Predictions for magnetic moments and β-decay transition strengths for mirror nuclei

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Predictions for magnetic moments and β-decay transition strengths of mirror nuclei can be made on the basis of empirical correlations between these quantities, and the technique is illustrated by deducing the magnetic moments of ^{23}Mg, ^{33}Cl, ^{37}Ar, ^{43}Ti, ^{51}Fe, and ^{55}Ni.

Strong correlations have been reported[1] between the magnetic moments and β-decay transition strengths of mirror nuclei. With correlation coefficients $|r| > 0.997$ these yield:

$$\gamma_p + (1.145\pm0.012)\gamma_n = (1.056\pm0.021) , \quad (1)$$

$$\gamma_p = (4.38\pm0.10)\gamma_\beta + (0.939\pm0.046) , \quad (2)$$

$$\gamma_n = (-3.82\pm0.10)\gamma_\beta + (0.101\pm0.043) , \quad (3)$$

with the definitions:

$$\gamma_\beta = \frac{1}{2}\left[\left(\frac{6170}{ft} - 1\right)\frac{1}{J(J+1)}\right]^{1/2} ;$$

$$\gamma_p = \mu_p / J, \quad \gamma_n = \mu_n / J ,$$

where ft is the ft value for the β transition between the mirror pair; μ_p and μ_n are the magnetic moments of the odd-proton and odd-neutron members of the pair, respectively; and J is their angular momentum. These correlations have been interpreted[1] as implying that the ratio of weak interaction constants in nuclei $|C_A/C_V| = 1.00 \pm 0.02$, considerably reduced from its free nucleon value. Less contentiously, we point out in this Brief Report that the empirical relations of Eqs. (1)–(3), and similar ones, have considerable predictive power.

We illustrate the technique by estimating the magnetic moments of ^{23}Mg, ^{33}Cl, ^{37}Ar, ^{43}Ti, ^{51}Fe, and ^{55}Ni. For each of these, the ft value of the mirror β transition and the magnetic moment of the mirror partner are known.[2] With the same data that were used[1] in obtaining Eqs. (1)–(3), a fit of the isoscalar quantity $(\gamma_p + \gamma_n)$ against γ_β yields, with a correlation coefficient $r = 0.969$,

$$\gamma_p + \gamma_n = (0.560\pm0.043)\gamma_\beta + (1.040\pm0.020) . \quad (4)$$

Equation (4) is better suited to the present purpose than either Eq. (2) or (3). This is best exhibited by fitting the isovector quantity $(\gamma_p - \gamma_n)$ against γ_β, which yields, with a correlation coefficient $r = 0.997$,

$$\gamma_p - \gamma_n = (8.20\pm0.19)\gamma_\beta + (0.838\pm0.089) . \quad (5)$$

A comparison of the uncertainties in the fitted values in Eqs. (2)–(5) shows that these uncertainties are predominantly of an isovector rather than an isoscalar nature.

Table I shows values of J, γ_p or γ_n, and γ_β for the mass numbers under study,[2] as well as the two independent estimates of the unknown gyromagnetic ratio obtained using Eqs. (1) and (4). The estimates are consistent with each other, and we conclude that the predicted magnetic moments of Table II should be accurate to better than 5%. Of these the only one which has been measured[2] is $\mu(^{37}\text{Ar}) = 0.95\pm0.20\ \mu_N$, to be compared with our estimate $\mu(^{37}\text{Ar}) = 1.22\pm0.02\ \mu_N$. Previous work of a similar nature[3] was based on the "odd-group" model,[4] in which the contributions to the magnetic moments from the even type of nucleon in odd-even nuclei, as well as those arising from non-nucleon degrees of freedom, are neglected. This yields, instead of Eq. (1),

TABLE I. Values of J, γ_p, γ_n, and γ_β as described in the text. The entries in the last column correspond to the predictions for the missing γ_p or γ_n using Eqs. (1) and (4), respectively.

A	J^π	γ_p	γ_n	γ_β	γ_p or γ_n
23	$\frac{3}{2}^+$	+ 1.4783(−)		+ 0.1389(15)	−0.369(19), −0.361(21)
33	$\frac{3}{2}^+$		+ 0.4292(−)	−0.0762(15)	+ 0.565(22), + 0.568(20)
37	$\frac{3}{2}^+$	+ 0.1355(−)		−0.1508(21)	+ 0.804(20), + 0.823(21)
43	$\frac{7}{2}^-$	+ 1.3200(114)		+ 0.1120(16)	−0.231(21), −0.217(23)
51	$\frac{5}{2}^-$	+ 1.4240(40)		+ 0.1442(62)	−0.321(19), −0.303(21)
55	$\frac{7}{2}^-$	+ 1.3780(11)		+ 0.0917(35)	−0.281(19), −0.287(20)

TABLE II. Predictions for magnetic dipole moments from Table I.

Nucleus:	^{23}Mg	^{33}Cl	^{37}Ar	^{43}Ti	^{51}Fe	^{55}Ni
μ (μ_N):	$-0.548(21)$	$+0.851(22)$	$+1.221(22)$	$-0.784(52)$	$-0.780(35)$	$-0.994(45)$

$$\gamma_p + 1.20\gamma_n = 1.00 , \quad (6)$$

with no estimate of the errors in the coefficients. As an example, Eq. (6) would result in $\mu(^{37}\text{Ar}) = 1.08\ \mu_N$.

We have shown how magnetic moments of nuclei can be predicted from information on the magnetic moment of the mirror partner, and on the β decay between the mirror pair. Other variants based on Eqs. (1)–(4) are easily envisaged, and should provide useful guides in experimental determinations of magnetic moments and β-decay transition rates of exotic mirror nuclei. Strictly speaking, these relations have been shown to hold for ground states of mirror nuclei, but it is possible that they hold for excited states as well, and it would be interesting to have the pertinent data.

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