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Total reaction cross sections of 50 and 65 MeV pions on nuclei

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Total reaction cross sections have been measured for 50 and 65 MeV $\pi^{\pm}$ on C, O, ^{18}O, S, Ca, and Zr. The motivation for this experiment is to obtain cross sections to act as constraints in optical model fits to elastic scattering data. Measurements using the "poor geometry" transmission method were made for exceptionally small angles with the aim of improving the accuracy of the extrapolation to zero solid angle. At these small solid angles the muon cone from pion decay contributes significantly and its effects were explicitly included. The accuracies of the elastic correction are evaluated by using error matrix techniques and an additional systematic error is included.

I. INTRODUCTION

Detailed experimental results for the elastic scattering of low energy pions by nuclei are now available for many target nuclei thanks to the operation of the various "meson factories." The quality of those data is usually good, extending typically over the angular range of $\approx 25°-140°$, which makes χ^2 fits of an optical potential quite feasible, as is the case with conventional low to medium energy nuclear particles (protons, deuterons, alpha particles). Total and total reaction cross sections are two observable quantities that are readily calculable from the optical potential or from the phase shifts obtained from fits to elastic scattering data. Detailed data for the elastic scattering with conventional particles uniquely determine the calculated total and total reaction cross sections and these observables, when measured experimentally, supply very little information on the optical potential that is not available from the elastic scattering itself.

The situation regarding total cross sections (σ_T) and total reaction cross sections (σ_R) for low energy pions on nuclei is very different from that for conventional particles. The interaction between low energy pions and nuclei is usually described in terms of the Kisslinger,[1] Ericson-Ericson[2] (EE), or the MSU (Ref. 3) potential where the last two contain about 13 adjustable parameters. Fits to the most extensive data for the elastic scattering of pions do not uniquely[3] determine all the parameters and it was shown by Seki and collaborators[4] that there exist correlations between several of the coefficients of the potential. Under such circumstances the two additional observables σ_T and σ_R provide useful information on the potential.

"Model-independent" analysis[5] of the elastic scattering for low energy pions by nuclei revealed that indeed the data determine only five to six parameters of the potential. Furthermore, σ_R could act as a constraint in optical model fits. This uncommon property was linked with the unusual shape of the local equivalent potential which results from the Kisslinger potential, or from the EE and MSU potentials (which are its more recent forms).[5] A large sensitivity of σ_T and σ_R to details of calculations using those potentials had been observed by several authors, in contrast to the experience with conventional particles. Sternheim and Yoo[6] pointed out that neutron density distributions in nuclei could be better determined if total cross sections and elastic scattering were analyzed together. Albanese *et al.*[7] stated, after analyzing very extensive data for the elastic scattering of pions, that accurate measurements of σ_R could distinguish different theories. The importance of reaction cross sections was stressed by the MSU (Ref. 3) group and Masutani and Yazaki[8] reported difficulties in fitting simultaneously angular distributions and total cross sections. Hence there is a case for measurements of total or total reaction cross sections for pions on nuclei to supplement existing data for the elastic scattering and to act as constraints in optical model analysis of these data.

There have been previous measurements of σ_T and σ_R for pions on nuclei[9,10] but at higher energies, mostly over the (3,3) resonance region. Very little σ_T and σ_R data are available at truly low energies (50–80 MeV) which is an interesting region where the link with pionic atoms (at zero energy) can be established. Several previous experiments were aimed at measuring the cross section for true absorption, thus excluding the inelastic scattering from the measured values. In the present context we are interested in σ_T and σ_R in the optical model sense, if these quantities are to act as constraints in optical model fits. In the case of σ_R it means that the inelastic scattering should be included.

This paper is concerned with the experimental details and analysis procedures for measuring reaction cross sections. The quantity we chose to measure was σ_R and not σ_T because σ_R could be measured to a higher experimental accuracy. This accuracy is due to the need to apply only elastic corrections to the partial results whereas in the case of measurements of σ_T it is necessary to explicitly apply[11] Coulomb nuclear interference corrections. A novelty in the present experiment is that we have measured pion transmissions to unusually small solid angles in order to extrapolate over shorter angular range. At the present energies that means measuring transmissions within the decay muon cone.

Section II describes the experimental procedure and Sec. III deals with the two main corrections to the raw data—the muon cone correction and the elastic correction. Another correction which is of an experimental origin, the back-scattering correction, is also discussed. The results are presented in Sec. IV with some examples of σ_R acting as a constraint in an optical model analysis of elastic scattering of pions. The full utilization of σ_R in optical model analyses will be described in a separate publication.

II. EXPERIMENT

Reaction cross sections were measured using the conventional "poor geometry" transmission technique.[9,12] The cross section for a pion not to be counted by a detector subtending a solid angle Ω at the target is

$$\sigma_{\rm att}(\Omega)=\sigma_{\rm abs^*}+\int_{\Omega}^{4\pi}\left[\frac{d\sigma}{d\Omega'}\right]_{\rm non}d\Omega' + \int_{\Omega}^{4\pi}\left[\frac{d\sigma}{d\Omega'}\right]_{\rm el}d\Omega' , \qquad (1)$$

where $\sigma_{\rm abs^*}$ is the cross section for all processes where an incident pion does not produce a detectable particle (in our case neutrons for example), $(d\sigma/d\Omega')_{\rm non}$ is the differential cross section for detectable particles other than those elastically scattered, and $(d\sigma/d\Omega')_{\rm el}$ is the differential cross section for elastic scattering.

Assuming the value of the integral for the elastic scattering to be known, then applying this elastic correction to the measured attenuation cross section $\sigma_{\rm att}(\Omega)$, one may define

$$\sigma_R(\Omega)=\sigma_{\rm att}(\Omega)-\int_{\Omega}^{4\pi}\left[\frac{d\sigma}{d\Omega'}\right]_{\rm el}d\Omega' = \sigma_{\rm abs^*}+\int_{\Omega}^{4\pi}\left[\frac{d\sigma}{d\Omega'}\right]_{\rm non}d\Omega' . \qquad (2)$$

The value of $\sigma_R(\Omega)$ for $\Omega=0$ is, by definition, the total reaction cross section in the optical model sense, i.e., the cross section for all processes except elastic scattering. The experimental problem is to extrapolate $\sigma_R(\Omega)$ to $\Omega=0$ and there could be a problem for such an extrapolation $\Omega\rightarrow 0$ if the elastic scattering were also involved. However, as its effects are removed from the measured $\sigma_{\rm att}(\Omega)$, one has to consider only the integral of the nonelastic processes. A sharp forward nonelastic cross section could invalidate this extrapolation procedure. However, this is most unlikely for $\theta<15°$ as can be seen from arguments based on the Fermi momentum in nuclei compared to the incident momentum, or from simple reaction models such as Coulomb excitation and diffraction inelastic scattering. Measurements of inelastic scattering of pions indeed show[13] that the differential cross section is well behaved for small angles and this is the case also for knockout of nucleons.[14]

This extrapolation procedure is obviously more accurate as the solid angles for which measurements are done are close to zero. However, for such small solid angles, the elastic correction and the muon cone correction (see below) become too large and a careful handling of these corrections is necessary in order to measure at small solid angles.

The experiment was performed on the $M13$ pion channel at the TRIUMF cyclotron. Both positive and negative pion beams were obtained with a typical momentum resolution of $\pm 0.4\%$. The thickness of the targets used in the present experiment had to be small in order to keep the energy losses in the target small compared to the pion energy. Coulomb multiple scattering had to be kept at a reasonable level as well. As a result, the transmission of pions through the targets was about 99.5%. That placed stringent requirements on the stability of the electronics and on the experimental procedure, where transmissions had to be measured to an accuracy of better than 10^{-4}.

Figure 1 shows the beam defining system consisting of two plastic scintillators B_1 and B_2 and an annular veto detector (with a hole radius of 1.4 cm) up-stream of the target. A time-of-flight (TOF) measurement for pion flight times between the production target and B_2 was used to discriminate against muons and electrons. A beam event was thus defined as

$$B=[{\rm TOF}\cdot(B_1\cdot B_2)]\cdot\bar{V} , \qquad (3)$$

with $\bar{V}$ indicating anticoincidence with the veto detector. Two identical transmission detectors S_1 and S_2 (with a radius of 10.16 cm) were used in coincidence. It was found essential to use two detectors in coincidence in order to reduce background. We preferred not to use a stack of transmission detectors[11,12] in favor of having identical conditions at each solid angle. This was possible because of the short time required for each transmission measurement (see below). An event of a detectable transmitted particle was defined as $S=B\cdot(S_1\cdot S_2)$. A smaller efficiency detector E, placed downstream of S_2,

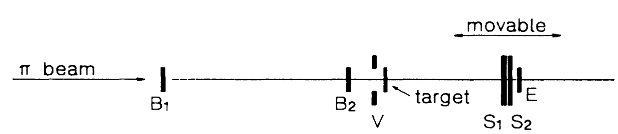

FIG. 1. Outline of the experimental setup showing beam defining detectors (B_1 and B_2), veto detector (V), two transmission detectors (S_1 and S_2), and efficiency detector (E).

was used to monitor the efficiency of the S detectors and their associated electronics. This efficiency was defined as

$$\eta=\frac{(B\cdot S)\cdot E}{B\cdot E} \tag{4}$$

and was found to be high ($\sim$99.8%) and stable throughout the experiment. The transmission was then defined as

$$T=\frac{S}{\eta B} . \tag{5}$$

It was found necessary to use throughout the system only twofold coincidences and to cascade them to form higher order coincidences. Otherwise, the stability was inadequate. In addition, twin scalers were used and their readings were compared for each measurement. The targets used were C (two thicknesses differing by a factor of 2), CH_2, H_2O, $H_2{}^{18}O$, S, Ca, and Zr. The thicknesses of the targets were about 0.5 g/cm^2.

The transmission and efficiency detectors were mounted on a cart whose position relative to the target determined the solid angle. A 12 position target wheel was used, and both solid angle and target position were changed by a remote control. The short counting time (several minutes) for each point helped to reduce effects due to long term changes in beam conditions. Measurements of targets and their corresponding empty frames were interleaved in such a way that frequent monitoring of beam stability was possible. Most measurements were repeated at least twice and a very good reproducibility of the results, well within statistics, was observed.

The pion energies at the channel were 54$\pm$0.2 and 68.5$\pm$0.2 MeV, so that after energy losses in the B detectors and in the 140 cm air gap, the energies at the center of the target were 50 and 65 MeV, respectively. For each pion energy and charge sign, eight solid angles in the range of 0.12 to 1 sr were measured.

The total number of beam events was about 7×10^6 in each measurement which ensured a statistical error of 2% to 4%. (See Ref. 12 for a discussion of statistical errors in such circumstances of correlated events.) With a B count rate of 3×10^4 sec^{-1} it was possible to measure 8–10 transmission values per hour. Dead time posed no problem at these counting rates with this 100% duty cycle accelerator cyclotron beam. The scalers were read into a PDP11 computer for an on-line analysis.

III. DATA REDUCTION

In order to obtain $\sigma_R(\Omega)$ [Eq. (2)] from the measured $\sigma_{att}(\Omega)$ three corrections have to be made: the muon cone correction, the correction due to elastic scattering, and a correction due to backscattering of particles into the veto detector.

A. Muon cone correction

At low pion energies and small solid angles, the S detectors are usually within the cone of muons from decays of pions in the beam and a fraction of the muons are therefore not detected, even without any target in. When the target is placed in the beam, the muon cone is opened up slightly due to the energy loss and multiple scattering (MS) of the pions in the target. In addition, muons which are produced upstream are scattered by the target. Since the S detectors do not distinguish between muons and pions, all the above processes cause additional transmission losses for the muons when the target is in, thus leading to spurious $\sigma_{att}(\Omega)$.

In order to correct for these effects, the differences between the muon transmissions when the target is out and when the target is in (Fig. 2) were calculated in two ways: (i) using a Monte Carlo simulation program and (ii) by an approximation method using interpolation and averaging procedures.

The Monte Carlo program traces the trajectories of all muons with the only approximation of assuming a zero width for the pion beam. For each muon, the program generates four parameters: the point of the pion decay, the MS angle of the relevant particle in the target,[15] and the two angles defining the direction of the muon, which is taken to be isotropic in the pion rest frame. These parameters are then used to determine whether or not the muon escapes the S detector. About 10^7–10^8 Monte Carlo pions are required in order to achieve a statistical accuracy of a few percent in the muon cone correction.

The approximation method involves two calculational steps. In the first one, muon transmission losses are calculated analytically including effects due to pion energy loss in the target but excluding MS. In the second step, this transmission-loss function is averaged over the MS angular distribution and over the target size with respect to the detector rim.

Figure 3 shows the muon cone corrections for 50.7 MeV pions on a Ca target calculated by the two methods. The bump, which is concentrated at the muon cone angle region, is due to MS of muons which are produced upstream of the target. In the calculation of this MS process an approximation is made where the distribution of muon energies is approximated by only two

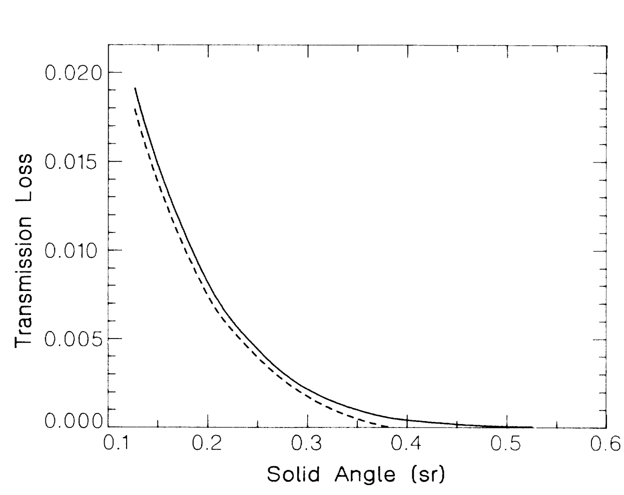

FIG. 2. Transmission losses due to muons missing the transmission detectors. The increased losses with the target in appear as a spurious cross section which must be corrected for.

values (high and low), according to the kinematics of the pion decay. This is the source of the discrepancy between the results of the two calculational methods in the above-mentioned bump. This discrepancy has a negligible effect on the extrapolated σ_R value since it increases the error on only the values of $\sigma_R(\Omega)$ at midrange solid angles spanned by this experiment. All error contributions are compared in Sec. IV.

The calculations also reproduced the experimental muon transmission losses observed without a target, as mentioned above. In practice, the second calculational method was used because it was about 10^3 less computer time consuming than the first. The corrections due to the muon cone were found to be essential at the smallest angles. Their contribution to the final errors was, however, small, compared to the error caused by the corrections due to elastic scattering.

B. Correction due to elastic scattering

As discussed above the experimental cross sections derived from the measured transmissions must be corrected by subtracting the cross sections for elastically scattered pions missing the transmission detector S. This is obtained by integrating the differential cross section for elastic scattering over the required angular range and the question arises as to how well this cross section is known. The present measurements are aimed at providing values of the total reaction cross sections to act as constraints in optical model analyses of elastic scattering, hence an experimental angular distribution for the elastic scattering is assumed to be available. The interpolation between measured values and the extrapolations at both ends of the range of integration were carried out in the present work with the help of an optical potential. This can be considered the most suitable method, thanks to the way the underlying physics is built into it.

Another possible way of using the experimental differential cross sections in the above integration is simply by a fit of one kind or another to a polynomial, either explicitly or implicitly. However, the important Coulomb effects at small angles are not taken into account properly in that way when the process involves extrapolations to angles smaller than those measured. In the optical model approach a best fit to the elastic scattering data is first made, where the Coulomb scattering amplitude certainly is added to the nuclear one. This potential is then used to predict the integrand of Eq. (2). For this measurement of reaction cross sections one subtracts elastic (Coulomb and nuclear) integrated cross sections and therefore no explicit use is made of the Coulomb or nuclear amplitudes separately. Rather their sum (squared) is used as represented by the potential which best fits the data. The situation is different when total cross sections are measured. In that case the Coulomb-nuclear interference must also be subtracted, causing large uncertainty.

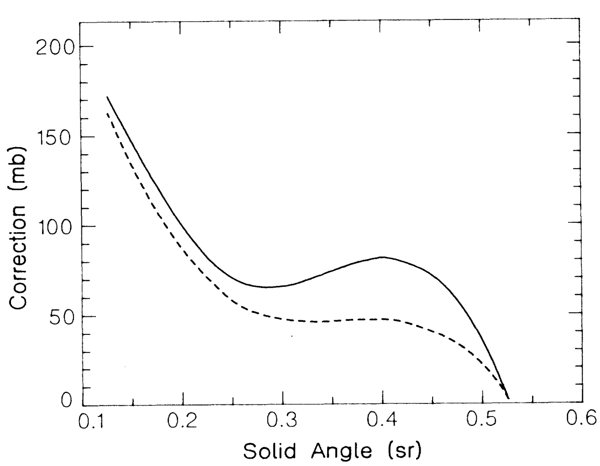

FIG. 3. Muon cone correction for 50 MeV pions on Ca target. Continuous curve—Monte Carlo method; dashed curve—numerical averaging method (see text).

The reliability of the above procedure and the error introduced by it into the final values for the reaction cross section are basic to the success of the "poor geometry" transmission method. Since the elastic correction is calculated with the help of optical potentials obtained from best fit to the data, the uncertainties are also estimated using similar techniques. This approach takes into account the quality of the data that forms the basis for the elastic correction.

Assume the optical potential contains N adjustable parameters a_i which are obtained by minimizing the χ^2 function $\chi^2(a_1, \ldots, a_N)$. The elastic correction for any angle θ_k is then a function of the parameters a_i, say $F_k(a_1, \ldots, a_N)$, calculated for the values a_i which minimize χ^2. The uncertainty in F_k due to the uncertainties δa_i is, therefore

$$\delta F_k = \left[\sum_{ij} \frac{\partial F_k}{\partial a_i} \frac{\partial F_k}{\partial a_j} \overline{\delta a_i \delta a_j} \right]^{1/2} , \tag{6}$$

where $\overline{\delta a_i \delta a_j}$ are obtained from the covariance matrix of the χ^2 fit,[5]

$$\overline{\delta a_i \delta a_j} = 2(M^{-1})_{ij} , \tag{7}$$

where

$$M_{ij} = \frac{\partial^2 \chi^2}{\partial a_i \partial a_j} \tag{8}$$

is evaluated at the best-fit point. The first order partial derivatives $\partial F_k / \partial a_i$ are obtained numerically by repeating calculations of the elastic correction F_k with the a_i values shifted.

The uncertainties in the elastic corrections were calculated using the above method, showing typical errors for the most forward angles of 2–3 % for the C target and about 1% for Zr, excluding systematic errors. Examining the nuclear and the Coulomb scattering amplitudes reveals that the latter are relatively more important as Z increases. Excluding from the χ^2 fit some forward angle data points led to very small increases in the errors of the elastic correction for the Zr target but to very large ones for the C target.

Figure 4 shows an example of a fit to elastic scattering data and the resulting elastic correction, obtained with

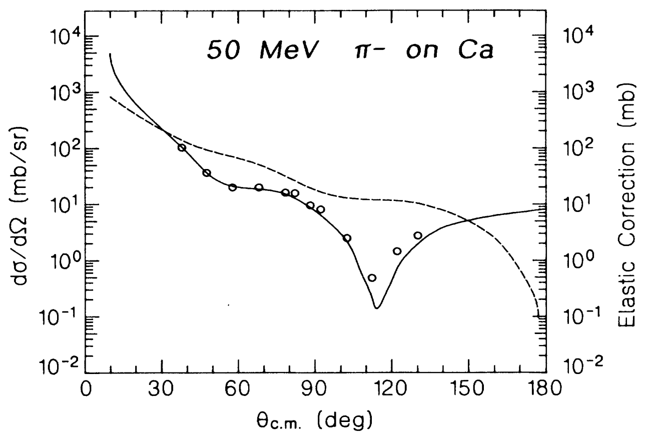

FIG. 4. Fit to angular distribution of elastic scattering using an MSU-type potential with adjusted parameters (continuous curve) and the elastic scattering correction (dashed curve).

an MSU-type[3] potential. Fits were also made with a "model-independent" potential[5] which led to almost identical results for the elastic corrections.

In order to take into account possible systematic errors in the elastic scattering data we have arbitrarily assumed that for each angle θ_k the uncertainty of the elastic correction was 10% of the corresponding calculated value. The effect of systematic error on the elastic scattering data to the final reaction cross section determination was studied by varying this error from 0% to 15%. The reaction cross section value changed only marginally but the reaction cross section error varied linearly with systematic elastic scattering correction error at $\frac{1}{2}$% reactions cross section error per 1% systematic elastic scattering error. All error contributions are summarized in Sec. IV.

The elastic correction must be self-consistent. It is calculated with the help of an optical model fit which also predicts the value of the total reaction cross section $\sigma_R^{(\text{calc})}$. This value is then compared with the final experimental reaction cross section $\sigma_R^{(\text{exp})}$ and if they differ by more than the estimated uncertainty the latter can then be used as a constraint in optical model fits to the elastic scattering data which, in turn, predict a new set of elastic corrections. These new elastic corrections are usually different from the ones used in the previous step and thus lead to a different value of $\sigma_R^{(\text{exp})}$. When the difference is significant one may have to continue the process until convergence is achieved. However, in most cases no more than a single iteration was required. This self-consistency procedure is particularly useful in identifying possible systematic errors in the experimental differential cross section which are not taken into account explicitly in the present analysis.

C. Correction due to backscattering

This correction is specific to the present experimental arrangement and is a consequence of the use of a veto detector upstream of the target. An acceptable pion event is conditional on anticoincidence with the veto detector, signaling an identified pion which passed through the hole in that detector. Genuine pion events that are associated with a backward going charged particle (of any kind) which is detected by the veto detector will therefore be vetoed. Such vetoed events consist of a part of the cross section which is to be measured and the nonelastic component of this "missing" cross section is, in fact, a part of the total reaction cross section that we wish to measure.

The above effect can be eliminated by placing the target inside the hole of the veto detector[9] thus causing the latter to have a vanishingly small solid angle at the target. For mechanical reasons, associated with the remotely operated target holder, the veto detector was placed upstream of the target in the present experiment and therefore a correction due to the self-vetoing by backward going particles had to be made.

This correction was made empirically in the following way. Remembering that a beam event is defined as $B=[(B_1 \cdot B_2)\cdot\text{TOF}]\cdot\bar{V}$, then the transmission of pions through the hole of the veto detector can be defined as

$$T_h=\frac{[(B_1 \cdot B_2)\cdot\text{TOF}]\cdot\bar{V}}{(B_1 \cdot B_2)\cdot\text{TOF}} .$$

TABLE I. Partial quantities and errors in the determination of total reaction cross section. The 50 MeV data for π^+ and π^- on carbon at $\Omega=0.213$ sr is taken as an example.

	π^+			π^-		
Quantity	Ratio of quantity to final σ_R	Error of quantity (%)	Contribution to error of $\sigma_R(\Omega)$ (%)	Ratio of quantity to final σ_R	Error of quantity (%)	Contribution to error of $\sigma_R(\Omega)$ (%)
$\sigma_{\text{att}}(\Omega)$	1.35	1.0	1.3	1.67	0.6	1.0
Elastic correction	0.10	2.0[a] 10.0[b]	0.1[a] 4.0[b]	0.67	2.0[a] 10.0[b]	1.3[a] 6.7[b]
Muon cone correction	0.11	10.0	1.1	0.11	10.0	1.1
Backscattering correction	0.11	15.0	1.6	0.04	29.0	1.1

[a]Error-matrix contribution (see text).
[b]Systematic error.

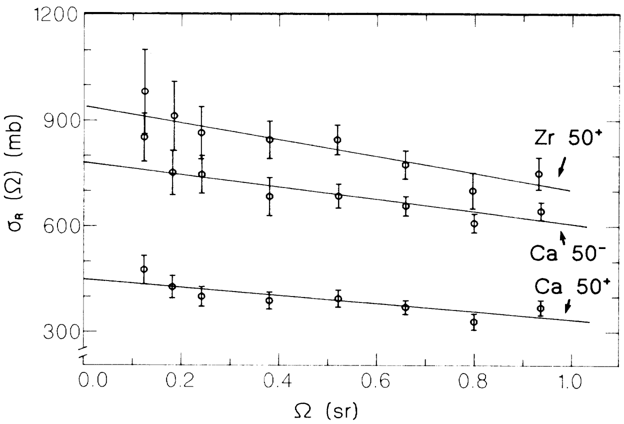

FIG. 5. Examples of a linear fit to $\sigma_R(\Omega)$ vs Ω and extrapolations to zero solid angle.

This ratio was available for all measurements and indeed it was found that the T_h was slightly reduced when a target was placed in the beam, compared to its value without a target. That reduction in transmission was due to the self-vetoing effect and for each target the cross section for this effect could easily be calculated. This cross section was added to the experimental reaction cross section as it is precisely the "missing" cross section. It is important to note that this correction is independent of the position of the transmission detector S and therefore averaging over many transmission measurements could be made leading to acceptably small errors for this correction. The magnitude of the backscattering correction was varied in several test cases by changing the position of the veto detector relative to the target. Although the backscattering correction changed by more than a factor of 2, the sum of this correction and $\sigma_{att}(\Omega)$ remained unchanged.

IV. RESULTS AND DISCUSSION

For each pair of transmission values for a target and its corresponding empty holder an attenuation cross section was derived. This cross section was then corrected for the three effects discussed in the previous section. As discussed in Sec. II, the statistical accuracy of each cross section value was in the range of 2–4 %. Other sources of error were the various corrections discussed above and the corresponding errors were combined quadratically with the statistical error. The error due to the elastic correction was assumed to be 10% of the correction itself as discussed in Sec. III B and the error due to the muon-cone correction was estimated from the scatter of calculated results, and was usually small compared with the other errors. The error of the correction due to the backscattering of particles into the veto detector was taken from the variance of the average cross section for self-vetoing by the target. This error was combined quadratically with the error of the final value of σ_R, discussed below. Table I summarizes the importance of these contributions.

The above procedure led, after applying the various corrections, to experimental values for $\sigma_R(\Omega)$ [Eq. (2)] together with their estimated uncertainties. As mentioned in the Introduction $\sigma_R(\Omega)$ is expected to extrapolate smoothly to $\Omega=0$ because the elastic scattering has been removed from it. Figure 5 shows examples of such extrapolations, where the straight lines were obtained by χ^2 fits. Linear dependence on the solid angle means a constant value of the differential cross section for nonelastic processes and this is not necessarily the expected behavior. Therefore, higher order polynomials were also tried but these were not really required by the data. Even for a second order polynomial the extra term was not well determined. In any case the extrapolated reaction cross sections agreed with those obtained from the linear fits. The χ^2 procedure also gave the estimated uncertainties of the fit parameters.

Table II summarizes the values of the total reaction cross section measured in this experiment. These values obviously depend on the data for elastic scattering used to calculate the elastic correction. In particular, the 10% systematic error assumed for the elastic scattering corrections is by far the largest error contribution as described in Table I. Some of the results, e.g., for π^- on O and Zr, must be regarded as preliminary because only unpublished data for the corresponding elastic scattering were available. It is interesting to note that although the errors of individual $\sigma_R(\Omega)$ points are of the order of 10%, the accuracy of the extrapolated σ_R is usually better as a consequence of the linear fit. Only results for C and Ca are given at 65 MeV because the elastic scattering data are not available for the other targets. As discussed in the Introduction, the motivation for the present measurements was the possible use of total reaction cross sections as constraints in optical model analyses of the elastic scattering of pions by nuclei. Examples of the effectiveness of such a constraint are shown in

TABLE II. Total reaction cross sections (mb). Reference numbers give the reference to the elastic scattering data used to calculate the elastic correction.

	Pion energy and charge			
Target	50^+ (Ref.)	50^- (Ref.)	65^+ (Ref.)	65^- (Ref.)
C	150±15 (16,18)	193±10 (16,19)	201±16 (17)	251±20 (17)
O	166±19 (18)	242±21 (19)		
^{18}O	179±31 (20)	272±20 (20)		
S	393±24 (16)	669±35 (16)		
Ca	439±36 (21)	781±42 (21)	563±43 (22)	772±53 (22)
Zr	949±61 (18)	1869±147 (19)		

TABLE III. Parameter values from χ^2 fits to elastic scattering of 50 MeV pions. (All data are from Ref. 16.)

Target	Charge	$\sigma_R^{(\mathrm{exp})}$ (mb)	$\sigma_R^{(\mathrm{calc})}$ (mb)	χ^2/F	Reb_0 (m_π^{-1})	Imb_0 (m_π^{-1})	Rec_0 (m_π^{-3})	Imc_0 (m_π^{-3})
C	+		137	2.9	-0.022 ± 0.039	-0.027 ± 0.006	0.199 ± 0.022	0.005 ± 0.070
C	+	150 ± 15	140	2.5	-0.021 ± 0.008	0.027 ± 0.003	0.198 ± 0.005	0.008 ± 0.016
C	−		216	0.3	-0.04 ± 0.05	-0.02 ± 0.08	0.215 ± 0.009	0.043 ± 0.014
C	−	193 ± 10	193	0.6	-0.026 ± 0.001	0.006 ± 0.003	0.200 ± 0.001	0.023 ± 0.003
S	+		308	2.2	-0.036 ± 0.007	-0.027 ± 0.015	0.187 ± 0.009	0.027 ± 0.011
S	+	393 ± 24	372	1.1	-0.038 ± 0.003	0.001 ± 0.005	0.204 ± 0.002	0.028 ± 0.007

Table III. The table shows parameter values for an MSU-type potential[3] obtained from χ^2 fits to the angular distribution of elastically scattered 50 MeV pions by C and ^{32}S targets. The parameters were kept at their pionic atom values[23] and only the complex one nucleon parameters b_0 and c_0 were varied. One set of parameters is obtained from a conventional fit whereas the second is obtained by using the σ_R value as a constraint. This is done simply by adding another term to the usually defined χ^2, namely

$$\chi_R^2 = \left[\frac{\sigma_R^{(\mathrm{calc})} - \sigma_R^{(\mathrm{exp})}}{\Delta\sigma^{(\mathrm{exp})}} \right]^2$$

in obvious notation. As is easily seen, this extra term acts as an efficient constraint, helping to reduce significantly the errors of the parameters obtained in χ^2 fits to elastic scattering.

In conclusion we have demonstrated the feasibility of measuring total reaction cross sections for 50 and 65 MeV pions on light and medium weight nuclei to an accuracy of 5–10%. Transmission measurements have been made at angles as small as 12° (0.13 sr) where the muon cone corrections are important but can be calculated reliably. The elastic corrections too do not introduce unacceptable errors and this error can be determined explicitly from the elastic scattering fitting procedures. The importance of total reaction cross sections as constraints in optical model analyses of pion scattering has also been demonstrated.

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