Modified Glauber model for the description of elastic scattering between heavy ions

A. Vitturi and F. Zardi
Dipartimento di Fisica "G. Galilei" and Istituto Nazionale di Fisica Nucleare, Padova, Italy
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The standard form of the optical limit of the Glauber model used for the description of elastic scattering is modified to account for the Coulomb distortion of the trajectory occurring in the case of heavy ion scattering at relatively low bombarding energies. We retain the description of the process in terms of nucleon-nucleon collisions, but we relate the nuclear density overlaps for each partial wave to the distance of closest approach of the associated Rutherford trajectory, rather than to the asymptotic value of the impact parameter. Parameter-free predictions for the elastic scattering of ^{13}C at 390 MeV and of ^{40}Ar at 1760 MeV from ^{208}Pb, based on experimental nucleon-nucleon total cross sections and standard densities, show a remarkable improvement for both differential and reaction cross sections. The consistency of the approach is strengthened by a correlated method for the inversion of the phase shifts, which, for the examined cases, yields optical potentials very similar to those obtained by optical-model fit analysis.

I. MODIFIED OPTICAL LIMIT

In the last few years several attempts have been made to describe elastic scattering processes between heavy ions in terms of the so-called optical limit to the Glauber model (see, e.g., Ref. 1 and references quoted therein). The basic point of the model is to express each partial wave phase shift as an integral, along straight-line trajectories, of quantities involving individual contributions of microscopic collisions weighted by the local matter density. In the optical limit this phase shift can be obtained in the factorized form

$$\delta_l=\tfrac{1}{2}\sigma_{\rm NN}(\alpha_{\rm NN}+i)\Omega_l \ , \tag{1}$$

where $\sigma_{\rm NN}$ is the total nucleon-nucleon cross section and $\alpha_{\rm NN}$ the ratio of real to imaginary part of the forward nucleon-nucleon scattering amplitude. Ω_l is the overlap integral of the nuclear densities along a straight line characterized by the impact parameter b

$$l+\tfrac{1}{2}=kb \ . \tag{2}$$

The optical limit factorization (1) is expected to hold in the limit of isotropy of the nucleon-nucleon scattering amplitude and in the hypothesis of completely independent microscopic collisions, each process taking place outside the range of the others.

The second condition is attained in the case of relatively small overlap between two colliding nuclei. Since the main features of heavy ion elastic scattering are essentially dominated by the grazing partial waves, the hypothesis is fulfilled in our case. Concerning the isotropy of the microscopic cross section, it can be assumed valid for energies not exceeding 100 MeV per nucleon.

On the other hand, due to the strong Coulomb field present in heavy ion collisions, the straight trajectory approximation breaks down in the above considered energy range. As a consequence the values of the grazing partial wave are raised, and this results in an overestimation of the reaction cross section and an underestimation of the grazing angle in the angular distribution. The geometry of the situation is schematized in Fig. 1.

We suggest a simple modification to mend these shortcomings, in a form such that the standard Glauber optical limit can be obtained as a particular case. By retaining the same general philosophy of the optical limit, we note that the superposition integral for a given "nearly straight" trajectory can be more profitably related to the value of the distance of closest approach r_l^0 rather than to value of the impact parameter. In the standard Glauber approach, l depends on r_l^0 according to Eq. (2) and one can write

$$l+\tfrac{1}{2}=kb=kr_l^0 \ . \tag{3}$$

For the strong Coulomb field case, l can be directly related to r_l^0 by the expression

$$kr_l^0=\eta+[\eta^2+(l+\tfrac{1}{2})^2]^{1/2} \ , \tag{4}$$

where η is the Sommerfeld parameter

$$\eta=Z_P Z_T e^2/\hbar v \ .$$

The superposition integral along straight trajectories of Figs. 1(a) and 1(b) clearly depends on r_l^0 only. In the case of Gaussian densities expressed as

$$\rho_i(r)=\rho_i(0)e^{-r^2/a_i^2} \quad (i=P,T)$$

for both target and projectile, the integral can be evaluated analytically, obtaining the expression

$$\Omega_l=\rho_T(0)\rho_P(0)\pi^2\frac{a_T^3a_P^3}{a_T^2+a_P^2}e^{-(r_l^0)^2/a_T^2+a_P^2} \ . \tag{5}$$

The scattering amplitude is then written in the standard form

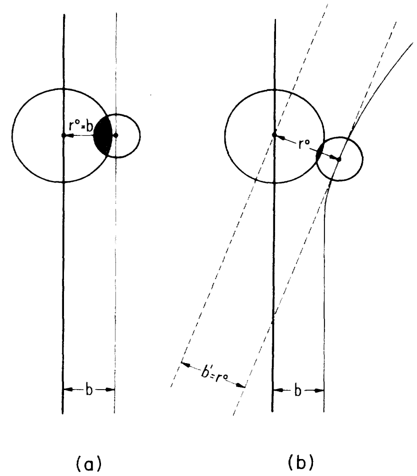

FIG. 1. Schematization of the nuclear density overlaps at the point of closest approach for trajectories with the same angular momentum, without and with the Coulomb distortion.

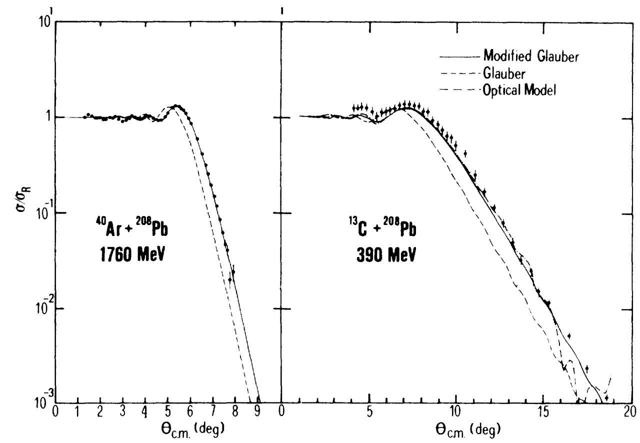

FIG. 2. Predicted elastic angular distribution for the reactions (a) ^{40}Ar + ^{208}Pb at 1760 MeV and (b) ^{13}C + ^{208}Pb at 390 MeV, in comparison with the experimental data (Refs. 2 and 3). The solid line gives the results of the modified Glauber model, the dashed line the results of the standard Glauber model. The dot-dashed line in (b) gives the results obtained through the optical model potential of Ref. 3 (cf. Table I).

$$f(\theta)=f_c(\theta)+\frac{1}{2ik}\sum_l (2l+1)e^{2i\sigma_l}(e^{2i\sigma_l}-1)P_l(\cos\theta) , \tag{6}$$

where the nuclear phase shift is given by Eq. (1) and σ_l is the usual Coulomb phase shift. The standard Glauber amplitude is obtained by correlating l and r_l^0 through Eq. (3), while our modified amplitude is obtained by Eq. (4).

II. APPLICATION TO ^{40}Ar and ^{13}C SCATTERING FROM ^{208}Pb

We have applied the above defined formalism to the elastic scattering of ^{40}Ar + ^{208}Pb at $E/A=44$ MeV (Ref. 2) and of ^{13}C + ^{208}Pb at $E/A=30$ MeV (Ref. 3). The values of the density parameters for ^{13}C [$\rho(0)=0.322$ fm^{-3}, $a=1.935$ fm] have been obtained from the ones of ^{12}C (Ref. 1) by rescaling according to the mass and taking the same value of $\langle r^2 \rangle$ (see, e.g., the discussion given in Ref. 4). For ^{40}Ar and ^{208}Pb we have assumed the Gaussian parametrization by Karol,[5] who adjusted the Gaussian function to reproduce the Fermi density distribution on the nuclear surface. The values σ_{NN} and α_{NN} associated with the microscopic nucleon-nucleon scattering have been obtained by interpolating at the corresponding energies the experimental values. All the relevant parameters used in the calculation are collected in Table I.

The predictions of the model for the elastic scattering cross sections are displayed in Figs. 2(a) and 2(b). The dotted line is obtained in the standard optical limit [Eqs. (1), (3), (5), and (6)], the full line is related to our modified optical limit [Eqs. (1), (4), (5), and (6)]. In view of the parameter-free nature of our calculation, the agreement of the modified Glauber model results with the experimental values is quite good, with a remarkable improvement with respect to the standard Glauber model. Note the better agreement with the experimental data in the Ar case, where the density distribution is better known. We also show in the Fig. 2(b) (dot-dashed) the results obtained through the optical potential of Ref. 3 (cf. Table I). In the Ar case the prediction of the optical model fit is practically indistinguishable from the modified Glauber model results.

A further insight on the situation can be gained by looking at the transparency coefficients $T_l = |S_l|^2$ [Figs. 3(a) and 3(b)]. As expected, our modified formula lowers the value of the grazing angular momentum, approach-

TABLE I. Values of the Sommerfeld parameter and of the microscopic collision parameters entering in formula (1). The quantities σ_{NN} and α_{NN} have been obtained by interpolating the experimental data (cf. Ref. 1). Also given are the optical potential parameters obtained by a best-fit procedure in Ref. 2 for ^{40}Ar + ^{208}Pb and Ref. 3 for ^{13}C + ^{208}Pb.

	η	σ_{NN} (fm^2)	α_{NN}	V (MeV)	R_V (fm)	a_V (fm)	W (MeV)	R_W (fm)	a_W (fm)
^{40}Ar+ ^{208}Pb ($E=1760$ MeV)	35.18	12.5	0.93	73.36	11.018	0.63	65.1	11.018	0.63
^{13}C+ ^{208}Pb ($E=390$ MeV)	14.20	19.6	0.87	50	9.68	0.551	25.7	9.576	0.558

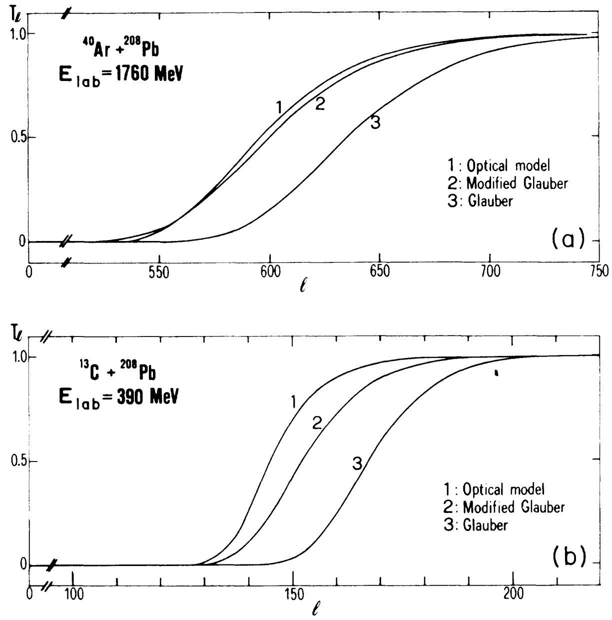

FIG. 3. Transparency coefficients $T_l = |S_l|^2$ for the reactions (a) ^{40}Ar + ^{208}Pb and (b) ^{13}C + ^{208}Pb, obtained in the optical model analysis (Refs. 2 and 3), in the modified Glauber model and in the standard Glauber model.

ing the results obtained in Refs. 2 and 3 through an optical potential fit (cf. Table I for the optical potential parameters). This improvement is also reflected in the values obtained for the reaction cross sections, as displayed in Table II.

III. INTERPRETATION IN TERMS OF AN OPTICAL POTENTIAL

Within the Glauber model one can obtain from the phase shifts a nucleus-nucleus optical potential $V(r)$ according to the integral transform[6]

$$V(r)=\frac{2\hbar v}{\pi r}\frac{d}{dr}\int_r^\infty \frac{\delta(b)}{(b^2-r^2)^2}b\,db \ . \tag{7}$$

We can apply this procedure to our modified Glauber phase shifts. As apparent from the results shown in the previous section they can in fact be considered for all practical purposes an equivalent to the experimental ones, although having a simple analytic form. Within the Glauber philosophy we are expected to correlate, in

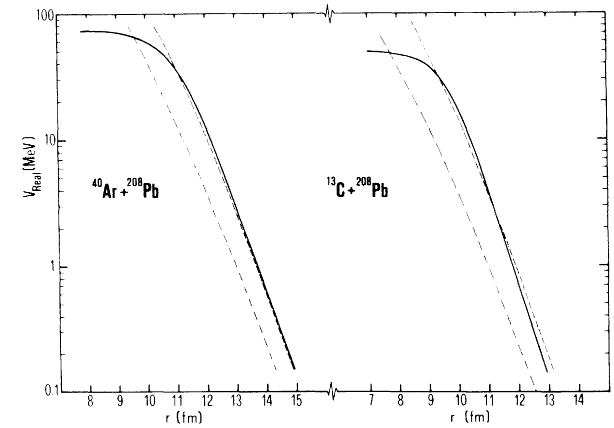

FIG. 4. Real part of the nucleus-nucleus potential obtained in the modified Glauber formalism (dashed line) compared with the phenomenological optical potential (solid line) from Refs. 2 and 3. For the meaning of the dot-dashed curve see the text.

(7), δ_l and b through $l+\frac{1}{2}=kb$. We obtain in this way the potential whose real part is shown as dot-dashed curve in Fig. 4, which in the tail region is decidely smaller than the phenomenological one (solid line). This fact has a simple explanation. According to (7) the value of $V(r)$ receives contributions from all values of $b \geq r$. Since our phases are associated with curved trajectories with turning points given by (4), we obtain a weak potential corresponding to the more external regions explored by the curved trajectories.

Along the approach developed in the previous section we have instead to interpret expression (7) according to

$$V(r)=\frac{2\hbar v}{\pi r}\frac{d}{dr}\int_r^\infty \frac{\delta_l(r_l^0)}{[(r_l^0)^2-r^2]^{1/2}}r_l^0 dr_l^0 \tag{8}$$

which, only in the case of straight trajectories, coincides with (7). At variance with expression (7), in the integral (8) receives contributions from values of the distance of closest approach for which $r_l^0 \geq r$. The resulting expression for the potential is

$$V(r)=-\frac{\hbar v \pi^{3/2}}{(a_T^2+a_P^2)^{3/2}}\frac{1}{2}\sigma_{\rm NN}(\alpha_{\rm NN}+i) \times \rho_T(0)\rho_P(0)a_T^3 a_P^3 e^{-r^2/a_T^2+a_P^2} \ . \tag{9}$$

In Fig. 4 the real part of the potential (dashed curve) is

TABLE II. Values of the reaction cross section obtained in the optical model analysis (cf. Table I), in our modified Glauber model, and in the standard Glauber model. Also given is the corresponding critical angular momentum $l_{1/2}$ ($T_{l_{1/2}}=\frac{1}{2}$).

	$\sigma_{\rm react}$ (fm^2) Optical model	Modified Glauber	Glauber	$l_{1/2}$ Optical model	Modified Glauber	Glauber
^{40}Ar + ^{208}Pb	484.1	485.8	545.3	596	600	635
^{13}C + ^{208}Pb	318.2	349.2	419.1	146	153	175

compared with the phenomenological optical potential, showing a fair agreement. A similar agreement also holds for the imaginary part, as expected from the transmission coefficients of Fig. 3.

One could observe that the same final expression for optical potential could have been obtained starting from the standard Glauber phase shift and using formula (7). We think however that our derivation of the potential (9) has the merit of the full consistency between phase shifts, potential and experimental data.

IV. CONCLUDING REMARKS

We have shown that it is possible to give a satisfactory account of the elastic scattering between heavy ions at relative low bombarding energy within the Glauber model, only releasing the straight line condition. The usual Glauber formula applies, but the overlap integral must be correctly evaluated taking into account the distortion of the trajectory due to the strong Coulomb field.

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