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Deformed nucleonic bags and nuclear magnetic properties

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Good fits to the magnetic moments of ^{3}He and ^{3}H are obtained when the nucleonic bags consisting of three quarks each are assumed to be deformed. Expressions for the magnetic dipole strengths residing in the low-energy and the Δ sectors are obtained. If, as manifested in the European Muon Collaboration effect, the bags expand in the nuclear medium, then the net deformation of a bag must also decrease in nuclei.

Several nagging problems of the nonrelativistic quark model are resolved when one takes the bag to be deformed with large D-state admixtures.[1-3] Deformed bags have been studied by various groups.[4,5] A recent analysis of the $E2/M1$ ratio in $\gamma N \rightarrow \Delta(1232)$ favors deformation.[6]

Here we wish to study the magnetic properties of nuclei within the context of the nonrelativistic quark model with deformed nucleonic bags.

We define the nucleon and the isobar wave functions as follows:[2]

$$|\mathrm{N}\rangle = [1-P_D(\mathrm{N})]^{1/2}|N_S\rangle + [P_D(\mathrm{N})]^{1/2}|N_D\rangle \ , \tag{1}$$

$$|\Delta\rangle = [1-P_D(\Delta)]^{1/2}|\Delta_S\rangle - [P_D(\Delta)/2]^{1/2}|\Delta_D^{(1)}\rangle + [P_D(\Delta)/2]^{1/2}|\Delta_D^{(2)}\rangle \ . \tag{2}$$

The subscripts S and D represent the spherical s-wave and the deformed D-wave parts, respectively. The superscripts (1) and (2) refer to the symmetry $(70,2^+)$ and $(56,2^+)$, respectively, for Δ.[7]

Let us assume that our nucleus consists of A such deformed nucleonic bags. We define the nuclear ground state by the Slater determinant of these nucleons:[8]

$$|0\rangle = \frac{1}{\sqrt{A!}}\mathcal{A}\prod_{a=1}^{A}|(qqq)_a\rangle \ , \tag{3}$$

where $|(qqq)_a\rangle$, $a=1,2,\ldots A$ are the nucleon wave functions (1). The antisymmetrizing operator $\mathcal{A}$ acts only within the nucleonic space.

We will assume that specific nuclear medium effects will modify the parameters P_D^N and P_D^Δ from their free nucleon values. Now we define the $M1$ operator as follows:

$$\hat{M}1 = \sum_{a=1}^{A}\left[\sum_{a\in i=1}^{3}\sum_{\mu}[\sigma_\mu(i)+l_\mu(i)]Q(i)\mu_q(i)\right] \ . \tag{4}$$

Here μ labels the spherical components and i the three quarks in the nucleon bag a. Q is the quark charge, σ and l represent the spin and the angular momentum parts, respectively, and $\mu_q = e\hbar/2m_qc$, where m_q is the mass of the quark. For $M1$ transitions we need an extra factor of $\sqrt{3/4\pi}$.

Using operator (4) with the wave function (1) for a single nucleon one obtains the following magnetic moments:

$$\mu(\mathrm{n}) = -\tfrac{2}{3}[1-P_D(\mathrm{N})]\mu_q \ , \tag{5}$$

$$\mu(\mathrm{p}) = [1-P_D(\mathrm{N})]\mu_q \ , \tag{6}$$

so $\mu(\mathrm{p})/\mu(\mathrm{n}) = -\frac{3}{2}$, i.e., not modifying the already successful SU(6) prediction.[1]

For the trinucleon system, one expects that the magnetic moment of the ground state is obtained from that of the odd nucleon.[9] So $\mu(^3\mathrm{He}) = \mu(\mathrm{n})$ and $\mu(^3\mathrm{H}) = \mu(\mathrm{p})$. The single parameter $P_D(\mathrm{N})$ is expected to be modified in the nuclear medium. Therefore,

$$\mu(^3\mathrm{He}) = -\tfrac{2}{3}[1-P_D(^3\mathrm{He})]\mu_q \ , \tag{7}$$

$$\mu(^3\mathrm{H}) = [1-P_D(^3\mathrm{H})]\mu_q \ , \tag{8}$$

where $P_D(A)$ refers to the deformation of the odd nucleon for a particular nucleus.

We now determine this $P_D(A)$ by fitting to the experiments. So,

$$\frac{\mu(^3\mathrm{He})}{\mu(\mathrm{n})} = \frac{1-P_D(^3\mathrm{He})}{1-P_D(\mathrm{N})}, \quad \frac{\mu(^3\mathrm{H})}{\mu(\mathrm{p})} = \frac{1-P_D(^3\mathrm{H})}{1-P_D(\mathrm{N})} \ . \tag{9}$$

Taking the experimental values $\mu(^3\mathrm{He}) = -2.12$, $\mu(\mathrm{n}) = -1.91$, $\mu(^3\mathrm{H}) = 2.98$, and $\mu(\mathrm{p}) = 2.79$ (all in units of μ_N) for the left-hand side and using $P_D(N) = \frac{1}{4}$, which gives good fit to various physical quantities,[1,2] we obtain $P_D(^3\mathrm{He}) = 0.168$ and $P_D(^3\mathrm{H}) = 0.199$.

Another headache for the theoreticians is to explain the magnitude of the deviations expressed by the following ratio:

$$\frac{\delta\mu(^3\mathrm{He})}{\delta\mu(^3\mathrm{H})} = \frac{\mu(^3\mathrm{He})-\mu(\mathrm{n})}{\mu(^3\mathrm{H})-\mu(\mathrm{p})} \ . \tag{10}$$

In our model it is

$$= -\frac{2}{3}\frac{[P_D(\mathrm{N})-P_D(^3\mathrm{He})]}{[P_D(\mathrm{N})-P_D(^3\mathrm{H})]} \ , \tag{11}$$

which, with the above values, is -1.08. This compares very well to the experimental value of -1.1. For the sake of comparison, note that Karl, Miller, and Rafelski[10] obtain $-\frac{2}{3}$ for the same ratio.

Can we understand these values of $P_D(A)$? One of the ways of explaining the European Muon Collaboration effect is to increase the effective confinement size of the nucleons in the nuclear medium.[11] If we accept this picture, then we can easily accommodate $P_D(A) < P_D(N)$. In a deformed bag there is a larger surface area in the flat region than at the edges of (say) the pumpkin. If the pull on the surface is uniform, greater expansion will take place perpendicular to the flat part and so the deformation will decrease.

It seems that ^{3}He is a more loosely bound system than ^{3}H is. [$B.E. = -7.77$ MeV, $r(\text{rms}) = 1.88$ fm for ^{3}He, compared to $B.E. = -8.48$ MeV, $r(\text{rms}) = 1.70$ fm for ^{3}H.] Since there is a greater space available for a bag to expand in ^{3}He than in ^{3}H, it is therefore reasonable that the change in $P_D(\text{N})$ is larger in the former case.

It has been discussed in literature[10] that in general the values of the magnetic moments of nucleons inside a nucleus are larger than in the free space. Our discussion of the magnetic moment of $A = 3$ system can be easily carried over to these cases.

Because it is strangeness conserving, the magnetic dipole operator can excite only N and Δ degrees of freedom in nuclei. The total strength is

$$S^{N+\Delta} = \sum_n |\langle n| \sum_{a=1}^{A} \theta(a) |0\rangle|^2$$
$$= \mathrm{Tr}(\overline{\theta^\dagger \theta}) + (\mathrm{Tr}\theta^+)(\mathrm{Tr}\theta) - \mathrm{Tr}(\theta^\dagger\theta) - (\mathrm{Tr}\theta)^2 , \tag{12}$$

where $\theta(a)$ refers to the large bracket part in (4). The trace

$$\mathrm{Tr}(\theta) = \langle 0| \sum_{a=1}^{A} \theta(a) |0\rangle \tag{13}$$

is taken with respect to the A-dimensional nucleon subspace.[8] $\overline{\theta^\dagger \theta}$ means it is for the same nucleon while in $\theta^\dagger\theta$ the nucleons may be different. We are ignoring the shell effects.[8] The excitations to the Δ sector are obtained by the standard procedure.[12] This taken away from $S^{N+\Delta}$ gives the strength sitting in the low-energy N sector. We obtain [in units of $\mu_q^2 \cdot (\frac{3}{4}\pi)$]:

$$S_{M1}^{\mathrm{N}+\Delta} = [\tfrac{8}{3} + \tfrac{5}{9} P_D(\mathrm{N}) - \tfrac{4}{3} P_D(\mathrm{N})^2](N+Z) + P_D(\mathrm{N})[\tfrac{17}{9} - \tfrac{5}{3} P_D(\mathrm{N})]Z$$
$$+ \tfrac{8}{9}[1 - P_D(\mathrm{N})]^2 \delta_{(N-1)/2,I} + 2[1 - P_D(\mathrm{N})]^2 \delta_{(z-1)/2,I} , \tag{14}$$

$$S_{M1}^{\Delta} = \frac{2}{3}\left[2\sqrt{[1 - P_D(N)][1 - P_D(\Delta)]} + \left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)\sqrt{P_D(\mathrm{N}) P_D(\Delta)/2}\right]^2 (N+Z) , \tag{15}$$

$$S_{M1}^{\mathrm{N}} = S_{M1}^{\mathrm{N}+\Delta} - S_{M1}^{\Delta} . \tag{16}$$

Here $I = \{0, 1, 2, 3, \dots\}$. The δ function ensures that these terms act only for the odd-N or odd-Z nuclei. Following Ref. 2, we can express $P_D(\Delta)$ in terms of $P_D(\mathrm{N})$,

$$P_D(\Delta) = 2P_D(\mathrm{N})/[1 + P_D(\mathrm{N})] , \tag{17}$$

to reduce the number of parameters to one. We thus obtain an expression for the magnetic dipole strength sitting in the low-energy sector in nuclei which depends upon one parameter $P_D(\mathrm{N})$.

For example, for $P_D(\mathrm{N}) = 0.1$ we obtain $23\mu_{\mathrm{N}}^2$ for ^{28}Si and $38\mu_{\mathrm{N}}^2$ for ^{48}Ca. These are higher than the present experimental values[13] which are $\sim 7\mu_{\mathrm{N}}^2$ and $\sim 6\mu_{\mathrm{N}}^2$, respectively, for the two nuclei. One has to go to $P_D(\mathrm{N}) \sim 0.04$ for ^{28}Si and $P_D(\mathrm{N}) \sim 0.02$ for ^{48}Ca to obtain values which are comparable to the experiments. From this, it appears that the nucleonic deformation goes down with the mass number.

In passing, let us mention that with the wave function (2) the magnetic moment of Δ^{++} is

$$\mu(\Delta^{++}) = 2 - P_D(\Delta) \tag{18}$$

in units of μ_q. Using Eq. (17) and $P_D(\mathrm{N}) = \frac{1}{4}$, we predict the $\mu(\Delta^{++})$ to be $1.6\mu_q$. This value is within the limits of the analysis of Ref. 14; however, an updated analysis is needed to clarify the point.

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