Δ production in proton-nucleus scattering at intermediate energies

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(Received 29 December 1986)

The present work is a study of Δ production in proton-nucleus collisions at intermediate energies. The theory is based on the Glauber multiple scattering theory. For the elementary $NN \rightarrow \Delta N$ process, we employ the amplitude decomposition by Silbar *et al.* and by Auger *et al.* The numerical values of the parameters involved are taken from the work of Kloet and Silbar. Two specific examples of nuclear targets are examined: Δ^{++} production in ^{3}He and ^{6}Li at 800 MeV. The effects of multiple scattering corrections on selected spin observables are studied in detail. In light of our results we discuss the possibilities of using nuclear reactions to obtain information on specific production amplitudes and on Δ propagation through the nuclear medium.

I. INTRODUCTION

The production of the nucleon isobar Δ(1232) in proton-nucleus interactions has up to now received little interest. On the experimental side there are a few investigations from Saturne, measuring the differential cross sections for Δ^{++} production on ^{6}Li (Ref. 1) and on ^{3}He (Ref. 2). On the theoretical side there are the investigations by Hennino[3] and by Jain[4] analyzing the Saturne data on ^{6}Li. These authors employ a meson exchange model to describe the elementary reaction $NN \rightarrow \Delta N$ and simple assumptions about the multiple scattering corrections.

In view of the importance of the Δ resonance in intermediate energy physics, it is surprising that so little interest has been attached to these studies. Recent analysis of experimental data taken at Argonne[5] constitutes a first study of the spin dependence of the elementary $N \rightarrow \Delta$ transition based on the production asymmetry. Some information is also contained in $NN \rightarrow NN\pi$ reactions.[6] However, we do not know of any systematic attempt trying to determine the structure of the elementary Δ production amplitude from interactions with nuclear targets.

The purpose of the present investigation is to draw attention to this peculiar situation, and to point out that valuable information can be obtained from nuclear Δ production. Since 16 amplitudes are needed to describe the general spin structure of the reaction $NN \rightarrow \Delta N$, there are a great variety of observables. An experimental determination of all of them seems at the moment ruled out. In this situation, production by nuclear targets has many advantages, since the spin and isospin quantum numbers of the target can be varied. In other words, nuclear targets could be used as amplitude filters.

It is quite obvious that multiple scattering effects can substantially influence the final results, and even invalidate naive predictions based on the single scattering approximation. Consequently, they must be handled with care. The appropriate framework for doing this is the multiple scattering model of Glauber.[7] However, we have found it convenient to start from an expression slightly different from the conventional one. It expresses the nuclear profile function as an expansion in terms of iterated commutators between the elementary production amplitude and the NN and ΔN elastic scattering amplitudes. Hence, it is essentially an expansion in powers of the difference between the NN and ΔN elastic amplitudes.

An important uncertainty is associated with the Δ propagation within the nucleus. In the absence of experimental information, the golden rule is to assume similar ΔN and NN scattering amplitudes. The virtue of this approximation is a simplification of the multiple scattering calculations, but it would be valuable if we could find a possibility to check this ansatz.

As an application of our formalism we have considered Δ production by light nuclei. Detailed calculations are performed for the reactions $p + {}^6\text{Li} \rightarrow \Delta^{++} + {}^6\text{He}$ and $p + {}^3\text{He} \rightarrow \Delta^{++} + {}^3\text{H}$.

For the numerical illustration we have chosen a model for the $NN \rightarrow \Delta N$ amplitude due to Kloet and Silbar,[8] based on unitarized iterated pion exchange. In this model all 16 amplitudes are nonvanishing, i.e., they simulate the full structure of the real production amplitude. This is an extremely important point. Models based on mixtures of π and ρ exchange predict vanishing asymmetries for the target and projectile nucleons, even though they reproduce the size of the differential cross section.

II. ELEMENTARY AMPLITUDES

In this section we shall specify our notation concerning the elementary amplitudes. There are three of them, corresponding to the production step $NN \rightarrow \Delta N$ and the elastic scattering steps $NN \rightarrow NN$ and $\Delta N \rightarrow \Delta N$.

There is only one possible isospin amplitude for the reaction $NN \rightarrow \Delta N$. We choose its isospin factor as

$$N_T = -\frac{1}{\sqrt{3}}\mathcal{T}_0\cdot\tau_1 , \qquad (2.1)$$

where index 1 refers to the target nucleon and index 0 to the projectile nucleon. The vector transition operator $\mathcal{T}_0$ is normalized so that

$$\langle \Delta^{++}\mathrm{n} \mid N_T \mid \mathrm{pp}\rangle = 1 .$$

There are 16 independent amplitudes needed to describe the reaction $\mathrm{pp}\rightarrow\Delta^{++}\mathrm{n}$. A convenient spin space decomposition of the production amplitude has been given by Silbar *et al.*[8] and by Auger *et al.*[9] We adhere to the choice of Auger *et al.*, i.e.,

$$M^{\Delta N}(\mathbf{k}_\Delta,\mathbf{k}) = \sum_{i=1}^{8} [f_i(q)Q_i(\hat{\boldsymbol{l}},\hat{\mathbf{m}}) + g_i(q)P_i(\hat{\boldsymbol{l}},\hat{\mathbf{m}})] , \qquad (2.2)$$

where $\mathbf{k}$ and $\mathbf{k}_\Delta$ are the projectile and Δ momenta and $(\mathbf{q},\Delta_\parallel) = \mathbf{k}-\mathbf{k}_\Delta$ the momentum transfer. When ambiguity could arise we write $\mathbf{q}_\perp$ for the orthogonal component $\mathbf{q}$ of the momentum transfer.

Furthermore, we define a triad of unit basis vectors

$$\hat{\boldsymbol{l}} = \hat{\mathbf{k}}, \quad \hat{\mathbf{m}} = -\hat{\mathbf{q}}_\perp, \quad \hat{\mathbf{n}} = \hat{\boldsymbol{l}}_\wedge\hat{\mathbf{m}} , \qquad (2.3)$$

in terms of which

$$P_i(\hat{\boldsymbol{l}},\hat{\mathbf{m}}) = Q_i(\hat{\boldsymbol{l}},\hat{\mathbf{m}})\boldsymbol{\sigma}_1\cdot\hat{\mathbf{n}}$$

and the eight spin operators Q_i are expressed as

$$Q_1(\hat{\boldsymbol{l}},\hat{\mathbf{m}}) = (\mathbf{S}_0\cdot\hat{\boldsymbol{l}})(\boldsymbol{\sigma}_1\cdot\hat{\boldsymbol{l}}) , \qquad (2.4a)$$

$$Q_2(\hat{\boldsymbol{l}},\hat{\mathbf{m}}) = i(\tfrac{4}{3})^{1/2}(\hat{\mathbf{m}}\cdot\overleftrightarrow{\mathrm{T}}_0\cdot\hat{\mathbf{n}})(\boldsymbol{\sigma}_1\cdot\hat{\boldsymbol{l}}) , \qquad (2.4b)$$

$$Q_3(\hat{\boldsymbol{l}},\hat{\mathbf{m}}) = (\mathbf{S}_0\cdot\hat{\mathbf{m}})(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{m}}) , \qquad (2.4c)$$

$$Q_4(\hat{\boldsymbol{l}},\hat{\mathbf{m}}) = -i(\tfrac{4}{3})^{1/2}(\hat{\boldsymbol{l}}\cdot\overleftrightarrow{\mathrm{T}}_0\cdot\hat{\mathbf{n}})(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{m}}) , \qquad (2.4d)$$

$$Q_5(\hat{\boldsymbol{l}},\hat{\mathbf{m}}) = (\mathbf{S}_0\cdot\hat{\mathbf{n}})(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{n}}) , \qquad (2.4e)$$

$$Q_6(\hat{\boldsymbol{l}},\hat{\mathbf{m}}) = i(\tfrac{4}{3})^{1/2}(\hat{\boldsymbol{l}}\cdot\overleftrightarrow{\mathrm{T}}_0\cdot\hat{\mathbf{m}})(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{n}}) , \qquad (2.4f)$$

$$Q_7(\hat{\boldsymbol{l}},\hat{\mathbf{m}}) = (\hat{\boldsymbol{l}}\cdot\overleftrightarrow{\mathrm{T}}_0\cdot\hat{\boldsymbol{l}}) , \qquad (2.4g)$$

$$Q_8(\hat{\boldsymbol{l}},\hat{\mathbf{m}}) = \frac{1}{\sqrt{3}}[(\hat{\mathbf{m}}\cdot\overleftrightarrow{\mathrm{T}}_0\cdot\hat{\mathbf{m}}) - (\hat{\mathbf{n}}\cdot\overleftrightarrow{\mathrm{T}}_0\cdot\hat{\mathbf{n}})] . \qquad (2.4h)$$

Here, $\mathbf{S}_0$ designates a vector transition operator with reduced matrix element[10] $\langle \tfrac{3}{2}\|\mathbf{S}_0\|\tfrac{1}{2}\rangle = (\tfrac{3}{2})^{1/2}$ and $\overleftrightarrow{\mathrm{T}}_0$ a spin-two tensor transition operator whose definition and properties are described in Appendix A.

A model for the reaction $\mathrm{pp}\rightarrow\Delta^{++}\mathrm{n}$, based on unitarized iterated pion exchanges, has been developed by Kloet and Silbar.[11] The amplitudes of this model, which are all nonvanishing, are utilized in our numerical illustrations.

In the Glauber model we work in impact parameter space. Hence, we need the impact parameter representation $\gamma^{\Delta N}(\mathbf{b})$ of the production amplitude. It is defined by

$$\gamma^{\Delta N}(\mathbf{b}) = \frac{1}{2\pi i k_\Delta}\int d^2q_\perp e^{-i\mathbf{q}_\perp\cdot\mathbf{b}}M^{\Delta N}(\mathbf{q}_\perp) , \qquad (2.5)$$

and has a spin decomposition identical to that of $M^{\Delta N}(\mathbf{q})$, i.e.,

$$\gamma^{\Delta N}(\mathbf{b}) = \sum_{i=1}^{8} [\tilde{f}_i(b)Q_i(\hat{\boldsymbol{l}}_b,\hat{\mathbf{m}}_b) + \tilde{g}_i(b)P_i(\hat{\boldsymbol{l}}_b,\hat{\mathbf{m}}_b)] , \qquad (2.6)$$

where now

$$\hat{\boldsymbol{l}}_b = \hat{\mathbf{k}}, \quad \hat{\mathbf{m}}_b = i\hat{\mathbf{b}}, \quad \hat{\mathbf{n}}_b = \hat{\boldsymbol{l}}_{b\wedge}\hat{\mathbf{m}}_b . \qquad (2.7)$$

The relation between the coordinate space and momentum space amplitudes is a linear one,

$$\tilde{f}_i(b) = \frac{1}{ik_\Delta}\int_0^\infty q\,dq \sum_{j=1}^{8} V_{ij}(qb)f_j(q) , \qquad (2.8a)$$

$$\tilde{g}_i(b) = \frac{1}{ik_\Delta}\int_0^\infty q\,dq \sum_{j=1}^{8} W_{ij}(qb)g_j(q) . \qquad (2.8b)$$

The matrices $[V_{ij}]$ and $[W_{ij}]$ are symmetric in their indices. Their only nonvanishing matrix elements are

$$V_{11} = V_{77} = J_0 , \qquad (2.9a)$$

$$V_{22} = V_{88} = J_2 , \qquad (2.9b)$$

$$V_{33} = V_{44} = V_{55} = V_{66} = \tfrac{1}{2}(J_2 - J_0) , \qquad (2.9c)$$

$$V_{35} = V_{53} = V_{46} = V_{64} = -\tfrac{1}{2}(J_2 + J_0) , \qquad (2.9d)$$

$$W_{11} = W_{77} = -W_{33} = -W_{44} = -W_{55} = -W_{66} = J_1 , \qquad (2.10a)$$

$$W_{22} = W_{88} = \tfrac{1}{2}(J_3 - J_1) , \qquad (2.10b)$$

$$W_{28} = W_{82} = \tfrac{1}{2}(J_3 + J_1) , \qquad (2.10c)$$

where the J functions are Bessel functions.

Our choice of basis vectors and operators $Q_i(\hat{\boldsymbol{l}}_b,\hat{\mathbf{m}}_b)$ has the advantage of yielding a symmetric relation between the pairs (f_i,g_i) and $(\tilde{f}_i,\tilde{g}_i)$ but the disadvantage of having the unconventional normalization $\hat{\mathbf{m}}_b^2 = \hat{\mathbf{n}}_b^2 = -1$. The inverse of Eq. (2.8) reads

$$f_i(q) = ik_\Delta\int_0^\infty b\,db \sum_{j=1}^{8} V_{ij}(qb)\tilde{f}_j(b) , \qquad (2.11a)$$

$$g_i(q) = ik_\Delta\int_0^\infty b\,db \sum_{j=1}^{8} W_{ij}(qb)\tilde{g}_j(b) . \qquad (2.11b)$$

The elastic scattering amplitudes $\mathrm{NN}\rightarrow\mathrm{NN}$ and $\Delta\mathrm{N}\rightarrow\Delta\mathrm{N}$ will not be treated in their full generality. We shall limit ourselves to amplitudes of the form

$$M^{\mathrm{el}}(\mathbf{q}) = f^{\mathrm{el}}(q) + g^{\mathrm{el}}(q)(\boldsymbol{\sigma}_{\mathrm{el}} + \boldsymbol{\sigma}_1)\cdot\hat{\mathbf{n}} , \qquad (2.12)$$

where $\boldsymbol{\sigma}_{\mathrm{el}} = \boldsymbol{\sigma}_0$ (or $\boldsymbol{\sigma}_\Delta$) is the Pauli spin operator for the projectile nucleon (or the Δ) and $\boldsymbol{\sigma}_1$ the Pauli spin operator of the target nucleon. For the Δ, the notation means $\boldsymbol{\sigma}_\Delta = 2\mathbf{S}_\Delta$, i.e., $\boldsymbol{\sigma}_\Delta$ stands for a set of 4×4 matrices. The impact parameter representation of the scattering amplitude $\gamma^{\mathrm{el}}(\mathbf{b})$ is defined as in Eq. (2.8) and has the decomposition

$$\gamma^{\rm el}(\mathbf{b})=\frac{1}{2\pi i k_{\rm el}}\int d^2q_\perp e^{-i\mathbf{q}_\perp\cdot\mathbf{b}}M^{\rm el}(\mathbf{q}_\perp)$$
$$=\tilde{f}^{\rm el}(b)+\tilde{g}^{\rm el}(b)(\boldsymbol{\sigma}_{\rm el}+\boldsymbol{\sigma}_1)\cdot\hat{\mathbf{n}}_b\ . \qquad (2.13)$$

The relation between the pairs $(f^{\rm el},g^{\rm el})$ and $(\tilde{f}^{\rm el},\tilde{g}^{\rm el})$ is identical to that between (f_1,g_1) and $(\tilde{f}_1,\tilde{g}_1)$.

The parameters of the ΔN interaction are poorly known. In the additive quark model $f^{\rm NN}=f^{\Delta\Delta}$ and $g^{\rm NN}=g^{\Delta\Delta}$.

III. NUCLEAR PROFILE FUNCTIONS

Before going on to detailed calculations for specific nuclei, it is useful to discuss the general nuclear profile function, i.e., the impact parameter production amplitude for nucleons at fixed positions $\mathbf{x}_1,\ldots,\mathbf{x}_A$.

On the nucleon level, we are dealing with a two channel problem, i.e., the nucleon-nucleon and the Δ-nucleon channels. It is convenient to account for this fact by introducing a matrix notation for the S matrix. Thus, for a target nucleon at position $\mathbf{x}=(\mathbf{s},z)$ the impact parameter representation of the S matrix becomes

$$S(\mathbf{b},\mathbf{x})=\begin{vmatrix} S^{\rm NN}(\mathbf{b}-\mathbf{s}) & \lambda S^{\rm N\Delta}(\mathbf{b}-\mathbf{s})e^{-i\Delta_\parallel z} \\ \frac{1}{\lambda}S^{\Delta\rm N}\left[\frac{\mathbf{b}}{\lambda}-\mathbf{s}\right]e^{i\Delta_\parallel z} & S^{\Delta\Delta}\left[\frac{\mathbf{b}}{\lambda}-\mathbf{s}\right]\end{vmatrix}\ . \qquad (3.1)$$

Here, $\Delta_\parallel=k-k_\Delta$ is the longitudinal momentum transfer and the parameter $\lambda=k_\Delta/k$ has been introduced in order to secure the unitarity of the S matrix.[12] The variable $\mathbf{b}$ is the impact parameter appropriate for a final NN channel and $\mathbf{b}/\lambda$ the corresponding one for a final ΔN channel. The submatrices of $S(\mathbf{b};\mathbf{x})$ are related to the corresponding profile functions by

$$S^{kj}(\mathbf{b})=\delta_{kj}-\gamma^{kj}(\mathbf{b})\ . \qquad (3.2)$$

When working with the Glauber multiple scattering theory, care must be exercised since the individual S-matrix elements in general do not commute. The collision operator must be properly time (or z) ordered. This operation yields the following nuclear S-matrix operator, appropriate for fixed nucleon positions,

$$S(\mathbf{b},\mathbf{x}_1,\ldots,\mathbf{x}_A)=\sum_{\rm perm}\theta(z_{i_A}-z_{i_{A-1}})\cdots\theta(z_{i_3}-z_{i_2})\theta(z_{i_2}-z_{i_1})S(\mathbf{b},\mathbf{x}_{i_A})\cdots S(\mathbf{b},\mathbf{x}_{i_1})\ , \qquad (3.3)$$

where the sum runs over all permutations $\{i_1,\ldots,i_A\}$ of $1,\ldots,A$. The nuclear profile function $\Gamma^{\Delta\rm N}(\mathbf{b})$ is obtained by sandwiching this operator between nuclear states. In particular, for the reaction $\rm N+A\to\Delta+A'$,

$$\Gamma^{\Delta\rm N}(\mathbf{b})=\langle\psi_f(\mathbf{x}_1,\ldots,\mathbf{x}_A)|-\lambda\{S(\lambda\mathbf{b},\mathbf{x}_1,\ldots,\mathbf{x}_A)\}^{\Delta\rm N}|\psi_i(\mathbf{x}_1,\ldots,\mathbf{x}_A)\rangle\ . \qquad (3.4)$$

Up to this point, the formalism, has been completely general. We shall now make the simplifying assumption that only terms linear in the NN→ΔN production amplitude be retained. For this case

$$\begin{aligned}\Gamma^{\Delta\rm N}(\mathbf{b},\mathbf{x}_1,\ldots,\mathbf{x}_A)=\sum_{\rm perm}&\theta(z_{i_A}-z_{i_{A-1}})\cdots\theta(z_{i_2}-z_{i_1})\\ &\times[\gamma^{\Delta\rm N}(\mathbf{b},\mathbf{x}_{i_A})S^{\rm NN}(\lambda\mathbf{b}-\mathbf{s}_{i_{A-1}})\cdots S^{\rm NN}(\lambda\mathbf{b}-\mathbf{s}_{i_1})\\ &+S^{\Delta\Delta}(\mathbf{b}-\mathbf{s}_{i_A})\gamma^{\Delta\rm N}(\mathbf{b},\mathbf{x}_{i_{A-1}})S^{\rm NN}(\lambda\mathbf{b}-\mathbf{s}_{i_{A-2}})\cdots S^{\rm NN}(\lambda\mathbf{b}-\mathbf{s}_{i_1})\\ &\ \vdots\\ &+S^{\Delta\Delta}(\mathbf{b}-\mathbf{s}_{i_A})\cdots S^{\Delta\Delta}(\mathbf{b}-\mathbf{s}_{i_2})\gamma^{\Delta\rm N}(\mathbf{b},\mathbf{x}_{i_1})]\ ,\end{aligned} \qquad (3.5)$$

where

$$\gamma^{\Delta\rm N}(\mathbf{b},\mathbf{x})=\gamma^{\Delta\rm N}(\mathbf{b}-\mathbf{s})e^{i\Delta_\parallel z}\ . \qquad (3.6)$$

In our analysis we do not consider the most general elastic NN→NN and ΔN→ΔN profile functions, but limit ourselves to those of type (2.12). For this case, a convenient rearrangement is possible, since we can assume the operators $S^{\rm NN}(\mathbf{b}-\mathbf{s}_i)$, $i=1,\ldots,A$, to commute amongst each other, and similarly for the operators $S^{\Delta\Delta}(\mathbf{b}-\mathbf{s}_i)$, $i=1,\ldots,A$. We stress that since in the end the elastic profiles will be averaged over spherically symmetric densities, this assumption does not entail any approximations. Under these provisions one derives the following alternative expression for the nuclear profile function:

$$\Gamma(\mathbf{b},\mathbf{x}_1,\ldots,\mathbf{x}_A)=\sum_{\text{perm}}\left\{\frac{1}{(A-1)!}\gamma^{\Delta N}(i_1)\prod_{k=2}^{A}S^{NN}(i_k)\right.$$
$$+\frac{1}{1!(A-2)!}[S^{el}(i_2),\gamma^{\Delta N}(i_1)]\prod_{k=3}^{A}S^{NN}(i_k)\theta(z_{i_2}-z_{i_1})$$
$$+\frac{1}{2!(A-3)!}[S^{el}(i_3),[S^{el}(i_2),\gamma^{\Delta N}(i_1)]]\prod_{k=4}^{A}S^{NN}(i_k)\theta(z_{i_3}-z_{i_2})\theta(z_{i_2}-z_{i_1})$$
$$\vdots$$
$$+\frac{1}{(A-1)!}[S^{el}(i_A),[S^{el}(i_{A-1}),\ldots,[S^{el}(i_2),\gamma^{\Delta N}(i_1)]\cdots]]$$
$$\left.\times\theta(z_{i_A}-z_{i_{A-1}})\cdots\theta(z_{i_2}-z_{i_1})\right\}, \tag{3.7}$$

where we have employed the notations

$$S^{el}(i)=\begin{cases}S^{\Delta\Delta}(\mathbf{b}-\mathbf{s}_i) & \text{when to the left of } \gamma^{\Delta N},\\ S^{NN}(\lambda\mathbf{b}-\mathbf{s}_i) & \text{when to the right of } \gamma^{\Delta N},\end{cases} \tag{3.8}$$

$$\gamma^{\Delta N}(i)=\gamma^{\Delta N}(\mathbf{b},\mathbf{x}_i)\ . \tag{3.9}$$

This formulation is particularly appealing since it provides a systematic expansion in powers of the spin amplitudes of $\gamma^{\Delta\Delta}$ and γ^{NN} and in the difference of their spin independent amplitudes.

As a simple illustration, consider the case of spin independent elastic amplitudes γ^{NN} and $\gamma^{\Delta\Delta}$. Then, a summation of the series (3.7) becomes possible. It leads to the well-known result,

$$\Gamma(\mathbf{b},\mathbf{x}_1,\ldots,\mathbf{x}_A)=\sum_i \gamma^{\Delta N}(i)\ ,$$
$$\prod_{k\neq i}[1-\gamma^{\Delta\Delta}(\mathbf{b}-\mathbf{s}_k)\theta(z_k-z_i)-\gamma^{NN}(\mathbf{b}-\mathbf{s}_k)\theta(z_i-z_k)]\ , \tag{3.10}$$

a result which could also have been obtained directly from the original expression (3.5).

We now consider the specific example of nucleon induced Δ production by light nuclei. A number of simplifying assumptions will be made. We shall try to present them as clearly as possible.

We assume the nuclear transition to be caused by the production step. The associated one body transition density $\rho_T(x)$ is assumed spherically symmetric. In the single scattering step, the elementary profile functions $\tilde{f}_j(b),\tilde{g}_j(b)$ [Eq. (2.8)] are consequently replaced by their folded counterparts

$$X_i(b)=\frac{1}{ik_\Delta}\int_0^\infty q_\perp dq_\perp \sum_j V_{ij}f_j(q_\perp)S_T(q_\perp^2+\Delta_\parallel^2)^{1/2}\ , \tag{3.11a}$$

$$Y_i(b)=\frac{1}{ik_\Delta}\int_0^\infty q_\perp dq_\perp \sum_j W_{ij}g_j(q_\perp)S_T(q_\perp^2+\Delta_\parallel^2)^{1/2}\ . \tag{3.11b}$$

The folded production profile is denoted $\hat{\gamma}(\mathbf{b},k)$ where the index k reminds us that $\hat{\gamma}$ still depends on the nucleon variables through its spin and isospin dependence. The assumption of spherical symmetry for ρ_T means, e.g., that quadrupole contributions are neglected. We shall come back to this point later. When folding the elastic scattering operators we attach the same average spherically symmetric density $\rho_0(x)$ to all the scattering steps. The correspondingly folded profiles are denoted

$$X^{el}(b)=\frac{1}{ik_{el}}\int_0^\infty q\,dq\,J_0(qb)f^{el}(q)S_0(q)\ , \tag{3.12a}$$

$$Y^{el}(b)=\frac{1}{ik_{el}}\int_0^\infty q\,dq\,J_1(qb)g^{el}(q)S_0(q)\ . \tag{3.12b}$$

The folded S matrix is denoted $\hat{S}^{el}(\mathbf{b},k)$.

In Eq. (3.7) we encounter the z ordering through the presence of θ functions. They substantially complicate the numerical evaluation. However, for light nuclei the densities are approximately Gaussian. We shall employ this property as basis for an approximate treatment of the product of θ functions. It consists of replacing them by

$$\prod_{k=2}^{n}\theta(z_k-z_1)\rightarrow\lambda_n(\Delta_\parallel R)=\frac{1}{S(\Delta_\parallel)}\frac{1}{(\pi R^2)^{n/2}}\int_{-\infty}^{+\infty}dz_1\cdots dz_n\exp\left[-\sum_1^n\left[\frac{z_i^2}{R^2}\right]+i\Delta_\parallel z_1\right]\prod_{k=2}^{n}\theta(z_k-z_1)\ . \tag{3.13}$$

When defining λ_n, we have divided by

$$S(\Delta_{\|})=\exp(-\Delta_{\|}^2 R^2/4)$$

in order to compensate for the fact that the longitudinal momentum transfer is taken into account when performing the folding in Eq. (3.11). The limiting values for $\Delta_{\|}=0$ are $\lambda_n(0)=1/n$. We stress that this procedure is exact for Gaussian densities and expected to be a good approximation for light nuclei.

We have not made any attempts to calculate all the terms in the expansion (3.7) but have limited ourselves to terms of second order or lower in the elastic spin amplitudes $Y^{\text{el}}(b)$. At the same time we have assumed the commutator $[S^{\text{el}}(i),\gamma^{\Delta\text{N}}(k)]$ to be of order $Y^{\text{el}}(b)$.

To the order considered, it is possible to obtain a relatively compact formula for the nuclear profile function. We write

$$\Gamma(\mathbf{b})=\langle \Delta,A' \,|\, \hat{\Gamma}(\mathbf{b}) \,|\, \text{N},A\rangle \;, \tag{3.14}$$

where the profile $\hat{\Gamma}(\mathbf{b})$ depends on the spin and isospin operators of all the nucleons involved, and the matrix elements are interpreted as matrix elements relative to these degrees of freedom. In the notation introduced above,

$$\begin{aligned}\hat{\Gamma}(\mathbf{b})=&\sum_{i_1}\hat{\gamma}^{\Delta\text{N}}(\mathbf{b},i_1)[\hat{S}_0(\mathbf{b})]^{A-1}+\sum_{i_1,i_2}\{\lambda_2[\hat{S}^{\text{el}}(\mathbf{b},i_2),\hat{\gamma}^{\Delta\text{N}}(\mathbf{b},i_1)]-Y^{\text{NN}}(b)\hat{\gamma}^{\Delta\text{N}}(\mathbf{b},i_1)(\boldsymbol{\sigma}_{i_2}\cdot\hat{\mathbf{n}}_b)\}[\hat{S}_0(\mathbf{b})]^{A-2}\\&+\sum_{i_1,i_2,i_3}\{\tfrac{1}{2}\lambda_3[\hat{S}^{\text{el}}(\mathbf{b},i_3),[\hat{S}^{\text{el}}(\mathbf{b},i_2),\hat{\gamma}^{\Delta\text{N}}(\mathbf{b},i_1)]]-\lambda_2 Y^{\text{NN}}(b)[\hat{S}^{\text{el}}(\mathbf{b},i_3),\hat{\gamma}^{\Delta\text{N}}(\mathbf{b},i_1)](\boldsymbol{\sigma}_{i_2}\cdot\hat{\mathbf{n}}_b)\\&\qquad+\tfrac{1}{2}[Y^{\text{NN}}(b)]^2\hat{\gamma}^{\Delta\text{N}}(\mathbf{b},i_1)(\boldsymbol{\sigma}_{i_3}\cdot\hat{\mathbf{n}}_b)(\boldsymbol{\sigma}_{i_2}\cdot\hat{\mathbf{n}}_b)\}[\hat{S}_0(\mathbf{b})]^{A-3}\;,\end{aligned} \tag{3.15}$$

with

$$\hat{S}_0(\mathbf{b})=1-X^{\text{NN}}(b)-Y^{\text{NN}}(b)(\boldsymbol{\sigma}_0\cdot\hat{\mathbf{n}}_b)\;. \tag{3.16}$$

In this expression it is understood that powers of $\hat{S}_0(\mathbf{b})$ be truncated at the proper power of $Y^{\text{NN}}(b)$, thereby making the formula consistent.

There is one important application where our expression for $\hat{\Gamma}$ can be further simplified. When the dependence on the target nucleon spin is the same in the S^{NN} and $S^{\Delta\Delta}$ matrices, i.e., when $g^{\Delta\Delta}=g^{\text{NN}}$, then the dependence on the target nucleon index i in $S^{\text{el}}(\mathbf{b},i)$ in the commutator terms can be dropped, since $(\boldsymbol{\sigma}_i\cdot\hat{\mathbf{n}}_b)$ commutes with $\hat{\gamma}^{\Delta\text{N}}(\mathbf{b},k)$ when $i\neq k$. As a result, the summation over the corresponding nucleon index i can immediately be performed.

IV. AMPLITUDES FOR $p^3\text{He}\to\Delta^{++}\,{}^3\text{H}$

Since the (^{3}He,^{3}H) system has the same spin-isospin content as the (p,n) system, the spin-isospin structure of the amplitude for the reaction $\text{p}+{}^3\text{He}\to\Delta^{++}+{}^3\text{H}$ is necessarily the same as that for the elementary reaction. Consequently, we define

$$M^{\Delta\text{N}}(\mathbf{q})=\sum_{i=1}^{8}[F_i(q)Q_i(\hat{l},\hat{\mathbf{m}})+G_i(q)P_i(\hat{l},\hat{\mathbf{m}})]\;, \tag{4.1}$$

where the operators Q_i and P_i are identical to those of Eq. (2.4) except that $\boldsymbol{\tau}_1$ and $\boldsymbol{\sigma}_1$ are replaced by $\boldsymbol{\tau}$ and $\boldsymbol{\sigma}$, the total Pauli isospin and spin operators of the nucleus. Similarly, the nuclear profile function has a decomposition identical to that of Eq. (2.6), with amplitudes that we now denote $\tilde{F}_i(b)$ and $\tilde{G}_i(b)$.

In the wave functions of the ^{3}He and ^{3}H nuclei we only retain their S-wave component. This implies that the transition density $\rho_T(x)$ and the average rescattering density $\rho_0(x)$ become identical. The properly antisymmetrized spin-isospin part of the nuclear wave function is

$$\begin{aligned}|\,\Psi(1,2,3)\rangle=\frac{1}{\sqrt{2}}[\,|\,\chi(1,2),3\rangle_T\,|\,\psi(1,2),3\rangle_S\\ -|\,\psi(1,2),3\rangle_T\,|\,\chi(1,2),3\rangle_S]\;,\end{aligned} \tag{4.2}$$

where indices T and S refer to ket vectors in isospin and spin space. Furthermore, $\chi(1,2)$ denote the wave function where particles 1 and 2 are coupled to total angular momentum zero, and $\psi(1,2)$ the one where they are coupled to total angular momentum one.

In our expression for the nuclear profile function $\hat{\Gamma}(\mathbf{b})$, Eq. (3.15), we encounter the spin-isospin matrix elements

$$\left\langle {}^3\text{H}\left|\sum_{i_1}(\tau_{i_1}\cdot\mathbf{a})\begin{Bmatrix}1\\ \boldsymbol{\sigma}_{i_1}\cdot\mathbf{b}\end{Bmatrix}\right|{}^3\text{He}\right\rangle=\left\langle \text{n}\left|(\boldsymbol{\tau}\cdot\mathbf{a})\begin{Bmatrix}1\\ -\boldsymbol{\sigma}\cdot\mathbf{b}\end{Bmatrix}\right|\text{p}\right\rangle\;, \tag{4.3a}$$

$$\left\langle {}^3\text{H}\left|\sum_{i_1,i_2}(\tau_{i_1}\cdot\mathbf{a})\begin{Bmatrix}1\\ \boldsymbol{\sigma}_{i_1}\cdot\mathbf{b}\end{Bmatrix}(\boldsymbol{\sigma}_{i_2}\cdot\hat{\mathbf{n}}_b)\right|{}^3\text{He}\right\rangle=\left\langle \text{n}\left|(\boldsymbol{\tau}\cdot\mathbf{a})\begin{Bmatrix}2\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}_b\\ -2\mathbf{b}\cdot\hat{\mathbf{n}}_b\end{Bmatrix}\right|\text{p}\right\rangle\;, \tag{4.3b}$$

$$\left\langle {}^3\text{H}\left|\sum_{i_1,i_2,i_3}(\tau_{i_1}\cdot\mathbf{a})\begin{Bmatrix}1\\ \boldsymbol{\sigma}_{i_1}\cdot\mathbf{b}\end{Bmatrix}(\boldsymbol{\sigma}_{i_2}\cdot\hat{\mathbf{n}}_b)(\boldsymbol{\sigma}_{i_3}\cdot\hat{\mathbf{n}}_b)\right|{}^3\text{He}\right\rangle=\left\langle \text{n}\left|(\boldsymbol{\tau}\cdot\mathbf{a})\begin{Bmatrix}2\hat{\mathbf{n}}_b^2\\ 2\hat{\mathbf{n}}_b^2(\boldsymbol{\sigma}\cdot\mathbf{b})-4(\mathbf{b}\cdot\hat{\mathbf{n}}_b)(\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}_b)\end{Bmatrix}\right|\text{p}\right\rangle\;, \tag{4.3c}$$

where the sum (i_1,i_2,i_3) runs over all permutations of (1,2,3). The matrix elements on the right-hand sides have been expressed as neutron-proton matrix elements only to emphasize that we are dealing with matrix elements of operators over the total system.

From Eq. (4.3a) it follows that the $(\tau\cdot\mathbf{a})$ and $(\tau\cdot\mathbf{a})(\sigma\cdot\mathbf{b})$ terms on the right-hand side come with opposite signs. Therefore, the single scattering contributions to the amplitudes $F_i(q)$ and $G_i(q)$ become

$$F_i(q)=\epsilon_i S(q^2+\Delta_\parallel^2)^{1/2} f_i(q) \ , \tag{4.4a}$$

$$G_i(q)=\eta_i S(q^2+\Delta_\parallel^2)^{1/2} g_i(q) \ , \tag{4.4b}$$

where $S(q)$ is the $(^3\text{He},{}^3\text{H})$ form factor, normalized to $S(0)=1$, and

$$\epsilon_i=\begin{cases}-1, & i=1,\ldots,6\\ +1, & i=7,8\end{cases} \tag{4.5}$$

$$\eta_i=\begin{cases}-1, & i=1,\ldots,4,7,8\\ +1, & i=5,6\ .\end{cases} \tag{4.6}$$

This means that due to the specific spin-isospin structure of the nuclei the single scattering approximation to the nuclear amplitudes $F_i(q)$ and $G_i(q)$ is not obtained by simply multiplying the corresponding elementary amplitudes by a nuclear form factor. Additional phase factors complicate the relations. As a result, many polarization observables will be different for the $(^3\text{He},{}^3\text{H})$ and (p,n) reactions. These remarks are strictly valid only when we limit ourselves to the S-wave part of the wave functions. When the S'- and D-wave parts are included, the transition form factor is no longer unique. As described in Appendix B, the relation between the nucleon and nuclear amplitudes becomes more involved.

With all the necessary nuclear matrix elements in our hands the evaluation of Eq. (3.15) becomes quite easy. We give the final result for the special case $\tilde{f}^{\,\Delta\Delta}(b)=\tilde{f}^{\,\text{NN}}(b)$ and $\tilde{g}^{\,\Delta\Delta}(b)=\tilde{g}^{\,\text{NN}}(b)$. Then, there are only two folded elastic amplitudes $X_0(b)$ and $Y_0(b)$ instead of the four of Eq. (3.12). Making systematic use of Eqs. (A7) and (A9) of Appendix A, one obtains, $i=1,\ldots,8$,

$$\begin{aligned}\tilde{F}_i(b)=\epsilon_i\{&(1-X_0)^2X_i\\&+2Y_0(1-X_0)[v_i(Y)+\lambda_2 t_i(Y)-\delta_i Y_i]\\&+2Y_0^2[\delta_i(v_i(X)+\lambda_2 t_i(X)-X_i)\\&\quad-\tfrac{1}{2}\lambda_3 t_i(t(X))-\lambda_2 v_i(t(X))]\}\ ,\end{aligned} \tag{4.7a}$$

$$\begin{aligned}\tilde{G}_i(b)=\eta_i\{&(1-X_0)^2Y_i\\&-2Y_0(1-X_0)[v_i(X)+\lambda_2 t_i(X)-\delta_i X_i]\\&+2Y_0^2[\delta_i(v_i(Y)+\lambda_2 t_i(Y)-Y_i)\\&\quad-\tfrac{1}{2}\lambda_3 t_i(t(Y))-\lambda_2 v_i(t(Y))]\}\ ,\end{aligned} \tag{4.7b}$$

where

$$\delta_i=\begin{cases}0, & i=1,\ldots,4\\ 1, & i=5,\ldots,8\ .\end{cases} \tag{4.8}$$

These expressions give the nuclear amplitudes in impact parameter space. Those in momentum space, $F_i(q)$ and $G_i(q)$, are calculated by the transformation (2.11).

It might be appropriate to remark that expressions (4.7) correspond to the complete multiple scattering series. The reason is that the truncation performed when deriving the nuclear profile function of Eq. (3.15) only affects multiple scattering terms of fourth or higher order.

The appropriate value of the parameter R entering the functions $\lambda_n(\Delta_\parallel R)$ is $R=1.66$ fm.

The observables of the reaction $\text{pp}\to\Delta^{++}\text{n}$ have been studied in great detail by Auger *et al.*[9] Since the observables of the reaction $\text{p}^3\text{He}\to\Delta^{++}\,{}^3\text{H}$ are the same, the expressions of Auger *et al.* apply without any change. The unpolarized differential cross section is

$$\frac{d\sigma}{d\Omega}=\sum_{i=1}^{8}(|F_i|^2+|G_i|^2)\equiv I\ . \tag{4.9}$$

Thus, in the single scattering approximation, Eq. (4.4), the nuclear cross section equals the nucleon cross section multiplied by the square of the nuclear form factor.

The vector analyzing power due to the target nucleus is

$$A_{0n}=\frac{2}{I}\sum_{i=1}^{8}\text{Re}(F_i^*G_i)\ . \tag{4.10}$$

Here, the nuclear and nucleon observables differ in the single scattering approximation, since $\epsilon_i\eta_i=1$ for $i=1,\ldots,4$ and $=-1$ for $i=5,\ldots,8$. The asymmetry A_{0n} is therefore an observable which is particularly interesting to measure.

The vector analyzing power due to the beam nucleon is

$$A_{n0}=\frac{2}{I}\sum_{i,j=1}^{8}\text{Re}(F_i^*G_j)U_{ij}\ , \tag{4.11}$$

with the matrix (U_{ij}) as defined in Ref. 9. Since this matrix satisfies $\epsilon_i U_{ij}\eta_i=U_{ij}$, the single scattering approximation yields an A_{0n} identical to that of the nucleon.

For a complete classification of all the observables of the reaction $\text{p}^3\text{He}\to\Delta^{++}\,{}^3\text{H}$ we refer to Ref. 9.

V. AMPLITUDES FOR $\text{p}^6\text{Li}\to\Delta^{++}\,{}^6\text{He}$

Since ^{6}He is a $J^P=0^+$, $T=1$ nucleus and ^{6}Li a $J^P=1^+$, $T=0$ nucleus, the spin structure of the nuclear production amplitude is different from that of the nucleon. In fact, there are only 12 rotationally invariant amplitudes. They are classified in the following way. In the isospin transition operator Eq. (2.1), we replace the nucleon τ_1 by a nuclear vector transition operator normalized so that $\langle\Delta^{++},{}^6\text{He}\,|\,N_T\,|\,\text{p},{}^6\text{Li}\rangle=1$. In the 16 spin space operators, Eq. (2.4), we replace the target nucleon spin operator σ_1 by a nuclear vector transition operator $\mathbf{V}$. We normalize $\mathbf{V}$ so that the matrix elements of its spherical components are[10]

$$\langle 0\,|\,V_q\,|\,1,M\rangle=\langle 00\,|\,11\,M\,q\rangle=\frac{(-)^{1-M}}{\sqrt{3}}\delta_{q,-M}\ . \tag{5.1}$$

Hence, the operators $Q_i(\hat{l},\hat{\mathbf{m}})$, $i=1,\ldots,6$, are obtained by replacing $\boldsymbol{\sigma}_1$ with $\mathbf{V}$. There are no transition operators corresponding to Q_7 and Q_8. The operators P_i are similarly obtained by first linearizing the $\boldsymbol{\sigma}_1$ dependence of $Q_i(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{n}})$ and then replacing $\boldsymbol{\sigma}_1$ by $\mathbf{V}$. In particular, there are no operators of type P_5 and P_6. The vanishing of Q_7, Q_8, P_5, and P_6 is directly connected to the scalar character of these operators in the target nucleon spin space. We summarize these elementary rules by writing

$$M^{\Delta N}(\mathbf{q})=\sum_{i=1}^{8}[\epsilon_i' F_i(q)Q_i(\hat{l},\hat{\mathbf{m}})+\eta_i' G_i(q)P_i(\hat{l},\hat{\mathbf{m}})] , \quad (5.2)$$

with

$$\epsilon_i'=\begin{cases}1, & i=1,\ldots,6\\ 0, & i=7,8\end{cases} \quad (5.3)$$

$$\eta_i'=\begin{cases}1, & i=1,\ldots,4,7,8\\ 0, & i=5,6 .\end{cases} \quad (5.4)$$

In the nuclear profile function $\hat{\Gamma}(\mathbf{b})$ we make the corresponding substitutions in $Q_i(\hat{l}_b,\hat{\mathbf{m}}_b)$ and $P_i(\hat{l}_b,\hat{\mathbf{m}}_b)$. Through this procedure we achieve that the relations between the pairs $[F_i(q),G_i(q)]$ and $[\tilde{F}_i(b),\tilde{G}_i(b)]$ become identical to that of the nucleons [Eqs. (2.8) and (2.11)].

The nuclear matrix elements involving ^{6}Li and ^{6}He have been evaluated under the assumption that these nuclei consist of an inert ^{4}He core with two added valence nucleons. Exchange contributions involving a core and a valence nucleon have been neglected. The J^π wave functions of the two valence nucleons have been taken to be[13]

$$|1^+\rangle=\alpha\,|(p_{3/2})^2,1^+\rangle+\beta\,|(p_{1/2})^2,1^+\rangle +\gamma\,|(p_{3/2},p_{1/2}),1^+\rangle , \quad (5.5a)$$

$$|0^+\rangle=\alpha'\,|(p_{3/2})^2,0^+\rangle+\beta'\,|(p_{1/2})^2,0^+\rangle , \quad (5.5b)$$

with normalizations $\alpha^2+\beta^2+\gamma^2=1$ and $\alpha'^2+\beta'^2=1$.

In our derivation of the nuclear production profile function we assumed the transition density $\rho_T(x)$ to be spherically symmetric. The one-body densities corresponding to the wave functions (5.5) contain a quadrupole component. As an illustration consider two valence nucleons in the $p_{3/2}$ state. In the single scattering approximation we encounter the matrix element

$$\langle {}^6\text{He}\,|(\sigma_1\cdot\mathbf{a})e^{i\mathbf{Q}\cdot\mathbf{x}_1}|\,{}^6\text{Li}\rangle=\langle 0\,|\{-(\tfrac{5}{3})^{1/2}(\mathbf{V}\cdot\mathbf{a})S_0(Q)-(\tfrac{3}{5})^{1/2}[(\mathbf{V}\cdot\hat{\mathbf{Q}})(\hat{\mathbf{Q}}\cdot\mathbf{a})-\tfrac{1}{3}\mathbf{V}\cdot\mathbf{a}]S_2(Q)\}\,|\,1,M\rangle , \quad (5.6)$$

with

$$S_k(Q)=\int_0^\infty r^2dr\,\rho_p(r)j_k(Qr) , \quad (5.7)$$

and $\rho_p(r)$ the radial p-wave density. Thus, as for scattering by deuterium, there is an additional quadrupole contribution. In the following, it shall be neglected on the grounds that the quadrupole moment of ^{6}Li is small. In Appendix B, we demonstrate how the single scattering terms are changed when it is included.

We now turn to the matrix elements needed for the determination of the nuclear profile function. In the single scattering term, we encounter, neglecting the quadrupole contribution,

$$\langle\Delta^{++}\,{}^6\text{He}\,|N_T(1)(\sigma_1\cdot\mathbf{a})\delta(\mathbf{r}-\mathbf{x}_1)|\,\text{p}^6\text{Li}\rangle=\frac{1}{\sqrt{2}}V_0\langle 0\,|\mathbf{V}\cdot\mathbf{a}|\,1,H\rangle\rho_T(r) , \quad (5.8)$$

with the reduced matrix element

$$V_0=\frac{1}{\sqrt{3}}(-\sqrt{5}\alpha'\alpha+\beta'\beta+\sqrt{2}\alpha'\gamma+2\beta'\gamma) . \quad (5.9)$$

The factor $1/\sqrt{2}$ arises from the isospin transition operator $N_T(1)$. The two-body matrix element appearing in the double scattering is again, neglecting quadrupole contributions,

$$\langle\Delta^{++}\,{}^6\text{He}\,|N_T(1)(\sigma_1\cdot\mathbf{a})(\sigma_2\cdot\hat{\mathbf{n}}_b)\delta(\mathbf{r}-\mathbf{x}_1)\delta(\mathbf{r}'-\mathbf{x}_2)|\,\text{p}\,{}^6\text{Li}\rangle=\frac{1}{\sqrt{2}}\xi V_0\langle 0\,|i\mathbf{V}\cdot\mathbf{a}_\Lambda\hat{\mathbf{n}}_b|\,1,M\rangle\rho_T(r)\rho_0(r') , \quad (5.10)$$

with the reduced matrix element

$$\xi V_0=\frac{1}{3\sqrt{3}}(-\sqrt{5}\alpha'\alpha-\beta'\beta+2\sqrt{2}\alpha'\beta-2\sqrt{10}\beta'\alpha+5\sqrt{2}\alpha'\gamma+\beta'\gamma) . \quad (5.11)$$

Given the two matrix elements (5.8) and (5.10), the remaining part of the determination of the nuclear profile function $\hat{\Gamma}(\mathbf{b})$ becomes straightforward. As for production by ^{3}He, we limit ourselves to the case of equal nucleon-nucleon and Δ-nucleon elastic scattering amplitudes, $f^{NN}(q)=f^{\Delta\Delta}(q)$ and $g^{NN}(q)=g^{\Delta\Delta}(q)$. In the notation of the preceding section the result can be written as, $i=1,\ldots,8$,

$$\tilde{F}_i(b)=\epsilon_i'\sqrt{2}V_0(1-X_0)^3\{(1-X_0)^2X_i+5Y_0(1-X_0)[v_i(Y)+\lambda_2t_i(Y)+\tfrac{1}{5}\xi_iY_i] +Y_0^2[-10X_i-20\lambda_2v_i(t(X))-10\lambda_3t_i(t(X))-4\xi_iv_i(X)-4\lambda_2\xi_it_i(X)+\tfrac{1}{2}\mathcal{Y}X_i]\} , \quad (5.12a)$$

$$\tilde{G}_i(b)=\eta_i'\sqrt{2}V_0(1-X_0)^3\{(1-X_0)^2Y_i-5Y_0(1-X_0)[v_i(X)+\lambda_2 t_i(X)+\tfrac{1}{5}\xi_i X_i]$$
$$+Y_0^2[-10Y_i-20\lambda_2 v_i(t(Y))-10\lambda_3 t_i(t(Y))-4\xi_i v_i(Y)-4\lambda_2\xi_i t_i(Y)+\tfrac{1}{2}\mathcal{Y}Y_i]\}\ , \quad (5.12b)$$

with

$$\xi_i=\begin{cases}\xi, & i=1,\dots,4\\ -1, & i=5,\dots,8\ ,\end{cases} \quad (5.13)$$

$$\mathcal{Y}=\left\langle {}^4\mathrm{He}\left|\sum_{i_1,i_2}(\boldsymbol{\sigma}_{i_1}\cdot\hat{\mathbf{n}}_b)(\boldsymbol{\sigma}_{i_2}\cdot\hat{\mathbf{n}}_b)\right|{}^4\mathrm{He}\right\rangle$$
$$=-2\hat{\mathbf{n}}_b^2=2\ . \quad (5.14)$$

Again the momentum space amplitudes $F_i(q)$ and $G_i(q)$ of Eq. (5.2) are obtained from $\tilde{F}_i(b)$ and $\tilde{G}_i(b)$ by the transformation (2.11).

The appropriate value for the parameter R entering the functions $\lambda_n(\Delta_\parallel R)$ is $R=2.07$ fm.

The observables of the reaction $p^6\mathrm{Li}\to\Delta^{++}\,{}^6\mathrm{He}$ differ from those of the nucleon. The unpolarized cross section is

$$\frac{d\sigma}{d\Omega}=\tfrac{1}{9}\sum_i(|F_i|^2+|G_i|^2)\equiv\tfrac{1}{9}I\ . \quad (5.15)$$

Since $F_7=F_8=G_5=G_6=0$ there are some amplitudes missing as compared with the nucleon case.

The vector analyzing power due to the beam nucleon is described by a formula identical to that for the nucleon,

$$A_{n0}=\frac{2}{I}\sum_{i,j=1}^{8}\mathrm{Re}(F_i^*G_j)U_{ij}\ . \quad (5.16)$$

The matrix U_{ij}, which is given in Ref. 9, couples F_5,F_6 to G_7,G_8 and F_7,F_8 to G_5,G_6. Therefore, only half the number of the terms with $i,j=5,\dots,8$ contribute for ^{6}Li targets.

Since ^{6}Li has spin 1 there are some new features in the observables measuring the target polarization. The vector analyzing power due to the target is

$$A_{0n}=\frac{\sigma(M=1)-\sigma(M=-1)}{\sigma(M=1)+\sigma(M=0)+\sigma(M=-1)}$$
$$=\frac{2}{I}\sum_{i=1}^{4}\mathrm{Re}(F_i^*G_i)\ . \quad (5.17)$$

This observable is intrinsically similar to A_{0n} in the nucleon or ^{3}He case. However, because of the vanishing of Q_7, Q_8, P_5, and P_6, the sum runs effectively only over the amplitudes with indices 1–4. Consequently, it confirms our expectation that A_{0n} is a potentially interesting observable, as was already remarked in the preceding section.

For a spin-1 target there is also the possibility of having a tensor analyzing power. We have chosen the component A_{nn}, where index n refers to the direction $\hat{\mathbf{n}}$,

$$A_{nn}=\frac{\sigma(M=1)+\sigma(M=-1)-2\sigma(M=0)}{\sigma(M=1)+\sigma(M=0)+\sigma(M=-1)}$$
$$=1-\frac{3}{I}\sum_{i=5}^{8}(|F_i|^2+|G_i|^2)\ . \quad (5.18)$$

It is an observable of great interest since it measures the relative size of

$$|F_5|^2+|F_6|^2+|G_7|^2+|G_8|^2\ .$$

VI. NUMERICAL ESTIMATES AND RESULTS

In the absence of experimental results concerning the spin observables, we shall essentially address here the question of the sensitivity of the calculations to various approximations. For this reason, the nuclear structure aspect of the problem will be kept in the lowest order compatible with known bulk experimental evidences.

In the case of ^{3}He, use is made of a single form factor, parametrized by

$$S(q)=e^{-a^2q^2}-ce^{-b^2q^2q^2}\ , \quad (6.1)$$

a functional form used by McCarthy *et al.*[14] to fit the charge form factor. The parameter values are derived from those obtained by electron scattering, correcting for the finite size of the proton and the center-of-mass correlations in the way proposed by Auger *et al.*[15]

The situation is slightly different in the $^6\mathrm{Li}\to{}^6\mathrm{He}$ transition, which is assumed to proceed through the valence nucleons only. On the other hand, the multiple scattering corrections take place on any nucleon, except the valence proton. Consequently, we must distinguish between two form factors.

The transition density $\rho_T(r)$ is given by the product of two $1p$ shell single particle orbitals. Its monopole part has a form factor we have parametrized by

$$S_T(q)=(1+x+y)e^{-A^2q^2}-xe^{-B^2q^2}$$
$$-ye^{-A^2q^2/2}+\tfrac{2}{3}B^2xq^2e^{-B^2q^2}\ . \quad (6.2)$$

The values $A^2=2.778$ fm^2, $B^2=0.7143$ fm^2, $x=-0.6438$, and $y=0.1947$ are so adjusted that in coordinate space the corresponding transition density $\rho_T(r)$ fits the values used by Jain.[4]

The multiple scattering corrections are calculated with a form factor of the same expression as (6.1). As for ^{3}He, the parameters are taken from electron scattering, measured by Li *et al.*,[16] corrected for the proton size and center-of-mass motion by the same prescription.[15] According to Li *et al.*[16] the form (6.1) does not allow us to fit electron scattering data above 6 fm^{-2} with accuracy. However, improvements to achieve a fit at higher momentum transfer modify only the very central part of the corresponding density $\rho(r)$ by 2–3 %. Such a small correction can be neglected.

In addition to the form factors, the $^6Li \rightarrow {}^6He$ transition requires the knowledge of the configuration mixings for the two valence particles distributed among the $1p_{3/2}$ and $1p_{1/2}$ single particle shells, possible higher configurations being ignored. We take the various weights from Donelly and Walecka,[13] who got their values by analyzing electromagnetic and weak interaction transitions in ^6Li.

Further refinements concerning the nuclear structure is not meaningful as long as we do not distinguish multiple scattering corrections in the entrance and exit channels, i.e., as long as the projectile and the ejectile are assumed to interact the same way with the target nucleons.

For the same reason, the center-of-mass constraints are handled in an approximate way, which has been checked and found satisfactory in proton-^4He elastic scattering up to momentum transfer of the order of 4 fm^{-1}.[15]

As far as the elementary amplitudes are concerned, the $N \rightarrow \Delta$ transition is described by a parametrization of the model amplitudes of Kloet and Silbar.[11] For the sake of convenience, the amplitudes $f_i(q)$ and $g_i(q)/q$ [see (2.2)] are reproduced by means of Gaussians or sums of Gaussians. The fit is made for an incident energy of 800 MeV and an invariant mass of the Δ of 1238 MeV. Two amplitudes, $f_2(q)$ and $f_8(q)$, are nonvanishing but small enough to be neglected. Some of the others vary so little over the 0°–90° range in the elementary center of mass that they can be considered as constant. The $f_i(q)$ and $g_i(q)$ used in the present calculations are listed in Table I. They fit the actual values of Kloet and Silbar up to 90° c.m. and somewhat beyond.

The elastic NN and ΔN amplitudes are assumed to be identical, and we retain only two components [see (2.13)]:

$$f^{el}(q) = 1.05(i - 0.23)e^{-0.35q^2/2}\ \text{fm}$$

and (6.3)

$$g^{el}(q) = -0.12q(1 - i1.9)e^{-0.4q^2/2}\ \text{fm}\ .$$

They correspond to a proton-proton and proton-neutron average of the 800 MeV amplitude of Bystricky *et al.*[17] parametrized by Auger *et al.*[18] The isospin dependence of the ΔN elastic amplitudes is ignored, which is legitimate at this stage.

The results of the present calculations are displayed in Figs. 1–3 for the $^3He \rightarrow {}^3H$ transitions, and in Figs. 4–6 for the $^6Li \rightarrow {}^6He$ case. For each observable, we compare the full calculations (solid lines) to the single scattering values (dotted lines), and to an approximate treatment of the multiple scattering corrections (dashed lines), in which the spin-dependent component of the elastic amplitudes $g^{el}(q)$ is neglected.

Only a very limited number of observables are displayed. They concern the differential cross section, the vector analyzing powers A_{n0} due to the beam and the target A_{0n}, a characteristic "depolarization tensor," D_{LL}, for ^3He, and the tensor analyzing power A_{nn} for ^6Li. Although this choice is somewhat arbitrary, it reflects our main findings and provides a sufficient basis for the present discussion. We shall list below a number of comments and remarks.

The observables have been plotted against the quadritransfer $|t|$. Because the calculations are restricted to the forward production angles, the longitudinal component has been kept to its minimal value, namely its value at 0°. We have verified at the single scattering term level that this approximation is valid over the considered range of transfers. For the same reason, kinematical effects arising from change of reference frame (c.m., lab, NN, or N-nucleus), which are impor-

TABLE I. Values of the elementary Δ-production amplitudes used in the present work. They correspond to an incident energy of 800 meV and a Δ mass of 1238 MeV (the variable q denotes the transverse momentum transfer).

$$f_1(q) = 43.3e^{-0.274q^2} + i5.5$$
$$f_2(q) \approx 0.0$$
$$f_3(q) = -7.2e^{-0.865q^2} - i4.55$$
$$f_4(q) = -4.0 - i4.8$$
$$f_5(q) = -7.2e^{-0.0575q^2} - i4.9e^{-0.0625q^2}$$
$$f_6(q) = -3.6 - i4.86$$
$$f_7(q) = -6.0(1.0 - e^{-0.24q^2}) - i5.5$$
$$f_8(q) \approx 0.0$$

$$g_1(q) = -iq(11.8e^{-0.71q^2} + 12.5e^{-0.125q^2})$$
$$g_2(q) = i0.75q$$
$$g_3(q) = 1.25q + iq13.1e^{-0.245q^2}$$
$$g_4(q) = -q(2.22e^{-0.1q^2} - 1.97e^{-0.2q^2}) - iq2.76e^{-0.029q^2}$$
$$g_5(q) = -q(16.2e^{-0.5q^2} - 25.4e^{-0.4q^2} + 9.5e^{-0.25q^2}) - iq(2.96e^{-0.1q^2} - 3.1e^{-0.29q^2})$$
$$g_6(q) = q(2.44e^{-0.1q^2} - 2.12e^{-0.2q^2}) + iq(10.4e^{-0.4}q^2 - 22.6e^{-0.25q^2} + 13.6e^{-0.15q^2})$$
$$g_7(q) = -q(1.06e^{-0.08q^2} + iq1.95e^{-0.025q^2})$$
$$g_8(q) = -iq(4.53e^{-0.05q^2} - 4.13e^{-0.1q^2})$$

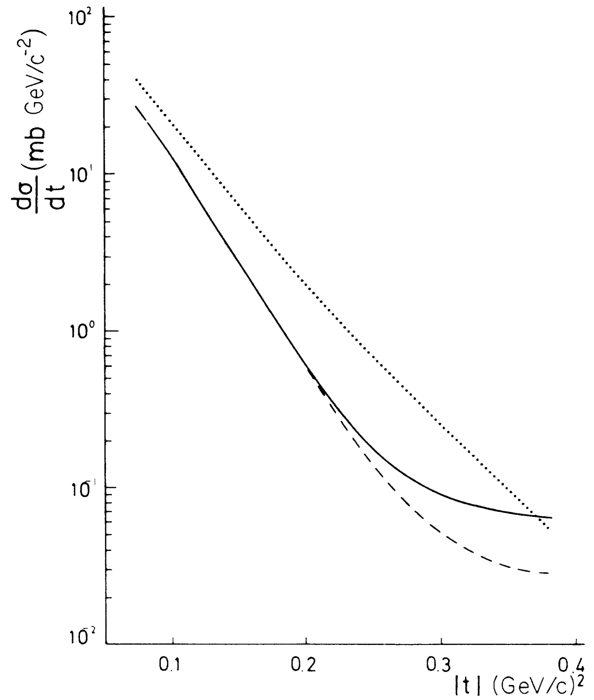

FIG. 1. $p+{}^3He \rightarrow \Delta^{++}+{}^3H$. Differential cross section plotted against the momentum transfer. The dotted line corresponds to the single scattering approximation. The dashed line denotes calculations in which only the spin independent part of the multiple scattering corrections are included. The results of the full calculation are displayed by the solid line.

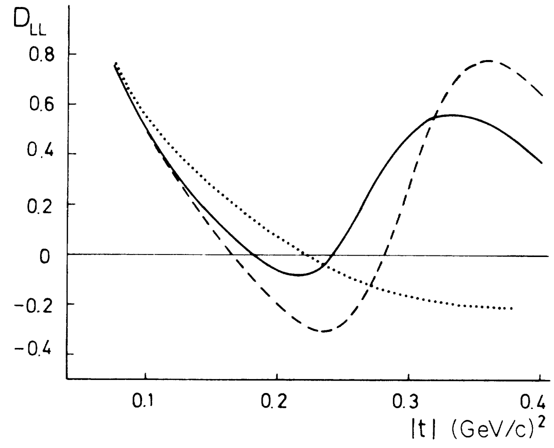

FIG. 3. $p+{}^3H \rightarrow \Delta^{++}+{}^3H$. Same as Fig. 1 but for D_{LL}.

tant in the present case, have been included through their first order expressions.

Except for the shape of the differential cross section, the single scattering approximation is never successful. Occasionally, for small momentum transfer, it may yield a crude average; see the analyzing powers A_{0n}, A_{n0}, and A_{nn} for the ^6Li case. However, predictions based on this approximation are hazardous.

Keeping only the spin-independent terms in the rescattering contributions is already a lot better. It ac-

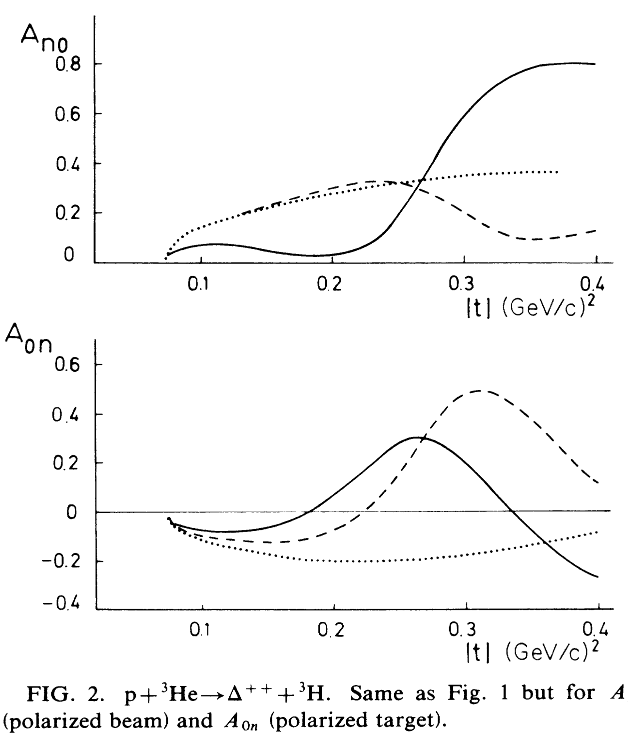

FIG. 2. $p+{}^3He \rightarrow \Delta^{++}+{}^3H$. Same as Fig. 1 but for A_{n0} (polarized beam) and A_{0n} (polarized target).

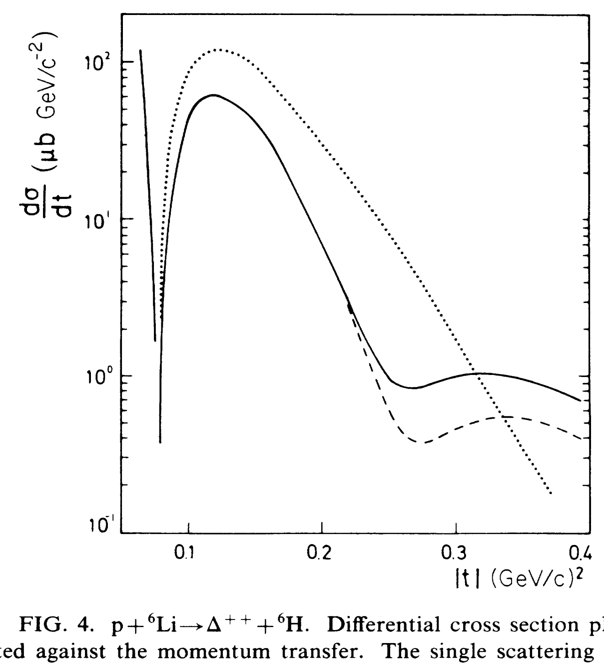

FIG. 4. $p+{}^6Li \rightarrow \Delta^{++}+{}^6H$. Differential cross section plotted against the momentum transfer. The single scattering approximation is given by the dotted line. The dashed line corresponds to calculations in which only the spin independent part of the multiple scattering corrections are included. The results of the full calculation are displayed by the solid line.

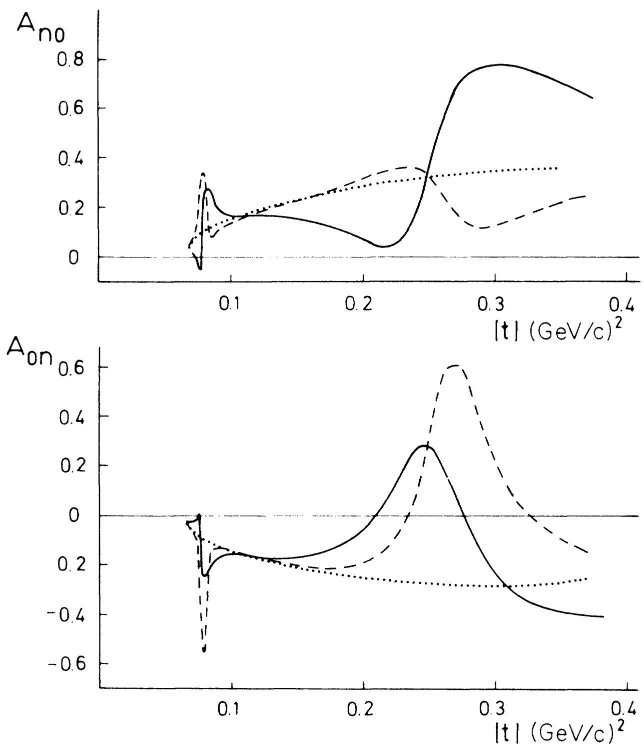

FIG. 5. $p+{}^6Li \rightarrow \Delta^{++} + {}^6He$. Same as Fig. 4 but for A_{n0} (polarized beam) and A_{0n} (polarized target).

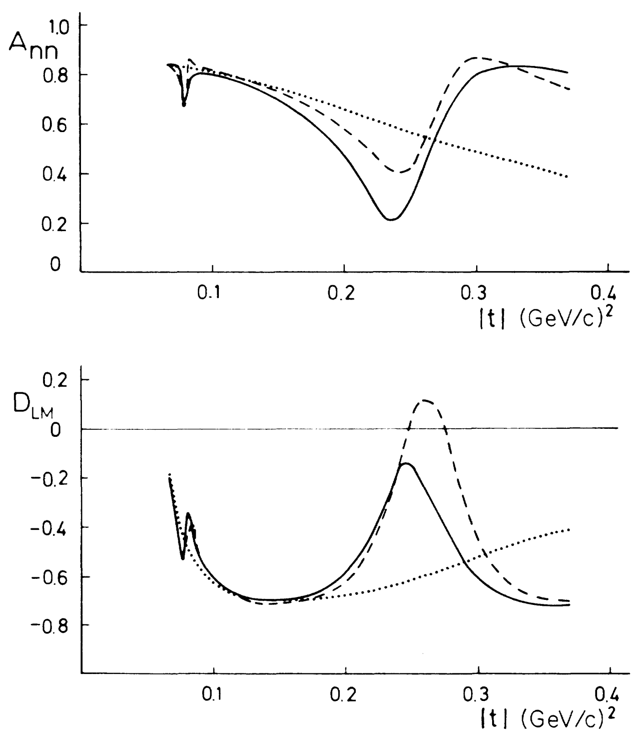

FIG. 6. $p+{}^6Li \rightarrow \Delta^{++} + {}^6He$. Same as Fig. 4 but for A_{nn} and D_{LM}.

counts partly for the nuclear structure aspect of the problem. It seems to be sufficient, over the range of momentum transfer investigated here, for observables involving sums or differences of squared amplitudes, like the differential cross section, D_{LL}, or A_{nn}. Observables built on interferences, like the asymmetries, are more sensitive to other contributions.

The spin-dependent rescattering contributions show their importance in various places. Most of the time they cannot be neglected, especially when the transverse momentum transfer reaches 2 fm^{-1}. The most dramatic case is the production asymmetry A_{n0} in the ${}^3He$-${}^3H$ transition. Actually only the terms linear in the spin-dependent amplitude play a role. Higher powers in $g^{el}(q)$ contributions can be ignored.

These spin-dependent corrections are somewhat sensitive to the terms involving the longitudinal momentum transfer [see Eq. (3.13)]. The effect is not large, so that it would be a bad approximation to throw them away, whereas the use of Gaussian densities at this level suffices to get a reasonable accuracy.

Since the rescattering contributions are important, our basic assumption concerning the propagation of the Δ through the nuclear matter may be too crude. This has the unpleasant feature of requiring tedious numerical exercises, but may bring valuable information.

VII. CONCLUSIONS

The present study has been devoted to Δ production in proton-nucleus collisions at intermediate energies. Within the Glauber model we have established practical formulas that account for the multiple scattering of the incident proton and the propagation of the Δ through the nucleus. The method is in our hands. It can be improved here and there but the main features are incorporated.

The two chosen examples clearly show the importance of multiple scattering corrections and the need for a rather refined treatment of the propagation, both projectile and ejectile. Predictions made within the single scattering approximation are unreliable. This aspect obscures our initial goal which was to use the nucleus and its spin selection rules as a means to isolate particular amplitudes. The situation is not hopeless, but the analysis of experimental data will require sophisticated methods to discriminate between amplitude effects and nuclear multiple scattering effects.

It is also important to notice that the differential cross sections are only sensitive to the spin-independent part of the elastic scattering amplitude. The spin-orbit component appears to have a decisive influence on the spin observables, in particular A_{n0}. This is a point of considerable interest.

ACKNOWLEDGMENTS

One of the authors (G.F.) would like to thank Prof. R. Vinh Mau for the kind hospitality at Orsay where this work was done. Division de Physique Théorique is an associated laboratory of CNRS.

APPENDIX A: PROPERTIES OF SPIN OPERATORS

Our spin transition operators $\mathbf{S}$ and $\overleftrightarrow{\mathrm{T}}$ are identical to those of Silbar *et al.*[8] Below we list some additional properties which are needed in the treatment of the multiple scattering series. We first introduce the Pauli spin operators $\boldsymbol{\sigma}_\Delta$ for the spin $\frac{3}{2}$ particle. They are 4×4 matrices related to the total spin operator by $\boldsymbol{\sigma}_\Delta=2\mathbf{S}_\Delta$ and given by the following (the Δ superscript is used when more convenient),

$$\sigma_x^\Delta=\begin{vmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{vmatrix},$$

$$\sigma_y^\Delta=i\begin{vmatrix} 0 & -\sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & -2 & 0 \\ 0 & 2 & 0 & -\sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{vmatrix}, \tag{A1}$$

$$\sigma_z^\Delta=\begin{vmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -3 \end{vmatrix}.$$

The vector transition operators $\mathbf{S}$ are 4×2 matrices, and their spherical components are normalized to

$$\langle \tfrac{3}{2} m_\Delta \mid S_\lambda \mid \tfrac{1}{2} m_{\mathrm{N}} \rangle = (\tfrac{3}{2})^{1/2} \langle \tfrac{1}{2}\, 1\, m_{\mathrm{N}} \lambda \mid \tfrac{3}{2} m_\Delta \rangle \;. \tag{A2}$$

One immediately verifies the following relations:

$$S_k\sigma_l - S_l\sigma_k = -i\epsilon_{klm}S_m \;, \tag{A3a}$$

$$\sigma_k^\Delta S_l - \sigma_l^\Delta S_k = 5i\epsilon_{klm}S_m \;. \tag{A3b}$$

The tensor transition operator T_{kl} is defined by

$$T_{kl}=\tfrac{1}{2}(S_k\sigma_l+S_l\sigma_k)=\tfrac{1}{2}(\sigma_k^\Delta S_l+\sigma_l^\Delta S_k) \;. \tag{A4}$$

From these definitions we derive

$$(\mathbf{S}\cdot\mathbf{a})(\boldsymbol{\sigma}\cdot\mathbf{b}) = -\frac{i}{2}(\mathbf{S}\cdot\mathbf{a}_\wedge\mathbf{b}) + \mathbf{a}\cdot\overleftrightarrow{\mathrm{T}}\cdot\mathbf{b} \;, \tag{A5a}$$

$$(\boldsymbol{\sigma}_\Delta\cdot\mathbf{b})(\mathbf{S}\cdot\mathbf{a}) = -\frac{5i}{2}(\mathbf{S}\cdot\mathbf{a}_\wedge\mathbf{b}) + \mathbf{a}\cdot\overleftrightarrow{\mathrm{T}}\cdot\mathbf{b} \;, \tag{A5b}$$

$$(\mathbf{a}\cdot\overleftrightarrow{\mathrm{T}}\cdot\mathbf{b})(\boldsymbol{\sigma}\cdot\mathbf{c}) = \tfrac{1}{4}[3(\mathbf{S}\cdot\mathbf{a})(\mathbf{b}\cdot\mathbf{c}) + 3(\mathbf{S}\cdot\mathbf{b})(\mathbf{a}\cdot\mathbf{c}) - 2(\mathbf{S}\cdot\mathbf{c})(\mathbf{a}\cdot\mathbf{b})] + \frac{i}{2}(\mathbf{a}\cdot\overleftrightarrow{\mathrm{T}}\cdot\mathbf{b}_\wedge\mathbf{c} + \mathbf{b}\cdot\overleftrightarrow{\mathrm{T}}\cdot\mathbf{a}_\wedge\mathbf{c}) \;, \tag{A6a}$$

$$(\boldsymbol{\sigma}_\Delta\cdot\mathbf{c})(\mathbf{a}\cdot\overleftrightarrow{\mathrm{T}}\cdot\mathbf{b}) = \tfrac{1}{4}[3(\mathbf{S}\cdot\mathbf{a})(\mathbf{b}\cdot\mathbf{c}) + 3(\mathbf{S}\cdot\mathbf{b})(\mathbf{a}\cdot\mathbf{b}) - 2(\mathbf{S}\cdot\mathbf{c})(\mathbf{a}\cdot\mathbf{b})] - \frac{3i}{2}(\mathbf{a}\cdot\overleftrightarrow{\mathrm{T}}\cdot\mathbf{b}_\wedge\mathbf{c} + \mathbf{b}\cdot\overleftrightarrow{\mathrm{T}}\cdot\mathbf{a}_\wedge\mathbf{c}) \;. \tag{A6b}$$

In particular, we get $(\boldsymbol{\sigma}_\Delta\cdot\mathbf{S})=(\mathbf{S}\cdot\boldsymbol{\sigma})=0$.

Our evaluation of the multiple scattering series makes repeated use of the commutator

$$(\boldsymbol{\sigma}_\Delta\cdot\hat{\mathbf{n}}_b)\sum_i[\tilde{f}_iQ_i(\hat{\mathbf{l}}_b,\hat{\mathbf{m}}_b)+\tilde{g}_iP_i(\hat{\mathbf{l}}_b,\hat{\mathbf{m}}_b)] - \sum_i[\tilde{f}_iQ_i(\hat{\mathbf{l}}_b,\hat{\mathbf{m}}_b)+\tilde{g}_iP_i(\hat{\mathbf{l}}_b,\hat{\mathbf{m}}_b)](\boldsymbol{\sigma}_0\cdot\hat{\mathbf{n}}_b)$$
$$= \sum_i[-t_i(\tilde{g})Q_i(\hat{\mathbf{l}}_b,\hat{\mathbf{m}}_b)+t_i(\tilde{f})P_i(\hat{\mathbf{l}}_b,\hat{\mathbf{m}}_b)] \;, \tag{A7}$$

with $t_i(z=(z_1,\ldots,z_8))$ given by

$$t_1(z)=-2z_3,\quad t_2(z)=2z_4,\quad t_3(z)=-2z_1,\quad t_4(z)=2z_2,\quad t_5(z)=0 \;,$$
$$t_6(z)=-2\sqrt{3}z_7-2z_8,\quad t_7(z)=-2\sqrt{3}z_6,\quad t_8(z)=-2z_6 \;. \tag{A8}$$

We also need

$$\sum_i[\tilde{f}_iQ_i(\hat{\mathbf{l}}_b,\hat{\mathbf{m}}_b)+\tilde{g}_iP_i(\hat{\mathbf{l}}_b,\hat{\mathbf{m}}_b)](\boldsymbol{\sigma}_0\cdot\hat{\mathbf{n}}_b) = \sum_i[-v_i(\tilde{g})Q_i(\hat{\mathbf{l}}_b,\hat{\mathbf{m}}_b)+v_i(\tilde{f})P_i(\hat{\mathbf{l}}_b,\hat{\mathbf{m}}_b)] \;, \tag{A9}$$

with

$$v_1(z)=-\frac{\sqrt{3}}{2}z_4-\tfrac{1}{2}z_3,\quad v_2(z)=\frac{\sqrt{3}}{2}z_3-\tfrac{1}{2}z_4,\quad v_3(z)=-\tfrac{1}{2}z_1+\frac{\sqrt{3}}{2}z_2,\quad v_4(z)=-\frac{\sqrt{3}}{2}z_1-\tfrac{1}{2}z_2 \;,$$
$$v_5(z)=\tfrac{1}{2}z_7-\frac{\sqrt{3}}{2}z_8,\quad v_6(z)=\frac{\sqrt{3}}{2}z_7+\tfrac{1}{2}z_8,\quad v_7(z)=\tfrac{1}{2}z_5+\frac{\sqrt{3}}{2}z_6,\quad v_8(z)=-\frac{\sqrt{3}}{2}z_5+\tfrac{1}{2}z_6 \;. \tag{A10}$$

APPENDIX B: *D* WAVES IN THE SINGLE SCATTERING TERMS

In Sec. IV we assumed the wave functions of the ^{3}He and ^{3}H nuclei to be pure *S* wave. In reality, there are also *S'*- and *D*-wave contributions. Below we show how the single scattering terms are changed when these components are added.

We define three transition form factors $\mathcal{F}_0$, $\mathcal{F}_1$, and $\mathcal{F}_1'$ as follows:

$$\langle {}^3\mathrm{H} \mid \boldsymbol{\tau}_1\cdot\mathbf{a} e^{i\mathbf{Q}\cdot\mathbf{x}_1} \mid {}^3\mathrm{He}\rangle = \langle \mathrm{n} \mid \boldsymbol{\tau}\cdot\mathbf{a}\mathcal{F}_0(Q) \mid \mathrm{p}\rangle \ , \tag{B1a}$$

$$\langle {}^3\mathrm{H} \mid \boldsymbol{\tau}_1\cdot\mathbf{a}\boldsymbol{\sigma}_i\cdot\mathbf{b} e^{i\mathbf{Q}\cdot\mathbf{x}_1} \mid {}^3\mathrm{He}\rangle = \langle \mathrm{n} \mid -\boldsymbol{\tau}\cdot\mathbf{a}\{\boldsymbol{\sigma}\cdot\mathbf{b}\mathcal{F}_1(Q) + [3(\boldsymbol{\sigma}\cdot\hat{\mathbf{Q}})(\hat{\mathbf{Q}}\cdot\mathbf{b}) - \boldsymbol{\sigma}\cdot\mathbf{b}]\mathcal{F}_1'(Q)\} \mid \mathrm{p}\rangle \ . \tag{B1b}$$

They are functions of $Q=(q_\perp^2+\Delta_\parallel^2)^{1/2}$ and have been calculated by Lazard *et al.*[19] The bra and ket vectors $\langle \mathrm{n}|$ and $|\mathrm{p}\rangle$ only serve as indicators of the corresponding quantum numbers.

The formulae given in Sec. IV correspond to the case $\mathcal{F}_0=\mathcal{F}_1=S_T(Q)$ and $\mathcal{F}_1'=0$. When all the form factors are present the single scattering approximation to the amplitudes of Eq. (4.1) reads

$$F_i(Q)=\epsilon_i\left\{ f_i(q)[\mathcal{F}_1(Q)-\mathcal{F}_1'(Q)+3\frac{\Delta_\parallel^2}{Q^2}\mathcal{F}_1'(Q)]+3i\frac{\Delta_\parallel q}{Q^2}g_i(q)\mathcal{F}_1'(Q)\right\} \quad \text{for } i=1,2 \ , \tag{B2a}$$

$$F_i(Q)=\epsilon_i\left\{ f_i(q)[\mathcal{F}_1(Q)-\mathcal{F}_1'(Q)+\frac{3q^2}{Q^2}\mathcal{F}_1'(Q)]-3i\frac{\Delta_\parallel q}{Q^2}g_i(q)\mathcal{F}_1'(Q)\right\} \quad \text{for } i=3,4 \ , \tag{B2b}$$

$$F_i(Q)=\epsilon_i f_i(q)[\mathcal{F}_1(Q)-\mathcal{F}_1'(Q)] \quad \text{for } i=5,6 \ , \tag{B2c}$$

$$F_i(Q)=\epsilon_i f_i(q)\mathcal{F}_0(Q) \quad \text{for } i=7,8 \ , \tag{B2d}$$

$$G_i(Q)=\eta_i\left\{ g_i(q)\left[\mathcal{F}_1(Q)-\mathcal{F}_1'(Q)+3\frac{q^2}{Q^2}\mathcal{F}_1'(Q)\right]-3i\frac{\Delta_\parallel q}{Q^2}f_i(q)\mathcal{F}_1'(Q)\right\} \quad \text{for } i=1,2 \ , \tag{B3a}$$

$$G_i(Q)=\eta_i\left\{ g_i(q)\left[\mathcal{F}_1(Q)-\mathcal{F}_1'(Q)+3\frac{\Delta_\parallel^2}{Q^2}\mathcal{F}_1'(Q)\right]+3i\frac{\Delta_\parallel q}{Q^2}f_i(q)\mathcal{F}_1'(Q)\right\} \quad \text{for } i=3,4 \ , \tag{B3b}$$

$$G_i(Q)=\eta_i g_i(q)\mathcal{F}_0(Q) \quad \text{for } i=5,6 \ , \tag{B3c}$$

$$G_i(Q)=\eta_i g_i(q)[\mathcal{F}_1(Q)-\mathcal{F}_1'(Q)] \quad \text{for } i=7.8 \ . \tag{B3d}$$

In Sec. V only the *S*-wave part of the transition density for $^6\mathrm{Li}\to{}^6\mathrm{He}$ was properly treated. The *D* wave is easily included, at least in the single scattering approximation. To this end we define the transition form factors $\mathcal{F}_1$ and $\mathcal{F}_1'$ through

$$\langle {}^6\mathrm{He} \mid (\boldsymbol{\tau}_1\cdot\mathbf{a})(\boldsymbol{\sigma}_1\cdot\mathbf{b})e^{i\mathbf{Q}\cdot\mathbf{x}_1} \mid {}^6\mathrm{Li}\rangle = (a_x+ia_y)\frac{V_0}{\sqrt{2}}\langle 0 \mid \mathbf{V}\cdot\mathbf{b}\mathcal{F}_1(Q)+[3(V,Q)(Q,b)-(V,b)]\mathcal{F}_1'(Q) \mid 1,M\rangle \ . \tag{B4}$$

The formulae of Sec. V correspond to the case $\mathcal{F}_1'=0$ and $\mathcal{F}_1(Q)=S_T(Q)$. The expressions for the single scattering approximation to the amplitudes of Eq. (5.2) are obtained from those of Eqs. (B2) and (B3). We set $\mathcal{F}_0=0$, make the replacements $\epsilon_i\to\epsilon_i'$, $\eta_i\to\eta_i'$, and apply a common multiplicative factor. Thus, for $i=1,2$ we have

$$F_1(Q)=\epsilon_i'\sqrt{2}V_0\left[f_i(q)\left[\mathcal{F}_1-\mathcal{F}_1'+3\frac{\Delta_\parallel^2}{Q^2}\mathcal{F}_1'\right]+3i\frac{\Delta_\parallel q}{Q^2}g_i(q)\mathcal{F}_1'\right] \tag{B5}$$

and similarly for $i=3,\ldots,8$.

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