Coupled-channel description of inelastic scattering from soft nuclei

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A coupled-channel formalism for describing inelastic hadron scattering from soft nuclei is derived. This formalism, applicable up to at least intermediate energies, employs collective many-body wave functions built from constrained Hartree-Fock or Hartree-Fock-Bogolyubov intrinsic states. Successive approximations are developed in order to derive simple expressions for transition form factors to be used in coupled-channel calculations. These form factors carry the information on the intrinsic structure of the nucleus since they depend explicitly on the intrinsic states $|q\rangle$ and $|q'\rangle$ as well as the overlap $\langle q|q'\rangle$. Under the narrow overlap approximation $\langle q|q'\rangle \sim \delta(q-q')$ this information gets partly lost, and the present formalism reduces to a more familiar form. An illustration is given in the context of phenomenological form factors for proton scattering from ^{196}Pt considered as a γ-unstable nucleus.

I. INTRODUCTION

For many years, nucleons and other hadrons have been used as probes of nuclear collective motion through inelastic scattering measurements. Usually these measurements have been interpreted in terms of simple collective models (harmonic vibrational model, rigid axial and triaxial rotor models, etc.) with variable success. Sometimes it happens that such models are inappropriate and therefore need to be extended. These extensions are usually achieved in an *ad hoc* manner, that is, for instance, by varying arbitrarily nuclear reduced matrix elements or by mixing vibrations carrying different phonon quantum numbers. These kinds of extensions clearly indicate that the simple collective models are actually too simple.

On the other hand, studies of nuclear collective motions have reached a higher level of sophistication. Schematically, these nuclear structure works may be classified into two categories. The first includes studies based on the mean-field theory: random phase approximation (RPA) theory[1] and generator coordinate method[2,3] for collective motions with small and large amplitudes, respectively. The second category deals with macroscopic models which are usually designed to describe large amplitude collective motions. This category includes the Bohr Hamiltonian expressed either in the laboratory coordinate system[4] or in the intrinsic frame.[5] It also contains the interacting boson model[6] Hamiltonian which may be written[7] (for the IBA-1) in the form of a Bohr Hamiltonian.

In principle these advanced nuclear structure models could be tested in hadron scattering studies. A straightforward manner in which to do this consists of taking nuclear reduced matrix elements as predicted by these models and selecting optical potential form factors from simple prescriptions. The drawback of this method is that collective wave functions and transition potentials are not treated in a consistent manner. In addition, ill-motivated assumptions regarding the radial behavior of potential form factors may lead to misinterpretations of scattering measurements.

A more consistent treatment of the whole scattering process is desirable in order to take advantage of the additional information that can be obtained about advanced collective models by scattering. These include details about the radial shape of form factors [or, alternatively, the behavior of the form factors in the momentum (Q) space beyond the low-Q region probed by γ decay or Coulomb excitation], and also the additional information (in comparison with electron scattering) gained by the fact that strong channel coupling implies interfering reaction paths.

It is the purpose of the present work to set up a general reaction formalism for inelastic scattering from large amplitude collective states, which treats potentials and wave functions in a consistent manner. This treatment is made in the framework of the coupled-channel (CC) formalism which is known to work reasonably well up to at least intermediate energies. In Sec. II we briefly derive CC equations and define collective (rotational and vibrational) states in the framework of the generator coordinate method (GCM). This section is introduced in the present paper for the sake for completeness. Section III illustrates how to generate transition potentials from the many-body wave functions defined previously. The actual calculations of these potentials represent a considerable numerical task. Consequently, in order to provide a manageable framework for the calculations, a semiadiabatic approximation for the many-body wave functions is used to generate the transition potentials. This is described in Sec. IV. Further simplifications are achieved by assuming that the overlap between GCM intrinsic states is a δ function. This approximation is used in Sec. V for the CC calculation of $(\vec{p},p')$ scattering from ^{196}Pt considered as a γ-unstable nucleus.

II. DEFINITIONS AND NOTATIONS

A. GCM wave functions

In the generator coordinate method[2] one considers a family of N-particle functions $\phi(x,q)$, where the symbol x denotes all particle coordinates. In the most general case, the parameter q stands for a set of n parameters $q_1,\ldots,q_n$. An approximate wave function $\psi(x)$ of the many-body system is obtained by making a linear combination of the functions ϕ as follows:

$$\psi(x)=\int f(q)\phi(x,q)dq \ . \tag{1}$$

The variational principle is then used to derive an integral equation for $f(q)$,

$$\int H(q,q')f(q')dq'=E_A\int I(q,q')f(q')dq' \ , \tag{2}$$

where E_A is an eigenvalue of Eq. (2), and $H(q,q')$ and $I(q,q')$ are the Hamiltonian and the overlap kernels defined as

$$H(q,q')=\langle\phi(q)\,|\,H_A\,|\,\phi(q')\rangle \ ,$$

and

$$I(q,q')=\langle\phi(q)\,|\,\phi(q')\rangle \ , \tag{3}$$

respectively, where H_A is the Hamiltonian of the interacting many-body system.

Equations (1)—(3) can be specialized to the description of nuclear collective spectra if one considers that the parameters q_i $(i=1,\ldots,n)$ are those which generate collective modes in nuclei.[2] Let us assume that the many-body wave functions $\phi(x,q)$ are made up of independent particles or quasiparticle states involved in Hartree-Fock (HF) or Hartree-Fock-Bogolyubov (HFB) calculations, and that q represents the set of constraint parameters. As is well known, the rotational symmetry can be spontaneously broken when use is made of the self-consistent field approximation; therefore, the HF or HFB solutions are degenerate with respect to rotations. In order to break the degeneracy on the angular momentum one replaces Eq. (1) with the following linear combination:

$$|\,\psi(x)\rangle=\int\int d\Omega f(q,\Omega)R(\Omega)\,|\,\phi(x,q)\rangle dq \ , \tag{4}$$

where $R(\Omega)$ is the rotation operator. The amplitude $f(q,\Omega)$ is determined through a minimization similar to that used to get Eq. (2), for each value of q. One can expand the amplitude f in the complete basis of Wigner functions D^{I*}_{MK},

$$f(q,\Omega)=\sum_{IMK}\frac{2I+1}{8\pi^2}f^I_{MK}(q)D^{I*}_{MK}(\Omega) \ ,$$

and introduce the Peierls-Yoccoz projection operator[8] P^I_{MK} to express $|\,\psi\rangle$ as

$$|\,\psi\rangle=\sum_{IMK}\int f^I_{MK}(q)P^I_{MK}\,|\,\phi(q)\rangle dq \ . \tag{5}$$

Since H_A is rotationally invariant, f is independent of M, and the physical state $|\,IM\rangle$ may be expressed[9] as

$$|\,IM,\nu\rangle=\sum_K\int f^I_{K,\nu}(q)P^I_{MK}\,|\,\phi(q)\rangle dq \ , \tag{6}$$

where ν is an index introduced to distinguish states having the same quantum numbers (I,M). The amplitudes $f^I_{K,\nu}(q)$ and eigenvalues E^ν_I of the states are obtained from the eigenvalue equations,

$$\sum_{K'}\int[H^I_{KK'}(q,q')-E^\nu_I N^I_{KK'}(q,q')]f^I_{K',\nu}(q')dq'=0 \ , \tag{7}$$

with the Hamiltonian and overlap kernels as defined[9] by

$$H^I_{KK'}(q,q')=\frac{2I+1}{8\pi^2}\times\int D^{I*}_{KK'}(\Omega)\langle\phi(q)\,|\,H_A R(\Omega)\,|\,\phi(q')\rangle d\Omega \ ,$$

and

$$N^I_{KK'}(q,q')=\frac{2I+1}{8\pi^2}\int D^{I*}_{KK'}(\Omega)\langle\phi(q)\,|\,R(\Omega)\,|\,\phi(q')\rangle d\Omega \ ,$$

respectively.

In the present work it is assumed that the wave functions $|\,IM,\nu\rangle$ are already known from the projection method outlined above. Therefore, Eq. (6) is used throughout in Sec. II B as the definition of the physical collective states [with $f^I_{K,\nu}(q)$ real].

B. CC formalism

In the center-of-mass (c.m.) system, the Hamiltonian H of the interacting particles can be expressed as

$$H=T+H_A(x_1,\ldots,x_n)+V(x_1,\ldots,x_n) \ ,$$

where T is the kinetic energy, H_A the Hamiltonian of the target nucleus, and V the interaction potential. It is assumed that V is the sum of two-body effective interactions between the incident particle and the target nucleons,[10]

$$V=\sum_{i=1}^{n}v(x,x_i) \ , \tag{8}$$

where x and x_i denote the particle coordinates. The effective interaction is in general nonlocal, complex and energy dependent. It contains a central term[10] as well as spin-orbit[11] and tensor[12] terms,

$$v_{12}=v^c+v^{LS}(\mathbf{L}\cdot\mathbf{s})+v^T S_{12} \ . \tag{9}$$

This interaction is assumed to be a local approximation to a t matrix[13] or G matrix.[14]

The Schrödinger equation

$$(H-E)\psi=0 \tag{10}$$

is solved using standard techniques described, for instance, in Ref. 10. These lead to the coupled equations for the radial functions $R^{J\pi}_c(r)$ regular at the origin,

$$\left[\frac{\hbar^2}{2m}\left[-\frac{d^2}{dr^2}+\frac{l(l+1)}{r^2}\right]-E+E^\nu_I\right]R^{J\pi}_c(r)=-\sum_{c'}\langle lsjIJ\pi\overline{M}\,|\,V\,|\,l'sj'I'J\pi\overline{M}\rangle R^{J\pi}_{c'}(r) \ , \tag{11}$$

where the ket $|lsjIJ\pi\overline{M}\rangle$ is a symbolic notation for the eigenfunction of the total angular momentum **J** and parity π, and m is the reduced mass of the system. Moreover, c is the label for a reaction channel defined in terms of the quantum numbers v, l, s, j, and I.

III. MATRIX ELEMENTS OF V: GENERAL CASE

In order to get an explicit form of the matrix elements (ME) of Eq. (11), let us introduce the following notation for the total angular momentum wave functions:

$$|lsjIJ\pi\overline{M}\rangle=\sum_K\int f^I_{K,v}(q)\,|lsjIK(q)J\pi\overline{M}\rangle dq\ , \quad (12)$$

where Eq. (6), along with the symbolic expression

$$|IMK(q)\rangle=P^I_{MK}\,|\phi(q)\rangle\ ,$$

is used. Inserting Eq. (12) into the ME of V shown in the right hand side (RHS) of Eq. (11), one obtains

$$\langle lsjIJ\pi\overline{M}\,|\,V\,|\,l'sj'I'J\pi\overline{M}\rangle=\sum_{KK'}\int\int f^I_{K,v}(q)f^{I'}_{K',v'}(q')\langle lsjIK(q)J\pi\overline{M}\,|\,V\,|\,l'sj'I'K'(q')J\pi\overline{M}\rangle dq\,dq'\ . \quad (13)$$

To proceed further, the potential V is expanded into multipoles. Rather than discuss the full effective interaction [Eq. (9)], we illustrate the general procedure with a very simple central interaction,

$$v(x,x_i)=(v_0+v_1\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}_i)g(\,|\,\mathbf{r}-\mathbf{r}_i\,|\,)\ , \quad (14)$$

where g is a radial form factor. Isospin-dependent terms have also been omitted for simplicity. Extensions to the full effective interaction including spin-orbit and tensor terms, and with appropriate approximations for exchange between projectile and target nucleons, may be found elsewhere.[15] The multipole expansion[16] of Eq. (14) yields

$$V=\sum_{LS\lambda}V_S(-1)^{L+S+\lambda}[T^{(LS)\lambda}(r,A)\cdot T^{(LS)\lambda}(\hat{\mathbf{r}},\sigma)]\ , \quad (15)$$

where $T^{(LS)\lambda}(r,A)$ is an irreducible tensor operator of degree λ,

$$T^{(LS)\lambda}(r,A)=\sum_{i=1}^{n}v_L(r,r_i)T^{(LS)\lambda}(\hat{\mathbf{r}}_i,\sigma_i)\ , \quad (16)$$

in which $v_L(r,r_i)$ is a multipole of the radial form factor g given in Eq. (14), and

$$T^{(LS)\lambda}(\hat{\mathbf{r}}_i,\sigma_i)=[\Sigma^{(S)}(\sigma_i)C^{(L)}(\hat{\mathbf{r}}_i)] \quad (17)$$

with $\Sigma^{(S)}$ and $C^{(L)}$ as defined in Ref. 10. The matrix element of the interaction shown in the RHS of Eq. (15) can be written as follows:[17]

$$\begin{aligned}\langle lsjIK(q)J\pi\overline{M}\,|\,V\,|\,l'sj'I'K'(q')J\pi\overline{M}\rangle&=\sum_{LS\lambda}V_S(-1)^{L+S+\lambda+j'+I+J}\begin{Bmatrix}jIJ\\I'j'\lambda\end{Bmatrix}\\&\times\langle lsj||T^{(LS)\lambda}(\hat{\mathbf{r}},\sigma)||l'sj'\rangle\\&\times\langle IK(q)||T^{(LS)\lambda}(r,A)||I'K'(q')\rangle\ . \quad (18)\end{aligned}$$

The first reduced matrix element appearing in Eq. (18) is just a geometrical factor.[10] The second one is responsible for the target nucleus transitions and is difficult to evaluate. Whatever is the complexity of the nuclear excited states, the reduced matrix element

$$\langle IK(q)||T^{(LS)\lambda}(r,A)||I'K'(q')\rangle \quad (19)$$

shown in the RHS of Eq. (18) may be written[10] in terms of products of a spectroscopic amplitude S_λ and a form factor F_λ. Using notations similar to those of Ref. 10, Eq. (19) may be expressed formally as

$$\langle IK(q)||T^{(LS)\lambda}(r,A)||I'K'(q')\rangle=\sum_{j_1j_2}S^{j_1j_2}_\lambda(q,q')F^{j_1j_2}_\lambda(r,q,q') \quad (20)$$

where (j_1,j_2) are indexes running, for instance, over the quasiparticle basis states of the HFB theory. The calculation of the spectroscopic amplitudes is quite involved and will not be shown here. However, that calculation can be performed using methods described by Amos *et al.*[18] On the other hand, the transition form factor $F^{j_1j_2}_\lambda$ may be written as

$$F^{j_1j_2}_\lambda(r,q,q')=R^{j_1j_2}_\lambda(r,q,q')\langle j_1||T^{(LS)\lambda}(\hat{\mathbf{r}},\sigma_i)||j_2\rangle\ , \quad (21)$$

where the reduced matrix element is a geometrical factor. The radial form factor is

$$R^{j_1j_2}_\lambda(r,q,q')=(2\lambda+1)^{-1}\int U_1(r',q)v_\lambda(r,r')\times U_2(r',q')r'^2dr'\ , \quad (22)$$

where U_i is the radial part of the wave function for a particle or quasiparticle involved in the constrained HF or HFB calculations mentioned earlier. In comparison with the usual[10] coupled-channel treatment of scattering from collective states, the essential complication arises from the introduction of the dynamic collective coordinates q in the double integral over q and q' in Eq. (13). That is, the nuclear matrix elements are defined in terms of quantities which are nonlocal in the collective coordinates.

IV. APPROXIMATIONS

A. Wave functions

The reaction formalism given in Sec. III implies a large computational effort which, for medium and heavy nuclei, is not feasible on most computers. Consequently, it

is necessary to develop some approximations in order to achieve manageable calculations.

We restrict the discussion to quadrupole collective motions. That is, the set of collective coordinates q only contains q_0 and q_2 defined as

$$q_0 \sim r^2 Y_0^2 ,$$

and

$$q_2 \sim r^2 (Y_2^2 + Y_{-2}^2) ,$$

which generate β and γ vibrations in nuclei. Going a step further, we also approximate the state $|IM,\nu\rangle$, Eq. (6), by $|\tilde{I}M,\nu\rangle$. This collective state, to within a normalization factor, is defined as follows:

$$|\tilde{I}M,\nu\rangle = \sum_K \int f^i_{K,\nu}(q)[D^{I*}_{MK}(\Omega) + (-1)^{I+K} D^{I*}_{M-K}(\Omega)] \times |\phi(q)\rangle dq , \quad (23)$$

and is invariant under time reversal. The physical assumption underlying Eq. (23) is that the characteristic time of rotations is longer than that of vibrations. This leads to a semiadiabatic approximation for the collective wave functions since the rotation and vibration components of the wave functions are factorized out under the integration symbol in Eq. (23). This approximation is of a different nature from that involved in the Bohr and Mottelson Hamiltonian since, in that model, the rotational, the vibrational, and intrinsic states are completely factorized.

B. Optical and transition potentials

A consequence of the approximation, Eq. (23), is that a new potential must be defined. In the truncated configuration space where the coupled equations are actually solved, this new potential is noted as $\tilde{\vartheta}$. Calculations of the matrix elements of $\tilde{\vartheta}$ are performed through a multipole expansion similar to that shown in Sec. III. Again, the matrix elements of $\tilde{\vartheta}$ contain a geometrical factor, a nuclear reduced matrix element, and a radial form factor. This form factor has a kernel which depends on the collective coordinates (q,q') as well as on the overlap $\langle q \mid q' \rangle$ between the intrinsic states $|\phi(q)\rangle$ and $|\phi(q')\rangle$.

Thus, the kernel K of the potential $\tilde{\vartheta}$ can be written formally as

$$K = U(\mathbf{r},\sigma,q,q')\langle q \mid q' \rangle ,$$

where $\mathbf{r}$ and σ denote the space and spin coordinates of the incident particle, respectively. After the multipole expansion of U has been performed, the matrix elements of $\tilde{\vartheta}$ can be written as

$$\langle lsj\tilde{I}J\pi\overline{M} \mid \tilde{\vartheta} \mid l'sj'\tilde{I}'J\pi\overline{M} \rangle = \sum_{\lambda KK'} U^{II'KK'}_{\lambda\mu\nu\nu'}(r,\sigma)(-1)^{I+j'+J} \begin{Bmatrix} jIJ \\ I'j'\lambda \end{Bmatrix} \times \langle j||Y^\lambda(\hat{\mathbf{r}})||j'\rangle \langle IK||Q^\lambda_{(\mu)}||I'K'\rangle , \quad (24)$$

with $\mu = |K-K'|$, and

$$U^{II'KK'}_{\lambda\mu\nu\nu'}(r,\sigma) = \int\int f^I_{K,\nu}(q) f^{I'}_{K',\nu'}(q') U_{\lambda\mu}(r,\sigma,q,q') \times \langle q \mid q' \rangle dq\, dq' . \quad (25)$$

The coupled-channel equations, Eq. (15), can be finally written as follows:

$$\left[\frac{\hbar^2}{2m}\left(-\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2}\right) - E + E^\nu_I + U^{II00}_{00\nu\nu}(r,\sigma)\right] R^{J\pi}_c(r) = -\sum_{KK'c'\lambda} U^{II'KK'}_{\lambda\mu\nu\nu'}(r,\sigma)\langle IK||Q^\lambda_{(\mu)}||I'K'\rangle \mathscr{A}(ljI,l'j'I',\lambda Js) R^{J\pi}_{c'}(r) , \quad (26)$$

where $U^{II00}_{00\nu\nu}$ is a diagonal term, $\mathscr{A}$ is a geometrical factor identical to that given in Ref. 19 except for a phase factor to be discussed later, and $\langle IK||Q^\lambda_{(\mu)}||I'K'\rangle$ is a nuclear reduced matrix element for rotations.[20]

The expression Eq. (26) is intended as a schematic description of the projectile-nucleus interaction. Of course, the following development also applies to a more complete description of the interaction, including Coulomb, spin-orbit, and tensor terms, and isospin as well as energy and density dependences in the context of the folding model.

C. Vibration amplitudes

The most important result of the present work is expressed in Eq. (25): the transition potential results from averaging a transition operator over the superposition amplitudes of the initial and final states. However, these amplitudes cannot *stricto senso* be considered as probability amplitudes for a proper quantum mechanical system.[3]

In order to get an object which can be interpreted as a vibrational wave function, it is necessary to transform f into g using the definition[3,21]

$$g^I_{K,\nu}(q) = \int (\langle q \mid q' \rangle)^{1/2} f^I_{K,\nu}(q') dq' . \quad (27)$$

The g's fulfill the correct orthonormality condition

$$\sum_K |g^I_{K,\nu}(q)|^2 dq = 1 . \quad (28)$$

When dealing with the Gaussian overlap approximation[3] (GOA), the Griffin-Hill-Wheeler equation (2) for f [or

Eq. (7) for $f^I_{K,\nu}$] can be expressed in the form of a collective Hamiltonian for g (or $g^I_{K,\nu}$), and the "true" vibration amplitudes $g^I_{K,\nu}(q)$, Eq. (27), can be inserted into Eq. (25) using the Gauss transform[22] of $f^I_{K,\nu}(q)$.

For the moment we discuss a more drastic approximation to Eq. (25), which can be readily used in actual CC calculations. This approximation consists of setting

$$\langle q \mid q' \rangle \sim \delta(q-q') \ . \tag{29}$$

In this case, f is identical to g, and Eq. (25) reduces to

$$U^{II'KK'}_{\lambda\mu\nu\nu'}(r,\sigma)=\int g^I_{K,\nu}(q)g^{I'}_{K',\nu'}(q)U_{\lambda\mu}(r,\sigma,q,q)dq \ . \tag{30}$$

It is easy to generate such form factors if one assumes that the collective optical potential is parametrized in terms of a "collective" radius[23] $\tilde{R}(\Omega')$ of the form [assuming $q \equiv (\beta,\gamma)$]

$$\tilde{R}(\Omega')=R_0\left\{1+\beta\cos(\gamma)Y^2_0(\Omega') + \frac{1}{\sqrt{2}}\beta\sin(\gamma)[Y^2_2(\Omega')+Y^2_{-2}(\Omega')]\right\} . \tag{31}$$

With an optical potential depending on the relative distance $r-\tilde{R}(\Omega')$, Eq. (30) becomes explicitly

$$U^{II'KK'}_{\lambda\mu\nu\nu'}(r,\sigma) = \int g^I_{K,\nu}(\beta,\gamma)g^{I'}_{K',\nu'}(\beta,\gamma)U_{\lambda\mu}(r,\sigma,\beta,\gamma)d\tau' \ , \tag{32}$$

where $d\tau'$ includes the metric of the Bohr Hamiltonian.[5] For the original Bohr Hamiltonian,[23]

$$d\tau'=\beta^4 \mid \sin(3\gamma) \mid d\beta d\gamma \ .$$

It is through Eq. (32) that various collective models of the nucleus can be tested in CC calculations, provided that these models are expressed in the intrinsic coordinate system. Such is the case of Bohr Hamiltonian which has been solved numerically by Kumar and Baranger.[5] In this case, the multipole of the potential given on the RHS of Eq. (32) is averaged[24] over a β-γ mesh involving 92 points.

Applications of Eq. (32) have been previously made to several nuclei.[25] Other collective models could indeed be tested. One of them could be the interacting boson model[6] (IBA-1) provided that it is expressed in terms of (β,γ) collective coordinates.[7] Using this model requires a different definition[26] of the collective radius $\tilde{R}(\Omega')$ since the IBA-1 deformation parameters (β,γ) describe the shape of valence nucleon distributions outside an inert core.

More schematic collective models could also be tested. For even-Z even-N rotors with axially and nonaxially symmetric rigid shapes, the vibration amplitude is

$$g^I_{K,\nu}(\beta,\gamma)=\delta(\beta-\beta_0)\delta(\gamma-0)\delta_{KK'}\delta_{K0} \ ,$$

and

$$g^I_{K,\nu}(\beta,\gamma)=A^I_{K,\nu}(\gamma)\delta(\beta-\beta_0)\delta(\gamma-\gamma_0) \ ,$$

respectively. These amplitudes, when inserted in Eq. (30), restore familiar forms of transition potentials for symmetric and asymmetric rotors.[19,20] Wave functions of the harmonic oscillator in five dimensions[5] can also be tested. In fact, these schematic models have been used to fix the phase factor appearing on the RHS of Eq. (26), when the CC calculations are performed with the computer program ECIS.[27]

V. ILLUSTRATION: γ-UNSTABLE MODEL

We demonstrate the use of the extended coupled-channel formalism, Eqs. (26)—(32), with a calculation of 35 MeV proton scattering from ^{196}Pt, in which the collective structure of the nucleus is described by the Wilets and Jean (WJ) model[28] for gamma-unstable nuclei. This model is similar to the O(6) limit of the IBA-1, which has been shown[29] to provide a useful first approximation to ^{196}Pt.

The collective wave functions may be expressed in the form

$$\mid IM,\nu\rangle=\sum_K A^I_{K,\nu}(\beta,\gamma) \mid IMK\rangle \ , \tag{33}$$

where the rotational eigenfunctions are

$$\mid IMK\rangle = \left[\frac{2I+1}{16\pi^2(1+\delta_{K0})}\right]^{1/2} \times[D^{I*}_{MK}(\Omega')+(-1)^{I+K}D^{I*}_{M-K}(\Omega')] \ ,$$

in which Ω' represents the Euler angles. For the WJ model, the β and γ dependences may be factored, so that

$$A^I_{K,\nu}(\beta,\gamma)=(6)^{1/2}\beta^2[\sin(3\gamma)]^{1/2}f_{n\Lambda}(\beta)\varphi^I_{\Lambda K}(\gamma) \ ,$$

in which the quantum numbers $\nu=(n,\Lambda)$ complete the description of the state. The normalization has been chosen so that

$$\langle IM,\nu \mid IM,\nu\rangle=\sum_K \int_0^\infty d\beta \int_0^{\pi/3} d\gamma \mid A^I_{K,\nu}(\beta,\gamma)\mid^2=1 \ .$$

The functions $\varphi^I_{\Lambda K}(\gamma)$ are available in closed form.[30] The functions $f_{n\Lambda}(\beta)$ may also be expressed in analytic form if a parabolic expansion is made about the minimum of the potential energy in the equation describing the β motion, as suggested by Wilets and Jean.[28] For the states coupled in the present calculations $(0^+_1,2^+_1,2^+_2,4^+_1)$, there are no excitations of the β degree of freedom (i.e., $n=0$), and consequently the β functions may be expressed as

$$f_{0\Lambda}(\beta)=\left[\frac{\hbar}{B\omega}\right]^{-1/4} \pi^{-1/4}\omega_\Lambda^{1/4}\beta^{-2} \times\exp\left\{-\frac{1}{2}\omega_\Lambda\left[\left[\frac{B\omega}{\hbar}\right]^{1/2}\beta-x_\Lambda\right]^2\right\} .$$

The constant $\hbar/B\omega$ and the parameter sets $(\omega_\Lambda,x_\Lambda)$ are determined from the low-lying level spacing and the experimental value[31] of the reduced transition probability $B(E2;0^+_1\rightarrow 2^+_1)$. Their values were $(\hbar/B\omega)^{1/2}=0.050$, $(\omega_0,\omega_1,\omega_2)=(1.063,1.141,1.212)$, and $(x_0,x_1,x_2)=(2.61, 2.78,2.96)$.

The diagonal and transition potentials were calculated by integrating multipoles of the deformed optical poten-

tial $U_{\lambda\mu}(r,\sigma,\beta,\gamma)$ with the initial- and final-state collective wave functions, Eq. (33), over the intrinsic variables β and γ, according to the Kumar ansatz.[5,24] The potential parameters used were those of Deason *et al.*,[32] who have made a coupled-channel analysis of (p,p′) scattering from ^{196}Pt using matrix elements from the IBA-1 and form factors from the symmetric rotor model. The spin-orbit and Coulomb parts of the potential were deformed and treated in the same way as the central parts, and the whole potential was expanded into spherical harmonics up to $\lambda=8$. In addition, the possibility of a *fixed* hexadecapole deformation β_4 as allowed for by including a term $R_0\beta_4\gamma_0^4$ in the expression for the nuclear radius $\tilde{R}(\Omega')$, Eq. (31). The CC calculations were performed with a computer code that accepts optical model parameters and the appropriate collective wave functions (given in terms of β and γ) as input, and generates as output a file that may be used directly as input to ECIS.[27] A copy of this program is available on request.

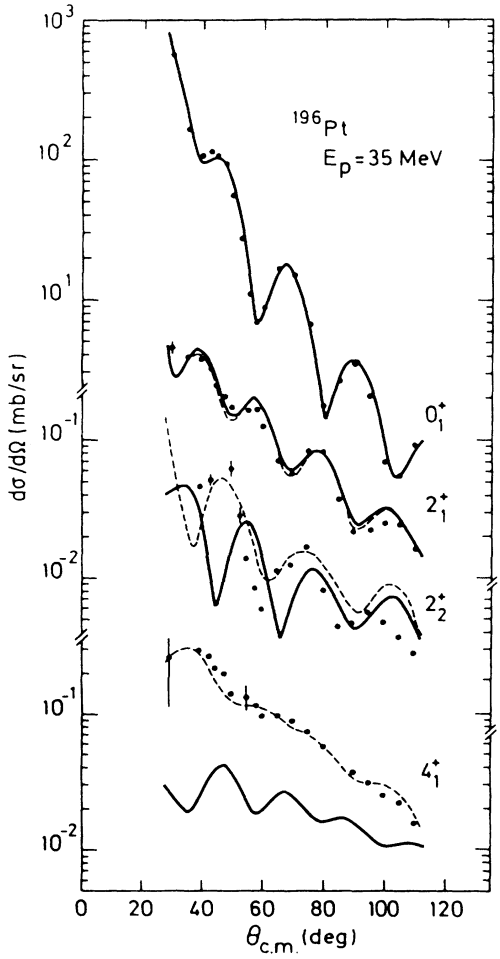

FIG. 1. Differential cross sections for elastic scattering and inelastic scattering of 35 MeV protons from the first and second 2^+ states as well as first 4^+ state of ^{196}Pt. The measurements are taken from Ref. 31. The full curves represent coupled-channel predictions based on the γ-unstable model (Ref. 28). For the dashed curves, see text.

With no further adjustments and no hexadecapole deformation, the WJ model (solid curves in Figs. 1 and 2) yields excellent results for the ground (0_1^+) and first-excited (2_1^+) state differential cross sections. There are obvious difficulties with the phasing of the calculated $(d\sigma/d\Omega)(2_2^+)$ and the magnitude of $(d\sigma/d\Omega)(4_1^+)$. At this point, it is not possible to recognize the origin of these difficulties; they may be tied with the very narrow overlap approximation, Eq. (29), or the WJ model, or both. However, the excitation of the 4_1^+ state is described satisfactorily (dashed curve in Fig. 1) by adding a small, static hexadecapole deformation ($\beta_4=-0.04$) that is consistent with other results in this mass region.[33] This additional parameter has negligible effect on the other transitions. The difficulty with the 2_2^+ transition may be remedied by a slight mixing of the 2_1^+ and 2_2^+ WJ collective wave functions, which implies a departure from the Wilets and Jean model. In the IBA language, this means that the O(6) dynamical symmetry is slightly broken, as found by Casten *et al.*[29] The dashed curves in Figs. 1 and 2 represent a further calculation in which the physical 2_2^+ state is given by

$$|\tilde{2}_2^+\rangle = 0.995\,|2_2^+\rangle - 0.100\,|2_1^+\rangle\ ,$$

where the states on the RHS are taken from the WJ model, and $|\tilde{2}_1^+\rangle$ is the orthogonal combination of them. It may be noted in Figs. 1 and 2 that both the differential cross section and the analyzing power observable predictions for the 2_2^+ state are very sensitive to a small perturbation of the wave functions. The admixture of the 2^+ states yields a very small static quadrupole moment for the 2_1^+ state ($Q_2=-0.22$ e b), which, however, is of the opposite sign from that determined by experiment[34] ($Q_2=0.63\pm0.06$ e b). We do not view this as a serious discrepancy, since the small quadrupole moment is a consequence of the cancellation of two large terms, and it may be anticipated that coupling to higher-lying states, as well as further mixing of other 2^+ states, may correct this problem.

Even within the restricted context of the δ-function overlap, Kumar ansatz,[5,24] and standard (i.e., Woods-Saxon) optical potentials, this example shows that the

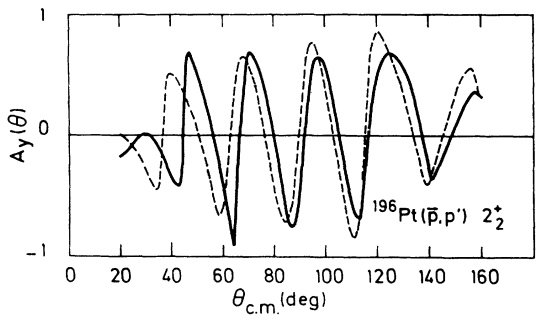

FIG. 2. Analyzing power for ($\vec{p}$,p′) scattering from the second 2^+ state of ^{196}Pt at 35 MeV. The full and dashed curves are CC predictions (see Fig. 1 and text for explanations).

treatment of scattering from "soft" nuclei described in this paper yields useful results. It has allowed a direct test (for the first time, to our knowledge) for the Wilets and Jean collective model. Also, the slight deviation from the model (state mixing) and the introduction of a hexadecapole deformation have a clear physical interpretation in terms of the nuclear geometry, which is not always true for CC calculations in which reduced matrix elements are varied arbitrarily to achieve an optimum fit to the data.

VI. CONCLUDING REMARKS

A coupled-channel formalism for inelastic scattering from soft nuclei has been developed. An important ingredient of this formalism is the definition of collective many-body wave functions in terms of constrained HF or HFB intrinsic states. Starting from a t—or G—matrix two-body effective interaction, an effective collective potential is formally built through successive approximations described in Sec. III and Sec. IV. These approximations lead to the definition, Eq. (26), of a system of coupled equations containing radial diagonal as well as off-diagonal potentials, Eq. (25), which result from the averaging of transition operators over the superposition amplitudes for initial and final states.

Equation (25) also involves the overlap $\langle q \mid q' \rangle$ between intrinsic states. This feature is a by-product of having adopted collective states such as built from the well-known generator-coordinate method (GCM). Since the intrinsic as well as collective motions are treated on the same grounds in the GCM framework, our description of the inelastic scattering from collective levels differs from, and is more complete than, that suggested earlier.[25] The significance of our formalism depends in particular on the narrowness of $\langle q \mid q' \rangle$. Whether $\langle q \mid q' \rangle$ is narrow for nuclei spread over a wide mass range has not been addressed yet. However, the example shown in Fig. 3 for ^{70}Ge illustrates that $\langle q \mid q' \rangle$ as determined from HFB calculations[35] is not very narrow, and suggests that our formalism is suitable for the treatment of inelastic scattering from this soft nucleus.[35]

CC calculations for $(\vec{p},p')$ scattering from ^{196}Pt considered as a γ-unstable nucleus have been performed at 35 MeV and compared with angular distribution measurements. In the present context, these calculations based on the assumption that $\langle q \mid q' \rangle \sim \delta(q-q')$, and shown in Figs. 1 and 2, should be viewed as the first step in our effort to treat the whole and rather complex scattering process.

The full treatment of Eq. (25) requires in particular that the potential kernels $U_{\lambda\mu}(r,\sigma,q,q')$ be known. These kernels are nonlocal with respect to collective coordinates and must be built explicitly. A possible approach to that goal consists in folding a two-body effective interaction with matter density distributions $\rho_{\lambda\mu}(r,\sigma,q,q')$ obtained from constrained HF or HFB calculations.[35] This issue is presently under consideration.

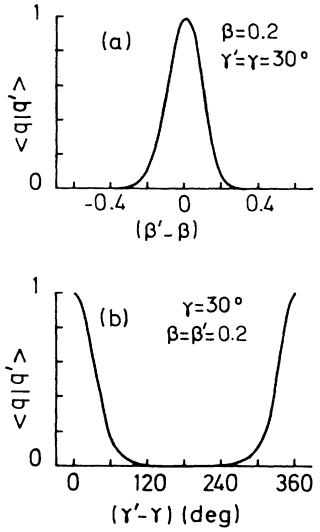

FIG. 3. Overlap kernel $\langle q \mid q' \rangle$ for ^{70}Ge as determined from constrained Hartree-Fock-Bogolyubov calculations.[35] The collective coordinates are $q=(\beta,\gamma)$ and $q'=(\beta',\gamma')$.

Finally, the general formalism developed in the present work may also be used (in principle) to describe consistently inelastic scattering from low-lying states and levels at high excitation energy (giant resonances). This is possible because the GCM can be applied to many problems of nuclear structure, such as giant resonances and shape vibrations, and to coupling mechanism between different modes of excitation.[36] In this case, the approximations developed in Sec. IV are no longer valid and, therefore, one faces the full complexity of the coupled-channel formalism developed in Sec. III.

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