Relativistic treatment of $0^+(p,p')0^-$ transitions

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$0^+ \to 0^-$ transitions by medium energy protons are studied in a relativistic impulse approximation treatment. We explicitly show the appearance of nonzero values for some spin observables, e.g., the analyzing power, which vanish in an equivalent nonrelativistic treatment. We compare our calculation with experiment and find qualitative agreement.

I. INTRODUCTION

In the present paper we continue our study of inelastic proton-nucleus spin observables within the framework of the Dirac equation. In previous publications[1,2] we have stated the importance of lower components of Dirac wave functions in the determination of spin observables. We have identified a particular combination of spin observables, the spin difference function,[1,2] as a very sensitive probe to differences between equivalent relativistic and nonrelativistic treatments. Although our treatment has been completely general we have, so far, restricted our calculations to $0^+ \to 1^+$ excitations which, we feel, constitute an excellent testing ground for these ideas. Even better, however, are $0^+ \to 0^-$ transitions. In this case the sensitivity between the two theoretical approaches does not lie in differences between spin observables, which enhance the uncertainties associated with experimental errors, but instead on the spin observables themselves. Furthermore, since both initial and final nuclear states have angular momentum zero, this particular transition shares the simplicity of elastic processes and also contains the rich structure characteristic of inelastic transitions. These $0^+ \to 0^-$ transitions are the main focus of the present work.

We start in Sec. II by writing the most general scattering amplitude, consistent with rotation and parity invariance, for $0^+ \to 0^-$ transitions and establish some well-known relations between spin observables. In Sec. III we perform equivalent nonrelativistic and relativistic calculations in plane wave impulse approximation and use the simplicity of the plane wave treatment to isolate the essential difference between these two approaches. In Sec. IV we perform a full relativistic distorted wave impulse approximation (RDWIA) calculation. Our results, together with experimental data and conclusions, then follow in Sec. V.

II. INVARIANT AMPLITUDES

In this section we derive the most general form for the $0^+ \to 0^-$ transition amplitude, by a spin $\frac{1}{2}$ probe, consistent with rotation and parity invariance. The only rotational invariant operators that we can form in the spin space of the projectile are 1, $\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}$, $\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}$, $\boldsymbol{\sigma}\cdot\hat{\mathbf{K}}$, where $\hat{\mathbf{n}}$, $\hat{\mathbf{q}}$, and $\hat{\mathbf{K}}$ are unit vectors in the direction of the normal to the scattering plane $\mathbf{k}\times\mathbf{k}'$, momentum transfer $\mathbf{k}-\mathbf{k}'$, and average momentum $(\mathbf{k}+\mathbf{k}')/2$, respectively, and they constitute, in the adiabatic limit, a right handed orthonormal coordinate system. If we further impose parity invariance, we note that the first two operators do not change sign under parity, and therefore should not appear in the scattering amplitude. It is only the last two (pseudoscalar) operators that can restore the parity change in the nuclear excitation.

Time reversal invariance is an exact symmetry of the strong interaction. It is very important to stress however, that for inelastic excitations this symmetry does not impose any constraints among the $0^+ \to 0^-$ amplitudes. Instead, however, time reversal invariance sets relations (e.g., the polarization-analyzing power theorem[3]), between the amplitudes of two different physical processes, the $0^+ \to 0^-$ reaction and its inverse, namely, the $0^- \to 0^+$ excitation. It is only for elastic scattering, where initial and final nuclear states are identical, that time reversal invariance constrains the amplitudes among themselves. In particular we know, even if we do not invoke parity invariance, that for elastic scattering the $\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}$ term must be absent from the time reversal invariant amplitude.

Therefore, the most general transition amplitude that one can write for the $0^+ \to 0^-$ excitation which respects rotation and parity invariance is given by

$$A(0^-)=[A_q(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})+A_K(\boldsymbol{\sigma}\cdot\hat{\mathbf{K}})]\Sigma(0^+ \to 0^-) , \tag{1}$$

where the A_q and A_K are scalar functions of energy and momentum transfer, and $\Sigma(0^+ \to 0^-)$ is the pseudoscalar nuclear operator defined by

$$\Sigma(0^+ \to 0^-)= | 0^- \rangle\langle 0^+ | . \tag{2}$$

In calculating spin observables we adopt the standard definition given by

$$\left[\frac{d\sigma}{d\Omega}\right]D_{\alpha\beta}=\tfrac{1}{2}\,\mathrm{Tr}(\sigma_\alpha A \sigma_\beta A^\dagger) , \tag{3a}$$

where $\alpha,\beta=(0,n,q,K)$, $\sigma_0 \equiv 1$, and $D_{00}\equiv 1$, so the unpolarized cross section is

$$\left[\frac{d\sigma}{d\Omega}\right]=\tfrac{1}{2}\,\mathrm{Tr}(AA^\dagger) \tag{3b}$$

and we note that eight of the possible sixteen independent spin observables vanish once we have assumed a parity invariant interaction.[4,5]

For our particular case of interest, namely, the $0^+ \to 0^-$ transition, there are, as in the case of elastic scattering, only three independent spin observables. We will choose them to be the cross section, the analyzing power A_y, and one of the spin rotation functions $B \equiv -D_{Kq}$.

The full set of spin observables, assuming only that the projectile spin is monitored, can be explicitly calculated in terms of the invariant A_q and A_K amplitudes and is given by

$$\begin{aligned}
\frac{d\sigma}{d\Omega} &= |A_q|^2 + |A_K|^2 , \\
\frac{d\sigma}{d\Omega} D_{nn} &= -|A_q|^2 - |A_K|^2 , \\
\frac{d\sigma}{d\Omega} D_{qq} &= |A_q|^2 - |A_K|^2 , \\
\frac{d\sigma}{d\Omega} D_{KK} &= -|A_q|^2 + |A_K|^2 , \\
\frac{d\sigma}{d\Omega} D_{0n} &= 2\,\mathrm{Im}(A_q A_K^*) , \\
\frac{d\sigma}{d\Omega} D_{n0} &= -2\,\mathrm{Im}(A_q A_K^*) , \\
\frac{d\sigma}{d\Omega} D_{qK} &= 2\,\mathrm{Re}(A_q A_K^*) , \\
\frac{d\sigma}{d\Omega} D_{Kq} &= 2\,\mathrm{Re}(A_q A_K^*) .
\end{aligned} \tag{4}$$

In particular we obtain, as many other authors have,[5,6] very simple relation between spin observables,

$$\begin{aligned}
D_{n0} &\equiv P = -A_y \equiv D_{0n} , \\
D_{qK} &\equiv Q = -B \equiv D_{Kq} ; \\
D_{nn} &= -1 .
\end{aligned} \tag{5}$$

Note that the deviations from the elastic scattering relations, $P = A_y$ and $Q = B$, are maximal in this case. In our previous study of the $0^+ \to 1^+$, we introduced the spin difference function $\Delta \equiv (Q-B) + i(P-A_y)$ as an effective way to quantify differences between equivalent nonrelativistic and relativistic treatments. We noted that at high energies where the contribution from exchange is expected to be small, the nonrelativistic treatment predicts a very small value for the spin difference function, while in the relativistic picture, large differences from zero appear in a natural way. In the $0^+ \to 0^-$ transition we will observe a similar behavior. The interesting part is that for the present case, the spin difference function is simply given by $\Delta = -2(B + iA_y)$ and deviations from zero amount to detect a nonzero value for the most easily measured spin observable, the analyzing power.

III. PLANE WAVE AMPLITUDES

The formal development of the concepts that we will use throughout the present work were previously developed in other papers.[2,7,8] We therefore refer the reader to those publications and only include some essential features to make the present treatment self-consistent.

The $0^+ \to 0^-$ transition amplitude in a nonrelativistic plane wave impulse approximation (PWIA) treatment is given by

$$A(0^-) = \left\langle 0^- \left| \sum_{n=1}^{A} e^{i\mathbf{q}\cdot\mathbf{r}_n} t_n \right| 0^+ \right\rangle , \tag{6}$$

where t_n is the nucleon-nucleon (NN) t matrix. The NN t matrix can be expressed, in the space of the target particles, in terms of the unit matrix and the three Pauli matrices, i.e.,

$$t_n = d_0 + \mathbf{d}\cdot\boldsymbol{\sigma}(n) . \tag{7a}$$

A convenient parametrization of the two-body t matrix in the c.m. frame is given by the Wolfenstein representation.[9] The above coefficients, which are operators in the projectile space, can then be written explicitly. The scalar operator is given by

$$d_0 = A(q) + iqC(q)(\sigma\cdot\hat{\mathbf{n}}) , \tag{7b}$$

while the axial vector operator by

$$\mathbf{d} = [B(q)\sigma + iqC(q)\hat{\mathbf{n}} + q^2 D(q)(\sigma\cdot\hat{\mathbf{q}})\hat{\mathbf{q}} + E(q)(\sigma\cdot\hat{\mathbf{K}})\hat{\mathbf{K}}] , \tag{7c}$$

where now σ is the spin operator of the projectile.

If we now perform a multipole decomposition of the projectile and target operators, and use a second quantization formalism we can express the transition amplitude as the product of a nuclear form factor and a projectile space operator in the following way:

$$A(0^-) = \sum_{ls} G_{ls}(q)[Y_l(\hat{\mathbf{q}})d_s]^*_{0,0} . \tag{8}$$

The precise form of the nuclear form factor is not relevant to the present discussion. It is sufficient to note that it is proportional to the parity Clebsch-Gordan (CG) coefficient $\langle l0; l_i0 \mid l_f0\rangle$. In the above expression s is the spin transfer in the reaction and takes only the values zero or one. Furthermore, since the final state of the target is $\mathbf{J}=0$, the l transfer in the reaction is constrained to be equal to s. Therefore, there are only two possible operators in the projectile space. These are given by

$$[Y_l(\hat{\mathbf{q}})d_s]^*_{0,0} = \frac{1}{\sqrt{4\pi}} \times \begin{cases} d_0 & (l=s=0), \\ -\mathbf{d}\cdot\hat{\mathbf{q}} & (l=s=1). \end{cases} \tag{9}$$

We note, as was mentioned before, that d_0 behaves as a scalar while $\mathbf{d}$, being an axial vector, makes $\mathbf{d}\cdot\hat{\mathbf{q}}$ a pseudoscalar. It is only this latter operator, which compensates the nuclear parity change, that is allowed to contribute to the transition amplitude. The scalar operator d_0 is guaranteed to be absent from the transition by using $\langle l=0,0; l_i,0 \mid l_f,0\rangle \equiv 0$, for $(l_i + l_f) = $odd. Hence, we recover the well-known result that an unnatural parity transition, treated in a nonrelativistic framework, must involve a spin transfer in the reaction.

A useful approximation in which to study these proton induced reactions is the adiabatic limit.[5] In this limit one assumes that the Q value of the reaction can be neglected when compared with the projectile's incident energy. For medium energy protons this constitutes a very good approximation and establishes the orthogonality of $\hat{\mathbf{q}}$ and $\hat{\mathbf{K}}$ away from the forward direction. (For medium energy reactions, $T^{\text{lab}} \sim 200$—800 MeV, and a typical excitation energy of 10 MeV, $\hat{\mathbf{q}}\cdot\hat{\mathbf{K}} \le 0.1$ for q beyond $\frac{1}{2}$ fm^{-1}, and becomes less than four percent at the point where the measured analyzing power, Fig. 2, has its largest deviation from zero.)

If we now use the value of $\mathbf{d}$ given in Eq. (7c), we observe that, in the adiabatic limit, the transition amplitude reduces to

$$A(0^-) = -\frac{1}{\sqrt{4\pi}} G_{l=s=1}(q)(B+q^2D)(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}) \ . \tag{10}$$

We immediately notice the absence of the $(\boldsymbol{\sigma}\cdot\hat{\mathbf{K}})$ term. This will make all spin observables, which consist of the interference of A_q and A_K, identically zero. In particular, we obtain

$$P = A_y = Q = B = 0 \text{ and } D_{nn} = D_{KK} = -D_{qq} = -1 \ . \tag{11}$$

in a nonrelativistic calculation with a local t matrix.

It is very important to stress that this result has nothing to do with time-reversal invariance. We previously emphasized that time-reversal invariance does not impose any constraints among the $0^+ \to 0^-$ amplitudes. In fact, we now show that by using a relativistic formulation, also based on a local parametrization of the NN interaction, all amplitudes allowed by invariance principles will be nonzero and this will lead to nontrivial values for the spin observables.

The $0^+ \to 0^-$ transition, in the context of a relativistic plane wave impulse approximation (RPWIA), is now given by

$$A(0^-) = u^\dagger(\mathbf{k}') \left\langle 0^- \left| \sum_{n=1}^{A} e^{i\mathbf{q}\cdot\mathbf{r}_n} \gamma^0 \gamma^0(n) t_n \right| 0^+ \right\rangle u(\mathbf{k}) \ , \tag{12}$$

where

$$u(\mathbf{k}) = \left[\frac{E+m}{2m}\right]^{1/2} \begin{bmatrix} 1 \\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{k}}{E+m} \end{bmatrix}$$

and (13)

$$u(\mathbf{k}') = \left[\frac{E'+m}{2m}\right]^{1/2} \begin{bmatrix} 1 \\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{k}'}{E'+m} \end{bmatrix}$$

are free Dirac spinors for the projectile, t_n is, aside from numerical factors, the Lorentz invariant nucleon-nucleon amplitude written in terms of scalar, vector, tensor, pseudoscalar, and axial-vector amplitudes,[10]

$$t_n = t_S + t_V \gamma^\mu \gamma_\mu(n) + t_T \sigma^{\mu\nu} \sigma_{\mu\nu}(n) + t_P \gamma^5 \gamma^5(n) + t_A \gamma^5 \gamma^\mu \gamma^5(n) \gamma_\mu(n) \tag{14}$$

and we are using the Bjorken-Drell[11] convention for the Dirac gamma matrices.

We continue to follow closely Ref. 2 and rewrite the RPWIA in a form that conveniently separates the spin dependence from the particular combination of upper and lower components involved; we write

$$\gamma^0 \gamma^0(n) t_n = \sum_{\nu=1}^{4} [f^\nu + g^\nu [\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}(n)]] \Gamma_\nu \Gamma_\nu(n) \ , \tag{15}$$

where the structure matrices are defined by

$$\Gamma_1 = 1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \Gamma_2 = \gamma^0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$
$$\Gamma_3 = \gamma^5 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \Gamma_4 = \gamma^0 \gamma^5 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \tag{16}$$

and

$$\begin{aligned} f^1 &= t_V, \quad g^1 = -t_A \ , \\ f^2 &= t_S, \quad g^2 = 2t_T \ , \\ f^3 &= t_A, \quad g^3 = -t_V \ , \\ f^4 &= t_P, \quad g^4 = 2t_T \ . \end{aligned} \tag{17}$$

From here on the formal development closely resembles the nonrelativistic treatment. We perform a multipole decomposition of the amplitude together with an expansion in second quantize operators to obtain, as in the nonrelativistic case, a transition operator that can be written as a nuclear form factor times an operator in the projectile space,

$$A(0^-) = u^\dagger(\mathbf{k}') \left[\sum_{\nu=1}^{4} \sum_{ls} G_{ls}^\nu(q) [Y_l(\hat{\mathbf{q}}) \sigma_s]_{0,0}^* \Gamma_\nu \right] u(\mathbf{k}) \ . \tag{18}$$

Again, the detailed form of the nuclear structure operator is not important to the present discussion. It suffices to mention its most important features. The nuclear form factor is, as in the nonrelativistic case, proportional to a parity CG coefficient. Note, however, due to the existence of lower components, that there are now four different types of parity CG coefficients, corresponding to the four different types of couplings, namely, upper-upper, lower-lower, upper-lower, and lower-upper. The last two couplings have no nonrelativistic counterpart. In particular, since lower components carry different parity than the corresponding upper components, these two parity CG coefficients will be nonzero in the $0^+ \to 0^-$ reaction, only for an (even) $l=0$ transfer in the reaction. This will allow the previously forbidden $l=s=0$ scalar operator to contribute to the transition and, as we will see, will give rise to the central difference between the nonrelativistic and relativistic approaches.

To determine the transition density, we must use four component Dirac bound states. To determine the lower components of these states, we assume an upper component $u_{Ejl}(r)$ given by a nonrelativistic shell model wave function. After all, we know that in the presence of Dirac scalar S and forth component of a vector V potentials, the upper component satisfies an equation which closely resembles a Schrödinger equation.[12] The lower component will then be constrained, by the Dirac equation, to satisfy[13]

$$w_{Ejl'}(r)=\frac{1}{(E+M+S-V)}\left[\frac{d}{dr}+\frac{(1+\kappa)}{r}\right]u_{Ejl}(r)\ , \tag{19a}$$

where

$$\kappa=\pm(j+\tfrac{1}{2})\ \text{ for } l=(j\pm\tfrac{1}{2}) \tag{19b}$$

and this will allow the complete determination of all the necessary transition densities.

We now concentrate on the evaluation of the transition amplitude. By making the appropriate changes in the projectile space operator (9), namely, $d_s\to\sigma_s$, we obtain

$$[Y_l(\hat{\mathbf{q}})d_s]^{*}_{0,0}=\frac{1}{\sqrt{4\pi}}\times\begin{cases}1 & (l=s=0),\\ -\sigma\cdot\hat{\mathbf{q}} & (l=s=1)\ .\end{cases} \tag{20}$$

The contribution from the spin dependent operator proceeds essentially as in the nonrelativistic case and does not reveal anything not previously known. In particular, the A_K contribution also vanishes and, if alone, will also lead to the trivial spin observable relations (11). The interesting contribution comes from the spin independent operator 1. Being a scalar operator it must only include upper-lower and lower-upper couplings in order to restore parity. This is exactly what the above discussion about selection rules implied. A scalar operator in the projectile space can contribute to the amplitude as long as it involves mixed coupling between Dirac components. To observe some of its consequences let us compute the $\nu=3$ (axial) contribution to the $l=s=0$ amplitude,

$$u^{\dagger}(\mathbf{k}')1\Gamma_3 u(\mathbf{k})\ ,$$

which, by explicitly substituting the free Dirac spinors (13), takes the following form:

$$\frac{E+m}{2m}\left[1,\ \frac{\sigma\cdot\mathbf{k}'}{E+m}\right]\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}1\\ \dfrac{\sigma\cdot\mathbf{k}}{E+m}\end{bmatrix}=\left[\frac{K}{m}\right](\sigma\cdot\hat{\mathbf{K}})\ . \tag{21}$$

This is a nonzero contribution to the A_K amplitude. This result shows that by allowing the dynamics of the process to be determined by the Dirac equation, we obtain, in contrast to an equivalent nonrelativistic calculation, nontrivial values for the spin observables without including nonlocal effects. The remaining amplitudes are all calculated in a similar way, and after some straightforward spin algebra we display our results in terms of the general structure given in Eq. (1),

$(l=s=0)$	$(l=s=1)$
$A_q(q)=\dfrac{q}{2m}t_P(q)\rho^{(+)}_{l=0}(q)$	$A_q(q)=\left[t_A(q)-2\left[\dfrac{E}{m}\right]t_T(q)\right]\rho^{(uu)}_{l=1}(q)$
$A_K(q)=\dfrac{K}{m}t_A(q)\rho^{(-)}_{l=0}(q)$	$A_K(q)\equiv 0$

(22a)

where we have defined

$$\rho^{(\pm)}_{l=0}(q)=\rho^{(ul)}_{l=0}(q)\pm\rho^{(lu)}_{l=0}(q)\ ,$$

$$\rho^{(uu)}_{l=1}(q)=\int_0^\infty x^2dx\ u_f(x)j_{l=1}(qx)u_i(x),\ldots\ , \tag{22b}$$

and we have neglected the small lower-lower contribution for bound state particles. The full amplitude consists of the coherent sum of both contributions.

IV. DISTORTED WAVE AMPLITUDE

In the present section we calculate the transition amplitude, in the framework of a relativistic distorted wave impulse approximation (RDWIA). The RDWIA transition amplitude is given, according to Ref. 7, by

$$A(0^-)=\int d\mathbf{r}\,\psi^{\dagger(-)}_{\mathbf{k}'s'}(\mathbf{r})\times\left\langle 0^-\left|\sum_{n=1}^{A}\gamma^0\gamma^0(n)t_n(|\mathbf{r}-\mathbf{r}_n|)\right|0^+\right\rangle\times\psi^{(+)}_{\mathbf{k}s}(\mathbf{r})\ , \tag{23}$$

where $t_n(|\mathbf{r}-\mathbf{r}_n|)$ is the Fourier transform of the fundamental NN interaction. We take $\psi^{\dagger(-)}_{\mathbf{k}'s'}(\mathbf{r})$ and $\psi^{(+)}_{\mathbf{k}s}(\mathbf{r})$ to be eikonal distorted waves[14]

$$\psi^{(\pm)}_{\mathbf{k}s}(\mathbf{r})=\left[\frac{E+m}{2m}\right]^{1/2}\begin{bmatrix}1\\ \dfrac{1}{(E+m+S-V)}(\sigma\cdot\mathbf{P})\end{bmatrix}\times e^{i\mathbf{k}\cdot\mathbf{r}}e^{iS^{\pm}(\mathbf{r})}\chi_S\ , \tag{24a}$$

where the Dirac eikonal phase

$$S^{\pm}(\mathbf{r})=-\frac{m}{K}\int_{\mp\infty}^{z}dz'\{V_c(r')+V_{so}(r')[\sigma\cdot(\mathbf{b}\times\mathbf{K})-iKz']\} \tag{24b}$$

is written in terms of the equivalent central and spin-orbit effective Schrödinger potentials

$$V_c(r)=\left[S(r)+\frac{E}{m}V(r)\right]+\frac{1}{2m}[S^2(r)-V^2(r)]\ ,$$
$$V_{so}(r)=\frac{1}{2m}\frac{1}{(E+M+S-V)}\frac{1}{r}\frac{d}{dr}[V(r)-S(r)]\ . \tag{24c}$$

TABLE I. 200 MeV optical potential parameters.

	Strength (MeV)	c (fm)	β (fm)	w
Scalar (real)	−390.00	2.240	0.508	0.206
Scalar (imaginary)	42.10	2.454	0.444	3.236
Vector (real)	329.70	2.335	0.519	0.020
Vector (imaginary)	−51.16	2.517	0.457	2.137.

The treatment of the transition is identical to the plane wave case. The only modifications arise because of the presence of distorted waves. This will make our previous analytic plane wave treatment no longer possible and will make us resort to numerical computations.

To compare with experiment we focus on the transition to the 10.975 MeV 0^- ($T=0$) state in ^{16}O by 180 MeV protons.[15] This is the largest proton energy for which spin observables have been measured. Ideally, of course, one would like to have data at higher energies where our theoretical approach is suited best.

The calculations were done with a relativistic parametrization of the NN interaction,[10] with the Lorentz invariant amplitudes evaluated at their optimal value.[10,16] This corresponds to an effective value for the laboratory energy given by

$$T_{\mathrm{lab}}^{\mathrm{eff}}(q^2) = \frac{1}{m}\left\{\left[\left(K^2+\frac{q^2}{4}+m^2\right)\left(\frac{K^2}{A^2}+\frac{q^2}{4}+m^2\right)\right]^{1/2} + \frac{K^2}{A}+\frac{q^2}{4}-m^2\right\} \geq T_{\mathrm{lab}}(q=0)\ . \qquad (25)$$

Note that for momentum transfers $q \sim 2\ \mathrm{fm}^{-1}$, it is already the 250 MeV NN amplitude which determines the behavior of the transition amplitude. For the nuclear structure part we simply assume a $p^{1/2} \rightarrow s^{1/2}$ single particle transition, and use harmonic oscillator wave functions with an oscillator parameter $\alpha=0.57\ \mathrm{fm}^{-1}$ for the upper components of Dirac wave functions. For the distortion we use three parameter Woods-Saxon Dirac optical potentials[17]

$$V(r)=V_0\frac{\left[1+w\left[\frac{r}{c}\right]^2\right]}{[1+\exp(r-c)/\beta]} \qquad (26)$$

with strengths and ranges fitted to best reproduce the (200 MeV) elastic cross section and analyzing power (Table I). Finally, we used a space independent effective mass for the bound nucleons of $M^*/M=0.830$, calculated by using the prescription of Ref. 18.

V. RESULTS AND CONCLUSIONS

We have calculated the cross section, analyzing power, and spin rotation function, both in plane wave and including distortions, and we now display our results together with the available experimental data. The plane wave cross section was multiplied by a factor of $\sim\frac{1}{2}$ in order to plot it together with the calculated and measured cross section, Fig. 1. We observe a qualitative agreement with

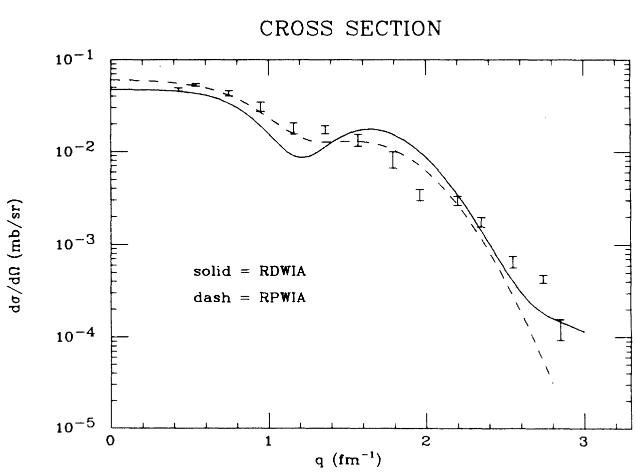

FIG. 1. Cross section for $T_{\mathrm{lab}}=180$ MeV proton excitation of the 0^-, $T=0$, 10.957 MeV state in ^{16}O. The parameters used are given in the text.

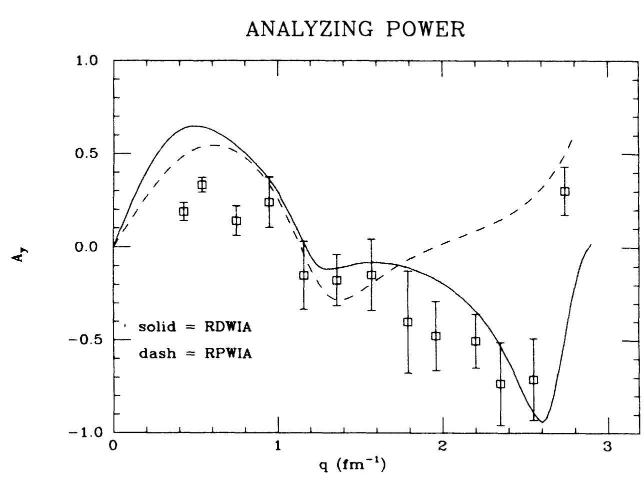

FIG. 2. Same as Fig. 1 for the analyzing power A_y.

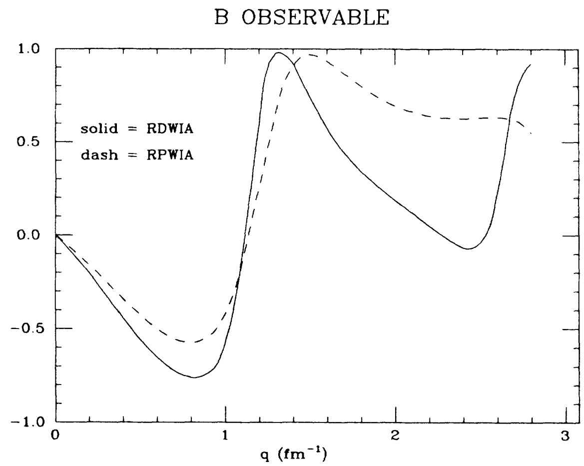

FIG. 3. Same as Fig. 1 for the spin rotation function B. No experimental data are available for this observable.

the data. Some discrepancies, however, exist for the analyzing power at small momentum transfer, Fig. 2. Since for small q the analyzing power can be written from (4) and (22) as

$$A_y = \frac{\left[\frac{2E}{3K}\right]\left[\frac{q}{2\alpha}\right]\left[\frac{M^*}{\alpha}\right]\text{Im}[2t_T(q)t_A(q)^*]}{|t_A(q)|^2}, \tag{27}$$

the large slope at the origin might indicate an overestimate of the bound nucleon mass or a fault in the NN amplitude. Of course, deficiencies in the model can never be completely ruled out. The experiment reveals very large values for the analyzing power. Some authors[6] have claimed that only by including the effects of exchange can the spin observables deviate from their trivial values (11). In this work we proved that there are additional descriptions, namely, a Dirac treatment, which will produce, even in a plane wave treatment, nontrivial values for the spin observables, without explicitly including exchange. Furthermore, since in the Dirac treatment these nontrivial values are not linked with exchange, they will survive at the higher energies where the effects of exchange should be small.

As we have previously said, inelastic spin observables constitute an excellent testing ground for these ideas, and careful study of these unnatural parity reactions, both from a theoretical and an experimental point of view, may help to clarify the status of all these theoretical models.

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[1]D. A. Sparrow *et al.*, Phys. Rev. Lett. **54**, 2207 (1985).
[2]J. Piekarewicz, R. D. Amado, and D. A. Sparrow, Phys. Rev. C **32**, 949 (1985).
[3]R. J. Blin-Stoyl, Proc. Phys. Soc. London, Sect. A **65**, 452 (1952).
[4]J. Piekarewicz, Ph.D. thesis, University of Pennsylvania.
[5]E. Bleszynsky *et al.*, Phys. Rev. C **27**, 902 (1983).
[6]S. S. M. Wong *et al.*, Phys. Lett. **149B**, 299 (1984).
[7]J. R. Shepard, E. Rost, and J. Piekarewicz, Phys. Rev. C **30**, 1604 (1984).
[8]J. Piekarewicz, Phys. Rev. C **32**, 1693 (1985).
[9]L. Wolfenstein, in *Annual Review of Nuclear Science* (Annual Reviews, Palo Alto, California, 1956), Vol. 6.
[10]J. A. McNeil, L. Ray, and S. J. Wallace, Phys. Rev. C **27**, 2123 (1983).
[11]J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).
[12]R. D. Amado, J. Piekarewicz, D. A. Sparrow, and J. A. McNeil, Phys. Rev. C **28**, 2180 (1983).
[13]J. J. Sakurai, *Advanced Quantum Mechanics* (Addison-Wesley, London, 1982).
[14]R. D. Amado, J. Piekarewicz, D. A. Sparrow, and J. A. McNeil, Phys. Rev. C **28**, 1663 (1983).
[15]J. J. Kelly, Ph.D. thesis, Massachusetts Institute of Technology, 1981.
[16]S. A. Gurvitz, J. P. Dedonder, and R. D. Amado, Phys. Rev. C **20**, 1256 (1979).
[17]A. M. Kobos *et al.*, Nucl. Phys. **A445**, 605 (1985).
[18]C. J. Horowitz and M. J. Iqbal, submitted to Phys. Rev. C.