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Vacuum instability for model field theories

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The perturbative vacuum for models in which fermions are coupled to scalar bosons and contain no derivative coupling for the scalar is shown to be unstable at the one-loop level. The instability is due to fluctuations at a sufficiently short-distance scale and is caused by the fermion loop contributions. Possible instabilities in models, which couple fermions to vector bosons only, are also discussed.

It is believed that quantum chromodynamics (QCD) is the correct theory to describe strong-interaction physics. Because QCD is difficult to study at small momentum transfers, a variety of model field theories has been introduced for the study of the properties of nuclear matter and finite nuclei,[1] and for the study of the properties of nucleons and other baryons.[2–6] These models, it is hoped, capture the essential aspects of QCD which are relevant for the particular kind of physics being studied. It is not completely clear how these model field theories should be interpreted. They can be thought of as either approximate *effective Lagrangians* for QCD or as approximate *equivalent Lagrangians*. Effective Lagrangians are designed to be used at some level of approximation (e.g., the *mean-field* level) and should, when studied at this level, reproduce the results of the underlying field theory exactly as calculated. In contrast, equivalent Lagrangians are designed to agree with the exact results of the underlying theory when one calculates to all orders in the equivalent field theory. In this note we will restrict our attention to the equivalent Lagrangian interpretation of these model theories.

Traditionally, the quark based models used to study baryons have been interpreted as effective theories.[2–6] In contrast, the nucleon based models used to study nuclear properties are often treated as candidates for (approximate) equivalent theories. Thus, these nuclear field theories are constrained to be renormalizable and it is suggested that this renormalizability constraint makes the models insensitive to the details of the high momentum behavior (i.e., no *ad hoc* cutoff prescriptions are needed).[1]

All of these model field theories couple fermions (either nucleons[1] or quarks[2]) to scalar bosons. In addition, the fermions may be coupled to pseudoscalars or vectors and there may be interaction terms between the various scalars, pseudoscalars, and vectors of the theory. However, these models contain no derivative couplings of the scalar boson. We will prove the following theorem: the perturbative vacuum for any theory of this type is unstable against quantum fluctuations at the one-loop level and that this instability occurs for all values of the coupling. This theorem is a generalization of a remarkable result derived recently by Soni,[7] and Ripka and Kahana[8] who showed that for the sigma model the vacuum is unstable against fluctuations with a sufficiently small distance scale at the one-fermion-loop level. The field configurations, which lead to the instability in Refs. 7 and 8, had a nontrivial topological winding number. We will show that the mechanism causing vacuum instability for the sigma model is general. It applies to all renormalizable models in which scalar or vector bosons are coupled to fermions. This instability is not related to any topological properties of the field configuration. We also show that in theories with scalar bosons coupled to fermions and without derivative couplings, the one-boson-loop contribution to the energy cannot stabilize the vacuum. The role of the boson loop in theories with only vector bosons coupled to fermions will be discussed at the end of this Communication. A somewhat different perspective on the problem has been provided by Perry who shows the relationship of this vacuum instability to the existence of tachyon poles and to the lack of asymptotic freedom.[9]

We consider a typical renormalizable Lagrangian which contains fermions, a scalar boson, and perhaps some additional scalars, pseudoscalars, and vectors: $\mathcal{L}=\mathcal{L}_{\sigma\psi}+\mathcal{L}'$, where $\mathcal{L}_{\sigma\psi}$ is

$$\begin{aligned}\mathcal{L}_{\sigma\psi}&=\tfrac{1}{2}\,\partial^\mu\sigma\partial_\mu\sigma-U(\sigma)+\bar{\psi}(i\not{\partial}-m+g\sigma)\psi\ ,\\ U(\sigma)&=c_0+\tfrac{1}{2}M\sigma^2+c_3\sigma^3+c_4\sigma^4\ ,\end{aligned}\tag{1}$$

and $\mathcal{L}'$ is the Lagrangian for any vectors, pseudoscalars and additional scalars, and for the coupling for these additional fields to σ and ψ. We shall begin by ignoring $\mathcal{L}'$ and will show at the end that the inclusion of $\mathcal{L}'$ will not affect our results so long as $\mathcal{L}'$ contains no derivative couplings of the scalar. The proof of vacuum instability follows closely the proof in Ref. 7 for the nonlinear sigma model. First, one integrates out formally the fermion field from $\mathcal{L}_{\sigma\psi}$ and obtains an equivalent Lagrangian. One can Wick rotate and express the equivalent Lagrangian in a Euclidean metric,

$$\begin{aligned}\mathcal{L}^{\mathrm{eq}}_{\sigma\psi}=&-\tfrac{1}{2}\,\partial_\mu\sigma\partial_\mu\sigma-U(\sigma)\\ &+\tfrac{1}{2}\,\mathrm{Tr}\ln[-\partial^2+(m-g\sigma)^2+ig\not{\partial}\sigma]\\ &-\tfrac{1}{2}\,\mathrm{Tr}\ln[-\partial^2+m^2]-\mathrm{CT}\ ,\end{aligned}\tag{2}$$

where CT stands for counter terms and the trace is over four-momentum, Dirac indices, and any internal coordinates. The two trace ln's may be combined to yield

$$-\tfrac{1}{2}\mathrm{Tr}\ln[1+GV]=-\tfrac{1}{2}\mathrm{Tr}[GV-\tfrac{1}{2}(GV)^2+\tfrac{1}{3}(GV)^3-\tfrac{1}{4}(GV)^4+\cdots] , \quad (3)$$

where $G=1/(-\partial^2+m^2)$ and $V=-2gm\sigma+g^2\sigma^2+ig(\not\partial\sigma)$. The GV and $GVGV$ terms are divergent. A subtraction of counter terms of the form $-\frac{1}{2}\mathrm{Tr}[GV-\frac{1}{2}GGVV]$ renders the expression finite. This subtraction is equivalent to a wave-function renormalization for σ and a renormalization of the potential U. Thus, the renormalized $\mathcal{L}^{\mathrm{eq}}_{\sigma\psi}$ is

$$\mathcal{L}^{\mathrm{eq}}_{\sigma\psi}=-\tfrac{1}{2}\partial_\mu\sigma\partial_\mu\sigma-U(\sigma)+\tfrac{1}{2}\mathrm{Tr}[-\tfrac{1}{2}(GVGV-GGVV)+\tfrac{1}{3}(GV)^3-\tfrac{1}{4}(GV)^4+\cdots] . \quad (4)$$

It will be shown that for σ fluctuations on a sufficiently small scale, the vacuum is unstable. We mean by this that the energy of a σ field configuration, which has fluctuations at this scale, is less than the energy of the perturbative vacuum. Consider a static field distribution $\sigma=f(\mathbf{x}/r)$, where f is some arbitrary functional form. R, the distance scale, will serve as a variational parameter. V consists of two parts, $-2gm\sigma+g^2\sigma^2$, which does not scale with R, and $ig(\not\partial\sigma)$, which goes like $(1/R)$. For sufficiently small R, GV scales like R, and we expect the expansion of the ln in Eqs. (3) and (4) to converge and to be dominated by the low order terms. We note that the expansion is related to the well-known derivative expansion. The applicability of this expansion for small R was noted by Soni.[7] To demonstrate a vacuum instability at the one-fermion-loop level, it is sufficient to show that the leading R behavior in the expansion (i.e., the $GVGV-G^2V^2$ piece) leads to a negative energy. In calculating the energy it is permissible to drop the $-2gm\sigma+g^2\sigma^2$ terms in V since these contributions are higher order in R. It is straightforward to derive an expression for the energy in this approximation, and one obtains an expression similar to that of Ripka and Kahana:[8]

$$E=g^2RN/(16\pi^2)\int\frac{d^3\mathbf{q}'}{(2\pi)^3}\left|\int d^3\mathbf{x}'f(\mathbf{x}')\exp(i\mathbf{q}'\cdot\mathbf{x}')\right|^2 q'^2A(q'/R)+E_{\mathrm{kin}} , \quad (5)$$

where N is the number of internal degrees of freedom (flavors, colors, etc.), E_{kin} is the kinetic energy, $E_{\mathrm{kin}}=R\int d^3\mathbf{x}'[\nabla' f(\mathbf{x}')]^2$, ∇' is the gradient with respect to the dimensionless variable $\mathbf{x}'=\mathbf{x}/R$, and $A(q)$ is given by

$$A(q)=2-(4m^2/q^2+1)^{1/2}\ln\left[\frac{(q^2+4m^2)^{1/2}+q}{(q^2+4m^2)^{1/2}-q}\right] .$$

For small R, the energy becomes

$$E=R\ln(mR)X+RY \quad (6)$$

with

$$X=g^2N/(8\pi^2)\int\frac{d^3\mathbf{q}'}{(2\pi)^3}\left|\int d^3\mathbf{x}'f(\mathbf{x}')\exp(i\mathbf{q}'\cdot\mathbf{x}')\right|^2 q'^2,$$

$$Y=\int\frac{d^3\mathbf{q}'}{(2\pi)^3}\left|\int d^3\mathbf{x}'f(\mathbf{x}')\exp(i\mathbf{q}'\cdot\mathbf{x}')\right|^2 \times q'^2[1+g^2N\ln(e^2q'^{-2})/(16\pi^2)] .$$

Note that the coefficient multiplying $R\ln(mR)$ is manifestly positive and thus the energy of such a configuration will be negative for sufficiently small R. Such a negative-energy configuration will occur for any value of the coupling constant. Minimizing the energy with respect to R gives $R=m^{-1}\exp(-a-16\pi^2/g^2N)$, where a is a constant of order unity which depends on the shape of the field configuration.

Let us now show that the one-sigma-loop contribution to the energy for such a field configuration will not stabilize the vacuum. The σ loop will also have a structure of $\mathrm{Tr}\ln(1+GV)$. In this case $G=1/(-\partial^2+M_\sigma^2)$ and $V=\partial^2U/\partial\sigma^2-M_\sigma^2$. Note that V for the σ loop does not scale with R, while V in the fermion loop goes like $(1/R)$ for small R. Since the leading contribution to the energy comes from a term quadratic in V, one sees that the σ loop contribution to the energy will go like $R^3\ln R$ for small R, i.e., it will be two powers of R suppressed compared to the fermion loop. Thus, for sufficiently small R the σ loop is negligible and cannot stabilize the vacuum. It is clear that the value of R, which actually minimizes the energy, need not be small enough so that the σ loop may be numerically ignored. However, it is also clear that the $R^3\ln(R)$ behavior of the σ loop implies that there must exist some value of R which leads to a negative energy.

Next let us consider whether $\mathcal{L}'$, the coupling to other scalars, pseudoscalars, and vectors can stabilize the vacuum. Let Φ denote the various fields in $\mathcal{L}'$ (excluding σ and ψ) and Φ_{vac} as their expectation values in the perturbative vacuum. The division of $\mathcal{L}$ into $\mathcal{L}_{\sigma\psi}$ and $\mathcal{L}'$ can always be accomplished in such a way that

$$\left.\frac{\delta\mathcal{L}'}{\delta\psi}\right|_{\Phi=\Phi_{\mathrm{vac}}}=\left.\frac{\delta\mathcal{L}'}{\delta\sigma}\right|_{\Phi=\Phi_{\mathrm{vac}}}=0 . \quad (7)$$

Since, to demonstrate vacuum instability, it is sufficient to show that *some* field configuration exists with negative energy, one can restrict one's attention to those configurations with $\Phi=\Phi_{\mathrm{vac}}$. If these configurations lead negative energies then instability has been established. Equation (7) implies that one-fermion-loop and one-σ-loop contributions to the energy will be unchanged. Of course, there can be contributions due to the one Φ loop. Such contributions, like the one σ loop, will go as $R^3\ln(R)$ and thus cannot stabilize the vacuum. If $\mathcal{L}'$ had contained derivative coupling of the scalar, a term which goes like $-R\ln(R)$ would have emerged. Such a term may stabilize the vacuum. However, for renormalization theories, such as those being considered here, one cannot include a scalar derivative coupling.

We have shown that there exist field configurations which at the one-loop level have an energy less than energy of the field of configuration associated with the perturbative vacuum. It is worth asking whether the existence of such configurations really demonstrates that the perturbative vacuum is unstable. To show that the perturbative vacuum is unstable, it is necessary to show that there exists a *quantum state* which has a lower energy. At this point all we have shown is that the classical energy of a field configuration for an effective Lagrangian, which includes one-loop effects, is negative. It is easy to show that this implies the existence of a negative energy quantum state at the one-loop level. Denote the Hamiltonian for the system described by the Lagrangian in Eq. (1) as H. The energy of a quantum state compared to the perturbative vacuum is $\langle\phi|{:}H{:}|\phi\rangle$, where the normal ordering is with respect to the perturbative vacuum. Next, consider as a quantum state a coherent state[10] of the form

$$|f(\mathbf{x})\rangle = \mathcal{N}\exp\left[-i\int d^3\mathbf{x} f(\mathbf{x})\pi(\mathbf{x})\right]|\text{vac}\rangle \ , \qquad (8)$$

where f is an arbitrary field configuration for the sigma field, π is the quantum operator conjugate to σ, $|\text{vac}\rangle$ is the perturbative vacuum, and $\mathcal{N}$ is a normalization constant. Coherent states have the property that

$$\langle f|{:}A(\sigma,\partial_x\sigma,\pi){:}|f\rangle = A(f,\partial_x f,0)$$

for an arbitrary functional A. The expectation value of a coherent state for a normal ordered operator is equal to the classical value of the functional for a time-independent configuration f. The expectation value of the normal ordered Hamiltonian at the one-loop level for a coherent state is therefore equal to the classical energy computed for the one-loop effective Lagrangian. Therefore, the existence of a negative-energy field configuration at the one-loop level implies the existence of a quantum-state with negative energy and hence an unstable vacuum.

The perturbative vacuum is unstable at the one-loop level for the class of models given in Eq. (1). It becomes a question of some importance to see whether this instability survives higher-order calculations. In one sense, to answer this question one must do more complete calculations.

The instability of the vacuum at the one-loop level is a very general result. Indeed, the instability due to the fermion loop also occurs for systems without scalar bosons, i.e., for systems of fermions coupled only to vectors. The proof is of the same form given above for coupling to scalars. However, in such a case, interactions among the vector bosons (in particular a three-boson interaction with derivative coupling) can give rise to $R\ln(R)$ terms and might stabilize the vacuum. It should be noted that QED is a theory of fermions interacting via vector bosons without any vector-vector interaction. Thus, the QED perturbative vacuum is unstable at the one-loop level. We wish to observe that the scale of fluctuations at which the instability occurs is $m\exp(1/\alpha)$. This is the same scale as the well-known Landau singularity in the renormalization group.[11] It is unclear whether the vacuum instability (or the Landau singularity) is real or whether it is a consequence of the one-loop approximation. Of course, if it is real, it violates the axioms of quantum field theory. In any event, it is not physically relevant because at the scales of the instability, QED by itself is not the appropriate theory. One must consider unified theories.

The formal properties of this vacuum instability may be related to the renormalization group as shown by Perry.[9] His arguments show that the vacuum instability will occur for all nonasymptotically free theories. Theories with scalar bosons coupled to fermions, and which have no derivative couplings, exhibit the vacuum instability at the one-loop level and are, of course, not asymptotically free. Theories with fermions coupled to vector bosons may or may not exhibit the vacuum instability depending on the nature and strength of the boson-boson coupling terms. Since it is precisely these coupling terms which can lead to asymptotic freedom, it is not surprising that the question of vacuum stability and asymptotic freedom are related.

Finally, we will make a few comments about the relevance of the instability of the perturbative vacuum at the one-loop level to the interpretation of the various phenomenological field theories used in the study of nuclei and nucleons. One possibility, of course, is that the instability will not generally survive higher-order corrections, in which case it might be argued that the one-loop instability is of no fundamental concern. The results are still disturbing as they suggest a limitation of the loop expansion. If, however, the instability is not an artifact of the one-loop calculation, it will probably not be useful to interpret these model field theories as approximate equivalent theories to the underlying QCD. Instead, it is probably more useful to interpret the theories as some sort of effective theory useful for a restricted class of calculations, such as mean-field calculations or perhaps in calculations with finite cutoffs. We wish to observe that the vacuum instability problem occurs at high momentum transfer in renormalizable field theories. Thus, the assumption that the renormalizable nature of the nucleon based models will make these theories insensitive to the treatment of high momentum processes should be viewed with considerable caution.

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