Isovector pairing collective motion: Generator-coordinate-method approach

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Isovector pairing collective motion is treated by means of the generator coordinate method. In this scheme, the isospin and number projection is performed analytically by the recognition of symmetry properties in the generalized Bardeen-Cooper-Schrieffer wave functions. Among the results obtained, our generator-coordinate-method values of energy and spectroscopic amplitude are shown to be comparable to those of shell model calculations. This is indeed encouraging, especially in view of the fact that they were reached using a simple approximation. The great simplicity of the present method, as compared with earlier complicated versions, suggests that they might prove useful in the study of isovector pairing collective states which are strongly populated by pair transfer reactions in medium weight nuclei.

I. INTRODUCTION

The main purpose of the present paper is to present a simple, although powerful, tool to deal with the isovector pairing collective motion.[1] A natural way to treat this kind of collective motion in a microscopic approach is to use the generator coordinate method (GCM). Following the simple phenomenological picture which was first formulated by Bohr and Mottelson, who described the pairing vibration states as harmonic vibrations around the minimum of a pairing potential, one uses BCS states with different pairing correlations as generating wave functions. Since the BCS wave functions are not eigenfunctions of the particle number operator, one has to perform a particle number projection in order to get a reliable description of the physical states for a specific nucleus. If, in addition, the effect of isovector (neutron-proton $T=1$) pairing correlations is included, one has to perform an isospin projection in order to describe the correct charge states.

As is well known, the simplicity of the BCS scheme is lost when one includes the charge degree of freedom.[2] If, additionally, one wants to do isospin and number projection simultaneously, one has to perform an integration in four angles (three Euler angles and one gauge), and in this case the treatment becomes more complicated than number projection. In fact, the number projection was first performed about 25 years ago[3] and the isospin projection was calculated only seven years ago.[4] The calculation is also more involved than the angular momentum projection for deformed nuclei, where the well known simplification of axial symmetry is made. Thus one is tempted to seek a possible counterpart of such axial symmetric in the isospin space. These axial symmetric wave functions must conserve the third component of the isospin. On the other hand, the generalized BCS wave functions are superposition of all isospin components and therefore they are not axial symmetric. Despite this difficulty, there is one way to construct axially symmetric wave function.

In the case of a charge independent pairing potential, the ground state energy is degenerate with regard to the isospin multiplet. For example, the ground state of the nucleus ^{56}Cr with $T=4$ has the same energy as the $T=4$ of ^{56}Ni, which is a self-conjugate nucleus. In other words, we may obtain the ground state of $N\neq Z$ nuclei from the yrast isospin state of the $N=Z$ nuclei. In the present case this is done breaking the symmetry in the isospin and gauge space. The wave function which shows these characteristics, and, additionally, are axial symmetric wave functions, in the isospin space, is the superposition of self-conjugate nuclei. Due to the projection of axial symmetric BCS wave function in definite isospin and number of particles, we can describe not only the ground state of the self-conjugate nucleus, but all the seniority 0^+ states of any nucleus in consideration. Another advantage of our procedure is that we obtain the isospin projection analytically and we can recover the simplicity of the number projection BCS description in the treatment of the isovector pairing collective motion. Furthermore, our analytical approach to isospin projection allows us to get also the spectroscopic amplitude, which is a relevant physical quantity for the analysis of two-nucleon transfer reactions.

It is worthwhile to mention here that the analysis of experimental data from two-nucleon transfer reactions suggests that the $T=1$ pairing force plays an important role in the description of the strongly populated 0^+ states in medium weight nuclei and that these 0^+ states can be understood in terms of pairing vibration and pairing rotations.[5] Bés and co-workers have had success in describing these new collective motions. On one hand, in analogy with the Bohr-Mottelson collective model of nuclear rotation, they consider the pairing deformation taking place in the four dimensional isospin and gauge space.[6] On the other hand, they[1] succeeded in describ-

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ing the pairing vibration in analogy with the shape vibration of nucleus. They[7] also describe these nuclear 0^+ states microscopically using the generalized quasispin formalism of Hecht.[8] This formalism has been employed to perform the exact model diagonalization in the space of only the 0^+ states. Even in this limited space, however, the size of matrices becomes very large if one wants to go beyond the three-level model. To overcome this kind of difficulty, the equation of motion technique has been used recently to treat the 0^+ states,[9] and good accuracy, when compared with exact results, was obtained. This led to the renewed interest in the neutron-proton pairing problem in which one of the major difficulties, recognized long ago,[10] was to know whether the BCS approach was an adequate tool to deal with it or not. In the present paper we show that the BCS formalism can be easily used to treat the isovector pairing collective states by means of GCM.

The next section contains a review of the GCM applied to the isovector pairing collective motion. In Sec. III we present the axial symmetric approximation. Second IV is devoted to the derivation of the analytical kernels of the GCM equations. The spectroscopic amplitude for two-nucleon transfer is derived in Sec. V and numerical results and the comparison of different solutions are presented in Sec. VI. Finally, some conclusions are drawn in Sec. VII.

II. THE GCM APPLIED TO THE ISOVECTOR PAIRING COLLECTIVE MOTION

If one wants to describe the isovector pairing collective motion in the GCM scheme with a BCS-type generating wave function and a set of collective coordinates $\Delta_1, \ldots, \Delta_i$ with $f(\Delta_i)$ a weight function of GCM, one has to solve the following Hill-Wheeler (HW) equations,[11]

$$\int d\Delta_A f^{N,T}(\Delta_A)[H^{N,T}(\Delta_B,\Delta_A)-EI^{N,T}(\Delta_B,\Delta_A)]=0 , \tag{2.1}$$

where $H^{N,T}(\Delta_B,\Delta_A)$ and $I^{N,T}(\Delta_B,\Delta_A)$ are the kernels of the GCM equations, given by

$$H^{N,T}(\Delta_B,\Delta_A)=\langle \Phi(\Delta_B)\,|\,\hat{H}\,\hat{P}_T\hat{P}_N\,|\,\Phi(\Delta_A)\rangle , \tag{2.2}$$

$$I^{N,T}(\Delta_B,\Delta_A)=\langle \Phi(\Delta_B)\,|\,\hat{P}_T\hat{P}_N\,|\,\Phi(\Delta_A)\rangle . \tag{2.3}$$

Here, P_T and P_N are, respectively, the projection operator for isospin and number of particles ($2N$). Both are written in the framework of Peierls-Yoccoz[12] projection technique.

In the present paper we consider the usual $T=1$ pairing Hamiltonian, given by

$$\hat{H}=\sum_{jmt}\epsilon_j c^{\dagger}_{jmt}c_{jmt}-G\sum_{\mu=0,\pm1}\sum_{jj'}A^{\dagger}_{j\mu}A_{j'\mu} . \tag{2.4}$$

Here, ϵ_j are the single particle energies; $A^{\dagger}_{j\mu}$ ($A_{j\mu}$) are pair creation (annihilation) operators in a spacial state j with total $J=0$, $T=1$, and $T_z=\mu$ explicitly written as

$$A^{\dagger}_{j\mu}=(\tfrac{1}{2}\Omega_j)^{1/2}\sum_{\substack{tm\\ t'}}(\tfrac{1}{2}t\tfrac{1}{2}t'\,|\,1\mu)(jmj-m\,|\,00)c^{\dagger}_{jmt}c^{\dagger}_{j-mt'} . \tag{2.5}$$

In the above formula $\Omega_j=j+\frac{1}{2}$ is the half-degeneracy of the level j and $c^{\dagger}_{jmt}$ (c_{jmt}) is the particle creation (annihilation) operator in the single particle angular momentum j, the magnetic number m, and the isobaric index t.

As we mentioned before, the adequate generator wave function for treating isovector pairing collective motion should be the BCS wave function, and $\Delta=\Delta_{\pi\pi},\Delta_{\nu\nu},\Delta_{\pi\nu}$ are the three generator coordinates which stand for the pairing energies for proton (π) -proton, neutron (ν) -neutron, and proton-neutron pairing, respectively. This BCS state is given by the product of quasiparticle annihilation operators acting on the vacuum

$$|\,\Phi(\Delta)\rangle=\left[\prod_{jm\tau}a_{jm\tau}\right]|\,0\rangle . \tag{2.6}$$

The quasiparticle operators $a_{jm\tau}$ are defined in terms of the basis states by a Bogoliubov transformation,

$$a_{jm\tau}=\sum_{t=\pi,\nu}(c^{\dagger}_{jmt}u^j_{t\tau}+s_{jm}c_{j-mt}v^j_{t\tau}) . \tag{2.7}$$

Here, $\tau=1,2$ reveals the existence of two kinds of quasiparticles. $u^j_{t\tau}$ and $v^j_{t\tau}$ are the generalized coefficients of the Bogoliubov transformation and, finally, $s_{jm}=(-)^{j-m}$ is a phase factor.

With all these considerations the energy and overlap kernel given in (2.2) and (2.3) can be written as

$$H^{N,T}(\Delta_B,\Delta_A)=\frac{2T+1}{16\pi^3}\sum_{KK'}\int d\Omega\, D^{T*}_{KK'}(\Omega)\int d\theta\exp(-iN\theta)h^{BA}(\Omega,\theta) \tag{2.8}$$

and

$$I^{N,T}(\Delta_B,\Delta_A)=\frac{2T+1}{16\pi^3}\sum_{KK'}\int d\Omega\, D^{T*}_{KK'}(\Omega)\int d\theta\exp(-iN\theta)n^{BA}(\Omega,\theta) , \tag{2.9}$$

where $D^T_{KK}(\Omega)$ is the Wigner D function and $\Omega=(\alpha,\beta,\gamma)$ are the three Euler angles in the three dimensional isospin space.

Due to the isomorphism between isospin and angular momentum, the overlap functions $h^{BA}(\Omega,\theta)$ and $n^{BA}(\Omega,\theta)$ can be derived in analogy to the angular momentum and particle number projected Hartree-Bogoliubov method, and need some length calculations,[13] and are given by

$$n^{BA}(\Omega,\theta)=\langle \Phi(\Delta_B)\,|\,\hat{R}(\Omega)\hat{S}(\theta)\,|\,\Phi(\Delta_A)\rangle =[\det X^{BA}(\Omega,\theta)]^{1/2} , \tag{2.10}$$

and

$$h^{BA}(\Omega,\theta)=\langle\Phi(\Delta_B)\,|\,\hat{H}\,\hat{R}(\Omega)\hat{S}(\theta)\,|\,\Phi(\Delta_A)\rangle\ ,$$
$$\frac{h^{BA}(\Omega,\theta)}{n^{BA}(\Omega,\theta)}=\sum_{jmt}\epsilon_j[\rho^{BA}_{tt}(\Omega,\theta)]^j-\frac{G}{2}\sum_{\substack{jm\\tt'}}[\rho^{BA}_{tt}(\Omega,\theta)]^j[\rho^{BA}_{t't'}(\Omega,\theta)]^j$$
$$-G\sum_{\substack{jm\\tt'}}[\rho^{BA}_{tt'}(\Omega,\theta)]^j[\rho^{BA}_{t't}(\Omega,\theta)]^j-G\sum_{\substack{jm\\j'm'\\tt'}}[\kappa^{BA}_{tt'}(\Omega,\theta)]^j[\sigma^{BA}_{t't}(\Omega,\theta)]^j\ . \tag{2.11}$$

The most convenient form of expressing the matrix $[X^{BA}(\Omega,\theta)]^j$, the generalized density matrix $[\rho^{BA}(\Omega,\theta)]^j$, and the pairing tensors $[\kappa^{BA}(\Omega,\theta)]^j,[\sigma^{BA}(\Omega,\theta)]^j$, for our purposes, are[14]

$$[X^{BA}(\Omega,\theta)]^j=U_j^{\rm tr}(\Delta_B)R^*(\Omega)U_j^*(\Delta_A)$$
$$+V_j^{\rm tr}(\Delta_B)R(\Omega)V_j(\Delta_A)e^{i\theta}\ , \tag{2.12}$$
$$[\rho^{BA}(\Omega,\theta)]^j=K(\Omega)V_j^*(\Delta_B)$$
$$\times\exp(i\theta)[X^{BA}(\Omega,\theta)]^{-1}V_j^{\rm tr}(\Delta_A)\ , \tag{2.13}$$
$$[\kappa^{BA}(\Omega,\theta)]^j=-\{R(\Omega)V_j^*(\Delta_B)$$
$$\times\exp(i\theta)[X^{BA}(\Omega,\theta)]^{-1}U_j^{\rm tr}(\Delta_A)\}^{\rm tr} \tag{2.14}$$

and

$$[\sigma^{BA}(\Omega,\theta)]^j=-R^*(\Omega)U_j^*(\Delta_B)[X^{BA}(\Omega,\theta)]^{-1}V^{\rm tr}(\Delta_A)\ . \tag{2.15}$$

Here, $U_j^{\rm tr}$ means the transposed of U_j. The V_j and U_j given in Eqs. (2.12)–(2.15) are the Bogoliubov 2×2 transformation matrices and the $R(\Omega)$ matrix follows the Edmonds convention.[15]

From here, we follow a prescription different from the traditional numerical method used to treat the angular momentum or isospin projection. In the numerical calculations, Chen *et al.*[4] approximate the integrals (2.8) and (2.9) by appropriate summations after calculating the matrix elements of (2.12) and (2.15) numerically. However, the integrand of (2.8) and (2.9) may present a highly oscillating behavior and the choice of mesh points to get an approximate four dimensional integral may not be very satisfactory. Hence, in this work we shall show that it is possible with certain approximations to perform the projection analytically after obtaining explicitly the matrix elements (2.12)–(2.15). This calculation is quite simple, as will become clear in the next section.

III. THE AXIALLY SYMMETRIC GENERALIZED BCS WAVE FUNCTION

In order to perform the isospin projection analytically, the first step is to look for axially symmetric wave functions in isospin space. For this purpose, we present the generalized BCS wave function (2.6) in the extended form

$$|\Phi\rangle=\prod_{jm>0}[\mathcal{C}^j_4c^\dagger_{j-m\pi}c^\dagger_{jm\pi}c^\dagger_{j-m\nu}c^\dagger_{jm\nu}+\mathcal{C}^j_{\pi\pi}s_{jm}c^\dagger_{j-m\pi}c^\dagger_{jm\pi}$$
$$+\mathcal{C}^j_{\nu\nu}s_{jm}c^\dagger_{j-m\nu}c^\dagger_{jm\nu}$$
$$+\mathcal{C}^j_{\pi\nu}s_{jm}(c^\dagger_{j-m\pi}c^\dagger_{jm\nu}+c^\dagger_{j-m\nu}c^\dagger_{jm\pi})+\mathcal{C}^j_0]\,|\,0\rangle\ , \tag{3.1}$$

where

$$\mathcal{C}^j_4=v^j_{\pi1}v^j_{\nu2}-v^j_{\nu1}v^j_{\pi2}\ ,$$
$$\mathcal{C}^j_0=u^j_{\pi1}u^j_{\nu2}-u^j_{\nu1}u^j_{\pi2}\ ,$$
$$\mathcal{C}^j_{\nu\nu}=u^j_{\pi1}v^j_{\nu2}-v^j_{\nu1}u^j_{\pi2}\ , \tag{3.2}$$
$$\mathcal{C}_{\pi\pi}=v^j_{\pi1}u^j_{\nu2}-u^j_{\nu1}v^j_{\pi2}\ ,$$

and

$$\mathcal{C}^j_{\pi\nu}=v^j_{\nu1}u^j_{\nu2}-u^j_{\nu1}v^j_{\nu2}\ .$$

The above expression (3.1) shows, quite obviously, that the wave function $|\Phi\rangle$ can neither be an eigenstate of $\bar{T}_z$ nor of $\bar{T}^2$, since it is constructed out of pairs of nucleons coupled to $T=1$ and $T_z=\pm1$ or 0. Its properties have been analyzed mainly by Ginochio and Weneser[2] and by Camiz, Covello, and Jean.[10] They concluded that the quasiparticle ground state wave function has no neutron-proton correlations. Then it reduces to the product of two BCS wave functions. In other words, the BCS ground state has $\mathcal{C}^j_{\pi\nu}=0$ with $\Delta_{\nu\nu}\neq0$, $\Delta_{\pi\pi}\neq0$, and $\Delta_{\pi\nu}=0$.

This BCS ground state wave function does have axial symmetry. However, if we use the fact that all the particles are in pairs (π–π, ν–ν, and π–ν), then due to charge independence they have the same energy, and the ground state of seniority, 0^+, is degenerate with respect to the isospin multiplet. For example, the ground state of ^{56}Cr with $T=4$ has the same energy as the $T=4$ state of ^{56}Ni, which is a self-conjugate nucleus. Therefore each self-conjugate nucleus is a representative of all nuclei within the same isospin multiplet, and we can use this fact in order to find an axially symmetric wave function.

In the case of a self-conjugate nucleus there are only two possibilities: one of them is that the nondiagonal terms of the 2×2 matrices U_j and V_j vanish. Then the coefficients $\mathcal{C}$ of the generalized wave function (3.1) become

$$\mathcal{C}^j_4=v_j^2,\quad \mathcal{C}^j_0=u_j^2,\quad \mathcal{C}^j_{\pi\pi}=\mathcal{C}^j_{\nu\nu}=u_jv_j,\ \text{and}\ \mathcal{C}^j_{\pi\nu}=0\ , \tag{3.3}$$

where, from (3.2), $v^j_{\pi1}=v^j_{\nu2}=v_j$ and $u^j_{\pi1}=u^j_{\nu2}=u_j$. The wave function (3.1) then takes the form

$$|\Phi_{\rm I}(\Delta)\rangle=\prod_{jm>0}(u_j+v_j s_{jm}c^{\dagger}_{jm\pi}c^{\dagger}_{j-m\pi})\times(u_j+v_j s_{jm}c^{\dagger}_{jm\nu}c^{\dagger}_{j-m\nu})|0\rangle\ . \quad (3.4)$$

The generalized BCS wave function (3.1) is here reduced to a BCS product form for neutrons and protons separately. It is obvious that this wave function is not axially symmetric. Furthermore, it describes only even-even nuclei, and then there must be another solution which applies to even-even as well as odd-odd nuclei. It is easy to prove that this solution does exist and is characterized by

$$\Delta_{\pi\pi}=\Delta_{\nu\nu}=0 \text{ and } \Delta_{\pi\nu}\neq 0\ . \quad (3.5)$$

The corresponding coefficients are

$$\mathcal{C}^j_0=-v_j^2,\ \mathcal{C}^j_4=u_j^2,\ \mathcal{C}^j_{\pi\pi}=\mathcal{C}^j_{\nu\nu}=0, \text{ and } \mathcal{C}^j_{\pi\nu}=u_j v_j\ , \quad (3.6)$$

and the generalized BCS wave function (3.1) becomes in this case

$$|\Phi_{\rm II}(\Delta)\rangle=\prod_{jm}(u_j+v_j s_{jm}c^{\dagger}_{jm\pi}c^{\dagger}_{j-m\nu})|0\rangle\ . \quad (3.7)$$

This wave function was suggested, long ago, by Banerjee and Parikh[16] to treat the $T=1$ pairing problem. If we apply the rotation operator $\hat{R}(\alpha)=\exp(i\alpha\hat{T}_z)$ to this type of wave function with any value of $\Delta_{\pi\nu}$, we find $\hat{R}\,|\Phi_{\rm II}(\Delta)\rangle=|\Phi_{\rm II}(\Delta)\rangle$, which implies that this class of wave function does indeed have the symmetry properties we were looking for.

Of course, for nuclei other than self-conjugate ones ($T_z\neq 0$), these properties are not good. However, due to the charge independence of the Hamiltonian, the fact remains that there are degeneracies in energy among the members of the isospin multiplet, and so it is sufficient merely to calculate the energy of one of its members, namely $T_z=0$, the self-conjugate nucleus.

As for the reaction transition rates, due to the Wigner-Eckart theorem, it is enough to calculate the reaction rates for a proton-neutron pair transfer from one self-conjugate nucleus to another. For these reasons we do not lose any generality if we formulate the theory of the collective motion by considering only self-conjugate nuclei. In doing so, we are fully utilizing the symmetry property of the class of wave functions which we have found.

IV. ANALYTICAL FORMS FOR THE ISOVECTOR PAIRING GCM KERNELS

By using the axially symmetric wave function (3.7) derived in the preceding section, we can obtain analytical forms for the GCM kernels (2.1) and (2.2) through the matrices $[\rho^{BA}(\Omega,\theta)]^j$, $[\kappa^{BA}(\Omega,\theta)]^j$, $[\sigma^{BA}(\Omega,\theta)]$, and $[X^{BA}(\Omega,\theta)]^j$, which are easily derived. If these matrix elements are substituted in (2.10) and (2.11), we get the following simple results for the overlap functions,

$$\frac{h^{BA}(\Omega,\theta)}{n^{BA}(\beta,\theta)}=\sum_j\left\{\frac{\Omega_j v_j^B u_j^A}{\det X_j^{BA}}[(4\epsilon_j-3G)v_j^B v_j^A e^{2i\theta}+2(2\epsilon_j+G\Omega_j)u_j^B u_j^A\cos\beta]-\frac{G\Omega_j(\Omega_j-1)}{[\det X_j^{BA}]^2}(u_j^B u_j^A v_j^B v_j^A e^{i\theta}\sin\beta)^2\right\}$$
$$-2G\sum_{j>j'}\frac{\Omega_j\Omega_{j'}u_j^B u_j^A v_{j'}^B v_{j'}^A}{\det X_j^{BA}\det X_{j'}^{BA}}[(v_j^B v_j^A u_{j'}^B u_{j'}^A+u_{j'}^B u_{j'}^A v_j^B v_j^A)e^{2i\theta}+(u_j^B u_j^A u_{j'}^B u_{j'}^A+v_j^B v_j^A v_{j'}^B v_{j'}^A e^{2i\theta})\cos\beta] \quad (4.1)$$

and

$$n^{BA}(\beta,\theta)=\prod_j[\det X_j^{BA}]^{\Omega_j}\ , \quad (4.2)$$

with

$$\det X_j^{BA}=(u_j^B u_j^A)^2+(v_j^B v_j^A e^{i\theta})^2+2u_j^B u_j^A v_j^B v_j^A e^{i\theta}\cos\beta\ . \quad (4.3)$$

We may notice that $h^{BA}(\Omega,\theta)$ is suitable for expansion in a binomial form because of its obvious dependence in the Euler angle β and the gauge angle θ. Some question may be raised because the third term of (4.1) for $h_{BA}(\beta,\theta)$ is proportional to

$$\prod_{j\neq j'}[\det X_{j'}^{BA}]^{\Omega_{j'}}[\det X_j^{BA}]^{\Omega_j-2}\ ,$$

and hence cannot be expanded in terms of $e^{i\theta}$ and $\cos\beta$ in the case of $\Omega_j=1$. However, the presence of the factors $\Omega_j(\Omega_j-1)$ removes this difficulty.

We now may use the procedure of repeated applications of Newton's binomial formula to the energy function $h^{BA}(\Omega,\theta)$ and obtain a polynomial in the angular variables $e^{i\theta}$ and $\cos\beta$. The way to get this polynomial form is presented in Appendix A. Here we simply give the final result

$$h^{BA}(\Omega,\theta)=\sum_{k,l}[Q_{kl}^{BA}(\cos\beta)^l+Q_{kl}^{BA'}(\cos\beta)^{l-2}]e^{i(2k-l)\theta}\ , \quad (4.4)$$

where

$$Q_{kl}^{BA}=\sum_{\substack{k_{j_1},\ldots,k_{j_m}\\ l_{j_1},\ldots,l_{j_m}}}\left[\prod_{j=j_1}^{j_m}F_j^{BA}\right]\left\{\sum_j[(4\epsilon_j-G)-4G(\Omega_j-k_j)](k_j-l_j)\right.$$
$$\left.+(2\epsilon_j-G\Omega_j)l_j+Gl_j-1)-G\sum_{j\neq j'}\frac{v_j^B v_j^A u_{j'}^B u_{j'}^A(\Omega_j-k_j)+u_j^B u_j^A v_{j'}^B v_{j'}^A(k_j-l_j)}{u_j^B u_{j'}^A v_j^B v_{j'}^A}l_j\right\} \quad (4.5)$$

and

$$Q_{kl}^{BA'}=-\frac{G}{2}\sum_{\substack{k_{j_1},\ldots,j_{j_m}\\ l_{j_1},\ldots,l_{j_m}}}\left[\prod_{j=j_1}^{j_m}F_j^{BA}\right]\left[\sum_{j\neq j'}\left[\frac{v_j^A u_{j'}^B}{v_j^A u_{j'}^B}+\frac{v_j^A u_{j'}^B}{v_j^A u_{j'}^B}\right]l_j l_{j'}\right], \tag{4.6}$$

with

$$l=\sum_j l_j,\quad k=\sum_j k_j,\quad \text{and}\quad F_j^{BA}=\begin{bmatrix}\Omega_j\\ k_j\end{bmatrix}\begin{bmatrix}k_j\\ l_j\end{bmatrix}(u_j^B u_j^A)^{2(\Omega_j-k_j)}(v_j^B v_j^A)^{2(k_j-l_j)}(2u_j^B u_j^A v_j^B v_j^A)^{l_j}. \tag{4.7}$$

From (4.6) we can easily see that $Q_{kl}^{BA'}=0$ when $l<2$, which guarantees the polynomial form for $h^{BA}(\beta,\theta)$ in (4.4).

Finally, we can make the last manipulation to get the energy kernel (2.8) and the overlap (2.9) of the GCM equations. The overlap (2.9) can be rewritten, after the binomial expansion and performing the integration over the gauge angle, as

$$I^{N,T}(\Delta_B,\Delta_A)=\frac{2T+1}{8\pi^2}\sum_{KK'}\sum_{\substack{k_{j_1},\ldots,k_{j_m}\\ l_{j_1}+l_{j_2}+\cdots+l_{j_m}=2k-N}}\left[\prod F_j^{BA}\right]I_{KK'}^{I}(2k-N), \tag{4.8}$$

with

$$I_{KK'}^{T}(2k-N)=\int d\Omega\, D_{KK'}^{T*}(\Omega)(\cos\beta)^{2k-N}. \tag{4.9}$$

Using the Wigner D matrix

$$D_{KK'}^{T}(\Omega)=\exp(iK\alpha)d_{KK'}^{T}(\beta)\exp(-K'\gamma),$$

the above integral has a nonzero value only if $K=K'=0$. Then, $d_{00}^{T}(\beta)=P_T(\cos\beta)$, where $P_T(\cos\beta)$ is the Legendre polynomial. Consequently, the integral (4.9) becomes

$$\mathcal{I}_T(n)=I_{00}^{T}(n)=\frac{n!\Gamma\left[\frac{n-T+1}{2}\right]}{2^{T-1}(n-T)!(n+T-1)!\Gamma\left[\frac{n+T+1}{2}\right]}. \tag{4.10}$$

Here, $n=2k-N$. From the existence condition for the integral, we have two restrictions, namely $n>T$ and $n-T$ even. Hence the overlap $I_{00}^{NT}(\Delta_B,\Delta_A)$ does not vanish only if

$$n=T,\ T+2,\ T+4,\ \ldots. \tag{4.11}$$

At this point we may notice that GCM kernels [(2.2) and (2.3)] exists, only if the values $n=T,\ T+2,\ T+4,\ \ldots,\ N$ are allowed ones. This means that axial BCS wave function does not contain components with parity $(-)^{T+N}=(-1)$. This selection rule means that we have components just with $T=N,\ N-2,\ N-4,\ \ldots,\ 0$ or 1 in the axial symmetric BCS wave function, and have only a mixture of yrast states of the self-conjugate nuclei instead of a superposition of wave functions of all the neighboring nuclei as in the traditional BCS treatment. These yrast states are totally charge symmetric; in other words, the isospin is invariant under the exchange of charge coordinates between two pairs with $T=0$ and 1. Therefore, the BCS axial symmetric wave function is the superposition of the lowest charge symmetric states of the neighboring self-conjugate nuclei. This fact puts the limitation on our model that we cannot describe states with other charge symmetries. For example, the $T=1$ states of ^{56}Ni cannot be included in our work.

Using (4.8) and (4.10), we get the following expression for the overlap kernel in the axially symmetric case,

$$I^{NT}(\Delta_B,\Delta_A)=\frac{2T+1}{2}\sum_{n=T}^{N}C_n^{BA}\mathcal{I}_T(n), \tag{4.12}$$

where

$$C_n^{BA}=\sum_{\substack{l_{j_1}+l_{j_2}+\cdots+l_{j_m}=n\\ k_{j_1}+k_{j_2}+\cdots+k_{j_m}=(n+N)/2}}\left[\prod_{j=j_1}^{j_m}F_j^{BA}\right].$$

The upper limit for n is determined from the binomial expansion which implies $0>l_j<k_j$. As by definition $\sum l_j=n$ and, due to the gauge angle integral, $n=2\sum k_j-N$, we conclude that $n\leq N$.

The calculation of the energy kernel (2.8) is easily to perform if we follow the procedure used to get the overlap kernel (4.12). The result is

$$H^{N,T}(\Delta_B,\Delta_A)=\frac{2T+1}{2}\times\sum_{n=T}^{N}[\mathcal{A}_n^{BA}\mathcal{I}_T(n)+\mathcal{B}_n^{BA}\mathcal{I}_T(n-2)], \tag{4.13}$$

where

$$\mathcal{A}_n^{BA}=\sum_{\substack{k=(n+N)/2\\ l=n}} Q_{kl}^{BA} \quad \text{and} \quad \mathcal{B}_n^{BA}=\sum_{\substack{k=(n+N)/2\\ l=n}} Q_{kl}^{BA'} . \tag{4.14}$$

Since $\langle \Phi(\Delta_B) | \{{}^H_I\}\hat{P}_T\hat{P}_N | \Phi(\Delta_A)\rangle$ was obtained analytically, it is possible, in principle, to solve the HW equation analytically.[17] However, in this work we solve the HW equation numerically following the traditional discretization prescription, and the numerical results are given in Sec. VI.

V. ANALYTICAL EXPRESSION FOR THE SPECTROSCOPIC FACTOR

Apart from the energy spectrum, another important quantity to test our model is the spectroscopic amplitude (β_j) of the two nucleon transfer reaction, which is discussed elsewhere[18] and is given by

$$\beta_j=\frac{\mathcal{N}_{T_f}\mathcal{N}_{T_i}}{2T_f+1}\int\int f(\Delta_f)f(\Delta_i)\langle\phi_{T_fN+1}(\Delta_f)\|A_j^\dagger\|\phi_{T_iN}(\Delta_i)\rangle d\Delta_f d\Delta_i , \tag{5.1}$$

where $f(\Delta_f)$ and $f(\Delta_i)$ are the weight functions for the final and the initial nuclei obtained by the GCM calculation, $\mathcal{N}_{T_f}$ ($\mathcal{N}_{T_i}$) is the normalization factor, and $|\phi_{T_fN+1}(\Delta_f)\rangle$ is the projected BCS wave function. Finally, $A_J^\dagger$ is the irreducible tensor of the operator given in (2.5).

In order to get an expression for the spectroscopic factor, we need to obtain the reduced matrix element of (5.1). This calculation begins with the computation of the matrix element of the two projected states, $\langle\phi_{T_fN+1}(\Delta_f)|A_{j\mu}^\dagger|\phi_{T_iN}(\Delta_i)\rangle$. The details of the calculation of these reduced matrix elements are presented in Appendix B and the following results are obtained before the integration over the Euler angle β and the gauge angle θ:

$$\begin{aligned}\langle\phi_{T_fN+1}(\Delta_f)\|A_j^\dagger\|\phi_{T_fN}(\Delta_i)\rangle=&C_{T_f}C_{T_i}4\pi\sqrt{\Omega_j}u_j^iu_j^f(-)^{T_f}\\ &\times\left\{\begin{bmatrix}T_f & 1 & T_i\\ 0 & 0 & 0\end{bmatrix}\frac{2}{\sqrt{2}}\int_0^{2\pi}d\theta\,e^{-iN\theta}\int_1^{-1}d(\cos\beta)P_{T_i}(\cos\beta)\frac{n^{fi}(\beta,\theta)}{\det X_j^{fi}}(u^fu^i+v^fv^ie^{i\theta}\cos\beta)\right.\\ &-\begin{bmatrix}T_f & 1 & T_i\\ 0 & 1 & -1\end{bmatrix}\left[\frac{T_i+1}{T_i}\right]^{1/2}2v_j^fv_j^i\\ &\left.\times\int_0^{2\pi}d\theta\,e^{-i(N-1)^\theta}\int_1^{-1}d(\cos\beta)[\cos\beta P_{T_i}(\cos\beta)-P_{T_i+1}(\cos\beta)]\frac{n^{fi}(\beta,\theta)}{\det X_j^{fi}}\right\},\end{aligned} \tag{5.2}$$

with $C_{T_f}C_{T_i}$ given in Appendix B.

Expanding all the terms in the above expression in terms of a polynomial in θ and β, and by the same index rearrangement used at arrive of the energy expression, we can perform the above integration exactly, and the final result is

$$\begin{aligned}\langle\phi_{T_fN+1}(\Delta_f)\|A_j\|\phi_{T_iN}(\Delta_i)\rangle=&4\pi\left[\frac{\mathcal{X}_{T_f}\mathcal{X}_{T_i}}{\Omega_j}\right]^{1/2}(-)^{T_f}\frac{v_j^f}{u_j^f}\sum_{n=T}^{N}\left[w^{(1)}(j)\begin{bmatrix}T_f & 1 & T_i\\ 0 & 0 & 0\end{bmatrix}\right.\\ &\left.+w_n^{(2)}(j)\begin{bmatrix}T_f & 1 & T_i\\ 0 & 1 & -1\end{bmatrix}\sqrt{T_i(T_i+1)}\right]\mathcal{F}_{T_i}(n) .\end{aligned} \tag{5.3}$$

To obtain the above expression, we have made use of the following identity for the isospin function (4.11):

$$\mathcal{F}_{T+1}(n-1)=\frac{n-T}{n}\mathcal{F}_T(n) . \tag{5.4}$$

Here,

$$\mathcal{X}_{T_f(T_i)}=\left[\frac{2T_f+1}{4}\sum_{n=T_i(T_f)}^{N(N+1)}\prod_j(F_j)_{fi}\mathcal{F}_{T_i(T_f)}(n)\right]^{-1} , \tag{5.5}$$

$$w_n^{(1)}(j)=\sum_{\substack{k=(n+N)/2\\ l=n}}\prod_j(F_j)_{fi}\frac{[2(\Omega_j-k_j)+l_j]}{\sqrt{2}} , \tag{5.6}$$

and

$$w_n^{(2)}(j)=-\frac{1}{n}\sum_{\substack{k=(n+N)/2\\ l=m}}\left[\prod_j(F_j)_{fi}\right]l_j , \tag{5.7}$$

where $(F_j)_{fi}$ is the same as (4.7) with the exchange

$A \rightarrow f$ and $B \rightarrow i$.

Substituting the reduced matrix element from (5.3) in (5.1), we have the final value for the spectroscopic amplitude of a two nucleon transfer reaction in the GCM scheme.

If the u_j's and v_j's are known, we can obtain the transition rates from (5.1). Although in our scheme it is easy to perform the calculation, it suffers from one restriction; that is, we can get the spectroscopic amplitude only in the charge symmetric states. This is due to the fact that our BCS wave function is axially symmetric and the transition $T \rightarrow T$ is not considered in the present work. Therefore the transitions that will be treated here are only $T_f = T_i + 1$ and $T_f = T_i - 1$.

VI. RESULTS AND DISCUSSION

We shall now discuss the several ways in which we tested the advantages and limitations of the present model. First, we compare our results with the exact results for the two level model. After that, we shall compare them with results obtained by shell model calculations in the fp shell. Furthermore, we shall present some results with five active levels, where the exact diagonalization cannot be handled. It is also necessary to say some words about the "discretized" method to solve HW equation (2.1). We use the prescription of Faessler and co-workers[19] for three points 0.5Δ, Δ, and 1.5Δ.

In order to show the simplicity of the present model, we shall compare our results in the system of two equally degenerate levels with pair degeneracy $\Omega = 20$ separated by a distance of $\epsilon = 10.0$ MeV and the total number of particles is $2N = 4\Omega$. This system is well known as the symmetric two level model and is also considered a possible course of testing the approximate solution of the pairing Hamiltonian. Using Δ as a generator coordinate, we get the ground state ($T=0$) and excited energies for several values of G; for each value of the energy we subtract the ground state ($T=0$) energy and we obtain Fig. 1, very close to the results obtained by Dussel and coworkers.[20] Therefore we see that the present scheme reproduces the exact results in the symmetric two level model. It is particularly interesting to notice that the present scheme gives similar results of the excited state of $T=4$, which means an excess of eight neutrons over protons, of the exact diagonalization method.

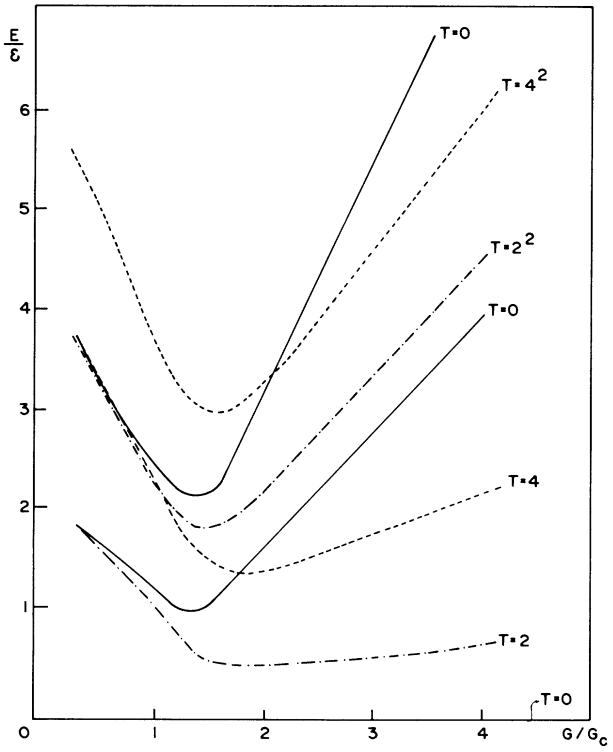

FIG. 1. The 0^+ energy spectra calculated by the GCM in the symmetric two level model as a function of pairing strength G divided by G_c. The ordinate gives the total energy relative ground state $T=0$ divided by the level spacing $\epsilon = 10.0$. The total number of particles is $N = 10$ and the pair degeneracy is $\Omega = 10$. T^a, where a denotes the first, second, etc., is the time that a state with isospin T appears.

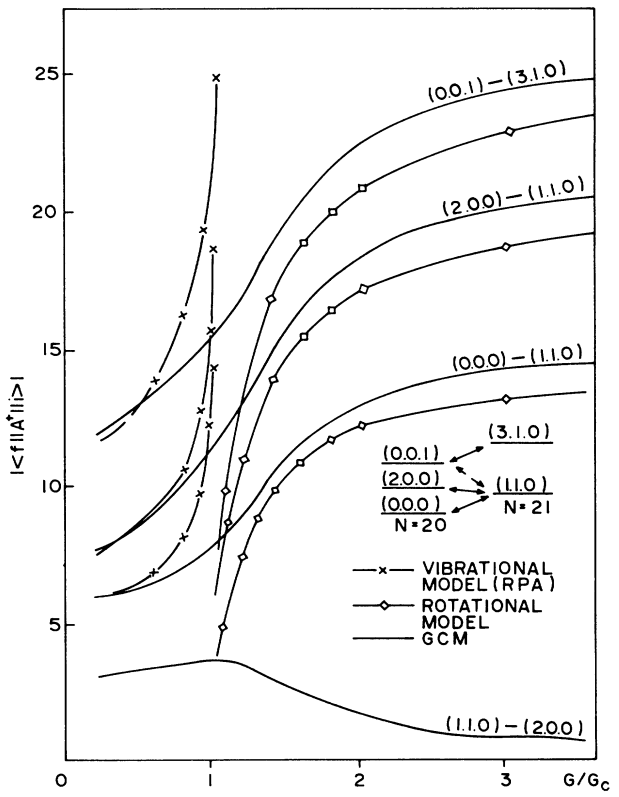

FIG. 2. The absolute value of the reduced transition matrix elements between $N=2$ and 21 as a function of G/G_c in a symmetric two level model with $\Omega = 10$ and level spacing $\epsilon = 10.0$ are presented in the several approximations calculated in this work. A level scheme clarifies the transitions that are considered. The double full arrows are allowed-allowed transition populations and the dotted arrows are the allowed-forbidden transition populations of the pairing vibrations for $N = 20$.

After comparison with the numerical results, we are convinced that the results obtained from our method are similar to those of the exact solution. We are now going to discuss the transition rates (5.1). In Fig. 2 the absolute value of the matrix element $|\langle f\|\sum A_j^+\|i\rangle|$ is plotted against the pairing strength $G/G_{\rm crit}$. The final state $|f\rangle$ and initial state $|i\rangle$ are obtained by the GCM. For the transitions considered the states have been labeled by (T,m,a), where a denotes the number of times that the state with isospin T and number of pairs $N=2\Omega+m$ has appeared. The double arrow means the allowed-forbidden transition which populates the pairing vibration state with $N=20$. In Fig. 2 the solid curve means that both $|i\rangle$ and $|f\rangle$ are GCM states and, as can be seen, it almost reproduces the exact results.[20] In the same figure the results from vibrational and rotational states are also presented. In the weak coupling limit $G/G_{\rm crit}\ll 1$ the GCM results are the same as those for the vibrational model. But in the other limit, that of the strong coupling limit, the difference between the GCM and rotational models is around 10%. The reason for this difference will be explained later. As expected, in the region of $G_{\rm crit}$ both the rotational and vibrational model fail. For $G_{\rm crit}$ the transition rates in the rotational model go to zero, while in the vibrational model it goes to infinity. This transition, however, is smooth in the GCM description. The physical meanings of GCM curves are the following: If we take the one point limit of the projected BCS wave function, and also take the rotation operators $R(\Omega)$ and $S(\theta)$ as unitary operators, we get the following wave function for the rotational model:

$$|NTKM_T;\Delta\rangle=\left[\frac{2T+1}{16\pi^3}\right]\times\exp(iN\theta)D^T_{M_TK}(\alpha\beta\gamma)\,|\,\mathrm{BCS},\Delta;K\rangle\ , \tag{6.1}$$

where α, β, γ, θ, and Δ are the generalized coordinates of the rotational model. Our projected wave function will be the solution of the Hill-Wheeler integral equation, if we use α, β, γ, and Δ as generator coordinates and the above wave function (6.1) with correlation between the intrinsic wave function and that of the rotational model adequately introduced by the operators R,S. The deformation of our system is measured by Δ_μ, where $\mu=(1,0,-1)$ mean $(\pi\pi,\pi\nu,\nu\nu)$, and has the same meaning of the intrinsic deformation in the collective model. The pair transfer operator in (6.1) could be the same as that of the traditional rotational model, i.e.,

$$\mathcal{P}_{\mu'}=e^{i\theta}\sum_{\mu'}\Delta_\mu D^1_{\mu'\mu}(\alpha\beta\gamma)\ . \tag{6.2}$$

In our case, we have $\Delta_{\pi\pi}=\Delta_{\nu\nu}=0$ and the pair transfer operator connects the states with same $K=0$. For the ground-ground transition we trivially get

$$\langle T_fN+1\|\mathcal{P}\|T_iN\rangle=\sqrt{2T_>}\sum_j\Omega_ju_jv_j=\sqrt{2T_>}\left[1-\left[\frac{G_{\rm crit}}{G}\right]\right]^{1/2}, \tag{6.3}$$

where $T_>$ is the largest value among the isospin transitions. Therefore it is easy to realize that the GCM curve is proportional to T, meaning that if we have a large T we have a large transition rate. It is interesting to note that if we take the one point limit in the expression (5.3) and consider $\alpha=\beta=\gamma=\theta=0$ and, if additionally we take $u_j^fu_j^i+v_j^fv_j^i=1$ and finally sum over the single particle levels, we get simply

$$\sum_j\langle f\|A\|i\rangle=\sqrt{2T_>}\sum_j\Omega_ju_j^fv_j^i\ , \tag{6.4}$$

which is the expression (6.3) of the rotational model if we approximate v_j^f by v_j^i. The above expression for the one point limit explains the 10% difference between the rotational model and the GCM for $G/G_{\rm crit}\gg 1$. Since in the GCM we do not replace v_j^f and v_j^i and we consider all the points in (5.3).

Finally, we are going to apply our formula to actual nuclei. Since our purpose it to establish the validity of the present method for studying the pairing isovector collective motion, we first compare our GCM results with the diagonalization of Bés and co-workers.[7] The calculated energy spectra for the region of $52<A<60$ is presented in Fig. 3 and compared with the exact results of Bés and co-workers and with the experimental data. From the spectra one can see that our results are quite satisfactory, in addition to the simplicity of our approximation, which maintains the axial symmetry for excited states. The main reason for the present approximation's validity for the excited $T=0$ states and for those states near the double magic nucleus is easily understood due to the fact that these states lie in or near the self-conjugate nuclei, where the axial symmetry ought to prevail. However, further investigation of the present approximation in all cases is needed to draw some definite conclusions about the symmetry properties of excited seniority zero states. One also can be surprised that this wave function may give a such result obtained in Figs. 1

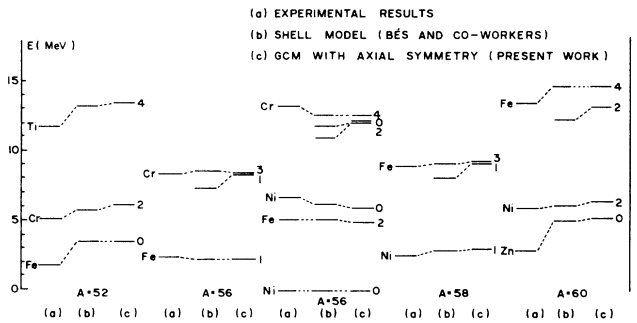

FIG. 3. Theoretical and experimental spectra of 0^+ states in the $52\le A\le 60$ region. The numbers to the right of each calculated level stand for the isospin. All the theoretical calculations are performed in the three active levels ($1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$) with single particle levels (0.00, 3.97, 4.22 MeV), and the pairing strength is $G=30/A$.

TABLE I. Results of "experimental" energies of isovector pairing collective states compared with present calculations in the five- and three-level models in the *fp* shell, where the inert cores are those of ^{40}Ca. The first column indicates the isotope in which the state is found. The second column is the isospin or isotope. Columns 3 and 4 show, respectively, the excitation energy (taken from Ref. 1) and the total binding energy (from Ref. 24) of the state. Column 5 displays "experimental" energies obtained from Eq. (6.5). The energies are calculated in the present work for the five-level model with values of −4.0, 0.00, 0.78, 1.08, and 4.00 MeV for the single particle energies modified for another nucleus according to Kisslinger and Sorensen (Ref. 23); the pairing strength is $G=24/A$. Column 7 displays the energies of Fig. 3 slightly modified to adjust the ground state levels of ^{54}Fe and ^{58}Ni.

A	T	E_x (MeV)	E_B (MeV)	E_e (MeV)	E_{t_1} (MeV)	E_{t_2} (MeV)
^{56}Ni	0	g.s.	484.01	0	0	0
^{54}Fe	1	g.s.	471.78	2.75	2.90	2.88
^{58}Ni	1	g.s.	506.48	2.30	2.48	2.46
^{52}Fe	0	g.s.	447.72	2.38	4.55	4.77
^{52}Cr	2	g.s.	456.36	5.62	7.01	7.30
^{56}Ni	0	5.23	478.78	5.23	6.36	5.94
^{56}Fe	2	g.s.	492.28	5.02	5.07	4.95
^{60}Zn	0	g.s.	515.02	2.36	3.93	4.01
^{60}Ni	2	g.s.	526.87	5.15	6.15	6.22
^{54}Fe	1	5.68	466.10	8.43	9.36	8.91
^{54}Cr	3	g.s.	474.02	8.67	9.08	9.01
^{58}Ni	1	3.55	502.93	5.80	9.00	8.48
^{58}Fe	3	g.s.	509.97	8.63	8.7	8.63
^{52}Fe	0	4.63	443.09	7.01	12.64	12.20
^{52}Cr	2	5.74	450.62	11.36	13.31	13.11
^{52}Ti	4	g.s.	451.98	12.71	14.03	14.68
^{56}Fe	2	5.63	486.65	10.65	12.73	12.01
^{56}Cr	4	g.s.	488.50	13.50	12.69	14.68
^{60}Ni	2	3.53	523.34	8.68	12.42	12.01
^{60}Fe	4	g.s.	525.39	13.18	13.44	13.52

and 3 and also in Table I due to the fact that simple ansatz such as (3.7) seems to contain only neutron-proton pairs. In reality this wave function contain all of Cooper's pairs due to the subtle fact that the magnetic number goes to m: $-j,\ldots,j$ instead of $m>0,\ldots,j$ as is usual in the BCS formalism; see formula (3.4).

Last but not least we should mention that in the realistic ($N=Z$) nuclei, unless we are dealing with isovector pairing collective states, the $J=0^+$ pairing does not dominate the spectrum and the $T=0$ pairing should be included and treated along with the usual like pairing as shown by Goodman.[21]

Next we present the GCM calculation for a system of five active levels. We do not expect crucial changes with the addition of two levels since the energy of these states are dominated by the lowest levels of the configuration. This fact can be verified in Table I, where in the first column we have listed the nuclei considered along with their mass number, isospin, and binding energy. This binding energy is used for the calculation of E_e,

$$E_e(k,A,T,M_T)=[E_B(56,0,0)-E_{BW}(56,0,0)] -[E_B(k,A,T,M_T)-E_{BW}(A,T,M_T)]\ . \tag{6.5}$$

Here, E_B is the experimental binding energy associated with the states $J=0^+$ with isospin T for the (A,M_T) nucleus. The ground state energy is labeled without the index k. E_{BW} is the semiempirical Weizsacker formula,

$$E_{BW}=b_{\rm vol}A-17A^{2/3}-2b_{\rm sym}T(T+1)/A -0.7Z^2(1-0.76/Z^{2/3})/A^{1/3}\ , \tag{6.6}$$

with $b_{\rm vol}=10.43$ MeV and $b_{\rm sym}=25.0$ MeV. The last two columns refer to the five active level and three active level calculations, respectively, where, as we see, the differences are small. In the five level calculation we choose the following values: (−4.0, 0.0, 0.78, 1.08, 4.0) for the single particle levels ($1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, $1g_{9/2}$) of ^{56}Ni, as considered earlier by Bayman and Hintz.[22] These values are slightly modified for other nuclei according to Kisslinger and Sorensen.[23] The pairing strength $G=24/A$ is selected in such a way that it reproduces the experimental data of the first $T=2$ state. This kind of choice is adequate for the detailed study of the region $40<A<70$. But for the moment the present calculation in the region of $52<A<60$ does not give any new characteristics. However, the importance of our method lies in its simplicity and this becomes obvi-

ous if we remember the rapidly growing complexity of shell model calculations when we increase the model space and include the isospin as a dynamic variable. In the present scheme, on the other hand, the projection can be performed within seconds on a normal computer.

The last step is to calculate the transition rates which connect the collective states of the isovector pairing states. Perhaps this kind of calculation is the most stringent test for our method. Then, by using the GCM wave functions we get spectroscopic amplitudes in the ^{56}Ni region. The results obtained are compared with those of an exact diagonalization of Bés and co-workers[7] and of the rotational model. In Table II, in the first column, we list the nuclear reaction considered; the energy of the state is given in the second column. In the third column we can see the spectroscopic amplitude of the GCM that reproduces the exact diagonalization results. As expected, we do not find any agreement with the rotational model because we are in the transition region.

VII. CONCLUSION

In this paper we present a new model with which to describe the collective 0^+ states in the medium weight nuclei. This model uses axially symmetric BCS-type wave functions in the charge space with a mixture of odd-odd and even-even nuclei as generating functions of the GCM. This axially symmetric wave function implies that we take only $T_z=0$ as being representative of all nuclei in the same isospin multiplet. This wave function also allows us to perform the isospin and number projection analytically. We, furthermore, show that in our formalism we recover in the isospin projection the simplicity of the number projected BCS theory.

The ground state of 0^+ isospin yrast states is obtained with the GCM for the two level model and in the ^{56}Ni region. The results are quite satisfactory compared with the exact results and experimental data. The same conclusion can be drawn with respect to the energy of excited states. It is worth mentioning that we assume the axially symmetric wave function to be good for the excited states. The spectroscopic factors have also been obtained by the approximation mentioned above, and similar conclusions were obtained.

The weak point of the present model is that we can describe only states which are totally symmetric in the isospin of the pairs of particles (they have even T for an even number of pairs and odd T for an odd number of pairs).

In the end we may hope to apply the results obtained here to treat the challenging problem which involves more than one shell. We can also apply this technique to the description of 0^+ states in real nuclei using more realistic interactions than a pure pairing force.

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TABLE II. The spectroscopic amplitudes obtained in the present work compared with the shell model approach. Numerical results for the pairing rotational model are also presented [see Eq. (6.4)]. E_{expt} is the energy of final nucleus populated by the reaction. T_f is the isospin of the final nucleus. For details of the nuclear system, see Fig. 3.

			Shell model			GCM			Pairing rotational model	
Reaction	E_{expt}	T_f	$1f_{7/2}$	$2p_{3/2}$	$1f_{5/2}$	$1f_{7/2}$	$2p_{3/2}$	$1f_{5/2}$	$1f_{7/2}$	$2p_{3/2}$	$1f_{7/2}$
^{54}Cr(p,t)^{52}Cr	0.00	2	0.489	0.713	0.780	0.604	0.697	0.810	0.795	0.357	0.414
^{54}Cr(p,t)^{52}Cr	2.48	2	0.871	0.038	0.083	0.748	−0.001	0.011			
^{54}Fe(^{3}He,n)^{56}Ni	0.0	0	1.741	0.457	0.531	1.700	0.471	0.531	1.191	0.517	0.581
^{54}Fe(^{3}He,n)^{56}Ni	6.58	0	−0.016	0.679	0.670	0.018	0.650	0.756			
^{54}Fe(t,p)^{56}Fe	0.0	2	0.470	0.702	0.776	0.548	0.686	0.787	0.753	0.328	0.367
^{54}Cr(t,p)^{56}Cr	0.0	4	0.476	0.829	0.960	0.512	0.814	0.968	0.794	0.346	0.386
^{58}Ni(p,t)^{56}Ni	0.0	0	0.402	0.669	0.768	0.410	0.664	0.738	0.627	0.489	0.558
^{58}Ni(p,t)^{56}Ni	6.58	0	0.537	−0.016	0.008	0.552	−0.027	−0.017			
^{56}Fe(t,p)^{58}Fe	0.0	3	0.456	0.827	0.955	0.454	0.813	0.957	0.511	0.507	0.560
^{58}Ni(^{3}He,n)Zn	0.0	0	0.734	1.089	1.121	0.876	1.026	1.155	0.641	0.951	1.063
^{58}Ni(t,p)^{60}Ni	0.0	2	0.399	0.832	0.939	0.341	0.797	0.951	0.405	0.601	0.672
^{58}Fe(t,p)^{60}Fe	0.0	4	0.405	0.820	1.029	0.576	0.704	0.859	0.427	0.632	0.708

APPENDIX A: DERIVATION OF THE OVERLAP FUNCTION $h(\beta,\theta)$

We present in this appendix the method used to obtain the overlap function $h(\beta,\theta)$ in the polynomial form of $(e^{i\theta})^k(\cos\beta)^l$.

Expression (3.2) may be replaced by the following form:

$$h^{BA}(\beta,\theta)=n^{BA}(\beta,\theta)\left[\sum_j\left[\frac{\Gamma_j^{BA}(\beta,\theta)}{|\det X_j^{BA}|^2}+\frac{\Lambda_j^{BA}(\beta,\theta)}{|\det X_j^{BA}|^2}\right]-\sum_{j>j'}\frac{\phi_{jj}^{BA}(\beta,\theta)}{\det X_j^{BA}\det X_{j'}^{BA}}\right], \tag{A1}$$

where

$$\Gamma_j^{BA}(\beta,\theta)=\Omega_j v_j v_j[(4\epsilon_j-3G)v_j^B v_j^A e^{2i\theta}+2(2\epsilon_j-G\Omega_j)u_j^B u_j^A\cos\beta], \tag{A2}$$

$$\Lambda_j^{BA}(\beta,\theta)=G\Omega_j(\Omega_j-1)(u_j^B u_j^A v_j^B v_j^A)^2 e^{2i\theta}\sin^2\beta, \tag{A3}$$

$$\phi_{jj'}^{BA}(\beta,\theta)=G\Omega_j\Omega_{j'}u_j^B u_j^A v_j^B v_j^A[(v_{j'}^B v_{j'}^A u_j^B u_j^A+u_{j'}^B u_{j'}^A v_j^B v_j^A)e^{2i\theta}+(u_j^B u_j^A u_{j'}^B u_{j'}^A+v_j^B v_j^A v_{j'}^B v_{j'}^A e^{2i\theta})\cos\beta]. \tag{A4}$$

Let us derive the third term, which we call $h_3^{BA}(\beta,\theta)$; $h_1^{BA}(\beta,\theta)$ and $h_2^{BA}(\beta,\theta)$ may be derived using the same method. Considering that $n^{BA}(\beta,\theta)=\prod[\det X_j^{BA}]^{\Omega_j}$, we have, for $h_3^{BA}(\beta,\theta)$,

$$h_3^{BA}(\beta,\theta)=\sum_{j>j'}[\det X_j^{BA}]^{\Omega_j-1}\sum_{j''\neq j'j}[\det X_{j''}^{BA}]^{\Omega_{j''}}\phi_{jj'}^{BA}(\beta,\theta). \tag{A5}$$

The binomial expansion may be written as

$$\prod_{j''}[\det X_{j''}^{BA}]^{\Omega_{j''}}=\prod_{j''}\sum_{k_{j''},l_{j''}}F_{j''}^{BA}e^{i(2k_{j''}-l_{j''})\theta}(\cos\beta)^{l_{j''}}, \tag{A6}$$

where

$$F_{j''}^{BA}=\begin{bmatrix}\Omega_{j''}\\ k_{j''}\end{bmatrix}\begin{bmatrix}k_{j''}\\ l_{j''}\end{bmatrix}(u_{j''}^B u_{j''}^A)^{2(\Omega_{j''}-k_{j''})}(v_{j''}^B v_{j''}^A)^{2(k_{j''}-l_{j''})}(2u_{j''}^B u_{j''}^A v_{j''}^B v_{j''}^A)^{l_{j''}}.$$

Then, (A5) becomes

$$\begin{aligned}h_3^{BA}(\beta,\theta)=-4G\sum_{j<j'}\left[\prod_{j''=j'j'}F_{j''}^{BA}\right]&\left[\begin{bmatrix}\Omega_j-1\\ k_j\end{bmatrix}\begin{bmatrix}k_j\\ l_j\end{bmatrix}(u_j^B u_j^A)^{2(\Omega_j-1-k_j)}(v_j^B v_j^A)^{2(k_j-l_j)}\right.\\&\times(2u_j^B u_j^A v_j^B v_j^A)^{l_j}\begin{bmatrix}\Omega_{j'}-1\\ k_{j'}\end{bmatrix}\begin{bmatrix}k_{j'}\\ l_{j'}\end{bmatrix}(u_{j'}^B u_{j'}^A)^{2(\Omega_{j'}-1-k_{j'})}(v_{j'}^B v_{j'}^A)^{2(k_{j'}-l_{j'})}\\&\times(2u_{j'}^B u_{j'}^A v_{j'}^B v_{j'}^A)^{l_{j'}}\exp\left\{i\left[2(k_j+k_{j'})-(l_j+l_{j'})+\sum_{j''}\chi_{j''}\right]\theta\right\}(\cos\beta)^{l_j+l_{j'}+\sum\chi_{j''}}\Bigg]\\&\times\Omega_j\Omega_{j'}u_j^B u_j^A v_j^B v_j^A[(v_j^B v_j^A u_{j'}^B u_{j'}^A+u_{j'}^B u_{j'}^A v_j^B v_j^A)e^{2i\theta}+(u_j^B u_j^A u_{j'}^B u_{j'}^A+v_j^B v_j^A v_{j'}^B v_{j'}^A e^{2i\theta})\cos\beta],\end{aligned}$$

with $\chi_{j''}=2k_{j''}-l_{j''}$.

With the help of some algebraic work and by using some binomial identities as below,

$$\begin{bmatrix}\Omega_j+1\\ k_j\end{bmatrix}\begin{bmatrix}k_j\\ l_j\end{bmatrix}\begin{bmatrix}\Omega_{j'}-1\\ k_j\end{bmatrix}\begin{bmatrix}k_{j'}\\ l_{j'}\end{bmatrix}=\frac{l_j+1}{\Omega_j}\begin{bmatrix}\Omega_j\\ l_j+1\end{bmatrix}\begin{bmatrix}k_j+1\\ l_j+1\end{bmatrix}\frac{l_j+1}{\Omega_{j'}}\begin{bmatrix}\Omega_{j'}\\ l_{j'}+1\end{bmatrix}\begin{bmatrix}k_{j'}+1\\ l_{j'}+1\end{bmatrix},$$

and also by some rearrangement of indices, we get a clear separation between angular variables and the coefficients in the following way:

$$\begin{aligned}h_3^{BA}(\beta,\theta)=-G\sum_{j<j'}\left[\prod_j F_j^{BA}\right]&\left[\frac{v_j^B v_j^A u_{j'}^B u_{j'}^A(\Omega_j-k_j)+u_j^B u_j^A v_{j'}^B v_{j'}^A(k_j-l_j)}{v_j^B u_j^A u_{j'}^B v_{j'}^A}l_{j'}+\frac{1}{2}\left[\frac{u_j^B v_{j'}^A}{v_j^B v_{j'}^B}+\frac{v_j^A u_{j'}^A}{u_j^A v_j^A}\right]\frac{l_j l_{j'}}{\cos^2\beta}\right]\\&\times\exp\left\{i\left[2(k_j+k_{j'})-(l_j+l_{j'})+\sum_{j''}\chi_{j''}\right]\right\}(\cos\beta)^{l_j+l_{j'}+\sum_{j''}\chi_{j''}}.\end{aligned} \tag{A7}$$

The other two terms are obtained using the same technique, and here we only present the final results,

$$h_1^{BA}(\beta,\theta)=\sum_{\substack{k_{j_1},\ldots,k_{j_m}\\ l_{j_1},\ldots,l_{j_m}}}\left[\prod_{j=j_1}^{j_m}F_j^{BA}\right]_j[(4\epsilon_j-G)(k_j-l_j)+(2\epsilon_j-G\Omega_j)l_j]\cos^l\beta e^{i(2k-l)\theta}\ , \tag{A8}$$

$$h_2^{BA}(\beta,\theta)=\sum_{\substack{k_{j_1},\ldots,k_{j_m}\\ l_{j_1},\ldots,l_{j_m}}}\left[\prod_{j=j_1}^{j_m}F_j^{BA}\right]_j[-4G(l_j-k_j)(k_j-l_j)+Gl_j(l_j-1)]\cos^l\beta e^{i(2k-l)\theta}\ . \tag{A9}$$

APPENDIX B: DERIVATION OF THE EXPRESSION OF THE REDUCED MATRIX ELEMENT OF THE TWO NUCLEON TRANSFER REACTION

To calculate the matrix element $\langle\phi_{T_fN+1}(\Delta_f)\mid A^\dagger_{j\mu}\mid\phi_{T,N}(\Delta_1)\rangle$, which is denoted by $\mathcal{S}_j(T_fN+1,\,T_iN)$, we use the projection integral (2.9) and (2.10) to obtain

$$\mathcal{S}_j(T_fN+1,\,T_iN)=C_{T_f}C_{T_i}\sum\int d\Omega'\int d\theta'\int d\Omega\int d\theta\,D^{T_f}_{M_{T_f}K'}(\Omega')D^{T_i^*}_{M_{T_i}K}(\Omega)e^{i(N+1)\theta}e^{-iN\theta'}$$
$$\times\langle\phi_f\mid\hat R^{\,\dagger}(\Omega)\hat S^{\,\dagger}(\theta)A_{j\mu}\hat S(\theta)\hat R(\Omega)\mid\phi_i\rangle\ , \tag{B1}$$

with

$$C_{T_f}C_{T_i}=\frac{2T_f+1}{16\pi^3}\frac{2T_i+1}{16\pi^3}\mid\langle\phi_f\mid\hat P_{T_f}\hat P_{N+1}\mid\phi_f\rangle\langle\phi_i\mid\hat P_{T_i}\hat P_N\mid\phi_i\rangle\mid^{1/2}\ . \tag{B2}$$

Notice, that to obtain $\mathcal{S}_j(T_fN+1,\,T_iN)$, the operator $A^\dagger_{j\mu}$, as the Hamiltonian operator, does not commute with $\hat R(\Omega)$ and $\hat S(\theta)$. If we use the following properties,

$$\hat S^{\,\dagger}(\theta')A^\dagger_{j\mu\mu}=e^{-i\theta}A_{j\mu}\hat S(\theta'),\quad \hat R^{\,\dagger}(\Omega)A^\dagger_{j\mu}=\sum_{\mu=0,\pm1}A^\dagger_{j\nu}D_{\nu\mu}(\Omega)\hat R(\Omega)\ , \tag{B3}$$

and with help of the properties of the Wigner D matrix $\hat R(\Omega)$ and $\hat S(\theta)$ operators, we have simply

$$\mathcal{S}_j(T_fN+1,\,T_iN)=C_{T_f}C_{T_i}\sum\int d\Omega''\int d\theta''(-)^{M_{T_f}-K}\begin{pmatrix}T_f&1&T_i\\-M_{T_f}&\nu&M_{T_i}\end{pmatrix}\begin{pmatrix}T_f&1&T_i\\-K&\mu&\xi\end{pmatrix}$$
$$\times D^{T_i^*}_{\xi K}(\Omega'')e^{-iN\theta}\langle\phi_f\mid A^\dagger_{j\mu}\hat R(\Omega'')\hat S(\theta'')\mid\phi_i\rangle\ . \tag{B4}$$

This result can be identified with the reduced matrix element of $\mathcal{S}_j(T_fN+1,\,T_iN)$ and, with the help of Clebsch-Gordan coefficient properties, we have

$$\langle\phi_{T_fN-1}(\Delta_f)\|A^\dagger_j\|\phi_TN(\Delta_i)\rangle=C_{T_f}C_{T_i}\sum\int d\theta\int d\Omega\,e^{-iN\theta}(-)^{T_f-K}\begin{pmatrix}T_f&1&T\\-K'&\nu&K'-\nu\end{pmatrix}$$
$$\times D^{T_i^*}_{K-\nu_1K}(\Omega)\langle\phi_f\mid A^\dagger_{j\nu}\hat R(\Omega)\hat S(\theta)\mid\phi_i\rangle\ . \tag{B5}$$

Then, using the definition (2.5) of the pair creation operator, we have

$$\langle\phi_{T_fN+1}(\Delta_f)\|A^\dagger_j\|\phi_{TN}(\Delta_i)\rangle$$
$$=C_{T_f}C_{T_i}\sum_{KK'\nu}\sqrt{1/2\Omega_j}(-)^{T_f-K}\begin{pmatrix}T_f&1&T_i\\-K'&\nu&K'-\nu\end{pmatrix}$$
$$\times\int d\theta\,e^{-iN\theta}\int d\Omega\,D^{T_i^*}_{K'-\nu}(\Omega)n^{fi}(\Omega,\theta)\left\{(\sigma^{fi}_{\pi\pi})^j\delta_{\nu_1}+(\sigma^{fi}_{\nu\nu})^j\delta_{\nu-1}+\frac{1}{\sqrt2}[(\sigma^{fi}_{\pi\nu})^j+(\sigma^{fi}_{\nu\pi})^j]\right\}\ , \tag{B6}$$

where $(\sigma^{fi})^j$ are the matrices from pairing tensors given in (2.15).

As in the case of energy, we are going to treat in the present paper only the axial symmetric case. Substituting in the above expression the matrix elements of $(\sigma^{fi})^j$, after little algebraic manipulation we may get expression (6.7).

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