Nucleon momentum distribution in ^{2}H from y-scaling analysis of inclusive electrodisintegration

C. Ciofi degli Atti,[a] E. Pace,[a),(b] and G. Salmè[a]
[a]Istituto Nazionale di Fisica Nucleare, Sezione Sanità, Istituto Superiore di Sanità, Physics Laboratory, I-00161 Rome, Italy
[b]Università "La Sapienza," Dipartimento di Fisica, I-00185 Rome, Italy
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The nucleon momentum distribution in the deuteron $n(k)$, for $k \leq 0.6$ GeV/c, is extracted from the inclusive process ^{2}H(e,e′)pn by a method based upon a theory of y scaling in which the nucleon momentum and binding are properly taken into account and the effects of final state interactions are also considered. The obtained momentum distribution remarkably agrees with the one extracted from the exclusive reaction ^{2}H(e,e′p)n as well as with predictions from realistic two-nucleon interactions.

In this Brief Report a method is presented which allows one to obtain the nucleon momentum distribution $n(k)$ in the deuteron from the analysis of the inclusive electrodisintegration process ^{2}H(e,e′)pn. Such an analysis is complementary to the analysis of the exclusive process ^{2}H(e,e′p)n, and is also able to provide information on high momentum components, which can hardly be investigated at present by exclusive reactions. The method is based upon a theory of y scaling in which the nucleon binding and momentum are correctly taken into account, and the effects of final state interactions (FSI's) on the experimental data are also considered.[1] Preliminary results of our analysis have already been presented in Ref. 2.

Our approach is similar to that of Ref. 3, but the results are different due to our improved theory of y scaling, whose basic elements will be recalled below. In plane wave impulse approximation (PWIA) and using relativistic kinematics, the quasi elastic (qe) cross section in the laboratory system for electron scattering by the deuteron is

$$\frac{d\sigma(q,\omega)}{d\omega\, d\Omega} = \sum_{i=\mathrm{p,n}} \int d^3k\, n(k)\sigma_{\mathrm{e}i}\delta\{\omega + M_{\mathrm{d}} - [M^2 + (\mathbf{q}+\mathbf{k})^2]^{1/2} - (M^2+\mathbf{k}^2)^{1/2}\} \ , \tag{1}$$

where M_{d} and M are the deuteron and nucleon masses, respectively; $\sigma_{\mathrm{e}i}$ is a relativistic electron-nucleon cross section (without the recoil factor); ω and q are the energy and trimomentum transfers;

$$\mathbf{k} = -\mathbf{k}_R = \mathbf{k}_{\mathrm{N}} - \mathbf{q}$$

is the nucleon momentum before interaction ($\mathbf{k}_{\mathrm{N}}$ being the nucleon momentum in the continuum and $\mathbf{k}_R$ the recoil momentum of the spectator nucleon); and

$$n(k) = |\psi(k)|^2 = |u(k)|^2 + |w(k)|^2$$

is the nucleon momentum distribution, $\psi(k)$ being the deuteron wave function. After integration over $\cos\alpha = (\mathbf{k}\cdot\mathbf{q})/(kq)$ and the polar angle ϕ, which describes the rotation of $\mathbf{k}$ around $\mathbf{q}$, one obtains

$$\sigma_2(q,\omega) = \frac{d\sigma(q,\omega)}{d\omega\, d\Omega} = (\overline{\sigma_{\mathrm{ep}}} + \overline{\sigma_{\mathrm{en}}}) \left| \overline{\frac{\partial\omega}{k\,\partial\cos\alpha}} \right|^{-1} F(q,\omega) \ , \tag{2}$$

where $\partial\omega/k\partial\cos\alpha$ is the phase space factor arising from the dependence of the energy conserving δ function upon $\cos\alpha$ and $F(q,\omega)$ is the deuteron nuclear structure function

$$F(q,\omega) = 2\pi \int_{k_{\min}(q,\omega)}^{k_{\max}(q,\omega)} k n(k) dk \ , \tag{3}$$

where $k_{\min(\max)}$ are the integration limits imposed by the energy conservation. In Eq. (2), the quantities with an overbar are evaluated at $k = k_{\min}$, since in obtaining Eq. (2) from Eq. (1) we have taken advantage of the fact that at high momentum transfer the quantity

$$(\sigma_{\mathrm{ep}} + \sigma_{\mathrm{en}}) \,|\, \partial\omega/k\partial\cos\alpha \,|^{-1}$$

depends very weakly upon k, so that it can be taken out of the integral and evaluated, e.g., at $k = k_{\min}$ (the error due to this approximation decreases with q, and at the lowest experimental value of q considered in our analysis is less than 5%). The upper limit of integration $k_{\max}$ in Eq. (3) rapidly increases with q, and consequently, because of the rapid falloff of $n(k)$ with k, one can safely consider $k_{\max}(q,\omega) \approx \infty$ even at moderate values of the momentum transfer. For such a reason, the q and ω dependence of $F(q,\omega)$ is practically governed only by the q and ω dependence of $k_{\min}$, which is determined from the energy conservation for $\cos\alpha = -1$, i.e., from

$$\omega + M_{\mathrm{d}} = (M^2 + q^2 + k_{\min}^2 - 2qk_{\min})^{1/2} + (M^2 + k_{\min}^2)^{1/2} \ . \tag{4}$$

Equation (4) shows that $k_{\min}$ depends only upon q and ω

and therefore, as explained in Ref. 1, can be assumed as a scaling variable y, i.e., $k_{\min}(q,\omega)=|y|$, with the following relation,

$$\omega+M_d=(M^2+q^2+y^2+2qy)^{1/2}+(M^2+y^2)^{1/2} , \quad (5)$$

defining the scaling variable. Equation (4) and the negative values of y correspond to $\omega<\omega_{\text{peak}}$, which is the region we are going to consider, since the effects from non-nucleonic degrees of freedom, e.g., exchange effects and Δ production, are expected to be of minor importance there.

The following definition of the nuclear *scaling function,*

$$F_1(q,y)=\frac{\sigma_2(q,\omega)}{(\bar{\sigma}_{ep}+\bar{\sigma}_{en})}\,\overline{\frac{\partial\omega}{k\,\partial\cos\alpha}} , \quad (6)$$

has been adopted in Ref. 1, for the trivial reason that, if the PWIA holds, F_1 reduces to the deuteron nuclear structure function (3) which, using Eq. (5), can be expressed in terms of q and y, i.e.,

$$F_1(q,y)=F(q,y)=2\pi\int_{|y|}^{k_{\max}(q,y)} kn(k)dk . \quad (7)$$

For large values of q, $k_{\max}$ can be replaced by infinity and $F_1(q,y)$ becomes independent of q, i.e., it scales in y and reduces to the longitudinal momentum distribution $f(y)$,

$$f(y)=2\pi\int_{|y|}^{\infty} kn(k)dk . \quad (8)$$

Equation (8) is a very useful one, since from it the nucleon momentum distribution in the deuteron can be obtained by a simple derivative,

$$n(k)=-\frac{1}{2\pi}\frac{1}{y}\frac{df}{dy}, \text{ with } k=|y| . \quad (9)$$

A plot of the quantity (6), corresponding to experimental cross sections σ_2 measured at different kinematical conditions, has therefore a great advantage over the usual analysis of inclusive data in terms of separate qe peaks; in fact, if scaling is observed, the experimental scaling function $f^{\text{ex}}(y)$ can be extracted from the data and $n(k)$ can in principle be obtained from Eq. (9). A word of caution is, however, necessary here. In order that such a procedure be free from any ambiguity related to FSI's, the effects of the latter on y scaling has to be carefully investigated. To this end, we take advantage of the results of Ref. 1, where it has been shown that the q behavior of $F_1(q,y)$, for fixed values of y, provides a model independent criterion to single out those data for which the PWIA breaks down. Indeed, because of the increase of $k_{\max}$ with q, $F_1(q,y)$ [Eq. (7)], *independently of the form of* $n(k)$, will *increase* with momentum transfer until it reaches its asymptotic value given by Eq. (8). Thus, if the PWIA holds, scaling should be approached from the bottom and any different approach, e.g., from the top, is *proof* of the breaking down of the PWIA and strong evidence of the relevance of FSI effects.

In order to quantitatively check the validity of the scaling hypothesis, we have plotted the experimental scaling function $F_1^{\text{ex}}(q,y)$ vs q for fixed, negative values of y. The results, which are shown in Fig. 1, have been obtained using in Eq. (6) the experimental inclusive cross sections σ_2 of Ref. 4, and the relativistic off-shell electron-nucleon cross section $\sigma_{ep(n)}$ given in Ref. 5, with the nucleon form factors of Ref. 7. The form of $\sigma_{ep(n)}$ does not strongly affect the values of $F_1^{\text{ex}}(q,y)$, since at high momentum transfer its k dependence is weak; for example, using the

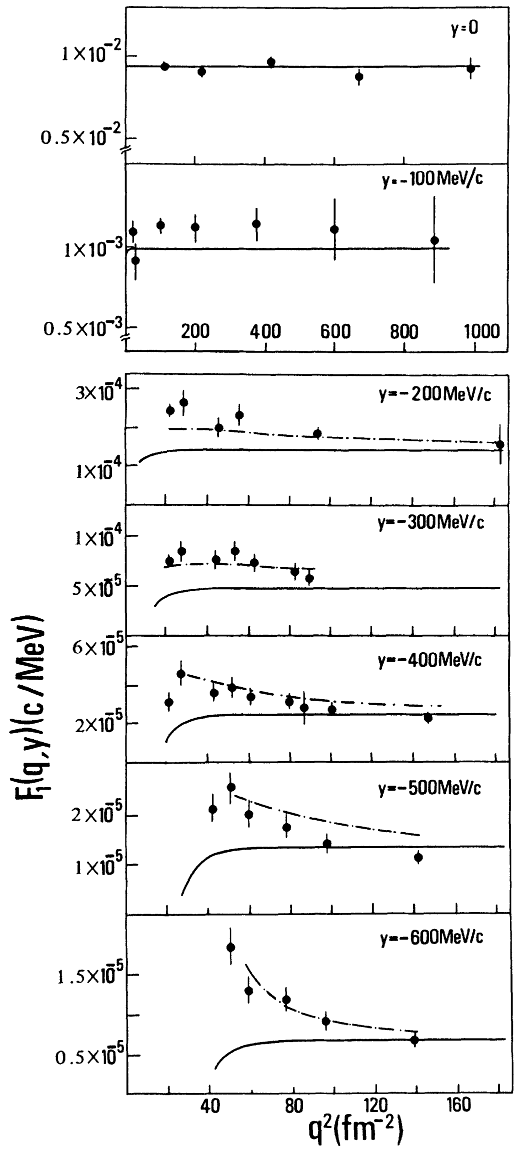

FIG. 1. Experimental scaling function of the deuteron $F_1^{\text{ex}}(q,y)$ [Eq. (6)] obtained from the Stanford Linear Accelerator Center (SLAC) inclusive cross section σ_2 (Ref. 4) and using the relativistic off-shell electron nucleon cross section σ_{eN} of Ref. 5. The full lines represent the theoretical scaling function $F_1^{\text{th}}(q,y)$ in PWIA obtained from Eq. (7) with $n(k)$ corresponding to the RSC interaction (Ref. 6). The dot-dashed lines include the effects of final state interactions according to the calculation of Ref. 8 (see the text). The two-nucleon center-of-mass energy in the final state, $E_{\text{c.m.}}$, varies from 40 to 240 MeV (at $y=-400$ MeV/c), and from 10 to 120 MeV (at $y=-600$ MeV/c), when the momentum transfer varies from $q^2=40$ fm^{-2} to $q^2=120$ fm^{-2}. The data are the average values of $F_1(q,y)$ in an interval of 100 MeV/c, centered on the indicated value of y. For $y=0$ and 100 MeV/c the curve with FSI is indistinguishable from the PWIA results.

cross section by free nucleons at rest changes the points in Fig. 1 by about 10%. In Fig. 1, the theoretical quantity F_1^{th} [Eq. (7)], obtained with $n(k)$ corresponding to the Reid soft core (RSC) interaction,[6] is also shown by the full lines. It can be seen that at the qe peak ($\omega=\omega_{peak}, y=0$) $F_1^{ex}(q,y)$ shows indeed a remarkable scaling behavior, thereby indicating that the PWIA works rather well [note that the effects due to meson production, which affect $\sigma_2(q,\omega)$ around the qe peak for the highest values of q, have been subtracted in Ref. 4]. At high negative values of y (i.e., for $\omega \ll \omega_{peak}$), scaling deteriorates and, more importantly, $F_1^{ex}(q,y)$ approaches its asymptotic value by *decreasing* with momentum transfer; such a behavior, according to our criterion given above, is a clear *model independent proof* of the breaking down of the PWIA. In fact, it can be seen that F_1^{th} rapidly *increases* to its asymptotic value with a behavior which is governed by k_{max} [cf. Eq. (7)]. Therefore, by means of the y-scaling plot shown in Fig. 1, we have singled out in a model independent way those values of q and y for which a description of the qe cross section in terms of the PWIA breaks down. More importantly, Fig. 1 provides the q behavior of such a breaking down, and therefore it can represent a stringent test for various calculations which take FSI effects into account. We have considered these effects by evaluating the theoretical scaling function F_1^{th} using in Eq. (6) a theoretical cross section σ_2 obtained[8] from deuteron bound and continuum states corresponding to the RSC interaction in all waves with $J \leq 2$, and to a regularized one pion exchange potential (OPEP) in higher partial waves. The results are represented by the dot-dashed lines in Fig. 1. It can be seen that such a calculation explains the q dependence of the experimental scaling function reasonably well, i.e., the q dependence of FSI effects. These are governed by the value of the two-nucleon center-of-mass energy in the final state, $E_{c.m.}$, which increases with q and decreases with $|y|$, so that, at high values of $|y|$ and low values of q, FSI effects are very large; however, with increasing values of q, they appreciably decrease and F_1^{ex} seems to approach a scaling value.

Having verified the qualitative correctness of the scaling hypothesis at large values of q, we have proceeded to a determination of $n(k)$ by means of Eq. (9). In order to minimize as much as possible the effects of FSI, the asymptotic scaling function $f^{ex}(y)$ has been obtained from $F_1^{ex}(q,y)$ by two different procedures: (i) for a given value of y, the points at the highest values of q (with and without corrections for FSI's), have been considered to directly represent $f^{ex}(y)$; (ii) for a given value of y all data for $F_1^{ex}(q,y)$ in the whole range of q have been used, and after correcting them for FSI's their average value has been assumed to represent $f^{ex}(y)$. It is very gratifying to see that these procedures yield very similar results, which give rise to the error bars shown in Fig. 2. This means in particular that the points at the highest value of q are not strongly affected by FSI's. The obtained $f^{ex}(y)$ has been used in Eq. (9) to get the momentum distribution represented by the filled squares in Fig. 3. In the same figure the momentum distribution extracted in Ref. 9 from Saclay exclusive ^{2}H(e,e'p)n data,[10] properly corrected for FSI and meson exchange current (MEC) effects, is also shown (triangles). It can be seen that exclusive and inclusive experiments are in very good agreement, which is a remarkable result in view of the totally different momentum transfer involved in the two processes ($q < 2.5$ fm^{-1} and $q > 5$ fm^{-1} in exclusive and inclusive experiments, respectively). To sum up, we have obtained $n(k)$ from inclusive data by adopting a scaling function which is free from the effects of FSI's, representing the main correction to the PWIA in the qe kinematics for $\omega < \omega_{peak}$; likewise, $n(k)$ has been obtained[9] from the (e,e'p) cross section after introducing proper corrections for the main effects in exclusive kinematics, i.e., FSI's and MEC's. Such a consistent treatment of the most relevant corrections to the PWIA is the main reason for the agreement between inclusive and ex-

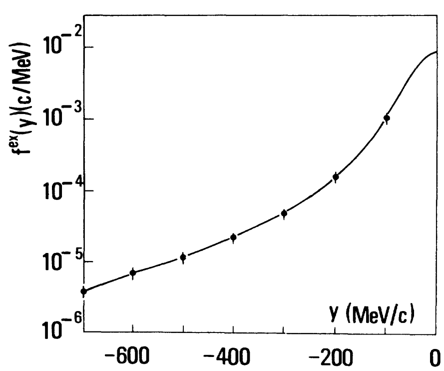

FIG. 2. The experimental asymptotic scaling function $f^{ex}(y)$ obtained from the scaling function $F_1^{ex}(q,y)$ shown in Fig. 1 (see the text). The solid line is a best fit to the data.

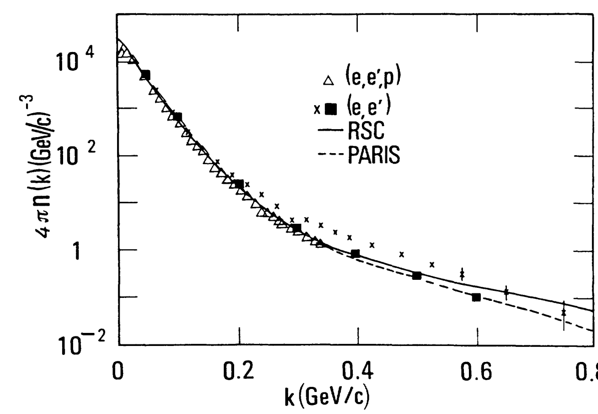

FIG. 3. The nucleon momentum distribution $n(k)=|\psi(k)|^2$ in the deuteron. Filled squares: $n(k)$ extracted in the present paper from the inclusive cross section ^{2}H(e,e')pn (Ref. 4), using Eq. (9) and $f^{ex}(y)$ shown in Fig. 2. Triangles: $n(k)$ extracted in Ref. 9 from the exclusive ^{2}H(e,e'p)n cross section (Ref. 10). Crosses: $n(k)$ extracted in Ref. 3 from the inclusive cross section (Ref. 4), disregarding the effects of final state interactions (see the text). The full and dashed lines represent $n(k)$ obtained from the RSC (Ref. 6) and Paris (Ref. 11) interactions, respectively. The normalization of $n(k)$ is

$$\int n(k) d^3k = 1 .$$

clusive experiments, which makes us confident that the quantity shown in Fig. 3 (squares and triangles) can indeed be interpreted as a nucleon momentum distribution.

The idea of obtaining the nucleon momentum distribution from a y-scaling analysis of the electrodisintegration data was first exploited in Ref. 3. The results of that analysis, based on the same experimental cross sections[4] we have used, are shown in Fig. 3 by the crosses, and it can be seen that they disagree with our results and with exclusive data. We have checked that such a disagreement, which has sometimes been interpreted as due to $6q$ admixture in the deuteron wave function,[12] is not a real one, but appears to be due to the version of y scaling[13] underlying the analysis of Ref. 3. In that version, besides the PWIA, an additional assumption has been made, namely that the component of the nucleon momentum perpendicular to the momentum transfer can be disregarded; such an approximation, which has been shown to be a poor one[1] (at least for the available experimental data), leads to a different phase space factor in Eq. (6) ($\partial\omega/\partial y$ instead of $\partial\omega/k\partial\cos\alpha$) and therefore to a different scaling function, which does not represent a nuclear structure function and exhibits a q behavior (cf. Fig. 2 of Ref. 3) such that nothing can be inferred from it about the possible presence of FSI's. Hence, in Ref. 3 the PWIA has been assumed to hold in the whole range of momentum transfer, and all data points, without any correction for FSI's, have been used to extract the momentum distribution. By such a procedure, which weights in the same way the data at low q (strongly affected by FSI's) and those at high q (where the FSI's are of minor relevance), the resulting "momentum distribution" contains the effects of FSI's, which are the origin of the disagreement between the crosses and the squares shown in Fig. 3.

The experimental momentum distributions are compared in Fig. 3 with the theoretical ones obtained from nonrelativistic wave functions corresponding to RSC and Paris interactions. It can be seen that the agreement with the latter interaction is impressive; however, the possibility of distinguishing, at high values of k, between different two-nucleon interactions is hindered by possible relativistic effects[14,15] and by the uncertainty related to the extraction of the momentum distribution from the experimental data. Such an uncertainty could in principle be reduced by new measurements of the inclusive cross section, such that the scaling function for $y \leq -300$ MeV/c could be investigated at higher values of the momentum transfer.

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