y scaling in inclusive scattering

S. A. Gurvitz
Department of Physics, Weizmann Institute of Science, Rehovot 76100, Israel and TRIUMF, Vancouver, British Columbia, Canada V6T 2A3

A. S. Rinat*
Commissariat de l'Energie Atomique, Centre d'Etudes Nucléaires de Saclay, Departement de Physique Nucléaires/Haute Energies and Laboratoire National Saturne, 91191 Gif-sur-Yvette, France and University of Maryland, College Park, Maryland 20742

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We investigate why and under what conditions y scaling occurs in inclusive scattering for large, but nonasymptotic q. We explore an exactly solvable model with a particle bound to an inert core, and study for it also the plane wave, Born, West, and eikonal approximations. For all, a function related to the approximate response manifestly scales in some scaling variable y. We then consider similar functions for the exact response and study the approach to the scaling limit for $q\rightarrow\infty$. Definite conclusions are reached as to quality, origin, and preferred scaling variables.

I. INTRODUCTION

Various experiments on inclusive electron scattering from nuclear targets presently enjoy particular interest.

In one category the energy and momentum transfer ω,q are in excess of several GeV. In this so-called deep inelastic region the virtual photon probes the parton substructure of a nucleus.[1] There, the extracted structure functions or responses approximately depend on one instead of two variables ω,q. The former is the x scaling variable

$$x=Q^2/2m\omega\ , \tag{1.1}$$

where $Q^2=q^2-\omega^2$ and m is the nucleon mass.

Experiments on A(e,e′)X have shown that the nuclear response $R^{\rm A}$ in the deep-inelastic region is not identical to the response of A free nucleons.[2,3] There is at present an on-going debate on the interpretation of this so-called EMC effect. Does the effect require QCD for its explanation,[4] or is simple nuclear dynamics sufficient?[5,6] Even if the latter is the case, deep inelastic scattering proceeds primarily by partons within nucleons, with only minor effects of the nuclear medium on these nucleons.

In a second class of inclusive scattering experiments, the energy imparted to the nucleon is small enough to disregard pion production. A description of the response in that case may be possible by means of conventional hadronic degrees of freedom. Indeed, the oldest data could be described by the response of a Fermi gas of (dressed) nucleons having no apparent dynamical correlations.[7]

A still limited but growing set of new data comprises mostly light and medium weight[8-10] nuclei and some of these data permitted a separation of $R^{\rm A}$ in longitudinal and transverse responses.[11-14] Their description requires more ingredients than the content of a Fermi gas. Some results can be accounted for by standard nuclear dynamics including exchange current effects,[15-30] but others defy a sufficiently accurate description. We mention, in particular, ratios of longitudinal and transverse responses and, further, the longitudinal response integrated over energy loss (Coulomb sum rule).[13] These discrepancies have led to several unconventional ideas,[31-34] but it is not yet clear that their introduction is unavoidable.

A different aspect of inclusive electron scattering data has been brought to the fore by West.[35] He considered the response of virtually any composite system for low ω and large q, where "low" and "large" depend on the size and spectra of the system. West considered, in particular, the response $R^{\rm A}$ of a nucleus which can be described by nonrelativistic dynamics with local interactions. His result (to which we shall return later) reads

$$qR^{\rm A}(q,\omega)=\frac{m}{4\pi^2}\int_{|y_W|}^{\infty}dk\,k\,n(k)\ , \tag{1.2}$$

where $n(k)$ is the nucleon momentum distribution. $qR^{\rm A}$ manifestly scales in the variable $y_W(q,\omega)$.

Actually, some form of y scaling escaped notice or emphasis decades ago. $qR^{\rm A}$ for a Fermi gas when $q\geq 2k_F$ (k_F is the Fermi momentum) scales exactly in the variable $y_W=y_W(q,\omega)$. The same is about true when $R^{\rm A}$ is described in a plane wave impulse approximation (IA). However, the appropriate scaling variable $y_{\rm IA}$ differs from y_W.[18,36-42] The two models mentioned have in common a neglect of final state interactions (FSI's) between the knocked-out proton and the spectator core.

West's paper[35] greatly contributed to a renewed interest in inclusive scattering. The new and partly high-quality data were shown to indeed scale approximately in y.[38-48] These observations started a vivid discussion as to the meaning of y scaling and the information one might be able to extract.[36-50]

Simultaneously, the same paper also caused confusion. The claim that $R^{\rm A}$ for a system in an interaction and for a Fermi gas scales in the same variable y_W has all too frequently been taken as an indication that FSI's are negligible. Yet this cannot be correct, in general, because FSI's are absent in the IA and, as already mentioned, $y_W\neq y_{\rm IA}$.

Indeed, West's derivation presented as a general one is, in fact, only an asymptotic result.[48,18,50] In that case, FSI's are indeed negligible and, in parallel, differences between various scaling variables vanish, e.g.,

$$y_W - y_{IA} \underset{\substack{y_W \text{ fixed} \\ q\to\infty}}{\rightarrow} O(q^{-1}) ,$$

and so do interaction effects.

Of course, this observation does not preclude significant effects for large but nonasymptotic q. This has actually been demonstrated for responses of the lightest targets, d and ^{3}He.[17,26] Without there being an *a priori* reason or theoretical clue, the original quest of West remains unanswered. Does R^A for an interacting system scale and, if so, what is the appropriate scaling variable?

The relevance of these questions has recently been put into focus by the observation that $R^{^3\text{He}}$, expressed by means of either y_W or y_{IA}, exhibits scaling of about equal quality.[49] This observation is in itself problematic. It is clear that (1.2) cannot hold in two approximations with different lower integration limits of the same integral. One might then hope that theory will favor one given candidate.

The authors of Ref. 49 therefore calculated $F(y)$ in the approximation (1.2) for several underlying two-body forces in ^{3}He. None reproduced the scaling function in the approximative form (1.2) beyond $|y| \sim 0.25$ GeV. Actually, this ought not come as a surprise, since Laget's calculations (Ref. 17) had shown the importance of FSI's, which are absent in (1.2).

One is presently in a situation where approximate y scaling is an established empirical fact, while, at least for ^{3}He, several options exist. In addition, one ought to envisage the possibility that the presence of FSI's will upset the immediate relation between R and $n(k)$. We also have to admit that even for a nucleus as simple as ^{3}He one can hardly answer crucial questions around y scaling. Yet, these seem vital for a motivated continuation of research in this field.

With a clear and limited purpose in mind we formulate below a model with a single degree of freedom, i.e., a particle bound to an inert core.[51] The model is simple enough in order to permit an exact evaluation and, consequently, answers all questions, in principle.

In the following we study that model in some detail. We are aware that some of our observations are known to some of our colleagues. However, we also ran into misgivings and erroneous conclusions, and therefore preferred a complete exposition over a possible, succinct communication.

In Sec. III below we define the model, indicate methods of calculating the exact response, and work out a standard approximation. We also discuss approximations which include parts of the FSI and emphasize, in particular, an eikonal approximation. Section IV contains numerical results for R. In Sec. V we recall the manifest scaling properties of approximations for R. We then study functions $\phi_i(y,q)$ constructed from the exact response for fixed y,q and investigate the approach to scaling. At least for the model, we reach definite conclusions and are also capable to select a preferred y variable, which, as ought to be the case, contains characteristics of the interaction.

II. THE NUCLEAR RESPONSE FUNCTION

Consider inclusive scattering of a weakly interacting scalar probe x from a target composed of A identical nucleons N,

$$\text{x}+\text{A}\rightarrow\text{x}'+\text{X} . \tag{2.1}$$

Only the outgoing probe x is detected, and for it one measures the losses q,ω in, respectively, the momentum $(\mathbf{k})$ and the energy (E); thus, $q=|\mathbf{k}-\mathbf{k}'|$, $\omega=E-E'$.

The cross section to lowest order in the elementary xN interaction reads

$$\frac{d^2\sigma_{\text{xA}}^{\text{incl}}}{dE'd\Omega}=\frac{d\sigma_{\text{xN}}}{d\Omega}R^{\text{A}}(q,\omega) , \tag{2.2}$$

where $d\sigma_{\text{xN}}/d\Omega$ is the elementary projectile-nucleon differential cross section. Equation (2.2) contains some binding corrections which remove off-shell effects inherent in the underlying impulse approximation.[52]

The quantity of major concern is, in this case, the purely longitudinal response $R^{\text{A}}(q,\omega)$ of the target to the weak scalar probe. The starting point for its construction is the operator $\rho_{\mathbf{q}}$ representing the Fourier transform of the single-particle density

$$\rho_{\mathbf{q}}=\sum_j e^{i\mathbf{q}\cdot\mathbf{r}_j} . \tag{2.3}$$

Matrix elements of $\rho_{\mathbf{q}}$ between states ϕ_m of the nuclear Hamiltonian H belonging to energies ϵ_m are the standard inelastic form factors

$$F_{0m}(q)=\langle\phi_0|\rho_{\mathbf{q}}|\phi_m\rangle . \tag{2.4}$$

Of many equivalent expressions for R, we cite

$$R^{\text{A}}(q,\omega)=\sum_m|F_{0m}(\mathbf{q})|^2\delta(\omega+\epsilon_0-\epsilon_m) \tag{2.5}$$

$$=-\pi^{-1}\text{Im}\langle\phi_0|\rho_{\mathbf{q}}^{\dagger}G(\omega+\epsilon_0+i\eta)\rho_{\mathbf{q}}|\phi_0\rangle$$

$$\times\pi^{-1}\text{Im}\langle\phi_0|\rho_{\mathbf{q}}^{\dagger}(\omega+\epsilon_0-H+i\eta)^{-1}\rho_{\mathbf{q}}|\phi_0\rangle . \tag{2.6}$$

We now study the nuclear response R in several models and approximations.

III. MODELS WITH A SINGLE DEGREE OF FREEDOM

Consider a nonrelativistic nucleon bound to an inert core. Without loss of generality, we shall assume the existence of only one bound state. In that case, Eq. (2.5) reads $(\epsilon_{\mathbf{p}}=\mathbf{p}^2/2m)$

$$R(q,\omega)=\int \frac{d\mathbf{p}}{(2\pi)^3}\,|F_{0\mathbf{p}}(\mathbf{q})|^2\delta(\omega+\epsilon_0-\epsilon_{\mathbf{p}})$$
$$=\int \frac{d\mathbf{p}}{(2\pi)^3}\left|\int d\mathbf{r}\,\phi_0(r)e^{i\mathbf{q}\cdot\mathbf{r}}\psi_{\mathbf{p}}^{(-)*}(\mathbf{r})\right|^2\delta(\omega+\epsilon_0-\epsilon_{\mathbf{p}}) \tag{3.1}$$
$$=-\pi^{-1}\int\int \frac{d\mathbf{p}}{(2\pi)^3}\frac{d\mathbf{p}'}{(2\pi)^3}\phi_0(\mathbf{p}-\mathbf{q})\mathrm{Im}\langle \mathbf{p}\,|\,G(\omega+\epsilon_0+i\eta)\,|\,\mathbf{p}'\rangle\phi_0(\mathbf{p}'-\mathbf{q})$$
$$=-\pi^{-1}\int\int d\mathbf{r}\,d\mathbf{r}'\,\phi_0(r)e^{i\mathbf{q}\cdot\mathbf{r}}\mathrm{Im}\langle \mathbf{r}\,|\,G(\omega+\epsilon_0+i\eta)\,|\,\mathbf{r}'\rangle e^{-i\mathbf{q}\cdot\mathbf{r}'}\phi_0(r')\ . \tag{3.2}$$

A specific interaction V between the nucleon and the core affects the response through the groundstate ϕ_0 and the scattering states $\psi_{\mathbf{p}}^{(-)}$. The latter are either explicit in form factors or implicit in the intermediate propagator $G\equiv(\omega-H+i\eta)^{-1}$. A standard expression for G reads $[H=H_0+V,\ G_0\equiv(\omega-H_0+i\eta)^{-1}]$

$$G=G_0+G_0VG=G_0+G_0tG_0\ , \tag{3.3}$$

where t satisfies the Lippmann-Schwinger equation $t=V+VG_0t$.

Calculation of the response R, Eq. (3.1), may be made in several ways. For instance, using a partial wave expansion of R, Eq. (3.1) becomes[18]

$$R(q,\omega)=m\zeta\sum_l(2l+1)\left|\int_0^\infty dr\,r^2\phi_0(r)j_l(qr)u_l(\zeta,r)\right|^2\ . \tag{3.4}$$

Here, u_l is the partial wave radial wave function of the knocked-out particle with momentum

$$\zeta=[2m(\omega+\epsilon_0)]^{1/2}\ , \tag{3.5}$$

and which is, as follows, normalized by means of the phase shift δ_l:

$$u_l(\zeta,r)\underset{r\to\infty}{\longrightarrow}\frac{e^{i\delta_l(\zeta)}\sin[\zeta r-l\pi/2+\delta_l(\zeta)]}{\zeta r}\ . \tag{3.6}$$

We now discuss some approximations to Eqs. (3.1) and (3.2) and follow similar steps as in Ref. 53 for the nucleon knock-out amplitude.

A. Plane wave approximation

The plane wave approximation (PW) is obtained when $V\to 0$, i.e., for $G\to G_0$, leading to

$$R^{\mathrm{PW}}(q,\omega)=\int\frac{d\mathbf{p}}{(2\pi)^3}\,|\phi(\mathbf{p}-\mathbf{q})|^2\delta(\omega+\epsilon_0-\epsilon_{\mathbf{p}})$$
$$=\frac{m}{4\pi^2q}\int_{|y_0|}^{y_{0,>}}dk\,k\,n(k)\ . \tag{3.7}$$

In Eq. (3.7), $n(k)=|\phi_0(k)|^2$ is the nucleon momentum distribution normalized as $\int[d\mathbf{k}/(2\pi)^3]n(k)=1$. The argument in the δ function in Eq. (3.7) expresses energy conservation with the difference between excited and ground state energies being furnished by ω. The integration limits are, respectively, minimal and maximal, possible values for p, and are

$$y_{0,>}=q+\zeta,\quad y_0=-q+\zeta\ . \tag{3.8}$$

At this point we mention that probability current is not conserved if part, or all of V is neglected in the FSI (i.e., here in $\psi_{\mathbf{p}}^{(-)}$), while no such approximation is made in ϕ_0. See Refs. 53—55 and references contained therein for numerical consequences of these inconsistencies in the calculation of $F_{0\mathbf{p}}(\mathbf{q})$.

Having treated the PW approximation, we define R^{FSI}, the final state interaction part with full V, by

$$R=R^{\mathrm{PW}}+R^{\mathrm{FSI}}\ . \tag{3.9}$$

B. Born approximation

If the FSI is weak, it suffices to describe it in Born approximation (BA). It has been shown in Ref. 53 [Figs. 3(a) and 3(b)] that the inelastic form factor $F_{0\mathbf{p}}(\mathbf{q})$ peaks for $\mathbf{p}\,||\,\mathbf{q}$, and that then

$$F_{0\mathbf{p}}^{\mathrm{BA}}(\mathbf{q})\sim\left[\frac{2q}{p+q}\right]F_{0\mathbf{p}}^{\mathrm{PW}}(\mathbf{q})\ . \tag{3.10}$$

Assuming (3.10) to hold in general, one finds

$$R^{\mathrm{BA}}(q,\omega)\sim\left[\frac{2q}{q+\zeta}\right]^2R^{\mathrm{PW}}(q,\omega)\ . \tag{3.11}$$

The BA respects the above-mentioned probability current conservation.

C. The West approximation

Here we just cite the result of the original attempt by West[35] (W) to account, in an approximate fashion, for FSI's:

$$R^{\mathrm{W}}(q,\omega)=\int\frac{d\mathbf{p}}{(2\pi)^3}\,|\phi(\mathbf{p}-\mathbf{q})|^2\delta(\omega+\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{p}+\mathbf{q}})$$
$$=\frac{m}{4\pi^2q}\int_{y_W}^{\infty}dk\,k\,n(k)\ , \tag{3.12}$$

with

$$y_W=\frac{-q}{2}+\frac{m\omega}{q}\ . \tag{3.13}$$

Contrary to y_0, Eq. (3.8), y_W is the minimal value of the momentum of a nucleus which absorbs the transferred energy and momentum ω,q while being and remaining on the energy shell.

D. Eikonal approximation

In Ref. 53 we studied an eikonal approximation (eik) for $F_{0p}(\mathbf{q})$ for large $|\mathbf{p}|$. Here we consider the same for the Green's function in (3.2), when $m\,|V|\,a/p \ll 1$ (a is the range of V). We may then write for the propagator G in Eq. (3.2) ($\mathbf{r}=\mathbf{b},z$; $v=|\mathbf{p}|/m$),[56]

$$G^{\text{eik}}(p^2/(2m);\mathbf{r},\mathbf{r}')=\frac{i}{v}\delta^{(2)}(\mathbf{b}-\mathbf{b}')\theta(z-z')\exp\left[-ip(z-z')+\frac{i}{v}\int_{z'}^{z}V(\mathbf{b},z'')dz''\right]. \tag{3.14}$$

Using ζ, Eq. (3.5), now substitute Eq. (3.14) into Eq. (3.2). Using the parity of the θ function, one obtains, for real V,

$$R^{\text{eik}}(q,\omega)=\frac{m}{2\pi\zeta}\int d^2\mathbf{b}\int\int dz\,dz'\,\phi_0(\mathbf{b},z)\cos\left[(q-\zeta)(z-z')+\frac{m}{\zeta}\int_{z'}^{z}V(\mathbf{b},z'')dz''\right]\phi_0(\mathbf{b},z'). \tag{3.15}$$

If, in addition, $|z|,|z'|<a$, with a the range of V, an average value $\langle V\rangle$ can be taken out of the z'' integral. Even if either $|z|$ or $|z'|$ lies outside the range a the procedure is still permissible in the case of strong binding. Contributions from $|z|,|z'|>a$ are then strongly suppressed by a rapidly decreasing $\phi_0(\mathbf{b},z)$. Thus,

$$R^{\text{eik}}(q,\omega)\sim\frac{m}{2\pi\zeta}\int d^2\mathbf{b}\int\int dz\,dz'\,\phi_0(\mathbf{b},z)\cos\left[\left[q-\zeta+\frac{m\langle V\rangle}{\zeta}\right](z-z')\right]\phi_0(\mathbf{b},z') \tag{3.16}$$

$$=\frac{m}{4\pi^2\zeta}\int_{|y_{\text{eik}}|}^{\infty}dk\,k\,n(k)\,, \tag{3.17}$$

with

$$y_{\text{eik}}=-q+\left[\zeta-\frac{m\langle V\rangle}{\zeta}\right]. \tag{3.18}$$

Notice that the derivation requires V to be *real*. An equation like (3.17) appears occasionally in the literature. It obtains when, in an *ad hoc* fashion, FSI's are accounted for, by inserting $\text{Re}V_{\text{opt}}(\mathbf{p}+\mathbf{q})$ in the δ function in Eqs. (3.7) or (3.12). The former leads to

$$R^{V}(q,\omega)\sim\frac{m}{4\pi^2 q}\int_{|y_V|}^{\infty}dk\,k\,n(k)\,, \tag{3.19}$$

with $[\text{Re}V_{\text{opt}}(\mathbf{p}+\mathbf{q})\to\langle V\rangle]$

$$y_V=-q+(\zeta^2-2m\langle V\rangle)^{1/2}$$
$$\xrightarrow[\zeta\gg(2m\langle V\rangle)^{1/2}]{}-q+\left[\zeta-\frac{m\langle V\rangle}{\zeta}\right]. \tag{3.20}$$

Equation (3.19), to be compared with (3.17), results from a prescription rather than from a derivation. Notice, moreover, the factor $1/q$ instead of $1/\zeta$. The difference of the two grows with $|y|$, the distance from the quasielastic peak (QEP).

We now make two remarks which seem to hold for all mentioned approximations R_i of the exact R^{ex}.

(i) All are of the form

$$R_i(q,\omega)=\xi_i^{-1}(q,\omega)\frac{m}{4\pi^2}\int_{|y_i|}^{y_{i,>}}dk\,k\,n(k)\,, \tag{3.21}$$

with ξ_i simple functions of the kinematical variables q,ω.

(ii) By definition the maximum of R corresponds to the QEP and is, for R_i, apparently attained for $y_i(q,\omega)=0$. For fixed q the latter equation defines ω_i, which all are close to $\omega\sim q^2/2m$.

IV. NUMERICAL RESULTS FOR A PARTICLE BOUND IN A WELL

We emphasized in the Introduction the need to study the response for an exactly solvable model. As may be expected, such a model will be a primitive one, yet it has the advantage that a meaningful comparison can be made between the exact response and approximations. Even more important will be the possibility of formulating a criterion for y scaling.

We thus investigate the response of a particle bound in a square well.[51] As in Ref. 53, we study two sets of parameters:

$$\text{“Strong binding”: } V_0=-52\text{ MeV},\ a=1.97\text{ fm}\,,\quad \epsilon=-23.52\text{ MeV}\,, \tag{4.1}$$

$$\text{“Weak binding”: } V_0=-21.54\text{ MeV},\ a=1.97\text{ fm}\,,\quad \epsilon=-2.225\text{ MeV}\,. \tag{4.2}$$

The kinematical range covered corresponds to $0.5\le E(\text{GeV})\le 2.0$, $\theta=30°,90°$. With E the incident energy and ω the energy transfer, the three-momentum transfer reads

$$q^2=\omega^2+4E(E-\omega)\sin^2(\theta/2)\,. \tag{4.3}$$

We first calculated R by means of Eqs. (3.1) and (3.4)–(3.6). In order to have a numerical check, we also proceeded from Eq. (3.2). Its partial wave expansion is readily shown to read

$$R(q,\omega)=\frac{m\zeta q}{8\pi^2}\sum_l(2l+1)\times\left|\phi_l(q,\zeta)+\int_0^{\infty}\frac{dk\,k^2\phi(q,k)t_l(k,\zeta)}{\epsilon_\zeta-\epsilon_k-i\eta}\right|^2, \tag{4.4}$$

with $[u=\cos^{-1}(\hat{\mathbf{p}}\cdot\hat{\mathbf{q}})]$

$$\phi_l(p,q)=\frac{1}{2}\int_{-1}^{+1}du\,P_l(u)\phi_0(\mathbf{p}-\mathbf{q}) . \quad (4.5)$$

The half off-shell partial wave t matrix in Eq. (4.4),

$$t_l(k,p)\equiv\langle k\mid t_l(\epsilon_p-i\eta)\mid p\rangle , \quad (4.6)$$

may be calculated using methods developed in Ref. 57.

In Figs. 1(a)–1(d) we display the results for $R(q,\omega)$ for varying E and fixed $\theta=90°$, as calculated with the strong binding (SB) parameters, Eq. (4.1). The dashed vertical lines indicate the kinematical limits $\omega=q$.

In each figure we compare the following:

(i) R^{ex}: Exact result (3.1) [calculated by means of (3.4)].

(ii) R^{PW}: plane wave approximation (3.7).

(iii) R^{BA}: Born approximation (3.11).

(iv) R^{W}: West approximation (3.12).

(v) R^{eik}: eikonal approximation (3.17).

In some figures we indicate below the ω axis three y values y_0, y_W and y_{eik}. Their absolute values are the lower limits of integrals characteristic of the approximation R_i (Sec. III). We shall return to these in Sec. V.

Only for $E=1.0$ and 1.5 GeV are parallel results shown for weak binding (WB) dynamics [Figs. 2(a) and 2(b)]. These are quantitatively similar to Figs. 1(b) and 1(c): a rapid and smooth transition from large negative to large positive y values, corresponding to an outstanding QEP at $y\sim 0$, with sharp falloff in the wings.

The pattern of R as function of ω displayed in Figs. 1 and 2 is not universal. For instance, for a fixed scattering angle $\theta=30°$, $E=1.5$ and 2.0 GeV, and when ω grows

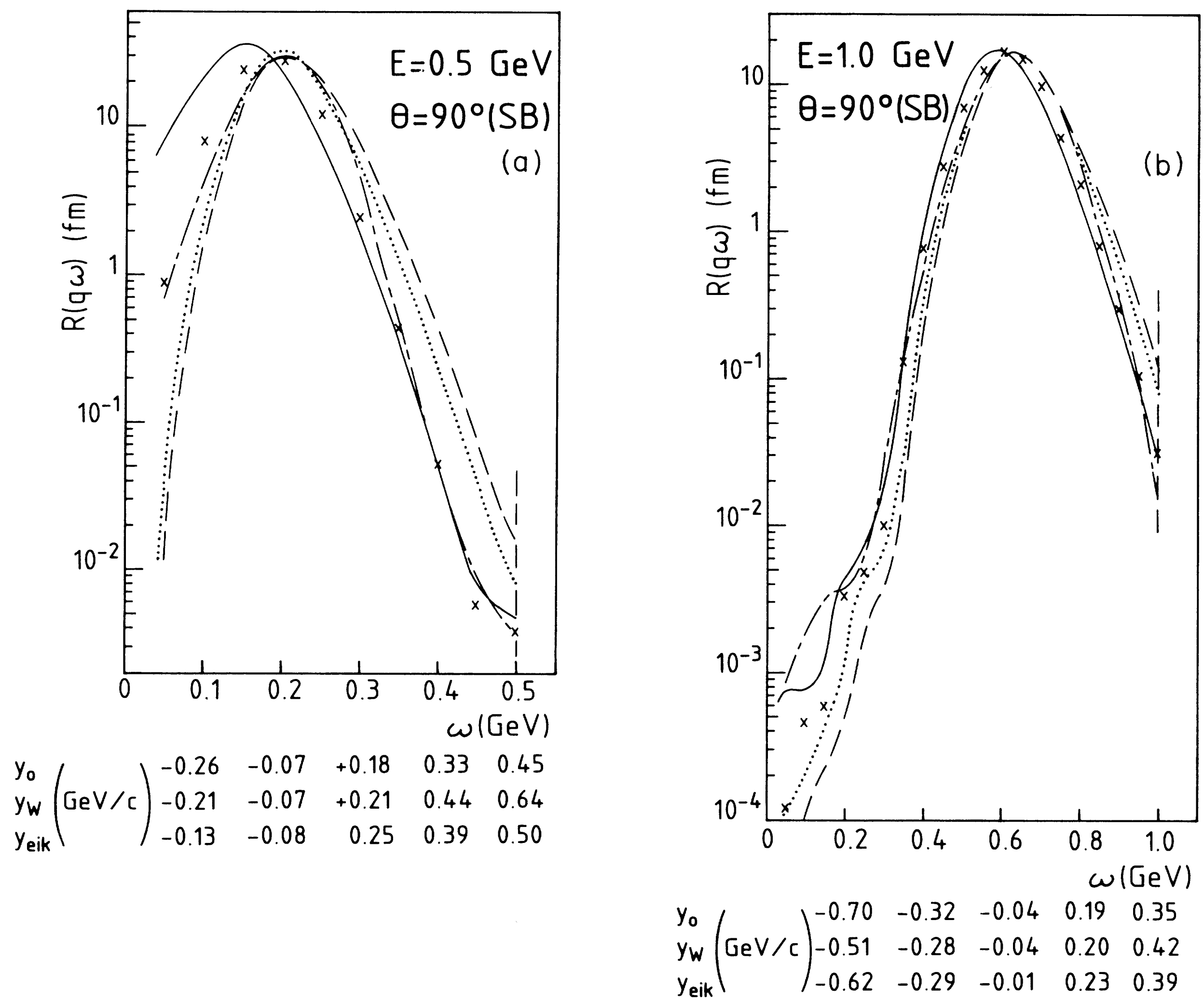

FIG. 1. (a) The response R for scattering from a square well with SB parameters (4.1) as functions of the energy loss ω. $E=0.5$ GeV and scattering angle $\theta=90°$. Crosses are exact results. Solid, dashed, dotted, and dotted-dashed lines correspond, respectively, to eikonal, plane wave, Born, and West approximations. Vertical dashed line marks kinematical limit $\omega=q$. We also give values of y_0, y_W, and y_{eik} [cf. Eqs. (3.8), (3.13), and (3.18)], the absolute values of which are approximation-dependent scaling variables used in the text. (b) Same as for (a) for $E=1.0$ GeV. (c) Same as for (a) for $E=1.5$ GeV. (d) Same as for (a) for $E=2.0$ GeV.

from threshold, the corresponding negative y, as usual, quickly reaches the QEP value $y \sim 0$. However, for a continuingly increasing ω, values of y only slowly increase and actually tend to oscillate. This immediately explains the unusual behavior of R as shown in Figs. 3(a) and 3(b).

One observes in all cases that around the QEP ($q \sim \xi$ or $y_i \sim 0$) contributions from the final state contributions (FSI's) to R are very small. Thus, for all approximations to FSI's, one has, for $y_i = 0$,

$$R = R^{\rm PW} + R^{\rm FSI} \approx R^{\rm PW} . \tag{4.7}$$

The regions away from the QEP are of greatest interest and there the quality of the various approximations is tested in a discriminating way.

Inspection of Figs. 1 and 2 shows that, with the exception of the low ω side of the QEP for the lowest E investigated, the eikonal approximation gives, over several decades, the best rendition of $R^{\rm ex}$.

Next, we consider the BA to the FSI, which, as expected, is universally superior to the PW approximation with $R^{\rm FSI} = 0$. One will not fail to observe the remaining discrepancy between $R^{\rm ex}$ and $R^{\rm IA}$. Apparently, $R^{\rm FSI}$ is *never* negligible.

This is illustrated in an alternative fashion in Table I. There we indicate (for SB results) the position of the QEP and q,ω values in the wings: FSI's are clearly of importance for kinematical situations represented in Figs. 1(a)–1(c).

Of particular interest are the results for the West approximation. While competing in quality with $R^{\rm eik}$ for $E = 0.5$ GeV, $R^{\rm W}$ otherwise seems to be a poor approximation on the elastic side of the QEP. At this point we recall the similarity between y_W and $y_{\rm FG}$, the y variable for a Fermi gas. This similarity has led to the statement that a fit by means of the West approximation implies that the system behaves as a noninteracting Fermi gas. It may be useful to recall that West originally intended to calculate R for a general system of interacting particles (the only assumption made in Ref. 37, namely locality of pair forces, hardly being a restriction). Therefore, had $R^{\rm W}$ included $R^{\rm FSI}$, it ought to be close to $R^{\rm ex}$ for all q,ω, which is not the case. In addition, one would expect y_W to depend on V, as does $y_{\rm eik}$.

It indeed turns out that West's original derivation is erroneous in general; only if y is fixed and $q \to \infty$ does it give the correct asymptotic expression. However, in that

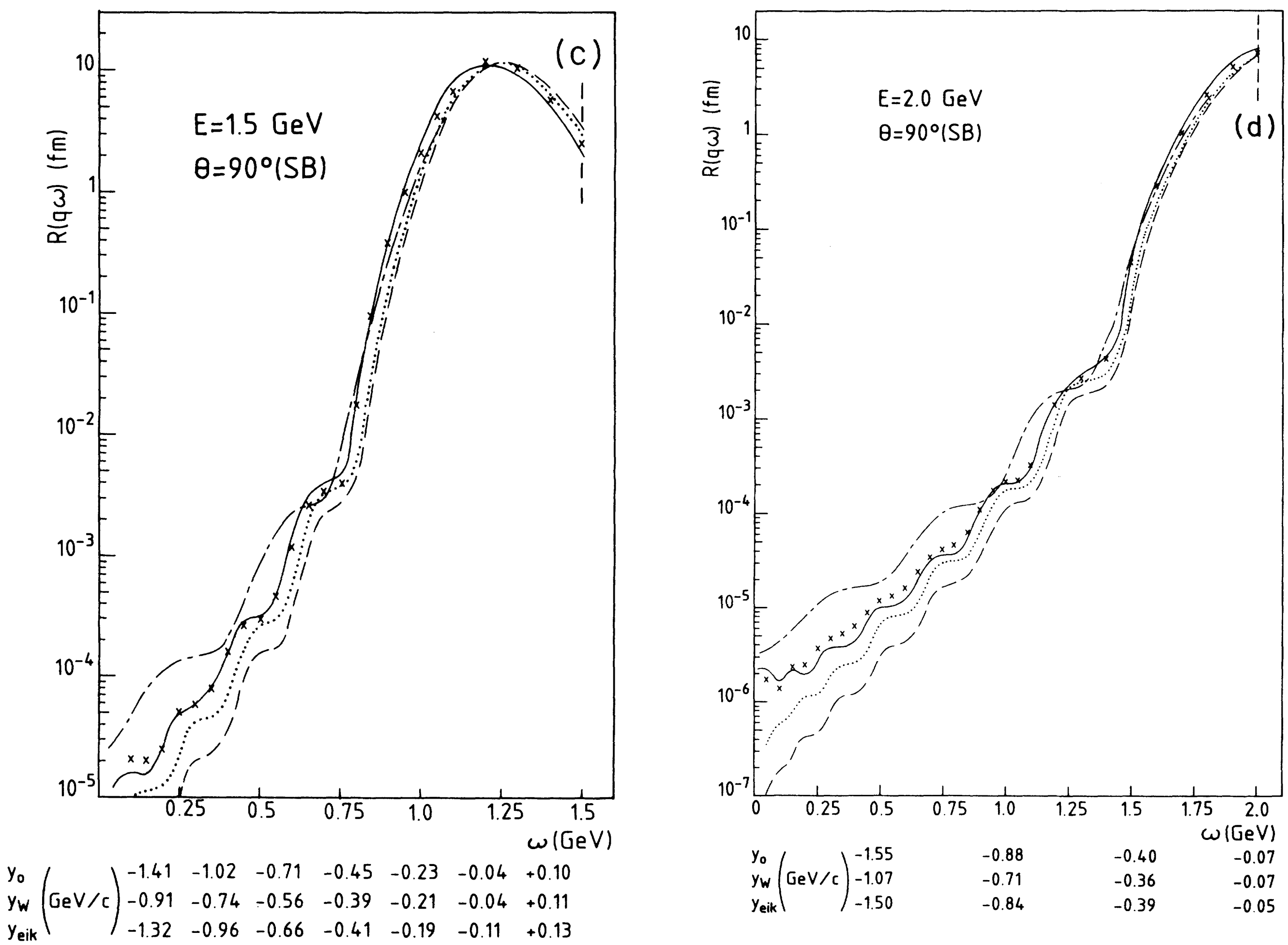

FIG. 1. (*Continued*).

limit it is indistinguishable from all other approximations and is just the PW result with $R^{\rm FSI}=0$ [cf. Eqs. (3.7) and (3.9)]. In other publications[48] (also see Ref. 18) V dependent terms have been retained. In that case Eqs. (3.12) and (3.13) will have to be amended and it is not evident that one will recover R in the form (3.13).

V. y SCALING

We start our discussion of y scaling from Eq. (3.21). For all R_i studied above—but, of course, not for all conceivable or reasonable approximations R_i to $R^{\rm ex}$—a form like (3.21) appears to hold. Since $n(k)$ rapidly falls with k, the upper limits of integrals $y_{i,>}$ in (3.21) either are, or may safely be replaced by ∞.

Strictly speaking, one ought to demand that the momentum distribution $n_i(k)$ shall be calculated in accordance with approximation i. Let us nevertheless assume

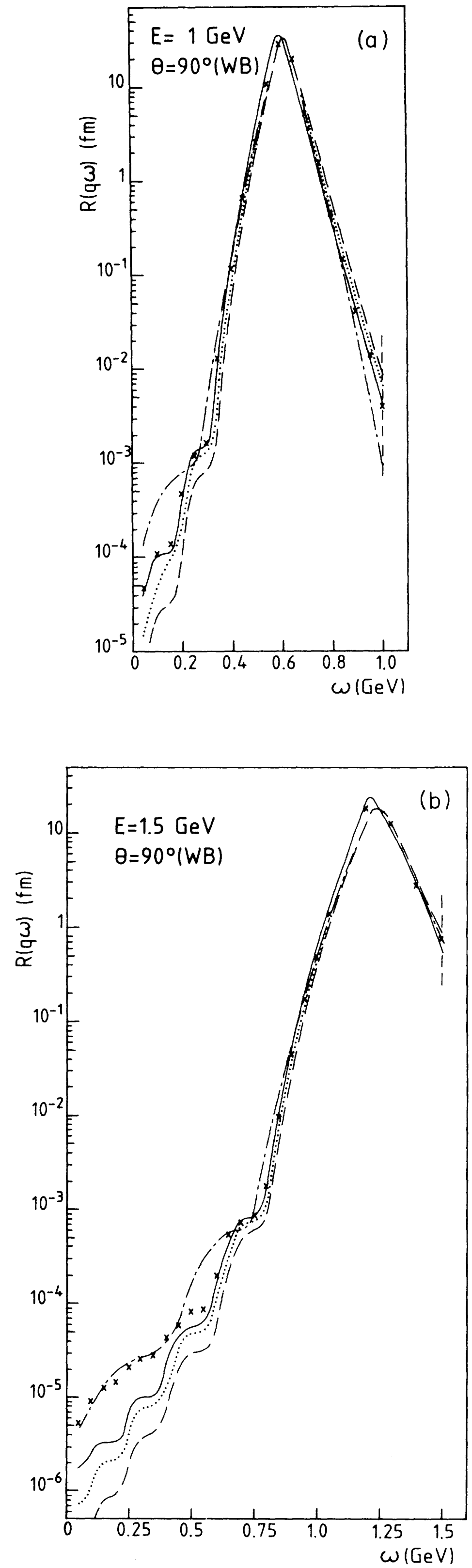

FIG. 2. (a) Legend same as for Fig. 1(b) for WB parameters (4.2). (b) Legend same as for Fig. 1(c) for WB parameters.

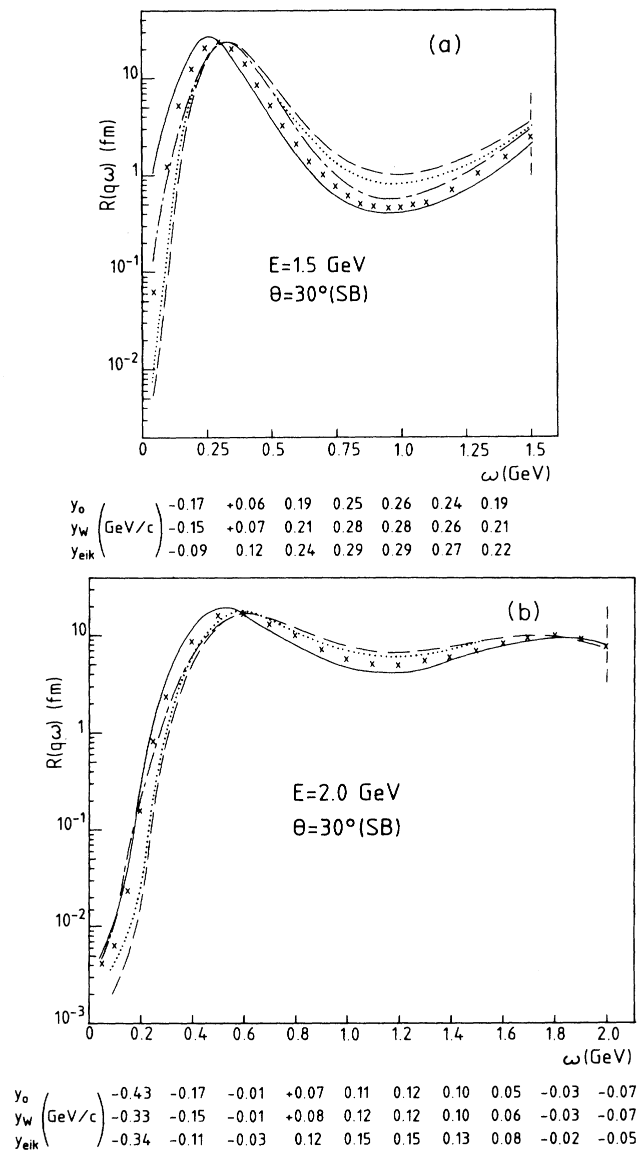

FIG. 3. (a) Legend same as for Fig. 1(c) for $\theta=30°$. (b) Legend same as for Fig. 1(d) for $\theta=30°$.

TABLE I. For $E=0.5$, 1.0, and 1.5 GeV ($\theta=90°$) we give, for the SB case, $\binom{q}{\omega}_{\rm QEP}$, the position of the QEP. In the subsequent line we give for each E two pairs $\binom{q}{\omega}$ corresponding to kinematical values well below and above the QEP. For the latter, the last line contains the ratio $R^{\rm FSI}/R^{\rm IA}$. Quantities with dimensions are in GeV.

	0.5		1.0		1.5	
$\begin{bmatrix} q \\ \omega \end{bmatrix}_{\rm QEP}$	0.59 0.19		1.07 0.61		1.52 1.23	
$\begin{bmatrix} q \\ \omega \end{bmatrix}_{\rm QEP}$	0.64 0.10	0.54 0.30	1.17 0.40	1.02 0.80	1.58 1.00	1.50 1.40
$\dfrac{R^{\rm FSI}(q,\omega)}{R^{\rm IA}(q,\omega)}$	3.89	−0.65	2.46	−0.45	0.67	−0.21

here that $n_i=n$ ($=n^{\rm exact}$). Equation (3.21) then becomes

$$\xi_i(q,\omega)R_i(q,\omega)=\frac{m}{4\pi^2}\int_{|y_i|}^{\infty}dk\,k\,n(k)\equiv F(y_i)\ . \quad (5.1)$$

The left-hand-side ξ_iR_i, though functions of q,ω, appear to depend on one variable $y_i=y_i(q,\omega)$ only, or differently stated, for the approximation i, ξ_iR_i shows perfect scaling. y and F are, respectively, scaling variable and scaling function. For reference, we collect in Table II expressions for y_i,ξ_i.

The interest in scaling in practice derives from (5.1): With the response equation (2.2) directly accessible from experiment and ξ_i known for a given approximation i, Eq. (5.1) leads to

$$n(y_i)=-\frac{4\pi^2}{m}\frac{1}{y_i}\frac{dF(y_i)}{dy_i}\ . \quad (5.2)$$

Consequently, the momentum distribution n appears related to R_i.

For what follows it will be more convenient to keep y and q fixed: $\omega_i=\omega_i(y,q)$ will then be an energy loss depending on the approximation. Thus, as an alternative to Eq. (5.1), one has

$$\xi_i(q,\omega_i)R_i(q,\omega_i)=F(y)\ . \quad (5.3)$$

At this point we emphasize the following:

(i) The scaling property expressed by Eq. (5.1) or (5.3) demonstrably holds for a number of *approximations* R_i to $R^{\rm ex}$ [Eqs. (3.7), (3.11), (3.12), and (3.17)].

(ii) Scaling is a useful tool only if, according to some well-defined criterion, a single approximation is definitely preferred.

In order to find such a criterion, we introduce functions of two variables

$$\phi_i(y,q)\equiv\xi_i(q,\omega_i)R^{\rm ex}(q,\omega_i)\ , \quad (5.4)$$

which, of course, do not scale. From their definitions (3.8), (3.13), and (3.18), one readily shows that, keeping one y_i fixed, for every y_i (also see Ref. 41),

$$y_i-y_j\underset{q\to\infty}{\longrightarrow}O(q^{-1})\ , \quad (5.5)$$

e.g.,

$$y_W=y_0(1+y_0/2q)-\frac{m\epsilon_0}{q}\ ,$$

$$y_{\rm eik}=y_0-m\langle V_0\rangle/q+O(q^{-2})\ .$$

One also checks that for all i (cf. Table II),

$$\xi_i(y,q)\underset{\substack{y\ \text{fixed}\\ q\to\infty}}{\longrightarrow}q\ , \quad (5.6)$$

e.g., $\xi_{\rm BA}=q(1+y_0/2q)^2\to q$, etc.

We now return to the well-defined functions $\phi_i(y,q)$, Eq. (5.4). From Eqs. (5.5) and (5.6), one finds

$$\phi_i(y,q)\underset{q\to\infty}{\longrightarrow}F(y)\ . \quad (5.7)$$

Equation (5.7) expresses the fact that for fixed y all asymptotic limits equal the PW result.[58] Thus, with (5.6),

$$R^{\rm FSI}\underset{q\to\infty}{\longrightarrow}0\ . \quad (5.8)$$

If a given approximation j is considered to be superior, Eq. (5.4) reads, for that j,

$$\phi_j(y,q)=\xi_j(q,\omega_j)R^{\rm ex}(q,\omega_j)$$
$$\sim\xi_j(q,\omega_j)R_j(q,\omega_j)=F(y)\ , \quad (5.9)$$

where $F(y)$ is the limit (common to all approximations)

$$\phi_j(y,q)\underset{q\to\infty}{\longrightarrow}F(y)\ . \quad (5.10)$$

Equations (5.9) and (5.10) enable the formulation of a criterion. Provided that the approach to the scattering limit is smooth, the faster the approach to scaling the better the quality. No precise mathematical criterion will be necessary for the model: the fastest approach of $\phi_j(y,q)$ to $F(y)$ is usually judged by the eye.

TABLE II. Scaling variables and characteristics kinematic variables [cf. Eqs. (3.21) and (5.1)] for various approximations i.

	y_i	ξ_i
PW	$\lvert -q+\xi\rvert$	q
BA	$\lvert -q+\xi\rvert$	$q\left[\dfrac{\xi+q}{2q}\right]^2$
West	$\left\lvert -\dfrac{q}{2}+\dfrac{\xi^2}{2q}-\dfrac{m\epsilon_0}{q}\right\rvert$	q
Eikonal	$\lvert -q+\xi-m\langle V\rangle/\xi\rvert$	ξ

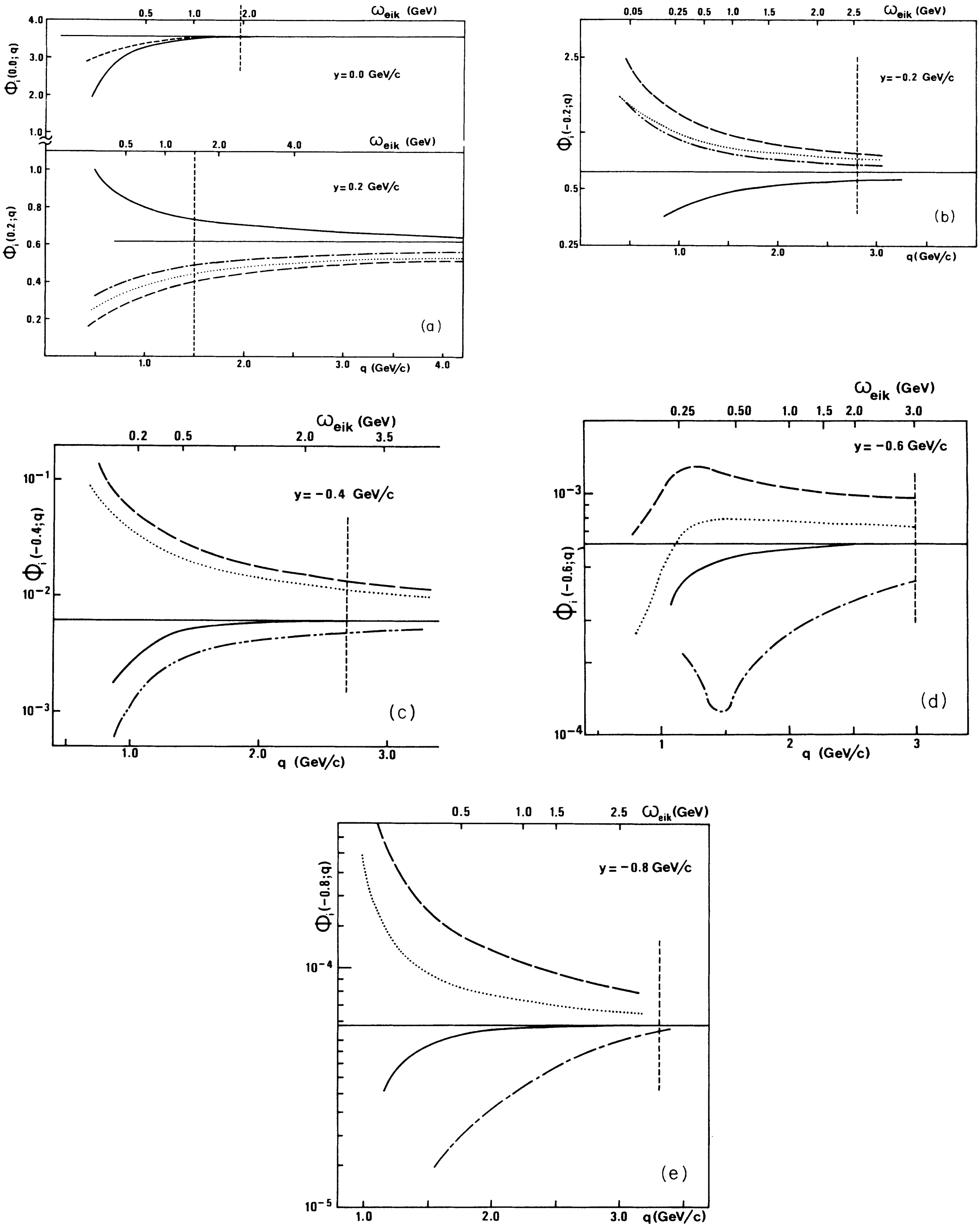

FIG. 4. (a) Functions of $\phi_i(y,q)$, Eq. (5.4), for $y = 0.2$ and 0.0 GeV. Horizontal solid lines correspond to the asymptotic limit $F(y)$, Eq. (5.7). Solid, dashed, dotted, and dotted-dashed lines correspond to kinematical factors in (5.9) (cf. Table II) for, respectively, the eikonal, PW, Born, and West approximations. (b) Same as (a) for $y = -0.2$ GeV. (c) Same as (a) for $y = -0.4$ GeV. (d) Same as (a) for $y = -0.6$ GeV. (e) Same as (a) for $y = -0.8.$ GeV.

We have implemented the program implicit in Eqs. (5.9) and (5.10) for the model. Formidable numerical problems had to be overcome. These are clearly due to large l values and, simultaneously, large arguments in the spherical Bessel and Hankel functions to be retained in the partial-wave sum (3.4).

For the SB parameters (4.1), we assembled in Figs. 4(a)—4(e) results for $\phi_i(y,q)$ for y (in GeV) $=0.2$, (-0.2), -0.8. Except for $y=0.2$ GeV, the investigated kinematical regions are on the small ω side of the QEP. Notice that, although for fixed y and q, the energy loss ω varies with the approximation, only $\omega_{\rm eik}$ has been entered in the figures. Also parallel to the horizontal axis, we give in each panel of Fig. 4 the asymptotic value $F(y)$, Eq. (59).

The following observations can be made.

(1) Except for $y=0$, and relatively small q, the eikonal approximation approaches the scaling limit $F(y)$ faster over the largest q range. The exception can be understood, since for $y=0$ and small q, also ω and thus ξ are small, which is contrary to the eikonal condition.

(2) The preference for $\phi_{\rm eik}$ becomes more pronounced for decreasing (negative) y, i.e., for regions away from the QEP. This proves the importance of the remnant FSI, which the eikonal approximation accounts for better than any other.

(3) All ϕ_i indeed tend to $F(y)$, but the approach to the asymptotic limit is slow. In some cases the approach is so slow that $\phi_i(y,q)$ for large q appears to be nearly a constant, which actually is not yet the true asymptotic limit $F(y)$. The consequent application of Eq. (5.2) then leads to a faulty extracted $n(y)$.

In our attempt to study the approach of scaling of $\phi_i(y,q)$ one apparently samples only a small kinematical region, where ω is small and q large. Only for large $|y|$ will large q leave ω relatively small. That region is usually the y-scaling regime: for those the eikonal model is vastly superior. For $|y|\lesssim 0.5$ GeV there hardly is a domain where ω is small and q is large.

We conclude this section with remarks bearing on the relation between y and x scaling. Since the latter is used in the relativistic regime, we should also use relativistic analogues of y_i, e.g.,[16]

$$|y^{\rm rel}|=\frac{q}{2}\left[1-\frac{\omega}{q}\left[1+\frac{4m^2}{Q^2}\right]^{1/2}\right],$$

where $Q^2=q^2-\omega^2$. Using Eq. (1.1),

$$|y^{\rm rel}|=\frac{m(1-x^2)}{x}\left[\left[1+\frac{4m^2}{Q^2}\right]^{1/2}+\left[1+\frac{4m^2x^2}{Q^2}\right]^{1/2}\right]^{-1}.$$

One further checks that also all relativistic variables $y_i^{\rm rel}$ tend to the same limit [cf. Eq. (5.5)], and thus, for fixed y,

$$|y^{\rm rel}|\underset{Q^2\to\infty}{\longrightarrow}\frac{m(1-x^2)}{2x}.$$

No numerical comparison could be made, since our model is intrinsically a nonrelativistic one.

Next, we remark that perfect x and y scaling are both asymptotic properties: For decreasing q, scaling becomes imperfect, which for deep-inelastic scattering is primarily due to gluon exchange. Even when using a different x variable, still of kinematical origin,[60] imperfect scaling is only somewhat remedied.

Now consider y scaling; for instance, in the PW variable y_0. For decreasing q, FSI will cause responses to become functions of y_0 and q, i.e., y scaling becomes imperfect. However, our model is simple enough to permit construction of a *dynamical* y variable which includes FSI effects. Consequently, scaling in that variable remains nearly perfect down to q values where y_0 scaling shows significant q dependence.

VI. CONCLUSIONS

Above we have described model studies of inclusive scattering from a system having only one degree of freedom, viz., a proton bound to an inert core. Together with exact calculations of the basic (longitudinal) response R, we also computed a number of approximations R_i. Except for low incident energies, $R^{\rm eik}$ appears to be superior over a wide range of kinematical conditions.

For all approximations studied we could easily establish a kinematical factor ξ_i which, when multiplying R_i, is a function of one variable y_i; that is, $\xi_i R_i=F(y_i)$, with F simply related to the single nucleon momentum distribution scales in the variable y_i. In contradistinction to these approximations R_i, there is no *a priori* reason why these factors multiplying the exact response $R^{\rm exact}$ will do the same, except asymptotically for $q\to\infty$ and apparently also at the QEP.

One can now ask how close one is to the scaling limit away from the QEP and the model answers this question. Again, we found that the exact response $R^{\rm ex}(y,q)$, now multiplied with the kinematical factor $\xi_{\rm eik}(y,q)$, approaches the scaling limit fastest, in particular for increasing $|y|$.

The result clearly establishes the following for the model.

(a) A criterion for y scaling.

(b) The importance of remnant FSI in virtually all kinematical regions except at the QEP.

(c) The superiority of the eikonal approximation to account for the FSI and *ipso eo*, the same for the scaling variable $y_{\rm eik}$. The latter statement implies that $y_{\rm eik}$ determines best high components of the single particle momentum distribution [cf. Eq. (5.2)].

A second point relates to the y-scaling phenomenon itself. Perfect scaling clearly holds when FSI are negligible. However, the converse is not true: the observation of approximate scaling in some y_i is not in any way proof that FSI are negligible. Indeed, we showed that in some cases an incorrect asymptotic limit is reached. From it one might derive an incorrect momentum distribution.

The above completes a program to study y scaling in a soluble model. Rather precise questions have been formulated and, within an intrinsically limited scope (only some approximations were tested), well-founded and conclusive answers could be given.

Some of our conclusions may well be approximately valid for actual many-body systems, but very little can be proved. For instance, only for an independent particle model is R known to scale. Not even the plane-wave impulse approximation to R scales rigorously for finite q, as does its counterpart R^{PW} in a model with only a single degree of freedom. Finally, we do not know of any general and reliable treatment of FSI's in A(e,e′p) which leads to scaling, as does the eikonal approximation for the model studied. Imperfect scaling apparently occurs in systems with short-range interactions and systems as diverse as nuclei and liquid atomic ^{4}He (and that sometimes in several y_i!). For these, one expects a clearly preferred y which undoubtedly will depend on the interparticle potential.

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*Permanent address: Weizmann Institute of Science, Rehovot 76100, Israel.

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