFL}$, between 1.5 and 3 MeV (c.m.); the maximal deviation is $\delta_{\rm CFL} - \delta_R = 1.7°$, which is below 1.5% of the maximum. In contrast, in the region of the resonance (0.96 MeV c.m.) and above 3 MeV (c.m.) the agreement is accurate. As a good phase shift equivalence between CFL and R is essential for the following, we mention this deviation; we cannot exclude the fact that it may distort our interpretation of off shell effects somewhat.

For energies between 18 and 40 MeV (c.m.) the phase shifts agree with very good quality;[6] for energies higher than 40 MeV (c.m.) the phase shifts of R and CFL start to differ distinctly; but in all cases the phase shifts are already small there and go, for increasing energy, smooth-

TABLE I. ^{6}Li calculations for different potentials indicated: CFL are potentials of Charnomordic, Fayard, and Lamot (Ref. 1), R are from Hahn *et al.* (Ref. 4), D is from Doleschall (Ref. 8), Paris is a separable representation of the Paris potential from Haidenbauer *et al.* (Ref. 9). The experimental value is Coulomb corrected according to Kukulin *et al.* (Ref. 7).

Nucleon-α potentials	n-p potentials	$E_{^6\rm Li}$	Calc. no.
$^2S_{1/2}$, $^2P_{1/2}$, $^2P_{3/2}$ (CFL)	$^3S_1$-$^3D_1$ (D)	-3.09 MeV	1
$^2S_{1/2}$ (R), $^2P_{1/2}$, $^2P_{3/2}$ (CFL)	$^3S_1$-$^3D_1$ (D)	-2.67 MeV	2
$^2P_{1/2}$ (R), $^2S_{1/2}$, $^2P_{3/2}$ (CFL)	$^3S_1$-$^3D_1$ (D)	-3.14 MeV	3
$^2P_{3/2}$ (R), $^2S_{1/2}$, $^2P_{1/2}$ (CFL)	$^3S_1$-$^3D_1$ (D)	-3.78 MeV	4
$^2S_{1/2}$, $^2P_{1/2}$, $^2P_{3/2}$ (R)	$^3S_1$-$^3D_1$ (D)	-3.66 MeV	5
$^2S_{1/2}$, $^2P_{1/2}$, $^2P_{3/2}$ (R)	$^3S_1$-$^3D_1$ (Paris)	-3.65 MeV	6
Experiment, Coulomb corrected (Ref. 7)		-4.54 MeV	

TABLE II. ^{6}He calculations; labels as in Table I; the D waves (D) are taken from Doleschall (Ref. 10), the neutron-neutron interaction is from Haidenbauer *et al.* (Ref. 9).

Nucleon-α potentials	n-n potentials	$E_{^6\mathrm{He}}$	Calc. no.
$^2S_{1/2}$, $^2P_{1/2}$, $^2P_{3/2}$ (CFL)	$^1S_0$ (Paris)	unbound	1
$^2S_{1/2}$, $^2P_{1/2}$, $^2P_{3/2}$ (R)	$^1S_0$ (Paris)	-0.27 MeV	2
$^2S_{1/2}$, $^2P_{1/2}$, $^2P_{3/2}$, $^2D_{3/2}$, $^2D_{5/2}$ (R,D)	$^1S_0$ (Paris)	-0.28 MeV	3
Experiment		-0.97 MeV	

ly to zero. This may be the reason that the calculated observables of the d-α scattering system at $E_\alpha = 15$ and 24 MeV (lab) are insensitive to the differences in the phase shifts beyond 40 MeV (c.m.), which were carefully tested for all three N-α partial waves.[4,6] For the calculation of the binding energies of ^{6}Li and ^{6}He an analogous test was not performed, as the low energy phases should dominate the binding. We therefore assume that the CFL and R potentials are for our purposes essentially phase shift equivalent, and we suppose that differences in the calculated observables of the three-particle system are mainly caused by differences in the short range behavior of the N-α subsystem waves, i.e., that those differences are essentially off shell effects.

The differences in the short range behavior of the two sets of N-α wave functions are useful in our case to characterize the off shell properties of the interaction, as their behavior is rather energy independent. Some waves are presented in Fig. 10 of Ref. 4. The behavior at 10 MeV (c.m.) is typical for a large energy region of the N-α system. Set R (solid lines) shows in the small distance region, (0–2) fm, of the $^2S_{1/2}$ waves a higher amplitude than CFL (broken lines); in contrast, in the P waves the amplitudes of R in this region are smaller

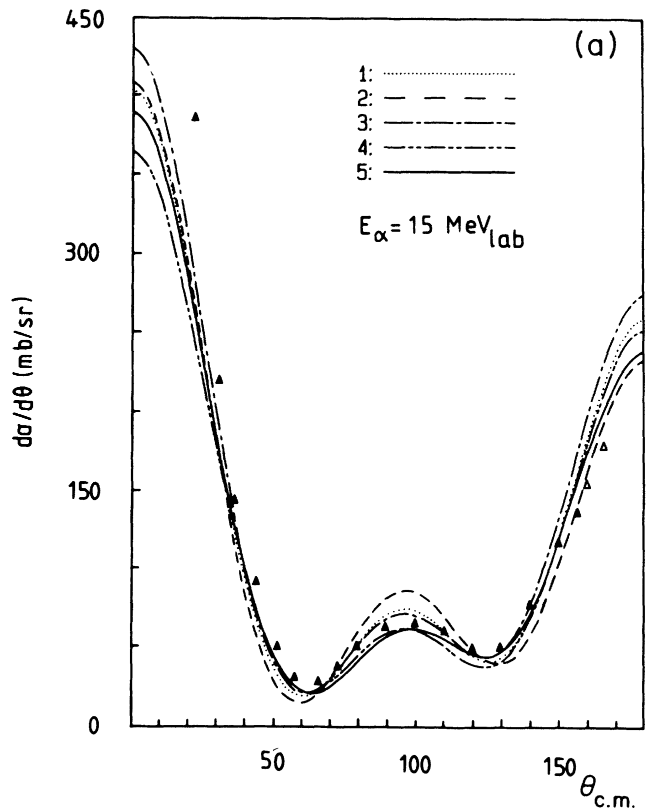

FIG. 1. Differential cross sections and analyzing powers of the elastic reaction d-α at the energy $E_\alpha = 15$ MeV (lab). Line markings are the same in all parts. Line labels are as follows: 1—N-α potentials from Charnomordic *et al.* (Ref. 1); 2—$^2S_{1/2}$ replaced by the resonating group potential (Ref. 4); 3—$^2P_{1/2}$ replaced by the resonating group potential; 4—$^2P_{3/2}$ replaced by the resonating group potential; 5—All N-α partial waves replaced by the resonating group potential. The experimental data for the cross sections and analyzing powers are taken from Senhouse *et al.* and Schmelzbach *et al.* at $E_\alpha = 15.72$ MeV (lab); filled triangles are measured directly; open triangles are derived by a phase shift fit (Ref. 11).

than those of CFL. According to this short range behavior of the low energy part of the spectrum, one may call the $^2S_{1/2}$ potential of CFL more repulsive than that of R; alternatively, the P wave potentials of CFL are more attractive than those of R.

III. OFF SHELL EFFECTS IN THE GROUND STATE ENERGIES OF ^{6}Li AND ^{6}He

The nuclei ^{6}Li and ^{6}He bind their constituents N-N-α only weakly. The bound state energy of the ^{6}Li ground state is ($J^\pi = 1^+$) -3.67 MeV, or -4.54 MeV when we subtract from the experimental value the repulsion which is caused by the Coulomb interaction.[7] The energy of the ^{6}He ground state ($J^\pi = 0^+$) is -0.97 MeV. To study relations between the bound state energy and the short range region of the N-α wave functions, we performed a series of Faddeev ground state calculations with the code of Doleschall. To check the accuracy of the numerical method we repeated a ^{6}He calculation of Parke and Lehmann[3] and quite accurately reproduced their value of -0.359 MeV with -0.361 MeV.

We started our ^{6}Li (1^+) calculations with the set of CFL potentials. For the n-p potential ($^3S_1$-$^3D_1$) we used

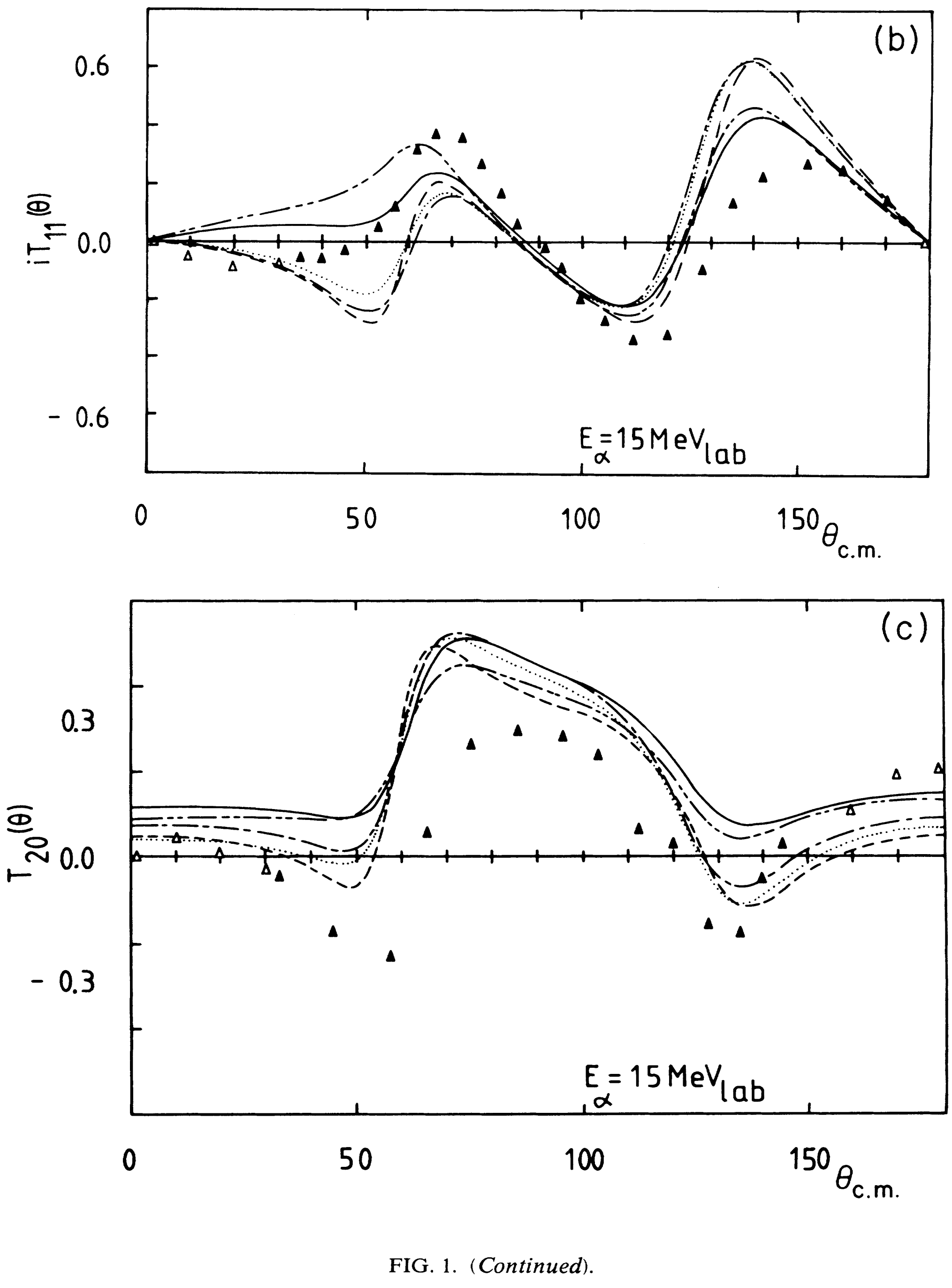

FIG. 1. (*Continued*).

a potential from Doleschall.[8] Next, we systematically substituted in any one N-α partial wave the corresponding potential from set R and performed three Faddeev calculations. In addition, we calculated the ground state energy with the complete set R; finally, we changed the tensor force and used a separable representation of the Paris potential[9] together with set R to study its influence on the binding energy. In Table I we give a summary of the results. We find (see calculations 1–4) that in all three N-α partial waves an increasing short range repulsion reduces the ^{6}Li ground state energy. Because of the strong subsystem resonance in the $^2P_{3/2}$ N-α interaction at 0.96 MeV (c.m.), this partial wave is dominating the three-particle system, and therefore its off shell effects (-0.69 MeV) are the strongest.

Calculation 5 is interesting in several respects:

(i) The transition from the CFL to the R model (which can be justified by the resonating group model[4]) improves the position of the ground state energy.

(ii) The energy of calculation 5 is different from the value of calculation 1 plus the summed off shell effects in calculations 2, 3, and 4. This expresses that off shell effects caused by one N-α partial wave are correlated with the residual channels. But this coupling is not too effective; without coupling we would expect -3.41 MeV, and in fact we find -3.66 MeV.

(iii) The choice of the tensor force has little influence on the result, as we see by calculation 6.

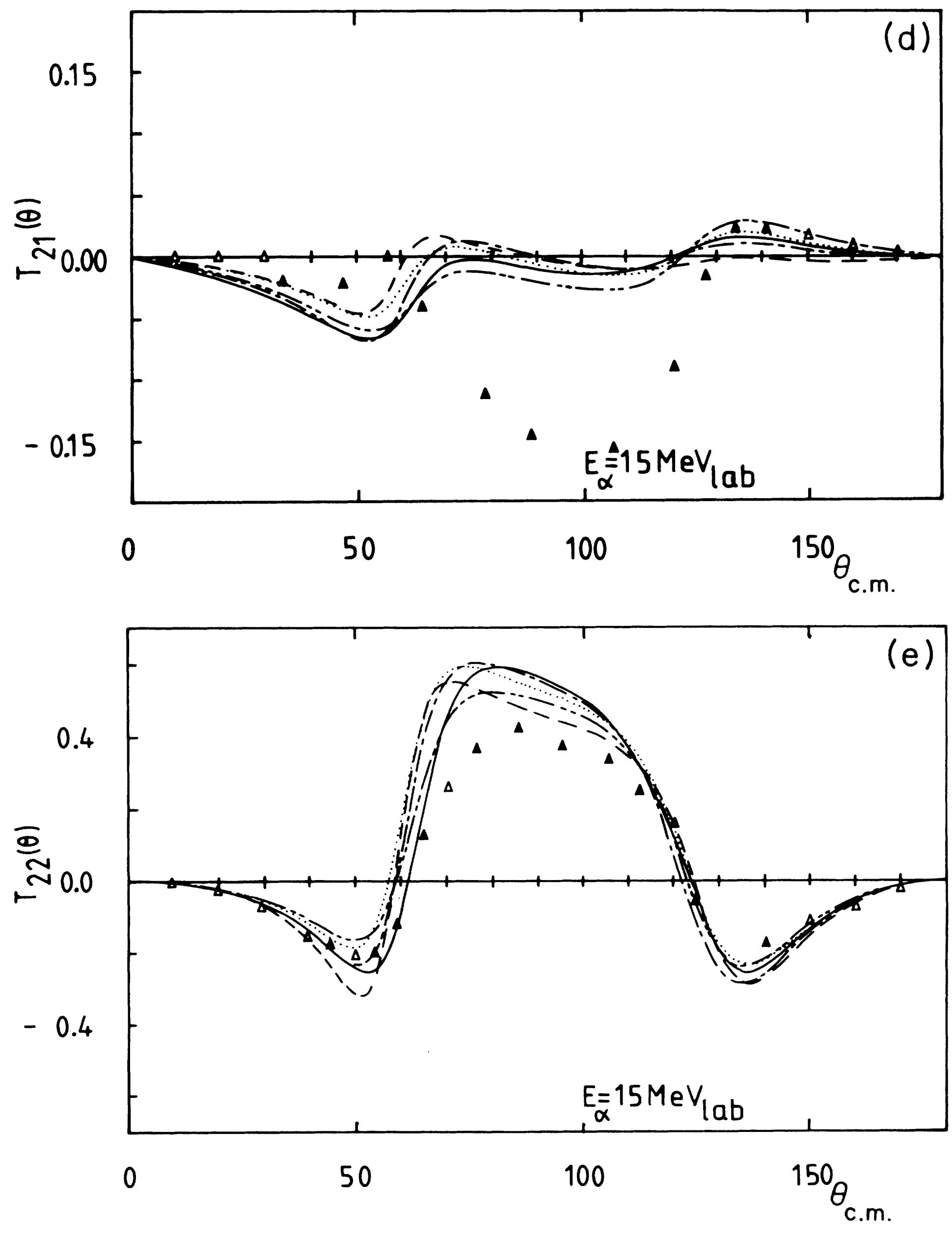

FIG. 1. (*Continued*).

The gap to experiment may come from experimental uncertainties in our N-α phase shifts, from an improper off shell behavior in the N-α interaction, or from three-body forces which are not included.

An analogous investigation for the ground state energy of ^{6}He (0^+) could not be performed, as for the CFL potential there is no bound state. In contrast, set R gives a binding with $E_{^6\mathrm{He}} = -0.27$ MeV; see Table II. Assuming that also in this system the $^2P_{3/2}$ N-α partial wave dominates, for ^{6}He the ground state energy also decreases when the dominating short range repulsion in the N-α system becomes stronger.

The ^{6}He calculations consume less computer time than the ^{6}Li calculations, as the tensor force is not involved. Therefore we could add a calculation where we included the $^2D_{3/2}$ and the $^2D_{5/2}$ N-α potentials.[10] But its influence on the bound state energy is small; see Table II. The reasons for the discrepancy with the experimental value may have the same origins as in the case of ^{6}Li.

IV. OFF SHELL EFFECTS IN THE ELASTIC DEUTERON-α SCATTERING

Parallel to the bound state calculations, we investigated also for the elastic deuteron-α (d-α) scattering system the dependence of the observables on the short range behavior of the N-α wave functions. The numerical accuracy of the code from Doleschall was tested by repeating the d-α calculations of Charnomordic, Fayard, and Lamot.[1] The agreement was very good; see Chap. II in Ref. 4 for details. We calculated the scattering at the energies $E_\alpha = 15$ and 24 MeV (lab); the Coulomb interaction was ignored; as n-p interaction we used the tensor force from Doleschall.[8] Again we started the calculations with the N-α potential set CFL and then systematically substituted the N-α partial waves by the potentials R; finally a calculation with the complete potential set R was performed. The results are shown in Figs. 1 and 2. Experimental data at $E_\alpha = 15$ MeV (lab) were not available, and therefore we show data points at $E_\alpha = 15.72$ MeV (lab).[11]

At all observables we find in wide angular intervals off shell effects which exceed the experimental inaccuracy of the data. The omission of the Coulomb forces in the calculations allows a detailed comparison with experiment only at the backwards angles ($\Theta_{\mathrm{c.m.}} > 140°$), because there the Rutherford cross section minimizes:

(i) Clear effects we find for $d\sigma/d\Omega$, iT_{11}, and T_{20}. An approach of $d\sigma/d\Omega$ to the data is at both energies

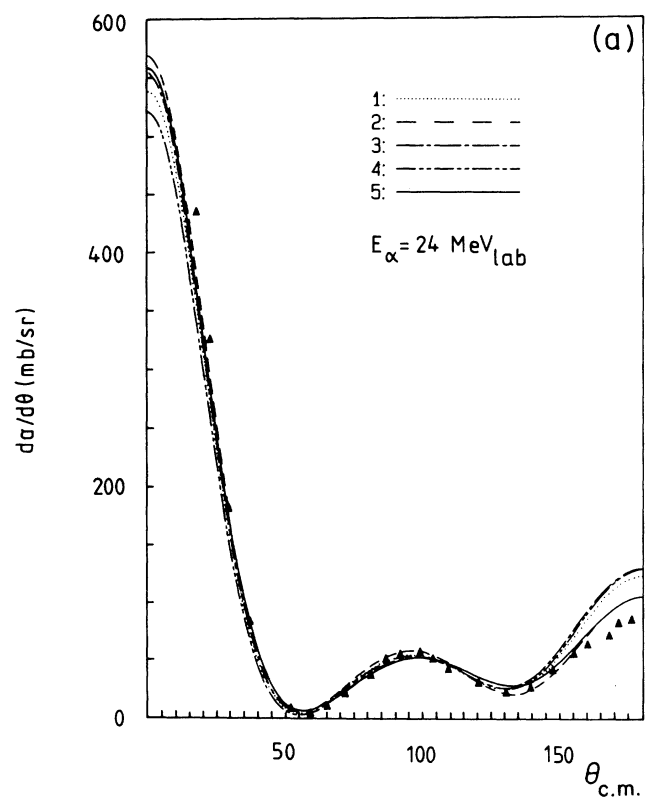

FIG. 2. As for Fig. 1 at the energy $E_\alpha = 24$ MeV (lab). The experimental data for the cross section are from Stewart *et al.* for the analyzing powers from Grüebler *et al.* (Ref. 12).

gained by increasing the short range amplitudes of the $^2S_{1/2}$ N-α spectrum (the R model). The $^2S_{1/2}$ partial wave seems to dominate the cross section at backwards angles, because the calculation with the $^2S_{1/2}$ substitution from R nearly coincides with the results for the complete set R. In the case of the analyzing powers iT_{11} and T_{20} at backwards angles, the $^2P_{3/2}$ N-α potential dominates (see Figs. 1 and 2), which produces an appreciable effect in the T_{20} observables. Like in the binding case, the experiments are approached here by more short range repulsion in the $^2P_{3/2}$ wave.

(ii) The T_{21} analyzing power is at backwards angles equally sensitive to the short range behavior of the $^2S_{1/2}$ and $^2P_{3/2}$ partial wave; as the off shell shifts are in different directions, the calculation with the complete set R is close to that with CFL.

(iii) For the analyzing power T_{22} the off shell effects are at backwards angles approximately of the order of

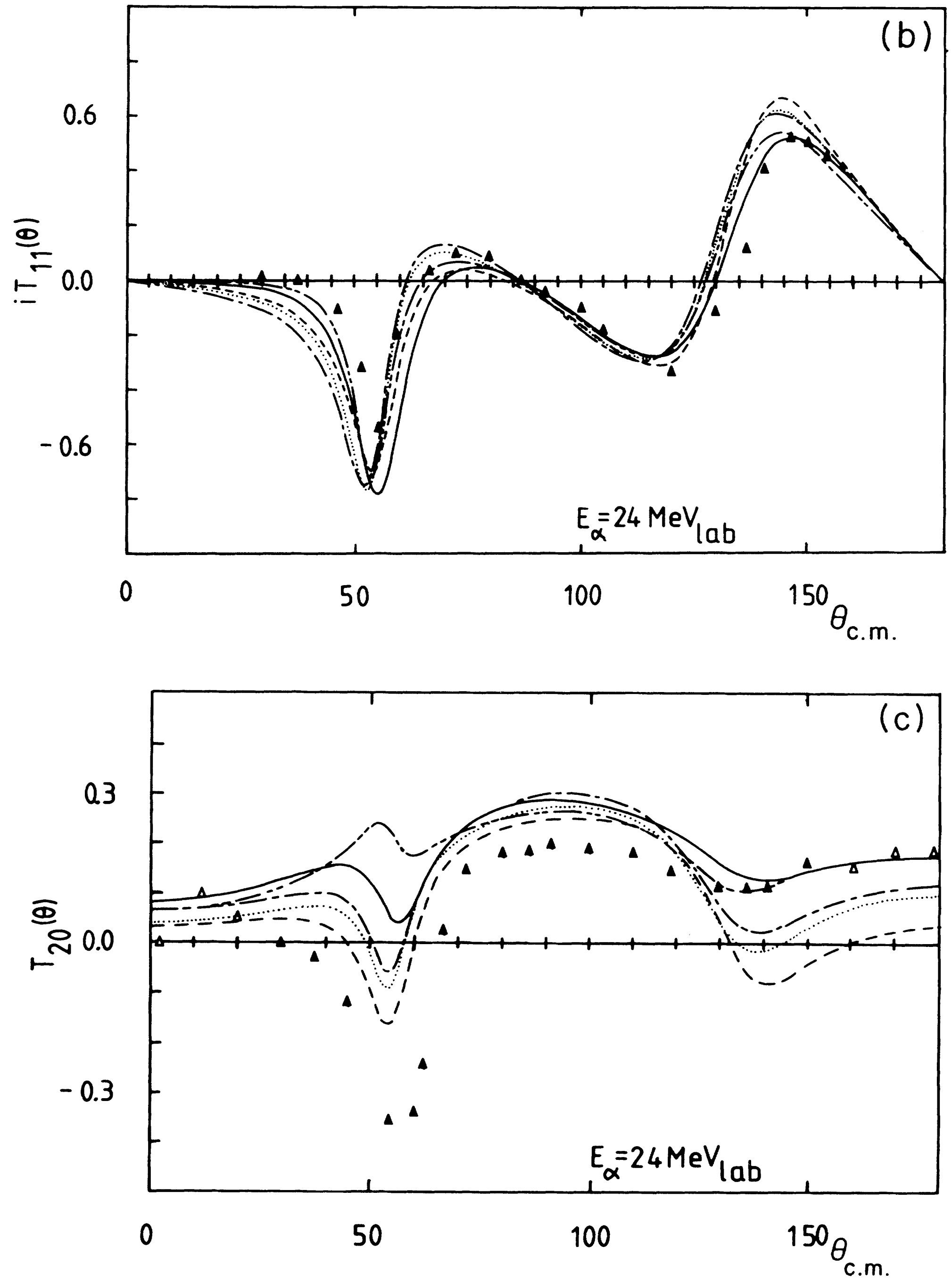

FIG. 2. (*Continued*).

the inaccuracy of the data, and the different results cannot be resolved by experiment.

(iv) Effects of the $^2P_{1/2}$ N-α wave are, corresponding to its close similarity for the CFL and R potentials, of minor importance in all observables.

Of course one should include also the d-α breakup channel in this study, but when the calculations were performed, we found that the agreement with calculations of Koike[2] at $E_\alpha = 15$ MeV (lab) was not too convincing and therefore do not represent the breakup. But these preliminary breakup results show, as expected, that also in this case the off shell effects exceed the inaccuracy of the data by far,[4,13] which should encourage further studies in these matters.

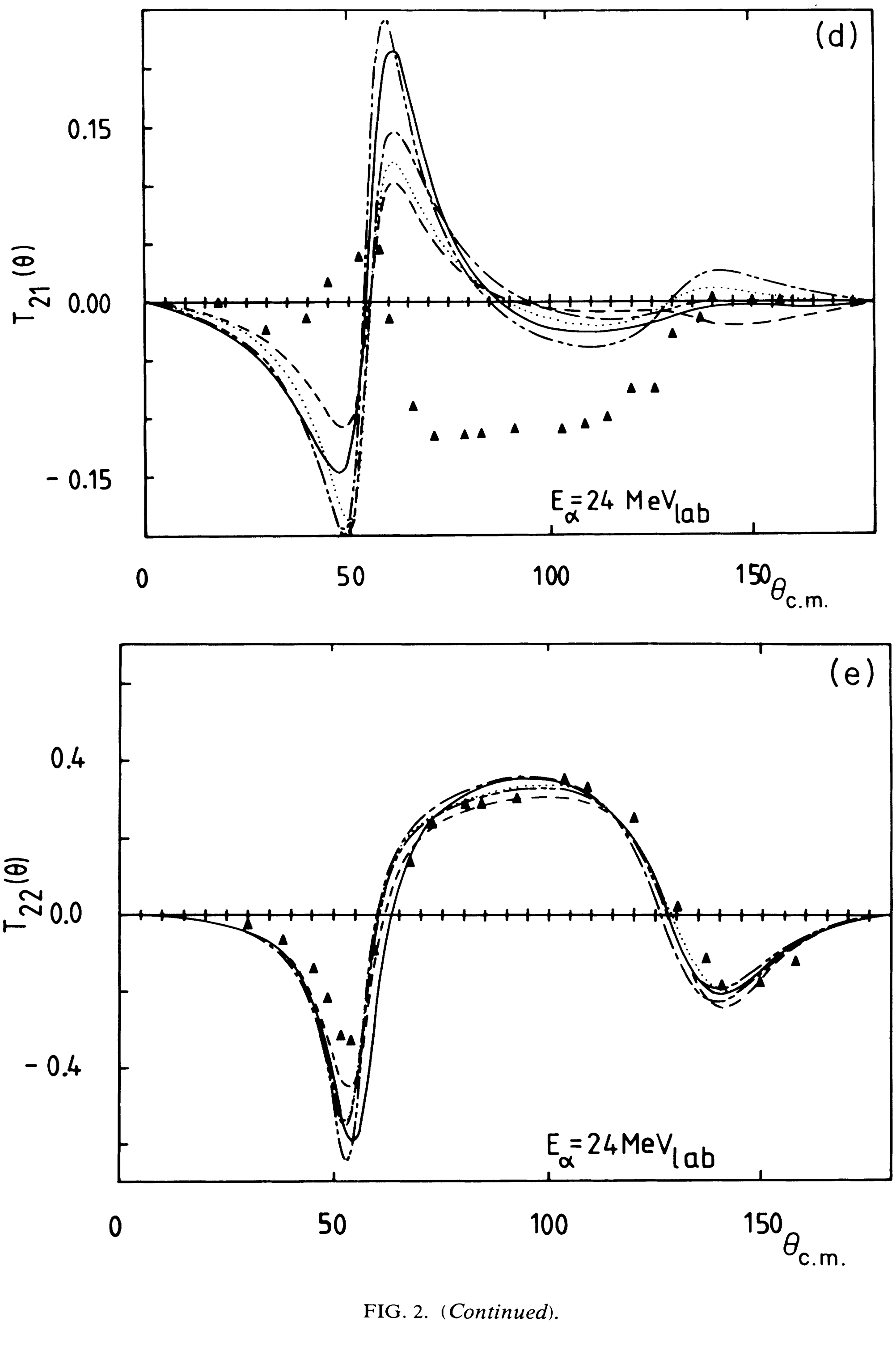

FIG. 2. (*Continued*).

V. SUMMARY

In a preceding paper[4] we showed that the Faddeev observables for elastic d-α scattering, calculated with the potentials of model R, are at backwards angles closer to experiment than the calculations with the CFL model. Now we add that also the ground state energies of ^{6}Li and ^{6}He are more realistic for the R model. A closer analysis reveals energy-independent regularities in the off shell effects for scattering and for binding. At both scattering energies chosen, $d\sigma/d\Omega$ is at backwards angles improved by increasing the short range amplitude in the $^2S_{1/2}$ N-α partial wave. In contrast, the analyzing powers iT_{11} and T_{20} demand a lower $^2P_{3/2}$ N-α amplitude than CFL. The ground state of ^{6}Li is approached whenever the short range amplitude of any N-α partial wave is lowered. For the $^2P_{3/2}$ partial wave the same could be shown for ^{6}He.

Comparing the binding and the analyzing power results of the R and CFL models with experiment one might ask if a N-α $^2P_{3/2}$ interaction which produces less short range amplitude than R is admissible in a microscopic N-α description. The backwards angle $d\sigma/d\Omega$ behavior indicates that the short range amplitude in the $^2S_{1/2}$ N-α wave is still too small in the R model.

We should be careful with absolute statements, however; the experimental and theoretical input in our calculations is in several respects incomplete: The magnitude of the Coulomb interferences may be important even at backwards angles; the uncertainty about the quality of the experimental data or about the importance of higher N-α partial waves may influence our interpretation of the Faddeev observables distinctly; the existence of three-body forces, which are postulated by the resonating group model, may be relevant, etc. All this may change our opinion about improvements in the off shell behavior for a specific model. But, at the least, our calculations indicate for a limited energy region what one has to change in the short range behavior of the N-α spectra, when special changes in the Faddeev observables are necessary.

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