Folding model for sub-barrier interaction between alpha-type nuclei

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We show that it is possible to derive the real parts of the low-energy optical potentials for the systems $^{16}O+^{16}O$, $^{12}C+^{12}C$, and $^{12}C+^{16}O$ simultaneously from a single effective interaction by means of the double-folding model. However, this effective interaction does not produce satisfactory results at high energies. In contrast, the effective interaction of Satchler ane Love gives satisfactory results in both regions and can be regarded as reference potential over a wide energy range.

I. INTRODUCTION

In the last few years there has been a great deal of discussion of the possibility that at energies below the Coulomb barrier the fusion and elastic scattering of heavy nuclei cannot be adequately derived by the optical model and that it is necessary to invoke explicitly the dynamics of the two interacting nuclei; see Ref. 1 for the extensive review. However, the evidence for this in the case of pairs of lighter nuclei ($Z_1 \cdot Z_2 \leq 80$) is less striking, and very recently Haider and Cujec[2] have shown that an excellent description of the sub-barrier fusion and elastic scattering of the three systems $^{12}C+^{12}C$, $^{16}O+^{16}O$, and $^{12}C+^{16}O$ can be made in terms of an optical potential based on the proximity potential.[3]

The underlying physical validity of this description is suggested by the fact that once the $^{12}C+^{12}C$ and $^{16}O+^{16}O$ systems have been fitted, the $^{12}C+^{16}O$ data are very well reproduced without any further adjustment of parameters at all. However, before drawing any conclusion in this respect, it is necessary to study the microscopic basis of the optical potentials of the three systems in question, and see whether they can be put on some common footing.

In the present paper we make this study in the simplest possible way: we see whether the real parts of the three optical potentials can all be derived by the double-folding model, starting from the same effective interaction. This model, as formulated by Satchler and Love,[4] for example, offers about the simplest way to express the phenomenological heavy-ion optical potential in more microscopic terms. However, it can in no way be regarded as the ultimate microscopic theory of the optical potential, essentially because the folding is made over the local density, rather than over the density matrix, i.e., over the wave functions, with the result that the model as it stands cannot be antisymmetrized, and the effective interaction of the model must be static and local. Furthermore, polarization of one nucleus by the other cannot be considered. For all these reasons it appears that the effective interaction of the model would be purely phenomological bearing little relation to the real N-N interaction.

Nevertheless, it has been possible to derive a simple effective interaction which is convenient to use in this model, but which at the some time bears some relation to the "real" N-N force and simulates to some extent the effect of the neglected antisymmetrization and polarization.[4,5] The application of the model with this interaction to the $^{12}C+^{12}C$ system in the energy range 35–60 MeV has been conspicuously successful,[6] so that our confidence in the underlying physical sense of the model is considerably strengthened. Even if one reverts to purely phenomenological interactions it is surely reasonable to insist that the same interaction should be applicable to different pairs of nuclei in the same energy range. This should be especially the case with the set of systems $^{12}C+^{12}C$, $^{12}C+^{16}O$, and $^{16}O+^{16}O$, since the pairs of mass numbers involved are not very different from the different systems, and there are similarities in the gross structure, also. In particular, since both nuclei have zero spin and $N=Z$, there will be a minimum of ambiguity associated with spin and isospin dependence.

In calculating these three systems we look first to the same energy range as Haider and Cujec,[2] i.e., below the Coulomb barrier. It is noteworthy that it is in this region that the possibility of providing some microscopic basis to the optical model is of direct practical interest: We are thinking of the importance for astrophysics of being able to use the optical model as reliably as possible to extrapolate measured fusion cross sections down to very low energies. However we also try to reproduce the available data in the high-energy region in order to see whether this effective interaction can be used over a wider energy range.

II. FOLDING MODEL

For a spin-independent, density-dependent, two-body effective interaction, $u(r,\rho)$, the double-folding model gives[6] for the optical potential

$$V(R)=\int d^3r_1 d^3r_2 \rho_1(r_1)\rho_2(r_2)u_{\text{eff}}(r,\rho_1+\rho_2)\ , \qquad (1)$$

where p_1 and p_2 are the master-density distributions of respective nuclei, assumed spherically symmetric, and

$$\mathbf{r}=\mathbf{r}_2+\mathbf{R}-\mathbf{r}_1 \ . \tag{2}$$

In a rigorous approach to the folding model the effective interaction $u_{\text{eff}}(r,\rho)$ will be derived from the G matrix by solving the Bethe-Goldstone equation, and can be complex. In the present more phenomenological approach the effective force is assumed to be real, so that only the real part of the optical potential can be derived; the imaginary part will have to be put "by hand," as in Refs. 4 and 7.

The form of effective interaction that we consider is fairly general, having two density-independent Yukawa terms of short range and long range, respectively, and a zero-field density-dependent term,

$$u_{\text{eff}}(r,\rho)=C_1\frac{e^{-\mu_1 r}}{\mu_1 r}+C_2\frac{e^{-\mu_2 r}}{\mu_2 r}+C_3\rho^{\sigma}\delta(\mathbf{r}) \ . \tag{3}$$

This actually has the same form as that of the Satchler-Love force,[4] except that our introduction of a density dependence allows a better conformity to a realistic G matrix.

Equation (1), for the real part of the optical potential, now becomes

$$V(R)=2\pi\int_0^{\infty} r_1^2 dr_1 \int_0^{\pi} u[r_1,R_1,r_2(\theta_1)] \times\rho_2[r_2(\theta_1)]\sin\theta_1 d\theta_1 \ , \tag{4}$$

where

$$r_2(\theta_1)=[(r_1^2+R^2-2r_1R\cos\theta_1)]^{1/2}$$

and

$$u[r_1,R,r_2(\theta_1)] =u_1(r_1)+u_2(r_1)+C_3\rho_1(r_1)\{\rho_1(r_1)+\rho_2[r_2(\theta_1)]\}^{\sigma} \ , \tag{5}$$

with

$$u_i(r_1)=\frac{4\pi C_i}{\mu_i^2}\frac{e^{-\mu_i r_1}}{r_1}\int_0^{r_1}\rho_1(r_1')r_1'\sinh(\mu_i r_1')dr_1' + \frac{\sinh\mu_i r_1}{r_1}\int_{r_1}^{\infty}\rho_1(r_1')r_1' e^{-\mu_i r_1'}dr_1' \ . \tag{6}$$

After a change of variables and some straightforward algebra, Eq. (4) becomes

$$V(R)=\frac{2\pi}{R}\int_0^{\infty} r_1 dr_1 \int_{|R-r_1|}^{R+r_1} u(r_1,R,r_2)\rho_2(r_2)r_2 dr_2 \ , \tag{7}$$

which is more suitable for calculations.

The matter densities that we took for ^{16}O and ^{12}C were obtained by a spherical Hartree-Fock (HF) calculation[8] with the effective force of Decharge and Gogny.[9] When the finite radius of the proton is taken into account, and a correction made for the c.m. motion, this force gives an excellent agreement (to within 0.01 fm) with the measured rms charge radii of all doubly magic nuclei. Thus the matter distribution calculated for ^{16}O (point nucleons were assumed) should be quite reliable.

For simplicity we also assume the nucleus ^{12}C to be spherical, noting, however, that the real equivalent spherical matter distribution that enters into the optical potential might have a smaller radius than that which emerges from our HF calculations: The rms radius that we obtain for point nucleon distribution is 2.35 fm. This is to be compared with the experimental value[10] of 2.46 fm.

III. RESULTS AND DISCUSSION

We first derived the folding-model potential for the Satchler-Love effective force.[4] It can be shown, see for example Fig. 1, that for all three systems this potential is quite different from the proximity potential. Then we determined a new force by fitting directly to the proximity potential for one particular system, i.e., the ^{12}C+^{12}C system. The fit was made linearly, by adjusting the coefficients C_1, C_2, and C_3 for fixed values of μ_1, μ_2, and σ. We selected the three points from the tail of the proximity potential, namely, at distances 6, 8, and 10 fm in order to reproduce its asymptotic behavior and fixed the values of μ_1 and μ_2 at the Satchler-Love[4] values of 4.0 and 2.5 fm^{-1}, respectively. We set $\sigma=0$ or $\sigma=1$ although the choice of σ was not crucial. Both classes of potentials had the same shape, with the $\sigma=1$ case producing strong repulsive cores, similar to the proximity potentials, almost a factor of 4.5 stronger than in the $\sigma=0$ case. The resulting values of C_1, C_2, and C_3 are given in Table I, where we have also shown, for purposes of comparison, the corresponding values for the Satchler-Love interaction.

With the effective interaction deduced by fitting to the proximity potential for the ^{12}C+^{12}C system, we deduce the folding-model potential for the other two systems as well. In all three cases the folding-model potential is very similar to the proximity potential beyond $r=4$ fm, leading us to expect that similar fits to the data will be obtained.

These folding potentials together with a Woods-Saxon imaginary potential were used in an optical code to calculate the elastic and the fusion cross sections. The latter are plotted in terms of the so-called S factor which is defined in the usual way by

$$S(E)=\sigma(E)\exp(2\pi n) \ , \tag{8}$$

TABLE I. Parameters of the effective interaction of Eq. (3).

	C_1 ($\mu_1=4.0$ fm^{-1})	C_2 ($\mu_2=2.5$ fm^{-1})	C_3	σ
	−19914.4 MeV	−3291.38 MeV	6574.08 MeV fm^3	0
Present	23692.88 MeV	−6829.50 MeV	9509.70 MeV fm^6	1
Satchler-Love	6315 MeV	−1961 MeV	−81 MeV fm^3	0

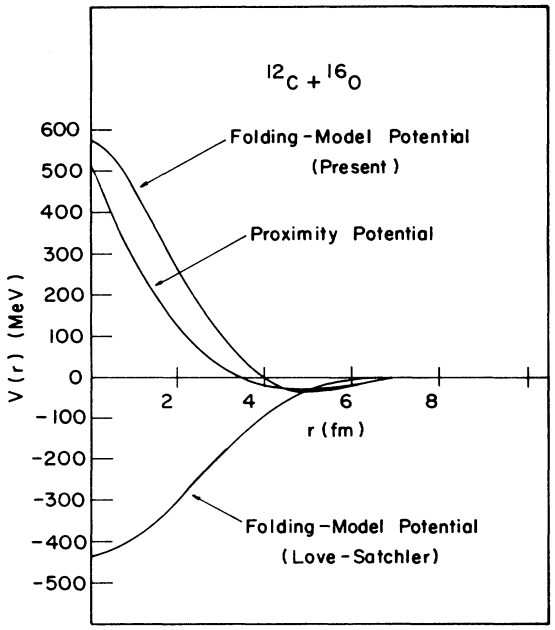

FIG. 1. Real parts of the present the proximity and the Satchler-Love optical potentials used in the calculations of the elastic and fusion cross-sections for the $^{12}C+^{16}O$ system. The potentials for the other two systems are qualitatively similar.

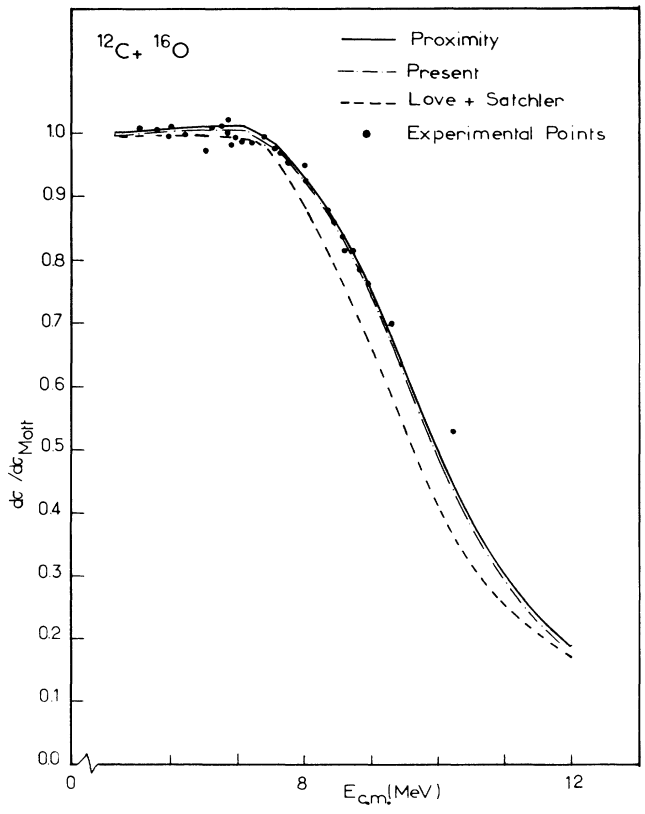

FIG. 2. 90° elastic cross sections normalized to the Mott cross section, for the $^{12}C+^{16}O$ system for the three potentials of Fig. 1. The data are taken from Ref. 2 and references cited there.

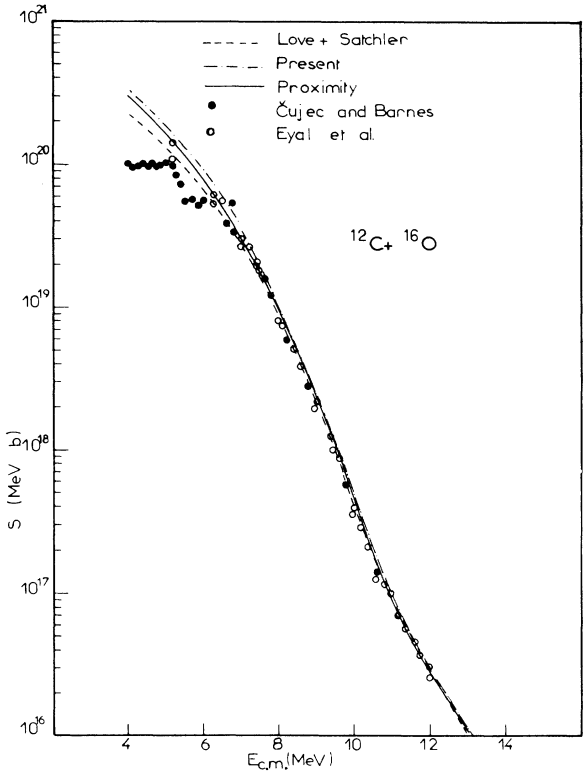

FIG. 3. Comparison of the S factors for the $^{12}C+^{16}O$ system. The data are taken from Ref. 2 and the references cited there.

where

$$n = Z_1 Z_2 e^2 (\mu/2h^2 E_{c.m.})^{1/2} . \tag{9}$$

Following Ref. 7, we took for the imaginary potential,

$$W(r) = \frac{W_v(E)}{1+\exp\left[\dfrac{r-r_w}{a_w}\right]} \tag{10}$$

an energy dependent depth of the form

$$W_v(E) = -0.5E^2 \text{ MeV} . \tag{11}$$

The geometrical parameters r_w and a_w as well as the Coulomb potential, are as in Ref. 2.

The resulting cross sections together with those of the proximity potentials (taken with the same imaginary parts as with our potentials), are compared with the experimental data in Figs. 2 and 7. Actually, since the $\sigma=0$ and the $\sigma=1$ potentials give very similar fits to the data, only the $\sigma=0$ case is presented, for clarity. It is seen that our folding potentials produce a good fit to the data for all three systems, very similar to the fits given by the proximity potentials. For the purpose of comparison, the fits given by the Love and Satchler folding potential taken with the same imaginary part, are also

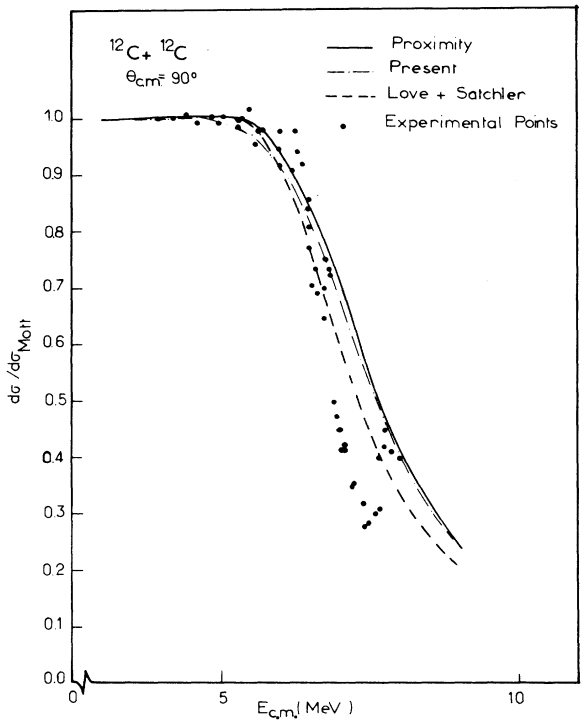

FIG. 4. Same as Fig. 2 but for the $^{12}C+^{12}C$ system.

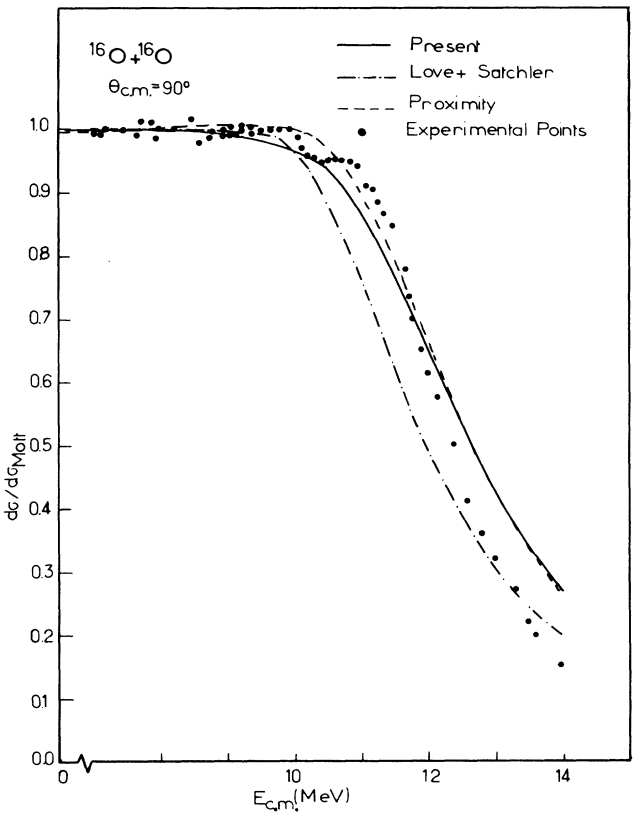

FIG. 6. Same as Fig. 2 but for the $^{16}O+^{16}O$ system.

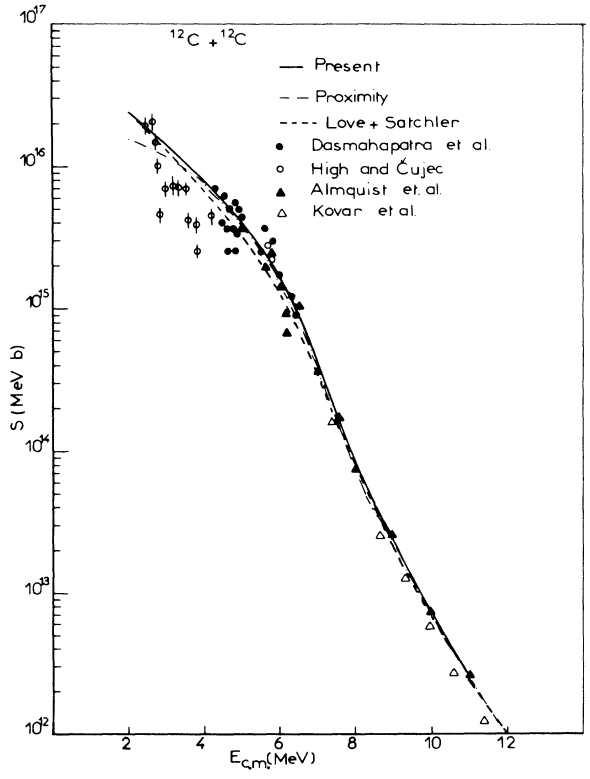

FIG. 5. Same as Fig. 3 but for the $^{12}C+^{12}C$ system.

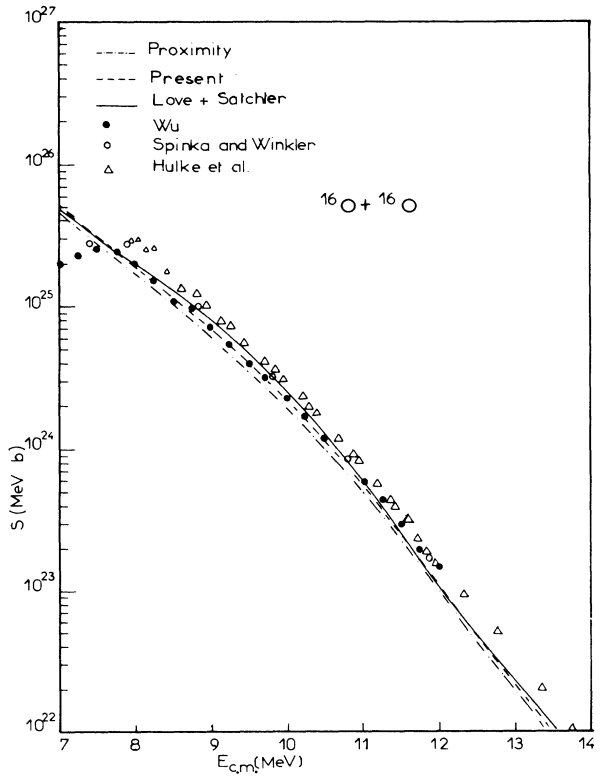

FIG. 7. Same as Fig. 3 but for the $^{16}O+^{16}O$ system.

shown. They are equally acceptable except perhaps for the elastic cross section of the $^{12}C+^{16}O$ system in which the energy dependence of the imaginary part seems to be more pronounced. However we also found a better quality fit for this system but with a different normalization of the imaginary part, namely, $W_v(E)=1.+0.2E^2+0.1E$ for low energies and $W_v(E)=0.1E^3+0.35E$ for high energies.

It is also possible to improve the fits by varying the normalization of the folded real potential parameters in the way prescribed in Ref. 11, with $V(r)=V_0(r)+\Delta V(r)$. In this case the polarization potential $\Delta V(r)$ might not have the same shape as $V(r)$. No single choice however leads to completely acceptable fits in all three cases. To improve our fits it is likely, that we have to adjust the real and the imaginary potentials separately in the three systems, and we are investigating this now. We may note in passing that the repulsive core of the $\sigma=1$ potential flattens the S factor in the $^{12}C+^{12}C$ system at energies below 4 MeV thus producing a slightly better fit at those energies.

We also turned our attention to the high energy region and tried to fit our potentials to the $^{12}C+^{12}C$ data of Refs. 6 and 12 and to the $^{16}O+^{16}O$ results of Refs. 7 and 13. No single satisfactory fit could be produced for these systems despite several variations of the imaginary parts of our potentials. It appears that our folding potentials derived from the proximity potentials cannot be used throughout the whole energy range.

To conclude, our calculations indicate that a unified description of the sub-barrier properties $^{12}C+^{12}C$, $^{12}C+^{16}O$, and $^{16}O+^{16}O$ systems can be obtained in terms of the double-folding model, using a common effective force. However, the potential derived by the Love and Satchler force are to be preferred, since they work over a much wider energy range. It appears that a more rigorous determination of the imaginary as well as the real part of the optical potential is required. In particular, the energy dependence is of crucial importance for the extrapolation to the low energies of astrophysical significance.

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