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Isospin sum rule for nuclear photoabsorption: Effect of retardation

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Motivated by the close similarity between a sum rule originally derived by Cabibbo and Radicati and a simplified version based on nonrelativistic nuclear physics in the long-wavelength limit, we have investigated the effect of retardation corrections. An account of the contributions due to higher multipolarities is presented, together with a physical interpretation of the results.

I. INTRODUCTION

As is well known, powerful relations between physical quantities can be obtained from considerations based on the fundamental commutation relations obeyed by transition operators such as the electromagnetic and weak currents.[1,2] One of the simplest examples is the Cabibbo-Radicati sum rule which applies to a system with isospin $\frac{1}{2}$ and imposes a restriction on the relative strengths of the (properly integrated) photoabsorption cross sections which lead to the excitation of states with isospin $\frac{1}{2}$ or $\frac{3}{2}$. For a proton target the sum rule is

$$\frac{1}{2\pi^2\alpha}(2\sigma_{-1}^{1/2}-\sigma_{-1}^{3/2})=\tfrac{1}{3}r_v^2-\left[\frac{\mu_v}{2m}\right]^2,$$
$$\sigma_{-1}^{T}=\int_{\omega_{\rm thr}}^{\infty}\frac{d\omega}{\omega}\sigma_v^T(\omega),\quad T=\tfrac{1}{2}\text{ or }\tfrac{3}{2} \qquad (1)$$

where $\sigma_v^T(\omega)$ is the absorption cross section for an isovector photon exciting a state with isospin T, α is the fine-structure constant, μ_v is the isovector magnetic moment, m is the mass, r_v defined by

$$\tfrac{1}{6}r_v^2=\left[\frac{dG_E^v(q^2)}{dq^2}\right]_{q=0}+\frac{1}{8m^2} \qquad (2)$$

is, essentially, the isovector mean-square radius, and $G_E^v(q^2)$ with $q^2<0$ is the conventional isovector Sachs form factor of the nucleon. The derivation of this relation Eq. (1), which was originally obtained by Cabibbo and Radicati[3] has been generalized by several authors[4] and can be found in most standard texts on current algebra.[1,2]

In the nuclear physics context a very similar sum rule (but without the magnetic moment term) was obtained independently[5–9] during the same period by several authors. The derivation, in this case, was both more general and more limited than that obtained in high-energy physics, which was based on current-algebra and infinite-momentum frame and/or dispersion-relation techniques. The generalization made it possible to consider also targets with isospin larger than $\frac{1}{2}$. This was significant for heavier nuclei but will not be of interest to us here. The limitation, which made the approach much simpler, was that it was based on the long-wavelength approximation, i.e., only electric dipole transitions were considered. This restriction was hardly severe at the time when the typical photonuclear cross-section measurements were performed in the energy region below 50 MeV, but would have been quite unsatisfactory for proton targets, where even the lowest excitations require photon wavelengths comparable to or smaller than the nucleon radius. In the latter case the more powerful approach based on relativistic current algebra which includes all retardation effects is, obviously, necessary.

The similarity of the results of the two approaches creates, however, a logical puzzle which we propose to investigate in this article: If we can obtain a result almost the same as Eq. (1) using a nonrelativistic description in the electric dipole approximation, then all the contributions due to relativity and/or retardation must either be contained in the magnetic term or cancel altogether! By "retardation" we mean here all multipolarities other than electric dipole as well as all finite-wavelength contributions which, obviously, become important at higher energies. These contributions are clearly important for $\sigma(\omega)$, but may cancel in the integral or when the difference between the two isospin components is taken in Eq. (1).

Our interest in the following will be confined to the study of the effect of retardation. In Sec. II we review the general formalism, which is applied in Sec. III for a brief recalculation of the sum rule in the electric dipole approximation. The study of the effects of retardation are presented in Sec. IV. Sections V and VI provide some additional illustrations and a summary of the main results.

II. THE INTEGRATED CROSS SECTIONS AND ISOSPIN

In this section we introduce some definitions and identify the terms which we propose to calculate. The cross section describing the absorption of photons of linear polarization $\hat{\epsilon}$, wave vector $\mathbf{k}$, and frequency $\omega=k$ by a system in the (ground) state $|0\rangle$ is

$$\sigma(\omega)=\frac{4\pi^2\alpha}{\omega}\sum_n{}'\,|\langle n\,|\,\hat{\epsilon}\cdot\mathbf{j}(\mathbf{k})\,|\,0\rangle\,|^2\,\delta(\omega-\omega_{n0})\ ,$$

where $\omega_{n0}=E_n-E_0$, the prime indicates that the ground state is excluded from the sum, and $\mathbf{j}(\mathbf{k})$ denotes the electromagnetic current operator. It is understood that $\sigma(\omega)$ will vanish if ω is below a threshold value, e.g., the excitation energy of the first-excited state. We also introduce the corresponding quantities associated with the isovector part $\mathbf{j}_3$ of the current, and specify them with an additional superscript v:

$$\sigma^v(\omega)=\frac{4\pi^2\alpha}{\omega}\sum_n{}'\,|\langle n\,|\,\hat{\epsilon}\cdot\mathbf{j}_3(\mathbf{k})\,|\,0\rangle\,|^2\,\delta(\omega-\omega_{n0})\ . \tag{3}$$

The inverse-energy-weighted cross section is

$$\begin{aligned}\sigma^v_{-1}&=\int_0^\infty\frac{d\omega}{\omega}\sigma^v(\omega)\\&=4\pi^2\alpha\sum_n{}'\frac{1}{\omega_{n0}^2}\,|\langle n\,|\,\hat{\epsilon}\cdot\mathbf{j}_3(\mathbf{k}_{n0})\,|\,0\rangle\,|^2\\&=\sigma^v_{-1}(\tfrac{1}{2})+\sigma^v_{-1}(\tfrac{3}{2})\ .\end{aligned} \tag{4}$$

Both the isospin and its third component in the state $|0\rangle$ are assumed equal to $\frac{1}{2}$ and the last line in Eq. (4) identifies terms in which the summation over n is restricted to isospin values $\frac{1}{2}$ and $\frac{3}{2}$, respectively. The isovector nature of the transition operator makes it clear that these are the only contributions. The current $\mathbf{j}_3$ contains, obviously, only the third component of the isospin operator, but we find it helpful to introduce the auxiliary strength functions

$$S(\lambda)=\sum_n{}'\frac{1}{\omega_{n0}^2}\,|\langle n\,|\,\hat{\epsilon}\cdot\mathbf{j}_\lambda(\mathbf{k}_{n0})\,|\,0\rangle\,|^2\ , \tag{5}$$

with $\lambda=\pm1$ refering to the remaining spherical components of the isospin vector. Assuming charge independence and applying the Wigner-Eckart theorem, we find

$$4\pi^2\alpha S(+1)=\tfrac{3}{2}\sigma^v_{-1}(\tfrac{3}{2})\ , \tag{6a}$$

$$4\pi^2\alpha S(-1)=\tfrac{1}{2}\sigma^v_{-1}(\tfrac{3}{2})+2\sigma^v_{-1}(\tfrac{1}{2}) \tag{6b}$$

and, consequently,

$$\begin{aligned}\frac{1}{4\pi^2\alpha}[2\sigma^v_{-1}(\tfrac{1}{2})-\sigma^v_{-1}(\tfrac{3}{2})]&=S(-1)-S(+1)\\&=\sum_n{}'\frac{1}{\omega_{n0}^2}[\langle 0\,|\,\hat{\epsilon}\cdot\mathbf{j}_{-1}(-\mathbf{k}_{n0})\,|\,n\rangle\langle n\,|\,\hat{\epsilon}\cdot\mathbf{j}_{+1}(\mathbf{k}_{n0})\,|\,0\rangle\\&\qquad-\langle 0\,|\,\hat{\epsilon}\cdot\mathbf{j}_{+1}(\mathbf{k}_{n0})\,|\,n\rangle\langle n\,|\,\hat{\epsilon}\cdot\mathbf{j}_{-1}(-\mathbf{k}_{n0})\,|\,0\rangle]\ ,\end{aligned} \tag{7}$$

where parity (or time reversal) has been used for the detailed rewriting in the last line. This result, which already contains the left-hand side of the Cabibbo-Radicati sum rule, i.e., Eq. (1), will be the starting point of the calculations presented in the next two sections. It is worth pointing out at this stage, however, that the currents labeled by $\lambda=\pm1$ are charge-transfer operators, and the states $|n\rangle$ do not belong to the system whose ground state is $|0\rangle$ but rather are isobaric analogs of such states. Since charge independence has been assumed, the restriction on the sum indicated by the prime in Eq. (7) excludes those states for which $\omega_{n0}=0$, i.e., the isospin counterparts of the ground state. Remembering, also, that we have chosen the isospin quantum numbers of $|0\rangle$ as $T=T_3=+\frac{1}{2}$, we conclude that what is excluded from Eq. (7) is the state

$$|a\rangle=T_-\,|0\rangle\ ,$$

or any other state differing from it by a spin rotation. The restriction on the summation affects, therefore, only the second term in the last line of Eq. (7).

III. THE LONG-WAVELENGTH LIMIT

The sum in Eq. (7) is, clearly, still impossible to evaluate. As a first step in obtaining an estimate of the right-hand side of the sum rule, we then apply the electric dipole approximation, i.e., assume that the wavelength $1/k_{n0}$ in all the relevant matrix elements is much larger than the size of the target. Letting $\mathbf{k}_{n0}=0$ and keeping in mind the long-wavelength relation

$$\mathbf{j}_\lambda(0)=i[H,\mathbf{d}_\lambda],\quad \mathbf{d}_\lambda=\int d^3x\ \mathbf{x}\rho_\lambda(\mathbf{x}) \tag{8}$$

where H is the Hamiltonian, $\rho_\lambda(\mathbf{x})$ denotes the isovector density, and $\mathbf{d}_\lambda$ the electric dipole operator, we find

$$S^{(0)}(-1)-S^{(0)}(+1)=\langle 0\,|\,[\hat{\epsilon}\cdot\mathbf{d}_{-1},\hat{\epsilon}\cdot\mathbf{d}_{+1}]\,|\,0\rangle$$
$$=\left\langle 0\left|\int d^3x\,(\hat{\epsilon}\cdot\mathbf{x})^2\rho_3(\mathbf{x})\right|0\right\rangle$$
$$=\left\langle 0\left|\sum_i(\hat{\epsilon}\cdot\mathbf{r}_i)^2 t_3(i)\right|0\right\rangle , \qquad (9)$$

where the superscript refers to the use of the long-wavelength approximation and $\rho_3(\mathbf{x})$ specifies the third component of the isovector density. In the derivation above, we applied closure, ignored the matrix elements of $\mathbf{d}_{-1}$ between $|0\rangle$ and $|a\rangle$ which vanish because of parity, and used the isospin commutation relation

$$[\rho_{-1}(\mathbf{x}),\rho_{+1}(\mathbf{y})]=\delta(\mathbf{x}-\mathbf{y})\rho_3(\mathbf{x}) . \qquad (10)$$

We note that if in Eq. (7) we replace the difference $S(-1)-S(+1)$ by the unretarded expression given by Eq. (9), we obtain the Cabibbo-Radicati sum rule without the magnetic term which, as noted in the Introduction, was obtained by several authors[5–9] more than 20 years ago.

IV. THE EFFECT OF RETARDATION

We now need to improve upon the approximation $k_{n0}=0$, i.e., to study the effect of terms appearing in an expansion of $\mathbf{j}(\mathbf{k})$ in powers of $\mathbf{k}$. Since parity implies that the only nonvanishing contributions to the products of the two current matrix elements in Eq. (7) will involve even powers of k_{n0}, we consider here the effect on the quantity

$$\Delta S=S(-1)-S^{(0)}(-1)-[S(+1)-S^{(0)}(+1)] \qquad (11)$$

of the quadratic terms only. These will include retarded-electric-dipole, magnetic dipole, electric and magnetic quadrupole , as well as octupole contributions, which we take into account collectively by writing

$$\Delta S=\sum_n{}'\frac{1}{\omega_{n0}^2}k_{n0}^2\frac{d}{dk^2}[\,|\langle n\,|\,\hat{\epsilon}\cdot\mathbf{j}_{-1}(\mathbf{k})\,|\,0\rangle|^2 - |\langle n\,|\,\hat{\epsilon}\cdot\mathbf{j}_{+1}(\mathbf{k})\,|\,0\rangle|^2]_{k=0}$$
$$=\frac{d}{dk^2}\{\langle 0\,|\,[\hat{\epsilon}\cdot\mathbf{j}_{-1}(\mathbf{k}),\hat{\epsilon}\cdot\mathbf{j}_{+1}(\mathbf{k})]\,|\,0\rangle - |\langle a\,|\,\hat{\epsilon}\cdot\mathbf{j}_{-1}(\mathbf{k})\,|\,0\rangle|^2\}_{k=0} , \qquad (12)$$

where we cancelled $k_{n0}=\omega_{n0}$ and used the definition of the analog state $|a\rangle$ given at the end of Sec. II. Strictly speaking, we should indicate that $|a\rangle$ may differ from $|0\rangle$ by a spin rotation in addition to isospin. We can always choose $\hat{\epsilon}$ and $\hat{k}$, however, in such a way as not to cause a spin flip transition. We will assume now that this choice has been made keeping in mind that, since at the end we average over spins and sum over the directions of polarization, this implies no loss of generality.

The calculation of the term containing

$$\langle a\,|\,\hat{\epsilon}\cdot\mathbf{j}_{-1}(\mathbf{k})\,|\,0\rangle=\sqrt{2}\langle 0\,|\,\hat{\epsilon}\cdot\mathbf{j}_3(\mathbf{k})\,|\,0\rangle \qquad (13)$$

is simple and can be done exactly. We find it convenient to use the decomposition[10]

$$\mathbf{j}_3(\mathbf{k})=i\left[H,\int d^3x\,\mathbf{x}\alpha(\mathbf{k}\cdot\mathbf{x})\rho_3(\mathbf{x})\right] -i\mathbf{k}\times\int d^3x\,\boldsymbol{\mu}_3(\mathbf{x})\beta(\mathbf{k}\cdot\mathbf{x}) , \qquad (14a)$$

with

$$\alpha(z)=\sum_{n=0}^{\infty}\frac{(iz)^n}{(n+1)!} ,$$
$$\beta(z)=\sum_{n=0}^{\infty}2(n+1)\frac{(iz)^n}{(n+2)!} , \qquad (14b)$$

where $\boldsymbol{\mu}_3(\mathbf{x})$ represents the isovector magnetization density, and note that the first term in Eq. (14a) will give zero when evaluated between states of the same energy. Since $\beta(0)=1$, we then find

$$\frac{d}{dk^2}\,|\langle a\,|\,\hat{\epsilon}\cdot\mathbf{j}_{-1}(\mathbf{k})\,|\,0\rangle|^2_{k=0} =2\left|(\hat{\epsilon}\times\hat{k})\cdot\left\langle 0\left|\int d^3x\,\boldsymbol{\mu}_3(\mathbf{x})\right|0\right\rangle\right|^2 =\frac{1}{2}\left[\frac{\mu_v(A)}{2m}\right]^2 , \qquad (15)$$

where $\mu_v(A)$ is the isovector magnetic moment of the A-particle system, i.e., the difference between the magnetic moments in the states $|0\rangle$ and $|a\rangle$. In agreement with our earlier discussion, we have chosen as an axis for the definition of the target spin the direction of the magnetic field of the incident photon, i.e., $\hat{k}\times\hat{\epsilon}$. It is worth emphasizing that the result of Eq. (15) is quite general and includes possible contributions from, e.g., mesonic currents. Our discussion of the first term on the right-hand side of Eq. (12), on the other hand, will be more restrictive. For the evaluation of the commutator, we will first adopt the "impulse approximation," i.e., write the current in terms of nucleon coordinates only, viz.,

$$\hat{\epsilon}\cdot\mathbf{j}_\lambda(\mathbf{k})=\sum_{i=1}^{A}\left[\frac{1}{m}\hat{\epsilon}\cdot\mathbf{p}_i e^{i\mathbf{k}\cdot\mathbf{r}_i} +\frac{i}{2m}\mu_v(\mathbf{k}\times\hat{\epsilon})\cdot\boldsymbol{\sigma}_i e^{i\mathbf{k}\cdot\mathbf{r}_i}\right]t_\lambda(i) , \qquad (16)$$

where $\mu_v/2m$ is the isovector magnetic moment of the nucleon and $[t_{-1},t_{+1}]=t_3$. In this case a straightforward calculation gives

$$[\hat{\epsilon}\cdot\mathbf{j}_{-1}(-\mathbf{k}),\hat{\epsilon}\cdot\mathbf{j}_{+1}(\mathbf{k})]=\sum_i\left[\frac{\hat{\epsilon}\cdot\mathbf{p}_i}{m}\right]^2+k^2\left[\frac{\mu_v}{2m}\right]^2 T_3 , \qquad (17)$$

where T_3 is the third component of the isospin operator of the entire system. Inserting the results of Eqs. (15) and (16) into Eq. (12), we then find

$$\Delta S=\frac{1}{2}\left[\frac{\mu_v}{2m}\right]^2-\frac{1}{2}\left[\frac{\mu_v(A)}{2m}\right]^2 , \qquad (18)$$

which, together with the unretarded result of Eq. (9), gives

$$\frac{1}{2\pi^2\alpha}[2\sigma^v_{-1}(\tfrac{1}{2})-\sigma^v_{-1}(\tfrac{3}{2})] = \frac{1}{3}\left\langle \sum_i \mathbf{r}_i^2\tau_3(i)\right\rangle_0 - \left[\frac{\mu_v(A)}{2m}\right]^2 + \left[\frac{\mu_v}{2m}\right]^2 \quad (19)$$

for a spin-$\frac{1}{2}$ or a spin-averaged system and with $\tau_3=2t_3$. Comparing the expression of Eq. (19) with the sum rule of Eq. (1), which refers to the nucleon, we notice that the retardation contributions not only introduce a magnetic moment term but, actually, bring in two such terms which, at first glance, might appear puzzling. In order to understand the origin of the extra term, we first note that the radius in Eq. (19) is the isovector mean-square radius of the point-particle system and not of the complete physical system, whose corresponding radius will be denoted by $r_v^2(A)$. This quantity is obtained from $\langle \sum r_i^2\tau_3\rangle_0$ by the addition of r_v^2, which is defined in Eq. (2) and represents the intrinsic size of the nucleon. With this notation we then rewrite Eq. (19) in the form

$$\frac{1}{2\pi^2\alpha}[2\sigma^v_{-1}(\tfrac{1}{2})-\sigma^v_{-1}(\tfrac{3}{2})] = \tfrac{1}{3}r_v^2(A) - \left[\frac{\mu_v(A)}{2m}\right]^2 - \tfrac{1}{3}r_v^2 + \left[\frac{\mu_v}{2m}\right]^2, \quad (20)$$

whose interpretation is relatively straightforward. The main point is the observation that Eq. (1) relates the absorption of photons by a nucleon which is associated with the production of particles, mainly mesons, to such physical properties of the nucleon as the magnetic moment and the mean-square radius. In the calculations described in this and the preceding section, on the other hand, the magnetic moment $\mu_v(A)$ was treated exactly, but the transition matrix elements included only nuclear and no subnucleon excitations. The electric dipole and magnetic dipole photomeson production by the nuclear target, for example, was entirely lost. It is not surprising, therefore, that the right-hand side of Eq. (19) contained only the point-particle radius, which is related, through the sum rule, to the excitation of the nuclear degrees of freedom only. A similar restriction is expressed by $\mu_v^2(A)-\mu_v^2$, which appears in the magnetic term and which contains the effect of the nuclear, but not of the subnucleonic, magnetic excitations. The left-hand side of Eqs. (19) and (20) should then be understood to refer to cross sections for the excitation of the nuclear degrees of freedom and nothing else. It is clear, now, that we can obtain the complete, integrated, photoabsorption cross section of the entire nucleus simply by moving the last two terms in Eq. (20) to the left-hand side. The result will then be

$$\frac{1}{2\pi^2\alpha}[2\sigma^v_{-1}(\tfrac{1}{2})-\sigma^v_{-1}(\tfrac{3}{2})]_{\rm tot} = \tfrac{1}{3}r_v^2(A) - \left[\frac{\mu_v(A)}{2m}\right]^2, \quad (21)$$

where the subscript tot indicates that the contributions due to particle production have, in a way, been included in the photoabsorption cross section. It is seen that Eq. (21) is in complete agreement with the Cabibbo-Radicati sum rule, even though the derivation was based on a nonrelativistic description and contains the effect of retardation only to the lowest nonvanishing order. The implication, clearly, is that all additional contributions, viz., relativistic corrections and higher-order retardation, should cancel exactly.

We conclude this section with some additional remarks: (a) The mass in the magnetic term of Eq. (1) is that of the target, i.e., the proton. The corresponding term in Eq. (21), however, still contains the nucleon mass and not that of the entire target. This is simply because all magnetic moments are measured in nuclear magnetons. (b) The derivation of Eq. (17) was based on Eq. (16), which includes only one-nucleon contributions to the nuclear current. Since two-body currents representing mesonic and other contributions were ignored, it is worth commenting on the limitations of this approach. If we rewrite the current of Eq. (16) in the space-coordinate representation, we easily find that the basic reason for the simplicity of the commutator of Eq. (17) is the locality of $\mathbf{j}_\lambda(\mathbf{x})$, i.e., the fact that it contains only delta functions of the form $\delta(\mathbf{x}-\mathbf{r}_i)$, and, possibly, derivatives such as $\hat{\epsilon}\cdot\nabla$ (in the convection term) and $\hat{\epsilon}\times\nabla$ (in the spin term). The delta function and the gradient $\hat{\epsilon}\cdot\nabla$ immediately imply the cancellation of all retardation terms since $\hat{\epsilon}\cdot\mathbf{k}=0$ by the transversality condition. This is not, however, quite true for $\hat{\epsilon}\times\nabla$, which gives $\hat{\epsilon}\times\mathbf{k}$, i.e., a nonvanishing term which specifies the direction of the magnetic field. As long as the current is local, therefore, we expect the cancellations to occur. The two-body mesonic currents are, of course, nonlocal, and they should be expected to introduce additional contributions. It should be kept in mind, however, that the nonlocalities are only a result of an incomplete (static) treatment of the mesonic effects and do not represent a physical feature of an ultimate theory. At this stage we find it preferable to avoid the possibly misleading complications associated with nonlocalities: The simple expression that we have used for the current adequately illustrates the main point in the derivation, viz., the local nature of the fundamental commutator and, for our purposes, it should suffice. (c) The reason why in Eq. (9) and elsewhere we found the point-particle rather than the full mean-square radius is that we simply chose not to include the size of the constituents in the definition of the isovector density. We could have chosen it otherwise, but it would have been inappropriate: allowing for the finite size of the nucleon without, at the same time, including the possibility of exciting the subnucleonic degrees of freedom, is obviously inconsistent. (d) The similarity of the results of Eqs. (1) and (21) suggests that, strictly speaking, the nuclear radius as it appears in, e.g., Eq. (9), should be associated with an expression of the type defined in Eq. (2) with m still the nucleon rather than the nuclear mass. Although numerically the mass term in Eq. (2) is clearly insignificant, it is interesting, but perhaps not surprising, to note that this expression corresponds exactly to a definition of r_v^2 given by

$$r_v^2 = 6\left[\frac{d\tilde{G}_E^v(q^2)}{dq^2}\right]_{q=0}$$

$$\text{with } \tilde{G}_E^v(q^2) = \frac{G_E^v(q^2)}{[1-(q/2m)^2]^{1/2}},\quad q^2<0 \qquad (22)$$

as discussed by Friar many years ago.[11] (e) In going from Eq. (20) to Eq. (21) we have essentially assumed that the photoproduction of mesons by the nucleus is the same as that of a system with the same number of nucleons with the same quantum numbers (in particular, the same isospin) as the actual nucleus. This does not exclude a redistribution of the production strength, and it is applied only to the integrated cross section σ_{-1} and not to the detailed energy dependence of the excitation function. This assumption, which is probably correct within the currently available experimental precision, is still rather simple-minded, but it is the best we can do in this approach. It is also true that the intrinsic magnetic moment and the radius of a nucleon were assumed to remain unaffected by the nuclear medium. We note that this is only a question of notation and not a physical conclusion: we could have changed both quantities at will in the definition of the nuclear current. It is worth remembering, however, that a modification of the nucleon properties in a nucleus will, in principle, have an effect on the value of the integrated total absorption cross section as long as the integration extends well into the region of particle production. (f) In the expressions for the nuclear current and the calculations that we described in this section we did not make any effort to separate the motion of the nucleons relative to the center of mass from that of the entire nucleus as a whole. The calculation, actually, can be and has been done more properly and the results remain unchanged. We did not exhibit the center-of-mass coordinates explicitly to avoid complicating the notation.

V. DETAILS

Our main objective, which has been the derivation of the sum rule shown in Eq. (21), was completed in the preceding section. At this point we consider it advisable to be somewhat more specific and illustrate in greater detail the nature of the cancellations which are responsible for the simplicity of the result. To avoid complications in the notation, we shall ignore all the spin terms and adopt for the remaining part of the current the simple form

$$\hat{\epsilon}\cdot\mathbf{j}_\lambda^c(\mathbf{k}) \simeq \frac{1}{m}\sum_i [\hat{\epsilon}\cdot\mathbf{p}_i + i\hat{\epsilon}\cdot\mathbf{p}_i\,\mathbf{k}\cdot\mathbf{r}_i - \tfrac{1}{2}\hat{\epsilon}\cdot\mathbf{p}_i(\mathbf{k}\cdot\mathbf{r}_i)^2]t_\lambda(i)\,, \qquad (23)$$

where the superscript identifies the convection part of the current. Since we are deliberately avoiding the use of Siegert's theorem for the unretarted electric dipole component [the first term in Eq. (23)] or the electric quadrupole component which is part of the second term, the above expression explicitly invites the addition of mesonic two-body contributions. As we noted earlier, however, such an addition would tend to destroy the locality of the current operator and, for our purposes, this is a price that we should be unwilling to pay. Defining the strength function $S^c(\lambda)$ by using Eq. (5) and the expression for the convection current of Eq. (23), we note that, apart from the ground-state terms, the retardation contributions to $S^c(\lambda)$ will be

$$\frac{d}{d\omega^2}\langle 0\,|\,[\hat{\epsilon}\cdot\mathbf{j}_\lambda^c(\mathbf{k})]^\dagger\hat{\epsilon}\cdot\mathbf{j}_\lambda^c(\mathbf{k})\,|\,0\rangle_{\omega=0} = \frac{1}{m^2}\frac{d}{d\omega^2}\left\langle 0\left|\sum_{ij}\Omega_i^\dagger\Omega_j t_\lambda^\dagger(i)t_\lambda(j)\right|0\right\rangle_{\omega=0} \equiv C_\lambda^{(1)} + C_\lambda^{(2)}\,, \qquad (24)$$

where Ω_i represents the factor inside the square brackets in Eq. (23), while $C_\lambda^{(1)}$ and $C_\lambda^{(2)}$ denote, respectively, the one-body ($i=j$) and two-body contributions to the expectation value appearing in Eq. (24). Keeping in mind the isospin-$\frac{1}{2}$ assignment of the ground state, we now conclude that the second-rank isotensor component of $t^\dagger(i)t(j)$ gives zero contribution. We note, in addition, that because of parity the product $\Omega_i^\dagger\Omega_j$ is symmetric in i and j and that this implies the vanishing of the isovector component also. We thus conclude that the correlation term $C_\lambda^{(2)}$ is independent of λ, i.e.,

$$C_\lambda^{(2)} = C^{(2)}\,. \qquad (25)$$

This observation leads to the first simplification that we wish to identify, viz, that when the difference between $S(-1)$ and $S(+1)$ is taken, the correlation terms for $\lambda=-1$ and $+1$ just cancel each other. These terms do not vanish individually; they contribute significantly to the total integrated absorption cross section but disappear from the isospin sum rule for $T=\frac{1}{2}$ targets. The second type of cancellations occurs in the one-body terms, $C_\lambda^{(1)}$, and has nothing to do with isospin. If we use the expression in Eq. (23) and calculate the expectation value of the product $\Omega_i^\dagger\Omega_i$, we easily find that it contains just two identical terms of order ω^2 with opposite sign which cancel each other, giving

$$C_\lambda^{(1)} = 0\,. \qquad (26)$$

One of the two terms mentioned above represents the $M1$-$E2$ contribution (positive), while the other—which is negative—results from the interference between unretarded and lowest-order retarded $E1$ (or $M2$) components. As already mentioned, this result is not directly associated with the sum rule and applies equally well to the one-body contributions to σ_{-1}. The reason why it is more pronounced here is that when the commutator between the two currents is taken, the two-body contributions disappear, and the vanishing of the retardation effects to lowest order is expressed by the result of Eq. (26). It is easy to see that the same type of argumentation applies to the retardation components of the spin-dependent part of the current. The situation becomes quite different, however, if the corresponding contributions to the absorption cross section and the Cabibbo-Radicati sum rule due to the isoscalar part of the current are considered. These components, obviously, can only lead to the excitation of states with isospin $\frac{1}{2}$, i.e., contribute only to $\sigma_{-1}(\frac{1}{2})$. Their effect cannot be expressed as an expectation value of a commutator, and this implies that the correlation terms no longer disappear. In addition, the fact that an unretarded isoscalar electric dipole operator does not exist modifies the influence of the retarded $E1$ components which appeared earlier through interference. The $E2$-$M1$ contribution is no longer cancelled and has to be evaluated explicitly for a specific problem. We shall not make an attempt here to include the effect of the isoscalar components to the Cabibbo-Radicati sum rule, which, rather surprisingly, have also been neglected in high-energy applications.[4] We only mention that a simple evaluation of the isoscalar contributions to the integrated absorption cross section for the case of the $A=3$ system indicates that they do not modify σ_{-1} by more than a few percent.

As a final item, we now present some numerical illustrations of the isospin sum rule both for the nucleon and for the $A=3$ system. Referring to the definition of r_v^2 of Eq. (2) and to particle data compilations,[12] we find, for the nucleon,

$$\tfrac{1}{3}r_v^2-\left[\frac{\mu_v}{2m}\right]^2=(2.8-2.5)\ \text{mb}=0.3\ \text{mb}\ . \tag{27}$$

The rather drastic cancellation between the electric (i.e., r_v^2) and the magnetic terms was noted many years ago by several authors.[1,2,4] For the trinucleon system we find[13]

$$\tfrac{1}{3}r_v^2(A=3)-\left[\frac{\mu_v(A=3)}{2m}\right]^2=(15-3)\ \text{mb}=12\ \text{mb}\ , \tag{28}$$

which is the appropriate value for the right-hand side of the sum rule of Eq. (21) for $A=3$. Even though the relative contribution of the magnetic term is predictably smaller in this case, it still introduces a 25% change in this case and cannot be considered negligible. Comparing also the results of Eqs. (27) and (28), we note that the relative contribution to the nuclear sum rule due to subnucleon excitations and meson production is very small. This is not only because the inverse-energy-weighted sum rule suppresses the strength of high-energy excitations, but, mainly, because of the cancellations between electric and magnetic terms shown in Eq. (27).

VI. SUMMARY AND DISCUSSION

We have investigated the validity of the Cabibbo-Radicati sum rule for a nonrelativistic nuclear system. In its full relativistic version this sum rule contains two terms on the right-hand side: a radius term and a magnetic moment term. We found the following: (a) The radius term is already contained in the simplest version of the theory, which is based on the nonrelativistic electric dipole approximation. (b) Inclusion of retardation corrections in lowest order leads to the appearance of the magnetic term. (c) All other (lowest-order) retardation contributions cancel out. The basic origin of this cancellation is the assumed locality of the electromagnetic current. (d) Statements (a) and (b) need qualification: what is obtained in (a) is only the point-particle radius, which is the physical radius minus that of the nucleon. Similarly, the result mentioned in (b) is the difference between two terms containing the magnetic moment of the nucleus and that of the nucleon, respectively.[14] The reason for the appearance of these differences is simple: Since what is studied is the excitation of the nuclear degrees of freedom and not subnucleonic excitations, the result should differ from the total by a term representing particle production by a free nucleon. The formalism yields this difference automatically because it contains a simple signature of these processes: the anomalous magnetic moment of the nucleon.

At this point it is probably sensible to ask, again, the following question: If lowest-order retardation is all we needed to reproduce the entire sum rule, should we expect that all higher-order retardation corrections, e.g., all higher multipolarities, will have no effect at all? That a positive answer should be inappropriate is already indicated by the second term on the right-hand side of the sum rule which is proportional to m^{-2}. It simply reminds us that we have been delinquent by not considering already the effect of relativity. Rather than elaborating on this point, we prefer to give here another argument which leads to the same conclusion: the complete derivation of the sum rule is based on, or is equivalent to, the use of dispersion relations. These relations allow us to express integrated cross sections in terms of low-energy, i.e., long-wavelength, amplitudes. Their applicability, to a nonrelativistic problem, is, however, questionable since their validity in such situations is known to be, at best, limited. We should conclude, therefore, that an attempt to introduce additional, higher-order, retardation corrections without the proper relativistic framework is at this point unwarranted. We might even wonder whether or not we were justified to study the lowest-order retardation without introducing also at least some relativistic corrections. A justification of the legitimacy of this procedure together with a discussion of the applicability of dispersion relations to the isospin sum rule will be presented elsewhere.

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