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Three-body potentials originating from cluster distortion

S. Nakaichi-Maeda* and E. W. Schmid
Institut für Theoretische Physik, Universität Tübingen, D-7400 Tübingen, Federal Republic of Germany
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The distortion of composite particles (clusters) during their mutual interaction leads to effective three-body forces. In a nonrelativistic theory, they originate from two sources. One is the coupling potential which couples a channel to a distortion channel. The second one is the energy denominator (propagator) which arises whenever a distortion channel is formally eliminated and represented by an elimination potential. In the present paper this second source is studied. A three-cluster system with only one distortion state in a two-cluster subsystem is considered. Two possibilities of defining a three-body potential from the three-cluster elimination potential are tested, namely (i) extraction of a two-body potential with "frozen" energy dependence and (ii) off-shell transformation and subsequent extraction of a two-body potential. The mathematical formalism is illustrated by a numerical example. In a simplified model of the triton, two Δ particles and a nucleon form the distortion channel. It is seen that the interaction of the third nucleon with the two Δ particles has a remarkable influence on the effective three-body potential. Considering the fact that excited nucleons like Δ particles tend to interact more strongly with nucleons than nucleons interact among themselves, we find the overbinding problem of the triton becoming more serious. From the present microscopic study it also becomes clear that an N-body Schrödinger equation with purely phenomenological energy-dependent two-body potentials is in general undefined because of insufficient information.

I. INTRODUCTION

Originally, the Schrödinger equation was written for pointlike particles which interact by classical potentials. In nuclear physics, however, it is used for nucleons, which are extended objects with internal structure. The price to be paid for this extension is effective interaction potentials of great complexity and ambiguity. One has to accept nonlocal potentials, energy-dependent potentials, and multibody potentials. Also one has to accept the fact that the potentials are not unique but can be transformed from one form to another without changing the observables. What is clearly needed in this jungle of potentials is a systematic study of the effective interaction of composite particles.

In nonrelativistic theory the transition from particle quantum dynamics to composite particle quantum dynamics is furnished by the resonating group method with full antisymmetrization[1] and orthogonalization of channel spaces.[2] Nonlocality of the potentials and multibody potentials arise by the Pauli principle, by short-range correlations of the particles, and by the formal elimination of channels. Some of these features have been studied in earlier papers.[3–6] In this paper we want to study an effective three-body potential which arises by the elimination of a cluster-distortion channel.

The elimination of distortion channels makes the effective interaction of clusters energy dependent. It modifies the two-cluster interaction by additional two-body potential terms which depend on the two-cluster relative motion energy. In a three-cluster system, it modifies the three-cluster interaction by additional potential terms which depend on the three-cluster energy. Energy-dependent two-body potentials are also used in phenomenological potential models. In such models, however, it remains unclear how to define the two-body energy variable in the potentials when the system consists of more than two bodies. A prescription given and applied in several papers[7–14] is in contradiction with the microscopic motivation of energy-dependent phenomenological potentials. The investigation of the present paper will give us more insight into this problem.

We will study a simple case. Two clusters get distorted by mutual interaction and enter into a distortion channel. A third cluster interacts with the first two clusters without participating in any distortion. We will see that even in this simple case we get a three-cluster elimination potential which differs from the two-cluster elimination potential by being a complicated three-body operator. Two ways of subtracting two-body potential components from this three-body operator will be studied. In both cases a three-body potential will remain. A numerical example will exhibit some special effects related to the presence of Δ particles in the triton.

The present paper is not meant to substitute for any of the valuable investigations on three-nucleon forces arising from meson exchange diagrams.[15] It should rather furnish some additional aspects coming from the theory of coupled-channel equations and make contact with the particular coupled-channel approach to three-nucleon forces of Ref. 16.

In Sec. II the three-body elimination potential arising by a two-cluster distortion is derived and two possibilities of interpreting it as an energy-independent two-body potential plus a three-body potential are formulated. In Sec.

III a simple numerical example is given and extensions of the model are discussed. Results are summarized in Sec. IV.

II. THEORY

We consider the simplest case of a three-cluster system in which cluster distortion plays a role: Clusters 1 and 2 can make a transition into a distorted configuration, while cluster 3 remains inert, i.e., it does not get distorted itself and it does not distort other clusters. The motion of clusters 1 and 2 is described by a coupled-channel equation,

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} |\chi_1\rangle \\ |\chi_2\rangle \end{bmatrix} = e \begin{bmatrix} |\chi_1\rangle \\ |\chi_2\rangle \end{bmatrix} . \tag{1}$$

The first channel is an open channel, with a channel Hamiltonian h_{11}, a relative motion energy e, and a relative motion state $|\chi_1\rangle$. The second channel is a closed distortion channel. In this channel, the two clusters are no longer in their ground states but in some excited ("distorted") state. In principle, there is a Hilbert space of such distortion states. But, in order to exhibit the systematics, we restrict this Hilbert space to only one element, called $|f\rangle$. In this case, $|\chi_2\rangle$ is not a state at all, but only an amplitude which tells us how much the compound configuration $|f\rangle$ is excited during the scattering process. The channel Hamiltonian h_{22} is just a number, namely the excitation energy e_f of the compound state $|f\rangle$, relative to the threshold of the first channel. The channel coupling Hamiltonians h_{12} and h_{21} couple the state $|\chi_1\rangle$ to the amplitude $|\chi_2\rangle$.

Formal elimination of the second equation in Eq. (1) leads to

$$h_{11}|\chi_1\rangle + h_{12}\frac{1}{e-h_{22}}h_{21}|\chi_1\rangle = e|\chi_1\rangle . \tag{2}$$

We change the notation slightly and write for Eq. (2)

$$(t+v)|\chi_1\rangle + \frac{|\lambda\rangle\kappa^2\langle\lambda|}{e-e_f}|\chi_1\rangle = e|\chi_1\rangle \tag{3}$$

in order to show that h_{11} consists of a kinetic energy operator t and a potential v, while h_{12} and h_{21} are just normalized form factors $|\lambda\rangle$ times coupling constants κ. The elimination potential appearing in the second term of Eqs. (2) and (3) is a rank-one separable potential with an energy-dependent strength parameter $\kappa^2/(e-e_f)$.

For the three-cluster system we use capital letters. The coupled-channel equation for the motion of clusters 1, 2, and 3 reads

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} |\chi_1\rangle \\ |\chi_2\rangle \end{bmatrix} = E \begin{bmatrix} |\chi_1\rangle \\ |\chi_2\rangle \end{bmatrix} . \tag{4}$$

Again, we can formally eliminate the second equation and write

$$H_{11}|\chi_1\rangle + H_{12}(E-H_{22})^{-1}H_{21}|\chi_1\rangle = E|\chi_1\rangle . \tag{5}$$

A closer inspection of the elimination potential $H_{12}(E-H_{22})^{-1}H_{21}$ and its decomposition into two- and three-body force components will be the main interest of the present paper.

Equations (1) and (4) are, in principle, derived by the resonating group method with orthogonalization of channel spaces,[2] where we start from a microscopic wave function, include a distortion channel, and rigorously observe the Pauli principle. The operators (h_{ij}) and (H_{ij}) become energy independent. The left-hand side of Eq. (5) defines an effective potential acting in the three-cluster system. When the similar effective potentials of the two-cluster subsystems are subtracted out, the remaining parts are three-body potentials. They arise from two origins: the Pauli principle and the elimination of the distortion channel. The operator H_{11} contains a three-body potential, caused by the Pauli principle, which has been investigated in a previous paper.[5] The Pauli principle introduces three-body components also in the coupling potentials H_{12}, H_{21}. We disregard these three-body interactions in the present paper. We want to study instead the three-body potential which arises from the denominator appearing in the second term of (5), because this three-body potential is a typical feature of cluster distortion. Disregarding the three-body potential of H_{11} and the three-body components of H_{12} and H_{21} is equivalent to neglecting antisymmetrization between constituent particles of different clusters and to using the coupled reaction channels equation,[17] instead of the resonating group equation.

The microscopic Hamiltonian of the two-cluster system, and similarly of the three-cluster system, is

$$H = \sum_i T_i + \sum_{i<j} V_{ij} - T_{\text{c.m.}} , \tag{6}$$

with single-particle kinetic energies T_i, pair potentials V_{ij}, and total center-of-mass energy $T_{\text{c.m.}}$; no three-body potentials are assumed to be present at the microscopic level. For the microscopic wave function of the (1,2) cluster system we make the ansatz

$$|\psi\rangle = |\phi_1\rangle|\phi_2\rangle|\chi_1\rangle + |f\rangle|\chi_2\rangle , \tag{7}$$

where $|\phi_1\rangle, |\phi_2\rangle$ are the ground states of the two clusters, $|\chi_1\rangle$ is the (1,2) relative motion state, $|f\rangle$ is a compound state of all constituents of clusters 1 and 2, and $|\chi_2\rangle$ is the excitation amplitude of $|f\rangle$. We take for the compound state $|f\rangle$ a product of two excited cluster states $|\tilde{\phi}_1\rangle, |\tilde{\phi}_2\rangle$ and a square integrable relative motion state $|\tilde{\chi}\rangle$,

$$|f\rangle = |\tilde{\phi}_1\rangle|\tilde{\phi}_2\rangle|\tilde{\chi}\rangle , \tag{8}$$

or a superposition of such products. Because

$$\langle\phi_1|\tilde{\phi}_1\rangle = \langle\phi_2|\tilde{\phi}_2\rangle = 0 , \tag{9}$$

the two terms on the right-hand side of Eq. (7) are orthogonal.

The microscopic state of the (1,2,3) cluster system is

$$|\Psi\rangle = |\phi_1\rangle|\phi_2\rangle|\phi_3\rangle|X_1\rangle + |f\rangle|\phi_3\rangle|X_2\rangle , \tag{10}$$

where $|\phi_3\rangle$ is the ground state of the third cluster, $|X_1\rangle$ is a (1,2,3) relative motion state (depending on two Jacobi vector variables), and $|X_2\rangle$ is the state of motion of the center of mass of the third cluster relative to the center of mass of the compound system described by $|f\rangle$.

The two-cluster coupled-channel equation (1) is obtained from

$$\langle \delta\psi \,|\, (H-e_t) \,|\, \psi \rangle = 0 \ , \tag{11}$$

together with Eqs. (6) and (7). The variation symbol means that the internal motion states $|\phi_1\rangle$ and $|\phi_2\rangle$, as well as the compound state $|f\rangle$, are kept fixed, while $|\chi_1\rangle$ and $|\chi_2\rangle$ are freely varied; e_t is the total energy. Similarly we get the three-cluster coupled-channel equation (4) from the microscopic Hamiltonian of the three-cluster system and the ansatz (10).

One can now see that the potential v appearing in Eq. (3) is just a (local) double folding potential. The form factor $|\lambda\rangle$ and the coupling strength κ are given by

$$\langle \phi_1 \,|\, \langle \phi_2 \,|\, H \,|\, f \rangle = |\lambda\rangle \kappa \ , \tag{12}$$

with the normalization condition $\langle \lambda \,|\, \lambda \rangle = 1$; the quantity $\langle \phi_1 \,|\, \langle \phi_2 \,|\, H \,|\, f \rangle$ is not a number but a state, since $|f\rangle$ depends on all microscopic variables (except total center of mass), while $\langle \phi_1 \,|\, \langle \phi_2 |$ depends only on cluster-internal variables.

The ground state energies of the two clusters are subtracted from e_t and from $\langle f \,|\, H \,|\, f \rangle$ to give e and e_f, respectively. The energy e_f is the excitation energy of the compound state $|f\rangle$, relative to the two-cluster threshold.

The ansatz (10) determines the embedding of the (1,2)-cluster system into the (1,2,3)-cluster system. We get for the Hamiltonian of the first channel

$$H_{11} = T_r + T_R + v_{12} + v_{23} + v_{31} \ . \tag{13}$$

The potentials v_{12}, v_{23}, v_{31} are double folding potentials of the respective two-cluster subsystems. The indices r and R refer to the (1,2) and (12,3) Jacobi variables. The operators T_r and v_{12} differ from t and v of Eq. (3) only by unit operators with respect to the motion of cluster 3.

In the derivation of the channel-coupling Hamiltonian H_{12}, several terms vanish because of Eqs. (8) and (9), and one gets

$$H_{12} = |\lambda\rangle \kappa 1_R \ . \tag{14}$$

The channel-coupling Hamiltonian has no three-body component because cluster 3 cannot excite both clusters 1 and 2 together. The situation is different when the distortion channel has only one excited cluster. But, as has been said before, we want to study the simplest case. For the Hamiltonian of the second channel we get

$$H_{22} = T_R + v_f + e_f \ . \tag{15}$$

The (local) potential v_f is the double folding potential calculated with the (1,2) compound state $|f\rangle$ and with the ground state $|\phi_3\rangle$ of the third cluster. As before, e_f is the excitation energy of $|f\rangle$ relative to the (1,2) threshold.

We can now rewrite Eq. (5) and get

$$(T_r + T_R + v_{12} + v_{23} + v_{31} + V_{\rm eli}) \,|\, X_1 \rangle = E \,|\, X_1 \rangle \ , \tag{16a}$$

$$V_{\rm eli} = |\lambda\rangle \frac{\kappa^2}{(E - T_R - v_f) - e_f} \langle \lambda | \ . \tag{16b}$$

The elimination potential $V_{\rm eli}$ is the quantity we want to study. It is essentially the same potential as the two-cluster elimination potential

$$v_{\rm eli} = |\lambda\rangle \frac{\kappa^2}{e - e_f} \langle \lambda | \ . \tag{17}$$

The crux is that $v_{\rm eli}$ depends on e, which is the kinetic energy of the asymptotic two-cluster relative motion. This quantity e is not readily available in the three-cluster system. What is available in the three-cluster system is the three-cluster relative motion energy E. There is an energy shift operator $(T_R + v_f)$, such that $(E - T_R - v_f)$ corresponds to e, as indicated by the parentheses in the denominator of Eq. (16b). The energy shift operator is a nonlocal operator (because it appears in the denominator) in the second Jacobi variable of the three-cluster system. In the discussion of the three-nucleon force in Ref. 16, it is split into three parts, i.e., (i) an energy-independent two-body potential obtained by "freezing" the energy dependence of the two-body elimination potential, (ii) an effective two-nucleon dispersive effect, and (iii) a three-nucleon force proper. We prefer to call (ii) as well as (iii) a three-nucleon potential.

At this point we should stop, for a moment, and think about the general meaning of energy-dependent two-body potentials. In Eqs. (16a) and (16b) we have a three-body equation with an energy-dependent "two-body" potential. The equation is well defined because we know the microscopic origin of the energy dependence. The origin is the compound state $|f\rangle$, and the knowledge of $|f\rangle$ allows us to calculate v_f, which is part of the energy shift operator. When two-body energy-dependent potentials are given without any microscopic basis, the energy shift operator is not known, and a multibody Schrödinger equation cannot be written because of insufficient information. Inside the interaction volume kinetic and potential energies tend to be large and of opposite sign. For this reason one should not omit v_f while keeping T_R. In this sense the prescription which has been adopted and used in Refs. 7–14 is unphysical. We shall also see in our numerical example that v_f is important.

Let us proceed by evaluating expression (16b). We are not interested in a situation where clusters 1 and 2 interact while cluster 3 is at asymptotic distances. Therefore we can use a truncated Hilbert space description for the spectator motion, in the elimination potential $V_{\rm eli}$. We diagonalize the energy shift operator $T_R + v_f$ in a finite Hilbert space to get the (approximate) eigenstate representation

$$T_R + v_f \approx \sum_{i=1}^{n} |u_i\rangle E_i \langle u_i| \ . \tag{18}$$

The elimination potential (16b) then becomes a rank-n separable potential,

$$V_{\rm eli} = \sum_{i=1}^{n} |\lambda\rangle \,|\, u_i\rangle \frac{\kappa^2}{(E - E_i) - e_f} \langle u_i \,|\, \langle \lambda | \ , \tag{19}$$

with a discrete set of values E_i for the energy shift; in each term of the sum, $(E - E_i)$ represents the energy e of the (1,2) subsystem.

Even though $V_{\rm eli}$ is a three-body potential in principle, its influence on the observables of the three-cluster system should not be called a three-body force effect. There are various possibilities to reduce the effect of a three-body potential. One can extract two-body potentials from it, or

carry out off-shell transformations, or do both things. In the following we will take a closer look at two of such possibilities.

Firstly, the most natural thing is to extract from the three-body elimination potential $V_{\rm eli}$ the subsystem elimination potential $v_{\rm eli}$ with energy dependence "frozen" at some fixed value e_0:

$$V_{\rm eli}=-|\lambda\rangle\frac{\kappa^2}{e_f-e_0}\langle\lambda|+V_{\rm I}^{(3)}\ . \tag{20}$$

We insert (20) into (16a). The first term on the right-hand side of (20), together with v_{12}, will then be an energy-independent Hermitian (12)-subsystem potential. We want this potential to be in agreement with effective range parameters of the subsystem. For this reason we choose $e_0=0$. Equation (16a) then contains three energy-independent Hermitian subsystem interactions and the three-body potential $V_{\rm I}^{(3)}$. The latter potential contains all the unpleasant features, like energy dependence, nonlocality, and infinite range. The choice of e_0 determines how much is subtracted out of $V_{\rm eli}$ and put into the energy-independent Hermitian two-body potential of the (12) subsystem. Once (16a) has been solved, for a certain energy and a certain boundary condition, one can in principle readjust e_0 such that the influence of $V_{\rm I}^{(3)}$ vanishes. But then one would have a different energy-independent Hermitian (12)-subsystem potential for each case under consideration, and this subsystem potential would not even reproduce the low energy data of the isolated subsystem. Therefore we stick to the choice $e_0=0$.

From (20), together with (19) and $e_0=0$ and with $\sum|u_i\rangle\langle u_i|$ as an approximate unit operator, we get

$$V_{\rm I}^{(3)}=\sum_i|\lambda\rangle|u_i\rangle\left[\frac{\kappa^2}{(E-E_i)-e_f}+\frac{\kappa^2}{e_f}\right]\langle u_i|\langle\lambda|\ . \tag{21}$$

We take a Taylor series expansion of the first term in the square brackets. The zero-order term cancels and we are left with

$$V_{\rm I}^{(3)}=-\frac{\kappa^2}{e_f}\sum_i|\lambda\rangle|u_i\rangle\left[\frac{E-E_i}{e_f}+\left[\frac{E-E_i}{e_f}\right]^2+\cdots\right]\times\langle u_i|\langle\lambda|\ . \tag{22}$$

When the two-cluster distortion state has a fairly high energy e_f while the three-cluster system is in a bound state or a low-energy scattering state, then the first power in $(E-E_i)$ dominates and the sign of the three-body potential depends on E_i. This then means that the interaction v_f of the third cluster with the two clusters in the distorted configuration $|f\rangle$ determines whether the three-body potential $V_{\rm I}^{(3)}$ is repulsive or attractive.

Secondly, for a further reduction of our three-body force we will try to get rid of the linear term in Eq. (22); a prescription was given earlier.[6] It is well known[18] that a potential which depends linearly on energy can be made energy independent by transforming the dynamical equation. In Eqs. (3) and (16) we reintroduce for brevity the notations h_{11} and H_{11} and expand the elimination potentials into a Taylor series,

$$\left\{h_{11}-\frac{\kappa^2}{e_f}|\lambda\rangle\left[1+\frac{e}{e_f}+\left[\frac{e}{e_f}\right]^2+\cdots\right]\langle\lambda|-e\right\}|\chi_1\rangle=0\ , \tag{23}$$

$$\left\{H_{11}-\frac{\kappa^2}{e_f}\sum_i|\lambda\rangle|u_i\rangle\left[1+\frac{E-E_i}{e_f}+\left[\frac{E-E_i}{e_f}\right]^2+\cdots\right]\langle u_i|\langle\lambda|-E\right\}|X_1\rangle=0\ . \tag{24}$$

The first powers of e and E have the operators

$$n=1+|\lambda\rangle\frac{\kappa^2}{e_f^2}\langle\lambda| \tag{25}$$

and

$$N=1+\sum|\lambda\rangle|u_i\rangle\frac{\kappa^2}{e_f^2}\langle u_i|\langle\lambda| \tag{26}$$

as factors. With these operators we perform the off-shell transformations

$$|\hat{\chi}_1\rangle=n^{1/2}|\chi_1\rangle\ , \tag{27}$$

$$|\hat{X}_1\rangle=N^{1/2}|X_1\rangle\ . \tag{28}$$

An equation for $|\hat{\chi}_1\rangle$ is obtained from Eq. (23) by inserting $n^{-1/2}n^{1/2}$ in front of $|\chi_1\rangle$ and by multiplying the equation with $n^{-1/2}$ from the left. Similarly, one gets an equation for $|\hat{X}_1\rangle$ from Eq. (24). The operators $n^{-1/2}$ and $N^{-1/2}$ are

$$n^{-1/2}=1-|\lambda\rangle c\langle\lambda|\ , \tag{29}$$

$$N^{-1/2}=1-\sum_i|\lambda\rangle|u_i\rangle c\langle u_i|\langle\lambda|\ , \tag{30}$$

with

$$c=1-e_f(e_f^2+\kappa^2)^{-1/2}\ . \tag{31}$$

The transformation leads from Eqs. (23) and (24) to

$$(h_{11}+\hat{v}_{\rm eli}-e)|\hat{\chi}_1\rangle=0 \tag{32}$$

and

$$(H_{11}+\hat{V}_{\rm eli}-E)|\hat{X}_1\rangle=0\ , \tag{33}$$

with new potentials

$$\hat{v}_{\rm eli}=-h_{11}|\lambda\rangle c\langle\lambda|-|\lambda\rangle c\langle\lambda|h_{11}+|\lambda\rangle c^2\langle\lambda|h_{11}|\lambda\rangle\langle\lambda|-|\lambda\rangle d\langle\lambda|+O(e^2)\ , \tag{34}$$

$$\hat{V}_{\rm eli} = -H_{11}\sum_i |\lambda\rangle|u_i\rangle c\langle u_i|\langle\lambda| - \sum_i |\lambda\rangle|u_i\rangle c\langle u_i|\langle\lambda|H_{11} + \sum_{i,j} |\lambda\rangle|u_i\rangle c\langle u_i|\langle\lambda|H_{11}|\lambda\rangle|u_j\rangle c\langle u_j|\langle\lambda| - \sum_i |\lambda\rangle|u_i\rangle d\left[1-\frac{E_i}{e_f}\right]\langle u_i|\langle\lambda| + \sum_i O((E-E_i)^2)\ , \tag{35}$$

where

$$d = e_f\kappa^2(e_f^2+\kappa^2)^{-1}\ . \tag{36}$$

Similar to what has been done in (20), we now extract from $\hat{V}_{\rm eli}$ the two-body potential $\hat{v}_{\rm eli}$ with its energy dependence "frozen" at $e=0$,

$$\hat{V}_{\rm eli} = \hat{v}_{\rm eli}(e=0) + V_{\rm II}^{(3)}\ , \tag{37}$$

which now defines the three-body potential $V_{\rm II}^{(3)}$. In the evaluation of $V_{\rm II}^{(3)}$ we use Eq. (18) to go back from the spectral representation to the operator and we replace $\sum |u_i\rangle\langle u_i|$ by the unit operator 1_R (which is omitted whenever its presence is trivial). A short calculation then leads to

$$V_{\rm II}^{(3)} = -|\lambda\rangle c\langle\lambda|(v_{23}+v_{31}-v_f) - (v_{23}+v_{31}-v_f)|\lambda\rangle c\langle\lambda| + |\lambda\rangle c\langle\lambda|(v_{23}+v_{31}-v_f)|\lambda\rangle c\langle\lambda| + \sum_i O((E-E_i)^2)\ . \tag{38}$$

The energy-dependent part of this three-body potential is a power series in $(E-E_i)$ starting with second order. The energy-independent part depends on the difference between the two potentials describing the interaction of cluster 3 with undistorted clusters 1,2 and the potential describing the interaction of cluster 3 with clusters 1,2 in the distortion state $|f\rangle$. The energy-independent part of the dominant matrix element $\langle\lambda|V_{\rm II}^{(3)}|\lambda\rangle$ vanishes when $(v_{23}+v_{31})$ folded with $|\lambda\rangle$ is equal to v_f, i.e., when the condition

$$\langle\lambda|(v_{23}+v_{31})|\lambda\rangle - v_f \equiv 0 \tag{39}$$

holds.

A physical explanation of this feature was given earlier.[6] An energy-dependent potential is not off-shell unitary: A wave packet coming in from infinity will change its norm in the interaction region. In the case of our elimination potentials the reason is clear. Transition to the second channel is responsible for the loss of probability in the first channel. And the physical meaning of the three-body force, which appears when the second channel is formally eliminated, is also clear. The three-body force has to compensate for that part of the interaction which has disappeared because the system has disappeared into another channel. This is why the interaction v_f plays such an important role in the three-body force. Now, the off-shell transformation (27) and (28) restores off-shell unitarity in the first channel, in the neighborhood of the energy around which the Taylor expansion has been made. Restoring off-shell unitarity reduces the three-body force *provided* that it does not make a big difference whether cluster 3 interacts with clusters 1,2 in a distorted or undistorted state. The numerical example of the next section will illustrate this situation.

III. NUMERICAL EXAMPLE AND DISCUSSION

As an example we choose a simple model of the triton, with nucleons considered to be composite particles. It should be clearly stated at the beginning that our model is not a realistic model of the triton. It is chosen to exhibit only one special feature which, however, may help to understand the real triton. This special feature is the binding effect of a three-body force arising from the simplest possible two-nucleon distortion channel.

The nucleons 1,2,3 of the triton play the role of clusters 1,2,3 in the mathematical treatment of Sec. II. By their mutual interaction nucleon 1 and nucleon 2 can go over into a square integrable state formed by two Δ particles. Nucleon 3 is considered to be inert, i.e., it cannot get excited into a Δ state. Also, by its interaction with nucleons 1 and 2, it cannot excite the $\Delta\Delta$ state of the (1,2) system. In defining the interaction potentials we introduce phenomenological parameters. The local potential v_{12} has the shape of the $^1S_0$ Reid soft core potential. Its strength is adjusted such that

$$\left[T_r + v_{12} - |\lambda\rangle\frac{\kappa^2}{e_f}\langle\lambda|\right]|\chi\rangle = 0 \tag{40}$$

yields the correct triplet scattering length. Here, e_f is the excitation energy of the $\Delta\Delta$ state. We will use $e_f = 600$ MeV. The form factor $|\lambda\rangle$ is chosen to be a normalized Gaussian in r space,

$$\lambda(r) \sim \exp[-3r^2/(4r_\lambda^2)]\ . \tag{41}$$

For its root mean square range r_λ, the values 0.6, 0.9, and 1.2 fm will be used. The coupling parameter κ^2 is then determined by the probability $P_{\Delta\Delta}$ for the presence of the $\Delta\Delta$ state in the deuteron. The latter has been discussed, for instance, by Haapakoski and Saarela.[19] We will consider several values of $P_{\Delta\Delta}$ in the range between 0% and 2%. The potentials v_{23} and v_{31} are chosen to be Gaussian potentials

$$v_{23}(r) = v_{31}(r) = -V_0\exp(-\alpha r^2) \tag{42}$$

with $\alpha = 0.46$ fm^{-2}. The potential depth $V_0 = 49.1$ MeV is fitted to yield the correct triton binding energy when v_{12} with $\kappa = 0$ yields the correct deuteron binding energy. In order to define v_f, we fold $(v_{23}+v_{31})$ with $\lambda(r_{12})$ and put a factor in front,

$$v_f = a\langle\lambda|(v_{23}+v_{31})|\lambda\rangle\ . \tag{43}$$

Setting $a=1$ means that condition (39) holds true. For $a=0$ we have no interaction between nucleon 3 and the distorted nucleons 1 and 2.

What we want to calculate is the effect of the three-body potentials $V_{\mathrm{I}}^{(3)}$ and $V_{\mathrm{II}}^{(3)}$, given by Eqs. (22) and (38), on the binding energy of our model triton, for various values of the phenomenological input parameters $P_{\Delta\Delta}$ and r_λ. Variational calculations are performed for the triton binding energy $(-E_{\mathrm{t}})$ with all potentials included, for the triton binding energy $(-E_{\mathrm{t,I}})$ with $V_{\mathrm{I}}^{(3)}$ switched off, and for the triton binding energy $(-E_{\mathrm{t,II}})$ with $V_{\mathrm{II}}^{(3)}$ switched off. In calculating E_{t} we applied a variational principle to the coupled-channel equation (4) directly. In this way, the evaluation of the series expansion of $V_{\mathrm{I}}^{(3)}$ and $V_{\mathrm{II}}^{(3)}$ can be avoided.

As variational principle we used the Euler-Lagrange method with iteration. The state $|X_1\rangle$ is approximated by a wave function of the form

$$X_1(\mathbf{r},\mathbf{R})=w(r_{12})u(r_{23})u(r_{31})\ , \tag{44}$$

and $|X_2\rangle$ becomes the function $X_2(R)$. In the first iteration step, $w(r_{12})$ and $X_2(R)$ are simultaneously calculated by solving a Euler-Lagrange integrodifferential equation, while a zero-order approximation for the function u is kept fixed. In the second iteration step, u and X_2 are calculated while w is kept fixed, and so on. With a reasonably well chosen zero-order approximation for u, rapid convergence of the iteration has been observed. In order to get $E_{\mathrm{t,I}}$, a single-channel three-body equation with potentials v_{12},v_{23},v_{31} together with the first potential on the right-hand side of Eq. (20) has to be solved. In this case, we employed the Ritz variational principle. A test function of the form of Eq. (44) has been used and small variations of w and u, in the vicinity of the (renormalized) converged solution of the first variational calculation, have been performed. As expected from arguments holding for perturbation theory, the variation did not improve the minimum in a noticeable way.

The energy $E_{\mathrm{t,II}}$ was obtained by the method used for $E_{\mathrm{t,I}}$. In this third case, the potentials acting in the three-body system are v_{12}, v_{23}, v_{31}, and $\hat{v}_{\mathrm{eli}}$ $(e=0)$. The latter potential is strong enough to have an influence on the wave function. We found the Ritz minimum with a function which is very close to a function given by Eq. (44), with the old u but with w being transformed according to Eq. (27), as expected by theory. This result for the variationally obtained wave function tells us that the variation principle is even catching terms of higher order perturbation theory. For this reason we are trusting our result for the energy.

The influence of the three-body potentials $V_{\mathrm{I}}^{(3)}$ and $V_{\mathrm{II}}^{(3)}$ on the binding energy of the triton is given by the energy differences

$$\Delta E_{\mathrm{I}}=E_{\mathrm{t}}-E_{\mathrm{t,I}}\ , \tag{45a}$$

$$\Delta E_{\mathrm{II}}=E_{\mathrm{t}}-E_{\mathrm{t,II}}\ . \tag{45b}$$

Figure 1 shows ΔE_{I} and ΔE_{II} as functions of $P_{\Delta\Delta}$, with $r_\lambda=0.6$ fm and $a=1$. For values of $P_{\Delta\Delta}$ below 1%, ΔE_{I} is small and ΔE_{II} is negligibly small.

The calculations have been repeated with $r_\lambda=0.9$ fm and $r_\lambda=1.2$ fm, because this phenomenological parameter to some degree reflects the mechanism by which a NN state is excited into a $\Delta\Delta$ state (gluon exchange or meson exchange). No drastic change of the results has been found. With $P_{\Delta\Delta}=0.5\%$ and $a=1$, ΔE_{I} becomes a little smaller by going over from $r_\lambda=0.6$ fm to $r_\lambda=1.2$ fm, while ΔE_{II} remains negligible.

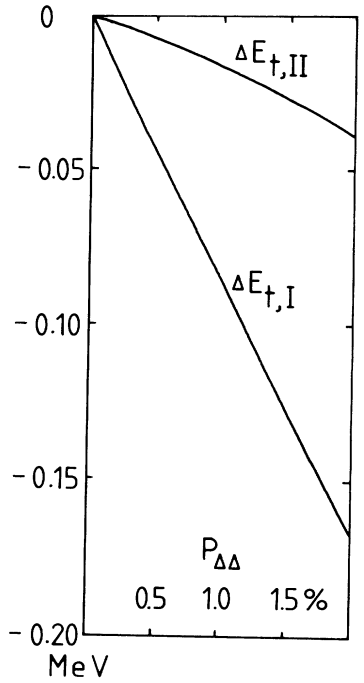

FIG. 1. Contributions $\Delta E_{\mathrm{t,I}}$ and $\Delta E_{\mathrm{t,II}}$ of the three-body potentials $V_{\mathrm{I}}^{(3)}$ and $V_{\mathrm{II}}^{(3)}$ to the triton energy E_{t}, as a function of the $\Delta\Delta$ probability $P_{\Delta\Delta}$ in the deuteron. The parameters e_f and r_λ are fixed at 600 MeV and 0.6 fm, respectively. In evaluating $\Delta E_{\mathrm{t,II}}$, assumption (39) has been used.

One gets the impression that the off-shell transformations (27) and (28), which restore off-shell unitarity in the vicinity of zero subsystem energy, reduce the three-body force to a negligible amount. This, however, is only true for $a=1$, i.e., when condition (39) is fulfilled. When Eq. (39) is not fulfilled because the NΔ interaction differs from the NN interaction, we get a quite different result. For $P_{\Delta\Delta}=0.5\%$ and $r_\lambda=0.6$ fm, Fig. 2 shows ΔE_{I} and ΔE_{II} as functions of a. For a weak NΔ interaction, ΔE_{II} becomes even larger than ΔE_{I} and reaches a value of 0.23 MeV at zero NΔ interaction. Conceptually, ΔE_{I} at $a=0$ corresponds to the dispersive effect (per nucleon pair) calculated in Ref. 16 where this effect was calculated with respect to the NN-NΔ transition by assuming no interaction between the Δ particle and the spectator nucleon.

In discussing the present result, and especially in drawing conclusions on the realistic triton, one has to be careful. We know that excited nucleons, like Δ particles, tend to interact more strongly than nucleons. Looking at coupling constants, we would expect the NΔ interaction to be stronger than the NN interaction by a factor of 4.[20] But the NΔ interaction has a partial wave dependence which is different from the partial wave dependence of the NN interaction. Our model is too simple to include this feature. We can only say that the NΔ interaction tends to be stronger than the NN interaction and that, for this

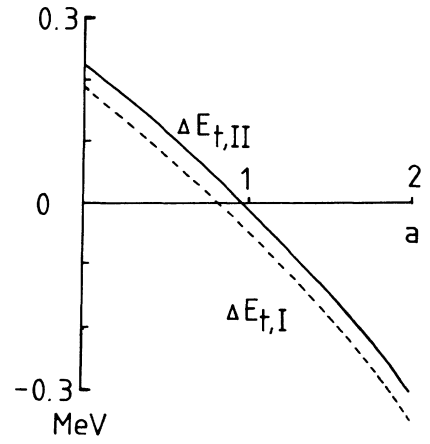

FIG. 2. Contributions $\Delta E_{t,I}$ and $\Delta E_{t,II}$ of the three-body potentials $V_I^{(3)}$ and $V_{II}^{(3)}$ to the triton energy E_t, as a function of the parameter a of Eq. (43). The parameters e_f, r_λ, and $P_{\Delta\Delta}$ are fixed at 600 MeV, 0.6 fm, and 0.5%, respectively.

reason, we should consider in Fig. 2 a values which are greater than one. Also, there are three nucleon pairs, not only one, which contribute to the effect. In this way, we may conclude that there is a non-negligible increase of the triton binding energy.

It is not the intention of the present paper to study the realistic triton and to give a quantitative estimation of this increase. The intention of this paper is to study a three-body potential arising from the elimination of a distortion channel. At first glance, one would not expect a three-body potential at all in the present case. We have seen that there is one arising from the energy denominator of the two-cluster elimination potential, and we have seen that it is not negligible.

The present investigation can be extended by including additional distortion channels, with different cluster excitations and with interchanged labels of clusters. The theory would become more complicated and one would see that also the channel coupling Hamiltonians can give rise to three-body forces. In the simple case considered in this paper, the presence of the third cluster is only felt in the energy denominator of the elimination potential, i.e., in the propagator, and not in the channel coupling Hamiltonians which determine the coupling form factors and coupling strengths. But elimination potentials will always have energy denominators and it is worthwhile to study, in isolation, their effect on three-body forces.

The increase of the triton binding energy, which we get from our qualitative study of effects arising from the energy denominator of the delta-delta elimination potential, should be added to the increase obtained by Friar *et al.*,[21] or by Sasakawa *et al.*,[22] from the inclusion of three-body forces originating from two-pion exchange. Our present study is thus increasing the problem of *overbinding* of the triton.

IV. SUMMARY

In a nonrelativistic theory of composite particles, effective three-body forces arise from three different sources: (i) from the indistinguishability of the constituent particles, (ii) from short-ranged particle correlations, and (iii) from the elimination of cluster-distortion channels. The elimination of distortion channels, again, produces effective three-body forces by two different mechanisms: (a) the channel coupling potentials, and (b) the energy denominators of the elimination potentials. The last mechanism has been studied in the present paper.

The simplest case of a three-cluster coupled-channels equation has been analyzed. Two of the three clusters can make a transition to a square integrable distortion state while the third cluster interacts with the first two clusters without being involved in the distortion. The effective three-body potential arises from the fact that the two-cluster elimination potential depends on the two-cluster subsystem energy which, in the full system, is a three-body operator. This three-body operator depends on the interaction of the third cluster with the two distorted clusters. It is seen that a multibody Schrödinger equation with energy-dependent phenomenological two-body interaction is undefined because of incomplete information. What are missing are the energy shift operators which lead from the energy of the full system to the subsystem energies. When all interactions are derived from a microscopic basis, the energy shift operators are, of course, defined.

The mathematical formalism has been illustrated by a numerical example. A simple model of the triton has been chosen to exhibit some special features which may be relevant also in the real triton. Two Δ particles and a nucleon are considered to form a distortion state of the three-nucleon system. From an estimation of the nucleon-delta interaction strength given by Huber *et al.*[20] we conclude that there will be an increase of the triton binding energy. This emphasizes the overbinding problem of the triton which has been encountered in recent calculations[21,22] with three-body potentials arising from two-pion exchange.

From the present investigation one can see again that effective two- and three-body potentials should be derived from the same microscopic basis because, to some extent, they are traded against each other by off-shell transformation of the dynamical equation.

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*Present address: Department of Physics, Hokkaido University, Sapporo, Japan.

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