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Coupled $K^\pi=0^+$ and $K^\pi=0^-$ vibrations in ^{234}U

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The multiphonon method based on $K^\pi=0^+$ and $K^\pi=0^-$ phonons is applied to ^{234}U. The anharmonicities obtained for the octupole vibrations are smaller than in the light Ra and Th isotopes and not significantly altered by the coupling with the $K^\pi=0^+$ mode. The $I=K=0^+$ and the $I=1$, $K=0^-$ states observed at 1044 and 1237 keV, respectively, cannot be interpreted as two-phonon states.

The explanation of the $K=0$ intrinsic states observed in even-even deformed actinides is still a challenging problem. The appearance of $K^\pi=0^-$ intrinsic states at an energy E well below the two quasiparticle gap 2Δ implies that octupole correlations play an important role. Similarly, the existence of excited 0^+ states at neighboring energies may arise from pairing and/or quadrupole interactions. The octupole and pairing correlations have been treated on an equal footing by Chasman[1] to study the ground state 0^+ and the first excited states 0^- and 0^+ below 2Δ. The present study deals with the same subject but places emphasis on the coupling between the different modes involved and pays special attention to the location of the next members of this family of intrinsic levels with $K=0$.

To this purpose, we apply the multiphonon method (MPM)[2] which uses a basis with reflection symmetry. In the simple version, where only the $K=0^-$ octupole is used as the building phonon, this method has been successfully applied[3] in the even Ra and Th with mass number $222 \leq A \leq 228$. In ^{234}U, where the first excited 0^+ and 0^- intrinsic states are observed nearly at the same energy, one needs, *a priori,* to introduce two building phonons characterized by the quantum numbers $K=0^+$ and $K=0^-$. Such a version of the MPM has been tested[4] in a simple model allowing an exact solution and has been found to be suited for the study of the anharmonicities of the coupled $K^\pi=0^+$ and $K^\pi=0^-$ vibrational spectra of deformed nuclei. The aim of this work is simply to evaluate the importance of the coupling of these two modes and the anharmonicities of the corresponding vibrations in a realistic case.

We briefly sketch the version of the MPM used in this work. A detailed presentation can be found in Ref. 5. We introduce two types of phonons as building blocks:

$$Q_1^\dagger = \tfrac{1}{2}\sum (X_1)_{mn} a_m^\dagger a_n^\dagger \ , \tag{1}$$

with $K^\pi=0^+$ and

$$Q_2^\dagger = \tfrac{1}{2}\sum (X_2)_{mn} a_m^\dagger a_n^\dagger \tag{2}$$

with $K^\pi=0^-$.

These phonons are of the Tamm-Dancoff (TD) type and defined as a superposition of two quasiparticles. The X matrices are antisymmetric. The entities (1) and (2) are no longer considered as bosons. We take fully into account their commutation rule

$$[Q_1, Q_2^\dagger] = -\tfrac{1}{2}\,\mathrm{tr}(X_1 X_2) + \sum (X_2 X_1)_{mn} a_m^\dagger a_n \ . \tag{3}$$

We built the multiphonon states

$$|p,q\rangle = \frac{Q_1^{\dagger p} Q_2^{\dagger q}\,|0\rangle}{p!q!} \ , \tag{4}$$

which do not form an orthogonal basis. Because of the Pauli principle, the calculation of the overlap matrix of the states (4) and the matrix elements of the model Hamiltonian H within these states is not simple. However, as shown in Ref. 2, this calculation can be done in a straightforward way either by use of a generalized Wick's theorem or by use of recursion formulas. Here, we use the latter approach which is more suited for the numerical calculations. The MPM aims to search for the eigenstates of H in the restricted space of the collective phonon (4) taking properly into account the Pauli principle (3). Once one has obtained the eigenstates as a superposition of states (4) one calculates the observable (e.g., electromagnetic transitions).

According to the limited goal of the present work, we chose a very simple model Hamiltonian and assume that the $Z=92$ protons and the $N=142$ neutrons of ^{234}U move in a deformed Nilsson field and interact through a short-range constant monopole pairing force and a charge independent, long-range quadrupole-quadrupole and octupole-octupole force. We use the Nilsson potential with the standard parameters of Lamm.[6] The calculations are made with a quadrupole deformation $\varepsilon_2=0.22$ as proposed by Löbner, Vetter, and Hönig.[7] No higher order deformations are assumed. The intrinsic matrix elements of the operators $r^l Y_{l0}$ are calculated according to the prescriptions of Boisson and Piepenbring.[8] The units are chosen so as to express the multipole strength parameters χ_2 and χ_3 in keV. The BCS gap parameters Δ_p and Δ_n are evaluated empirically from the experimental masses.[9] Their values may be obtained by use of different methods.[10-12] We adopt $\Delta_p=970$ keV and $\Delta_n=660$ keV from the method in Ref. 12, with a possible uncertainty evaluated to $\cong 100$ keV. The BCS equations are solved using 20 active levels equitably distributed on each side of the Fermi surface, and taking, as usual, the $-Gv^2$ single particle renormalization into account. The corresponding

pairing strength parameters,

$$G_p = 159 \text{ keV}, \quad G_n = 125 \text{ keV} , \tag{5}$$

are fixed that way. To get the collective phonons (1) and (2), i.e., the matrices X_1 and X_2, the secular equation of the TD approximation (TDA) is solved. For the 0^+ phonon, we retain the second solution which looks like the physical random-phase approximation (RPA) solution. For a given set of pairing parameters G, we stay with two free parameters: the quadrupole χ_2 and octupole χ_3 strengths. In principle, their values are adjusted so as to get, within the MPM, the lowest 0^+ excited energy $E(0_2^+)$ at $\cong 810$ keV and the lowest 0^- energy $E(0_1^-)$ in the neighborhood of the intrinsic band head energy E_{int}. A rough estimate of the rotational part contained in the observed $E(1^-) = 786$ keV leads to $E_{\text{int}} \cong 775$ keV. The maximum value of the numbers of phonons $n = p + q$ in the basis (4) has been limited here to $n_{\max} = 8$. With this choice, the stability of the energies $E(0_1^-)$ and $E(0_2^+)$ is obtained within $\cong 3$ keV and that of the $E(0_3^+)$, $E(0_2^-)$, and $E(0_4^+)$ within $\cong 10$ keV. We mainly focus our attention to the location of the 0_3^+, 0_2^-, and 0_4^+ "two phonon states" and to their de-excitation through $E1$ and $E2$ transitions. The subindices used here correspond to the order of the calculated levels obtained within the presently used version of the MPM.

In Table I, we give the values of the energies ω_{TDA} of the building phonons, the energy ω_{RPA} one gets in the RPA, and the values of $E(0_2^+)$ and $E(0_1^-)$ obtained in the MPM. We note that the results concerning $E(0_2^+)$ and $E(0_1^-)$ are quite insensitive to the value of χ_2, so that in the practical situation encountered in ^{234}U the problem has only one effective free parameter χ_3. To get a fine adjustment of $E(0_2^+)$ we may change a little the pairing gap Δ_n within the uncertainties of its empirical value. As a consequence, we found that the first excited 0^+ state in ^{234}U is certainly not of a quadrupole (β) nature, as also demonstrated previously by Chasman.[1] We may also add that if one uses $n_{\max} < 8$ the energies $E(0_2^+)$ and $E(0_1^-)$ are strongly overestimated.

In Fig. 1 we present the calculated energy spectrum up to 2 MeV. On the left-hand side we give the $E1$ transitions. The numbers at the right of the arrows linking the initial intrinsic ψ_i to the final intrinsic state ψ_f give the values of $|\langle \psi_f | E1 | \psi_i \rangle|^2$. As usual, the effective charges are

$$e_p = \frac{N}{A} e, \quad e_n = -\frac{Z}{A} e . \tag{6}$$

TABLE I. Values (in keV) of the energies ω_{TDA}, ω_{RPA}, and E_{MPM} for $\chi_2 = 1$ keV and $\chi_3 = 9.1$ keV.

	0^+	0^-
ω_{TDA}	1257	1143
ω_{RPA}	1323	
E_{MPM}	781	768

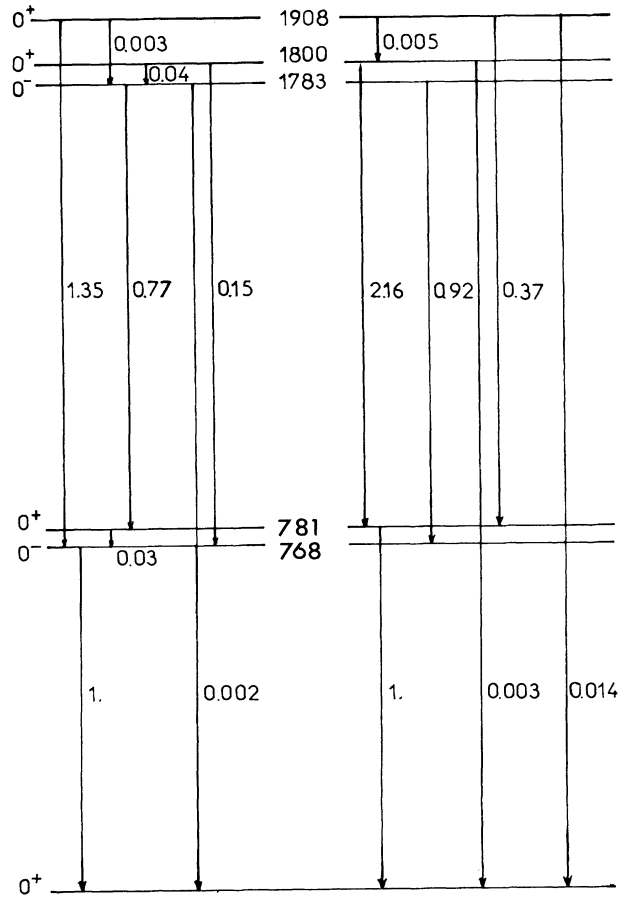

FIG. 1. Energy spectrum obtained for ^{234}U in the multiphonon method. The intrinsic $E1$ transitions are given on the left-hand side, the intrinsic $E2$ on the right-hand side.

The units are chosen so that

$$|\langle 0_1^+ | E1 | 0_1^- \rangle|^2 = 1 . \tag{7}$$

On the right-hand side we give, in a similar way, the information concerning the possible $E2$ transitions. Here, the effective charges are

$$e_p = e(1+\varepsilon), \quad e_n = e\varepsilon .$$

The calculations have been done with $\varepsilon = 0$ and $\varepsilon = 0.1$. The numbers given in Fig. 1 correspond to $\varepsilon = 0$. The results presented in Fig. 1 appeal the following comments.

The $E1$ and $E2$ transitions clearly indicate the nature of the states 0_3^+, 0_2^-, and 0_4^+ obtained between 1780 and 1910 keV.

The 0_3^+ state is strongly related by $E2$ to the 0_2^+ state and therefore appears to be mainly the two-$(K=0^+)$-phonon state.

The 0_2^- state is equally connected by $E1$ to 0_2^+ and by $E2$ to 0_1^- and is a good candidate for a $(K=0^+ + K=0^-)$-phonon state.

The 0_4^+ state decreases mainly to the 0_1^- state by $E1$ and is, by no doubt, principally of octupole nature.

The anharmonicities of the "two-phonon states" obtained in the MPM appear smaller for 0_2^+ and for 0_3^+ than

for 0_4^+ since

$$R_1=\frac{2E(0_2^-)}{E(0_1^-)+E(0_2^+)}=2.30\ , \tag{8}$$

$$R_2=\frac{E(0_3^+)}{E(0_2^+)}=2.30\ , \tag{9}$$

$$R_3=\frac{E(0_4^+)}{E(0_1^-)}=2.48\ . \tag{10}$$

The energy spectrum is evidently dilated, but to a lesser extent compared to the Ra and Th isotopes,[3] where the ratios of Eq. (10) were greater than 3. As a consequence, none of the calculated two-phonon states can explain the 0^+ and 0^- levels observed,[13] respectively, at 1044 and 1237 keV for which the anharmonicity ratios (8) and (9) are 0.6 and 0.8 and would lead to a compressed energy spectra. These observed states correspond by no doubt to levels out of our restricted collective basis (4), and hence are of a noncollective nature. According to our simple choice of H, we would like to remind the reader not to use the calculated values of $E(0_3^+)$, $E(0_2^-)$, and $E(0_4^+)$ for a precise prediction of the location of the two-phonon states in ^{234}U. We know also, on the other hand, that the coupling to the noncollective states may slightly alter these calculated values, however without changing the order of magnitude of the ratios R.

We observe also that the cross-over transitions (with $\Delta n=2$) or the transitions $\Delta n=0$ are strongly hindered compared to the favored $\Delta n=1$ transitions mentioned before.

Furthermore, these conclusions are not changed if one uses $\varepsilon=0.1$ instead of $\varepsilon=0$ for the $E2$ effective charges.

It also becomes clear that the large anharmonicities of the octupole vibrations are not fundamentaly affected by the presence of a second building phonon with $K^\pi=0^+$. A calculation within the frame of the MPM version with only one building phonon,[3] where the χ_3 has been (slightly) readjusted so as to reproduce E_{int}, leads to $R_3=2.44$ compared to 2.48 of (10). The ratio of the first two $|\langle\psi_f|E1|\psi_i\rangle|^2$ with $\Delta n=1$ is 1.55, compared to 1.35 in Fig. 1.

In Table I we observe that the MPM energies $E(0_2^+)$ and $E(0_1^-)$ are much smaller than the energies ω_{TDA} obtained in the TDA. This clearly shows the importance of the parts of H not taken into account in the latter approximation. We may also add that the fitted value of χ_3 is just slightly larger than the critical value $\chi_c=8.9$ keV where the RPA has no more a physical solution.

TABLE II. Values of the deviation of the particle number for the $K^\pi=0^+$ and $K^\pi=0^-$ multiphonon states.

0^+	ΔN	0^-	ΔN
0_1^+	−0.41	0_1^-	−1.11
0_2^+	−0.47	0_2^-	−1.16
0_3^+	−0.77		
0_4^+	−1.41		

We have also checked that the results presented in Table I and Fig. 1 are qualitatively not altered if we change slightly the gaps Δ_p and Δ_n within the uncertainties of their empirical values. Furthermore, as in Ref. 3, the trick which consists of choosing χ_2 values slightly different in the model Hamiltonian than in the construction of the TDA phonons does not change the R ratios and the electric transitions in a significant way. Finally, to insure that the nonconservation of particle number inherent to the BCS treatment does not bring in too many spurious effects, we have calculated the deviation $\Delta N=|\langle\psi|\hat{N}|\psi\rangle-N|$. It was found that for the five first excited states we considered here $\Delta N\lesssim 1$, as shown in Table II.

To summarize the main conclusions of this work we can say that, in ^{234}U, the octupole vibrations are not as anharmonic as in the light Ra and Th isotopes. Furthermore, they are not stongly affected by the $K=0^+$ mode. The two-phonon states are pushed into the energy region ($E\cong 2$ MeV) of the noncollective two quasiparticle states, but keep strong favored electric transitions to the one-phonon states.

These last conclusions are in contrast with the prediction of Soloviev.[14] This author uses, *a priori*, a basis where only two phonons of the same type are introduced (i.e., $n_{\max}=2$). He therefore certainly overestimates strongly the energies of the two-phonon states.

A systematic application of this version of the MPM to other nuclei of the same mass region is in progress[15] and will be published later on.

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